Electromagnetic Mass Shift in Light-Cone Algebra

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Within the framework of light-cone algebra, it is shown that it is possible to obtain the electromagnetic mass shift to be finite by requiring that the contribution from the leading light-cone singularity of the current commutator to the absorptive part of the electroproduction be manifestly gauge-invariant to order $1/q^2$.

It has been known for some time that on the assumptions that (i) the inelastic structure functions $W_2(q^2,\nu)$ and $W_L(q^2,\nu)$ of the electroproduction have scaling behavior

$$\nu W_2(q^2,\nu) \sim F_2(\xi), \tag{1}$$

$$W_L(q^2,\nu) \sim F_L(\xi) + \frac{H_L(\xi)}{q^2} \tag{2}$$

as q^2, $\nu \to \infty$ with $\xi = q^2/(2m\nu)$ finite; (ii) the amplitudes T_2 and T_L, whose absorptive parts are, respectively, W_2 and W_L, satisfy unsubtracted dispersion relations; and (iii) $F_L(\xi)=0$; the divergent part of the electromagnetic mass shift of a nucleon is given by[1]

$$(\Delta m)_\infty = -\frac{3e^2}{16\pi^2}\left(\int_{q_m{}^2}^{\infty}\frac{dq^2}{q^2}\right) \times \left[\int_0^1\left(F_2(\xi) - \frac{H_L(\xi)}{\xi}\right)d\xi\right]. \tag{3}$$

Thus unless

$$\int_0^1 F_2(\xi)d\xi = \int_0^1 \frac{H_L(\xi)}{\xi} \tag{4}$$

the mass shift is logarithmic divergent. It is well known that the light-cone algebra[2] implies the assumption (iii), namely,[3] $F_L(\xi)=0$ [this avoids a quadratic divergence in the mass shift]. It also implies Eq. (1), and it has been noted[4] that the leading light-cone singularity has implications for nonleading behavior in the deep-inelastic region and that Eq. (2) follows from it with $F_L(\xi)=0$. Assumption (ii) is also consistent with it. The purpose of this paper is to point out that the condition (4) which leads to finite electromagnetic mass shift is also satisfied if in the light-cone-algebra approach we insist that the contribution to the absorptive part of the electroproduction from the leading light-cone singularity of the current commutator be manifestly gauge-invariant to order $1/q^2$ in the deep-inelastic region.

To show this let us define the absorptive part of the electroproduction amplitude:

$$A_{\mu\nu} = \tfrac{1}{2}(2\pi)^3\frac{p_0}{m}\int d^3z\, e^{-iq\cdot z}\langle p|[J_\mu^{\text{em}}(z), J_\nu^{\text{em}}(0)]|p\rangle = \frac{2\pi}{m^2}W_2(q^2,\nu)P_\mu P_\nu - 2\pi W_l(q^2,\nu)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right), \tag{5a}$$

where

$$P_\mu P_\nu = p_\mu p_\nu + (p_\mu q_\nu + p_\nu q_\mu)/2\xi + \frac{q^2}{4\xi^2}\delta_{\mu\nu},$$
$$W_l(q^2,\nu) = \frac{\nu^2}{q^2}W_2(q^2,\nu) - W_1(q^2,\nu),$$
$$W_L(q^2,\nu) = W_2(q^2,\nu) + W_l(q^2,\nu),$$
$$\nu = -\frac{p\cdot q}{m}, \quad \xi = \frac{q^2}{2m\nu}. \tag{5b}$$

In the quark model the leading light-cone singularity of the current commutator is given by

$$[J_\mu^{\text{em}}(z), J_\nu^{\text{em}}(0)] \underset{z^2=0}{\sim} 2[s_{\mu\nu\rho\sigma}V_\sigma^{Q^2}(A;z,0) + \epsilon_{\mu\nu\rho\sigma}A_\sigma^{Q^2}(S;z,0)]\frac{\partial}{\partial z_\rho}D(z), \tag{6a}$$

where

$$s_{\mu\nu\rho\sigma} = \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma}, \quad D(z) = -\frac{1}{2\pi}\epsilon(z_0)\delta(z^2), \tag{6b}$$

Q is the charge matrix, and $V_\sigma^{Q^2}(A;z,0)$ and $A_\sigma^{Q^2}(S;z,0)$ are, respectively, antisymmetric (with respect to $z \leftrightarrow 0$) vector and symmetric axial-vector bilocal operators. Only the vector bilocal operator contributes to the spin-summed matrix elements, which are defined as

$$\tfrac{1}{2}(2\pi)^3\frac{p_0}{m}\langle p|\, V_\sigma^{Q^2}(A; z, 0)\,|p\rangle = \tilde{G}^A(p\cdot z)\frac{p_\sigma}{m} + i\tilde{h}^A(p\cdot z)z_\sigma + \cdots , \tag{7a}$$

where $\cdots$ denotes terms which vanish at $z^2=0$. Define the Fourier transforms

$$\begin{aligned} h^A(\xi) &= \frac{1}{2\pi}\int d(p\cdot z)e^{i\xi(p\cdot z)}\tilde{h}^A(p\cdot z), \\ \tilde{h}^A(p\cdot z) &= \int_{-1}^{+1} d\xi' e^{-i\xi'(p\cdot z)}h^A(\xi), \end{aligned} \tag{7b}$$

and similarly for $G^A(\xi)$ and $\tilde{G}^A(p\cdot z)$. Thus using Eqs. (5)–(7), in the light-cone approach, $A_{\mu\nu}$ (in the deep-inelastic region $\nu\to\infty$, $q^2\to\infty$, ξ finite) is given by

$$A_{\mu\nu} \to 2\left(\frac{1}{m}s_{\mu\nu\rho\sigma}p_\sigma L_\rho + s_{\mu\nu\rho\sigma}I_{\sigma\rho}\right), \tag{8a}$$

where

$$L_\rho = i\int d^4z\, e^{-iq\cdot z}\tilde{G}^A(p\cdot z)\frac{\partial}{\partial z_\rho}D(z) = 2\pi(q+\xi p)_\rho\frac{\xi}{q^2}G^A(\xi), \tag{8b}$$

$$\begin{aligned} I_{\sigma\rho} &= i\int d^4z\, e^{-iq\cdot z}\tilde{h}^A(p\cdot z)z_\sigma\frac{\partial}{\partial z_\rho}D(z) = -\frac{\partial}{\partial q_\sigma}\left[2\pi(q+\xi p)_\rho\frac{\xi}{q^2}h^A(\xi)\right] \\ &= -2\pi\frac{\xi}{q^2}\left[\left(\delta_{\sigma\rho} + \frac{2\xi}{q^2}(2\xi p_\sigma p_\rho + p_\sigma q_\rho + q_\sigma p_\rho)\right)h^A(\xi) + \frac{2\xi}{q^2}[(q+\xi p)_\sigma(q+\xi p)_\rho]h'^A(\xi)\right], \end{aligned} \tag{8c}$$

where $h'^A(\xi) = d/d\xi\, h^A(\xi)$. Then from Eq. (8a)

$$\begin{aligned} A_{\mu\nu} = 2\pi\Bigg\{&2\xi\left[\frac{2\xi}{m}\left(\frac{P_\mu P_\nu}{q^2}\right)G^A(\xi) - \frac{\xi}{m}\frac{p^2}{q^2}\delta_{\mu\nu}G^A(\xi)\right] \\ &- 2\xi\left[-\frac{2\delta_{\mu\nu}}{q^2}h^A(\xi) + \frac{8\xi^2}{q^2}\left(\frac{P_\mu P_\nu}{q^2}\right)h^A(\xi) - \frac{4\xi}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)h'^A(\xi) + \frac{8\xi^3}{q^2}\left(\frac{P_\mu P_\nu}{q^2}\right)h'^A(\xi) + \frac{2\xi}{q^2}\delta_{\mu\nu}h'^A(\xi)\right] + O\left(\frac{1}{q^4}\right)\Bigg\}. \end{aligned} \tag{9}$$

In Eq. (9) the first square bracket is the contribution of the leading term. Also we note that within this contribution the first term, that is, $(2\xi/m)[(P_\mu P_\nu)/q^2]G^A(\xi)$, is gauge-invariant, while the second term containing $\delta_{\mu\nu}$ is not but it behaves like $1/q^2$ compared to the first term. The second square bracket is the contribution of the next-to-leading term, and it also behaves like $1/q^2$ compared to $(2\xi/m)[(P_\mu P_\nu)/q^2]G^A(\xi)$. Now we have gauge invariance to $O(1)$, and if we insist that the contribution to $O(1/q^2)$ should also be gauge-invariant, then the terms which behave like $1/q^2$ and are not individually gauge-invariant should combine to give a gauge-invariant contribution to order $1/q^2$. This means (with $p^2=-m^2$) we must have

$$m\xi G^A(\xi) + 2h^A(\xi) = 2\xi h'^A(\xi). \tag{10}$$

The condition (10) is our main result and we next show that it implies the condition (4). With the relation (10) we have

$$A_{\mu\nu} = 2\pi\frac{2\xi}{q^2}\left[\left(\frac{2\xi}{m}G^A(\xi) - \frac{8\xi^2}{q^2}[h^A(\xi) + \xi h'^A(\xi)]\right)P_\mu P_\nu + 4\xi h'^A(\xi)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\right] + O\left(\frac{1}{q^4}\right). \tag{11}$$

This gives

$$\begin{aligned} \nu W_2(q^2, \nu) &= 2\xi G^A(\xi) - \frac{8m\xi^2}{q^2}[h^A(\xi) + \xi h'^A(\xi)] \\ &\quad + O(1/q^4), \\ W_t(q^2, \nu) &= -\frac{8\xi^2}{q^2}h'^A(\xi) + O(1/q^4). \end{aligned} \tag{12}$$

Now with the usual definitions

$$\begin{aligned} \nu W_2(q^2, \nu) &= F_2(\xi) + O(1/q^2), \\ W_t(q^2, \nu) &= \frac{1}{m}\left[F_L(\xi) + \frac{m^2}{q^2}H_t(\xi)\right] + O\left(\frac{1}{q^4}\right), \\ W_L(q^2, \nu) &= \frac{1}{m}\left(F_L(\xi) + \frac{m^2}{q^2}H_L(\xi)\right) + O\left(\frac{1}{q^4}\right) \end{aligned} \tag{13}$$

we have from Eq. (12)

$$F_2(\xi)=2\xi G^A(\xi),$$
$$F_L(\xi)=0, \tag{14}$$
$$H_l(\xi)=-\frac{8\xi^2}{m}h'^A(\xi),$$

and using Eqs. (5b) and (13)

$$H_L(\xi)\equiv 2\xi F_2(\xi)+H_l(\xi)=4\xi^2\left(G^A(\xi)-\frac{2}{m}h'^A(\xi)\right). \tag{15}$$

Now from the condition (10) we have

$$m\int_0^1 2\xi G^A(\xi)d\xi+4\int_0^1 h^A(\xi)d\xi=4\int_0^1 \xi h'^A(\xi)d\xi. \tag{16}$$

But it is easy to see that [integrating by parts and noting that at threshold $h^A(\xi=1)=0$]

$$\int_0^1 \xi h'^A(\xi)d\xi=-\int_0^1 h^A(\xi)d\xi. \tag{17}$$

Thus the condition (16) becomes

$$\int_0^1 2\xi G^A(\xi)d\xi=\frac{8}{m}\int_0^1 \xi h'^A(\xi)d\xi, \tag{18}$$

which on using Eqs. (14) and (15) gives

$$\int_0^1 F_2(\xi)d\xi=\int_0^1 \frac{H_L(\xi)}{\xi}d\xi, \tag{19}$$

that is, the condition for the vanishing of the logarithmic divergence of the electromagnetic mass shift. The same conclusion was recently reached[5] by using a null-plane current commutator algebra.

In the end we want to emphasize that our conclusion about the finiteness of electromagnetic mass difference is based on two assumptions: (i) The leading light-cone singularity also implies the nonleading behavior. (ii) The gauge invariance is imposed on the light-cone algebra to order $1/q^2$. Assumption (ii) is reasonable; and as far as assumption (i) is concerned, it is testable experimentally as pointed out in Ref. 4. If this assumption is borne out by experiments, probably no additional contribution to the nonscaling structure function will be needed. Moreover, such additional contributions would be of a different nature, as within the framework of the light-cone-algebra approach such contributions would involve quark mass explicitly.

Our assumptions about the nonleading light-cone terms are not valid in the free-field case. The free-field theory suggests the presence of manifestly gauge-invariant nonleading terms. According to our assumptions no such terms are present, as the whole of nonleading part comes from the second term on the right-hand side of Eq. (7a).

While it is true that light-cone algebra is abstracted from free-field theory (so far as singularity and internal symmetry structure are concerned), yet one is not fully committed to the free-field theory; otherwise there would be no point in abstracting the algebra. The free-field model fixes $H_L(\xi)$ to be $2\xi F_2(\xi)$. The light-cone algebra, on the other hand, in general gives no information about $H_L(\xi)$ if one confines oneself to the leading part. Only by the two assumptions stated above does one have the result

$$\int_0^1 F_2(\xi)=\int_0^1 \frac{H_L(\xi)}{\xi}$$

giving the finite mass difference.

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