[1]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970.

[2]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3]K. G. Wilson, Phys. Rev. 129, 1499 (1969).

[4]R. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

[5]H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970).

[6]R. Jackiw, R. Van Royen, and G. West, Phys. Rev. D 2, 2473 (1970).

[7]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1.

[8]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[9]M. O. Taha, Nuovo Cimento 60A, 651 (1969); Phys. Rev. 162, 1694 (1967).

[10]M. O. Taha and A-M. M. Abdel-Rahman, Univ. of Khartoum report, 1972 (unpublished).

[11]M. O. Taha, Univ. of Khartoum report, 1972 (unpublished).

[12]R. Jackiw, invited talk at the 1972 Annual Meeting of the American Physical Society, San Francisco, 1972 (unpublished).

[13]Duane A. Dicus, Roman Jackiw, and Vigdor L. Teplitz, Phys. Rev. D 4, 1733 (1971).

PHYSICAL REVIEW D VOLUME 8, NUMBER 2 15 JULY 1973

O(4) Gauge Theory with Three-Triplet Quarks*

T. P. Cheng

Rockefeller University, New York, New York 10021†

(Received 2 April 1973)

A theory of electromagnetic and weak interactions based on the O(4) gauge group is formulated along the general program put forward recently by Pais. In our scheme the hadron symmetry $SU(3) \times SU(3)'$ is incorporated in such a way that the model has a (kinematical) $\Delta I = 1/2$ rule for nonleptonic weak decays. The lepton assignment may be made to produce CP-violation effects which are more "convergent" without enlarging the gauge group. There is no lower bound to the masses of the intermediate vector bosons, while the upper limit for one of the charged W's is only 18 GeV. The elastic reaction cross sections $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ are about 1.4 times the usual $V-A$ values. Other physical features of this model are also discussed in some detail.

I. INTRODUCTION

Following the pioneering works of Weinberg and Salam, a number of renormalizable theories of of weak and electromagnetic interactions have been constructed in the framework of spontaneously broken gauge symmetries.[1-9] Looking over the totality of the physically viable models proposed so far, we may broadly distinguish the following two classes: (i) Most of the existing models are based on the gauge group $SU(2)\times U(1)$. They differ from each other in their lepton and hadron multiplet assignments. [In this class we include those theories based on larger groups but in which some of the intermediate vector bosons are endowed with superheavy masses so that the group is effectively reduced, in the first approximation, down to the subgroup $SU(2)\times U(1)$ (Ref. 4); we also include here the Georgi-Glashow model[2] which is based on SO(3) or SU(2)—namely it may be viewed as an $SU(2)\times U(1)$ model with all multiplets transforming trivially under the U(1) group.] (ii) Recently Pais has advocated the exploration of a class of theories based on the gauge groups $O(4)\times \mathcal{G}$.[8,9] [They include $SU(2)\times SU(2)$, $O(4)\times U(1)$, $O(4)\times O(4)$, etc.] The central idea, as proposed first by Segrè,[9a] is that *two* sets of charged intermediate vector bosons of *comparable* masses are made to mediate separately the $\Delta S=0$ and $\Delta S=1$ weak decays. This also opens up the possibility of giving neutral leptons maximal CP-violating phases and yet, Pais shows, the effects on physical processes will be superweak. In this paper we shall present a model built around this central conception but based on a strict O(4) gauge group, namely the group $SU(2)\times SU(2)$ plus R, "parity" of the group O(4). (The relevance of this R symmetry will be discussed in Sec. II.) Compared with the O(4) theory of Refs. 8 and 9, it involves different lepton and hadron multiplet assignments: A larger number of neutral particles are needed here but all multiplets in this theory are R-symmetric.

Another motivation for our model originates from questions concerning hadron structures. The attractive simple picture of all hadrons made out of three fractionally charged $\mathcal{P}$, $\mathcal{N}$, and λ quarks may prove to be inadequate on a number of accounts. In this connection we have in mind the $\pi^0 \to 2\gamma$ rate problem,[10] and the question of allowing quarks in a relative S-wave state for the 56-plet of static SU(6). There are also some suggestions that deep-inelastic electroproduction experiments may be better described by picturing protons as composed of three "valence" quarks in an SU(3) singlet "sea."[11] Gauge theories shed new lights on these questions since quarks are assumed to be the fundamental hadron fields appearing in the renormalizable Lagrangians. On the simplest level, certain models can fit only with integrally charged quarks and the requirement of sufficiently suppressing the $\Delta S = 1$ neutral-current events also demands the existence of extra quark fields.[12] Again among the proposed quark schemes we can broadly distinguish the following two classes: (i) The first may be labeled as "one-triplet" theories where there is only one SU(3) triplet and the rest of the quarks are assumed to be singlets. A notable model in this class is the one with eight integrally charged quarks as discussed in Refs. 2, 3, 6, 8, 9, 13, and 14. It has been shown that gauge theories with such quarks may embody the $\Delta I = \frac{1}{2}$ rule for nonleptonic decays. This emerges via the assumed dominance of scalar Higgs exchanges over vector exchanges in the SO(3) model[14]; and via purely neutral-vector-meson exchanges in the $O(4) \times \mathcal{G}$ models.[8,9] With such integrally charged one-triplet quarks (the fractionally charged one-triplet model runs into trouble with the $\pi^0 \to 2\gamma$ rate, among others) one must then face the problem of "embedding" these quarks in some higher-hadron-symmetry group and see how such a hadron picture, being necessarily unconventional, may be made compatible with the known phenomenology. Some of these questions have already been discussed in Refs. 14 and 9.[15] (ii) On the other hand, it is well known that the three-triplet models based on SU(3)×SU(3)′ have enjoyed general phenomenological successes.[16] This motivates us to incorporate instead such quarks as fundamental hadrons in an O(4) gauge theory of electromagnetic and weak interactions.[17] However, we shall demonstrate that with the nine integrally charged quarks of Han and Nambu[18] this is not possible. Nevertheless, the slight modification of our hadron picture by an addition of two SU(3) ×SU(3)′ singlets overcomes all the difficulties and it also leads naturally to a gauge model that has a kinematical $\Delta I = \frac{1}{2}$ rule for nonleptonic decays.[19,20] All this will be discussed in Sec. III. It suffices to note here that the $\Delta I = \frac{1}{2}$ rule may be incorporated in a particularly simple manner if the basic O(4) strategy is followed. Here with the ($\Delta S = 0$) and ($\Delta S = 1$) currents, $\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{N}$, $\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\lambda$, being coupled to separate intermediate vector bosons, there will be no need for an elaborate cancellation scheme of the unwanted $\Delta I = \frac{3}{2}$ piece which inevitably appears in any theory with only one set of charged intermediate bosons.[9a]

As it turns out the two motivations stated above [the O(4) gauge group and SU(3)×SU(3)′ hadron symmetry] are compatible with each other; the alteration in the lepton section necessitated by the incorporation of SU(3)×SU(3)′ quarks in the hadronic sector is just the one that allows us to assign all particles into R-symmetric representations of O(4). In particular all fermions (both left- and right-handed ones) will be assigned to the $(\frac{1}{2}, \frac{1}{2})$ quartets and (0, 0) singlets. In connection with this R invariance, we mention here a related feature of the CP-violation effects in this type of models. In the particular scheme of CP violation as discussed by Pais, the principal effects (i.e., the imaginary off-diagonal elements of the $\bar{K}^0K^0$ mass matrix) come about through the higher-order mixing of the two afore-mentioned intermediate vector bosons. In Ref. 9 a detailed argument has been given to show that divergences (before renormalization) involved in such higher-order diagrams are reduced in a R-symmetric theory. The divergence is in general quadratic in SU(2)× SU(2) and at most logarithmic in O(4) models. Lepton assignments and this CP problem are discussed in Sec. IV.

Like a number of existing theories this model is free of anomalies. With respect to ordinary hadrons it has no strangeness-changing neutral current and amplitudes for $\bar{\mathcal{N}}\lambda \to \mu^+\mu^-$ via box diagrams are also sufficiently suppressed. Similarly higher-order $\Delta S = 2$ amplitudes are not of a magnitude that will contradict experimental facts. As in the O(4)×U(1) model of Ref. 9, the hadronic neutral current being "charmed" the reaction ($\nu + p \to \nu +$ ordinary hadrons) is suppressed; having no ($\bar{e}e$) term in the leptonic neutral current, the $\nu_\mu e \to \nu_\mu e$ process is forbidden in the lowest order; the cross sections for $\nu_e e \to \nu_e e$ and $\bar{\nu}_e e \to \bar{\nu}_e e$ are fixed at values about 1.4 times those in the standard $V-A$ theory.

One notable feature of the present model is that it sets no lower bound on the masses of the intermediate bosons, and their upper limits are 18 and 37 GeV for the two charged W's and 41 GeV for the neutral vector boson.[21]

Summarizing, we present a model that brings together the strategy of Refs. 8, 9, and 9a with a hadron symmetry group SU(3)×SU(3)′. Our scheme is based on a strict O(4) gauge symmetry (i.e., with R-reflection invariance) and it implies a conventional hadron picture of baryons being bound states with the triplet "valence" quarks superimposed on a "sea" which behaves like a SU(3) singlet. (This singlet "sea" is needed, as in Refs. 8, 9, and 14, to implement the kinematical $\Delta I = \frac{1}{2}$ rule.) The price paid for doing this is the introduction of a small number of additional neutral particles. The relatively low value for the upper bound of one of the charged W's and the prediction for $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ means that it may be possible to test such a model with the existing experimental facilities. While a negative result in the search for intermediate bosons of mass up to 18 GeV will surely destroy this scheme, a discovery of a low-mass charged W together with a measurement of $\sigma(\nu_e e)$ or $(\bar{\nu}_e e)$ with results in the neighborhood of 1.4 times those of the $V-A$ theory should be considered as encountering a signature characteristic of the present model.

II. THE O(4) FORMALISM

The group O(4) being isomorphic to SU(2)×SU(2), its six generators may be represented conveniently by η_i and ζ_i $(i=1,2,3)$ satisfying the commutation relations

$$\begin{aligned}&[\eta_i, \eta_j] = i\epsilon_{ijk}\eta_k ,\\&[\zeta_i, \zeta_j] = i\epsilon_{ijk}\zeta_k ,\\&[\eta_i, \zeta_j] = 0 .\end{aligned} \tag{2.1}$$

The gauge-covariant derivative is then

$$D_\mu = \partial_\mu - ig_1\vec{B}_\mu \cdot \vec{\eta} + g_2 \vec{C}_\mu \cdot \vec{\zeta} ,$$

where $\vec{B}_\mu$ and $\vec{C}_\mu$ are the six vector gauge fields. We shall also demand that the symmetric Lagrangian be invariant under the transformation R: $\vec{\eta} \leftrightarrow \vec{\zeta}$, $\vec{B}_\mu \leftrightarrow \vec{C}_\mu$, namely the reflection operator of O(4). This discrete symmetry requires that (i) $g_1 = g_2 \equiv g$ and (ii) all particles must belong to R-symmetric representations: (0,0), $(\frac{1}{2}, \frac{1}{2})$, (1,0)+(0,1), etc. It is important to note that if (ii) is not satisfied and the interactions are only SU(2)×SU(2)-invariant then there is no way to maintain the equality in (i) (not even in the approximate sense). It comes about because higher-order divergences will require $g_1 \neq g_2$ counterterms. In other words, renormalizibility requires that we have *all* terms consistent with the symmetry group of the Lagrangian. The present model is based on a strict O(4) [i.e., R-symmetric SU(2)×SU(2)] gauge group and its physical consequences depends crucially on this feature. For example, our prediction for definite values of upper bounds for W masses and of $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ will be lost in an approximate O(4) theory.

Anticipating that the gauge symmetry will be broken spontaneously and five of the vector particles will become massive through the Higgs mechanism we reexpress the above covariant derivatives in terms the normal modes, namely, the gauge fields with respect to which the zeroth-order mass matrix will be diagonal:

$$D_\mu = \partial_\mu - ie\left[A_\mu Q + Z_\mu(\eta_3 - \zeta_3) + \frac{1}{\sqrt{2}} W_\mu^1(\eta_+ - \zeta_+) + \frac{1}{\sqrt{2}} W_\mu^2(\eta_+ + \zeta_+) + \text{H.c.}\right], \tag{2.2}$$

where

$$\begin{aligned}&e = g/\sqrt{2}\,, \quad Q = \eta_3 + \zeta_3 ,\\&\eta_\pm = \eta_1 + i\eta_2 , \quad \zeta_\pm = \zeta_1 \pm i\zeta_2 ,\\&A_\mu = \frac{B_\mu^3 + C_\mu^3}{\sqrt{2}}, \quad Z_\mu = \frac{B_\mu^3 - C_\mu^3}{\sqrt{2}},\\&W_\mu^1 = \tfrac{1}{2}[B_\mu^1 - C_\mu^1 - i(B_\mu^2 - C_\mu^2)],\\&W_\mu^2 = \tfrac{1}{2}[B_\mu^1 + C_\mu^1 - i(B_\mu^2 + C_\mu^2)].\end{aligned}$$

We note that under R reflection W_μ^1, Z_μ are odd and W_μ^2, A_μ are even. All fermions will be assigned to the $(\frac{1}{2}, \frac{1}{2})$ quartets [or to the trivial (0,0) singlets]

$$f = (f^+, f_1, f_2, f^-) . \tag{2.3}$$

With respect to their (η_3, ζ_3) eigenvalues (recall $Q = \eta_3 + \zeta_3$) they are[22]

$$\begin{aligned}&f^+ = f(\tfrac{1}{2}, \tfrac{1}{2}),\\&f_1 = [f(-\tfrac{1}{2}, \tfrac{1}{2}) - f(\tfrac{1}{2}, -\tfrac{1}{2})]/\sqrt{2} ,\\&f_2 = [f(-\tfrac{1}{2}, \tfrac{1}{2}) + f(\tfrac{1}{2}, -\tfrac{1}{2})]/\sqrt{2} ,\\&f^- = f(-\tfrac{1}{2}, -\tfrac{1}{2}).\end{aligned}$$

Clearly under R, f_1 is odd and $f^\pm, f_2$ are even. The O(4)-invariant coupling term of f to the gauge field is

$$-ig\bar{f}\gamma^\mu(\vec{B}_\mu \cdot \vec{\eta} + \vec{C}_\mu \cdot \vec{\zeta})f = A^\mu j_\mu^{\text{em}} + Z^\mu j_\mu^{(0)} + W^{1\mu} j_\mu^{(1)} + W^{2\mu} j_\mu^{(2)} + \text{H.c.},$$

with

$$j_\mu^{(0)} = -ie(\bar{f}_1\gamma_\mu f_2 + \bar{f}_2\gamma_\mu f_1) , \tag{2.4}$$

$$j_\mu^{(1)} = -ie(\bar{f}^+\gamma_\mu f_1 + \bar{f}_1\gamma_\mu f^-) , \tag{2.5}$$

$$j_\mu^{(2)} = -ie(\bar{f}^+\gamma_\mu f_2 + \bar{f}_2\gamma_\mu f^-) . \tag{2.6}$$

We shall need Higgs scalars transforming as $(\frac{1}{2}, \frac{1}{2})$ and (1,0)+(0,1) representations. Let H be the $(\frac{1}{2}, \frac{1}{2})$ scalars,[23] $\vec{T}$ be (1,0), and $\vec{R}$ be (0,1). In the symmetric limit the Lagrangian is invariant under R reflection so only the combination

$$X = \vec{T} \cdot \vec{\eta} + \vec{R} \cdot \vec{\zeta} \tag{2.7}$$

will appear. The self-interaction among the scalars $P(H, X)$ is assumed to be of such a nature that desired scalar fields will develop vacuum expectation values. This breaks the original O(4) symmetry down to a U(1) group and leaves only A_μ as the massless photon field. For this we require only the neutral members to develop nonzero vacuum expectation values, denoting them by H_1, H_2 [from $(\frac{1}{2}, \frac{1}{2})$ in a notation similar to that of (2.3)] and T, R [from (1,0) and (0,1), respectively]. Consequently $|D_\mu H|^2$ and $|D_\mu X|^2$ contain the following terms:

$$\mathcal{L}_{00} = -e^2 Z_\mu^{\ 2}(H_1^{\ 2} + H_2^{\ 2}), \tag{2.8}$$

$$\mathcal{L}_{11} = -2e^2(W_\mu^1)^2(H_1^{\ 2} + T^2 + R^2), \tag{2.9}$$

$$\mathcal{L}_{22} = -2e^2(W_\mu^2)^2(H_2^{\ 2} + T^2 + R^2), \tag{2.10}$$

$$\mathcal{L}_{12}^{(-)} = 2e^2(W_\mu^{1\dagger} W_\mu^2 - W_\mu^{2\dagger} W_\mu^1)(H_1^\dagger H_2 - H_2^\dagger H_1), \tag{2.11}$$

$$\mathcal{L}_{12}^{(+)} = 2e^2(W_\mu^{1\dagger} W_\mu^2 + W_\mu^{2\dagger} W_\mu^1)(T^2 - R^2). \tag{2.12}$$

Thus in order to have W_μ^1 and W_μ^2 as normal modes we must demand $\mathcal{L}_{12}^{(\pm)} = 0$, which can be achieved by having $R = T$ and by having H_1 be relatively real with respect to H_2. [Clearly if we have more than one scalar $(\frac{1}{2}, \frac{1}{2})$ we only demand the sum of their contribution to $\mathcal{L}_{12}^{(-)}$ to be zero.[23]] The resulting zeroth-order masses $M_{0,1,2}$ for Z, W^1, and W^2, respectively, can be read off directly from $\mathcal{L}_{00,11,22}$. We note that as a zeroth-order mass inequality

$$M_0^{\ 2} < M_1^{\ 2} + M_2^{\ 2}. \tag{2.13}$$

III. THE HADRON SECTOR AND THE $\Delta I = \frac{1}{2}$ RULE

As fundamental hadrons we shall use the Han-Nambu three-triplet quarks.[17,18] The nine quarks transform as members of the (3, 3) representation of the global symmetry group SU(3)×SU(3)′. For a given subscript i (= 1, 2, 3) the three quarks $\mathcal{P}_i$, $\mathfrak{N}_i$, and λ_i form a triplet under the ordinary SU(3) with the usual quark isospin and hypercharge assignments. On the otherhand $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ are members of an SU(3)′ triplet with $Y'(\mathcal{P}_1) = Y'(\mathcal{P}_2) = \frac{1}{3}$ and $Y'(\mathcal{P}_3) = -\frac{2}{3}$, similarly for $\{\mathfrak{N}_i\}$ and $\{\lambda_i\}$. The charge formula is modified to be $Q = T_3 + \frac{1}{2}Y + Y'$. Consequently the quarks are integrally charged with

$$Q(\mathcal{P}_1, \mathcal{P}_2) = -Q(\mathfrak{N}_3, \lambda_3) = 1$$

and

$$Q(\mathcal{P}_3, \mathfrak{N}_1, \mathfrak{N}_2, \lambda_1, \lambda_2) = 0.$$

As usual the known hadrons are assumed to be singlet under SU(3)′ (hence for these states the charge formula is automatically reduced to the normal Gell-Mann–Nishijima relation).

Given such a hadron structure, the O(4) gauge theory of electromagnetic and weak interactions of hadrons we construct must satisfy the following basic criteria:

(i) It must contain in its SU(3)′ singlet sector the correct $\Delta S = 0$ and $\Delta S = 1$ currents associated separately with the 2 sets of charged intermediate bosons W^1, W^2:

$$J^{(1)} \sim \sum_{i=1}^{3} (\overline{\mathcal{P}}_i \mathfrak{N}_i)_L + \cdots$$

and

$$J^{(2)} \sim \sum_{i=1}^{3} (\overline{\mathcal{P}}_i \lambda_i)_L + \cdots,$$

where the shorthand notation $(\overline{\mathcal{P}}_i \mathfrak{N}_i)_L$ means $\overline{\mathcal{P}}_i \gamma_\mu [(1+\gamma_5)/2] \mathfrak{N}_i$ and $\cdots$ stands for SU(3)′ nonsinglet pieces.

(ii) Furthermore one very important physical constraint which the model must satisfy is the severe suppression of strangeness-changing neutral currents. This implies that we cannot tolerate any $(\overline{\lambda}_i \mathfrak{N}_j)$ terms in our currents even for $i \neq j$. Namely, it will not be reasonable to expect that SU(3)′ is such an exact symmetry as to provide the needed 10^{-8}, or better, suppression factor. In practice, this means that no λ_i's may be put into the same multiplet with any of the $\mathfrak{N}_i$'s.

(iii) There being two pairs of oppositely charged particles $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathfrak{N}_3$, λ_3 the desire to have an R-invariant theory leads us naturally to explore first the assignment [with respect to the O(4) gauge group] of two $(\frac{1}{2}, \frac{1}{2})$ quartets and one (0, 0) singlet.

For the left-handed quarks, we can try to satisfy (i)–(iii) by the assignment [in the notation of (2.3)] of two quartets:

$$(\mathcal{P}_1, \mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3)_L \text{ and } (\mathcal{P}_2, \lambda_1, \lambda_2, \lambda_3)_L; \tag{3.1}$$

one singlet = $\mathcal{P}_{3L}$. However, it is not hard to check that none of the currents resulting from the above multiplets can produce in the lowest order an effective interaction for nonleptonic decay which is $\Delta S = 1$ *and* SU(3)′ singlet. Moreover, there is no way to make an acceptable [in the sense of (i)–(iii)] assignment for the right-handed quarks altogether.

This first resounding failure teaches us that a sensible formulation of O(4) gauge theory combined with SU(3)×SU(3)′ as the hadron group will need more than the usual three-triplet quarks. The question is whether one can accomplish this with a minimal modification of our hadronic picture. We shall demonstrate in the following that an addition of two SU(3)×SU(3)′ singlets, call them r and s, will not only solve all the above-mentioned difficulties but will naturally lead us to a *kinematical* $\Delta I = \frac{1}{2}$ rule for nonleptonic decays. Thus the picture of hadron structure we have in mind is the following one: The usual Han-Nambu 3-triplets will act as "valence" quarks in a "sea" which contains r and s and which transforms as singlet under SU(3) and SU(3)$'$. So for all practical purposes when we come to think of hadron symmetries these extra modifications may be completely ignored.

We now present our model: The $(\frac{1}{2},\frac{1}{2})$ quartets are

$$Q_1^L=[\mathcal{P}_1,\mathcal{N}_1(\alpha),r(\beta),\mathcal{N}_3]_L\,, \tag{3.2}$$

$$Q_2^L=[\mathcal{P}_2,s(\beta),\lambda_2(\alpha),\lambda_3]_L\,, \tag{3.3}$$

$$Q_1^R=[\mathcal{P}_1,\mathcal{N}_2,r,\mathcal{N}_3]_R\,, \tag{3.4}$$

$$Q_2^R=[\mathcal{P}_2,s,\lambda_1,\lambda_3]_R\,, \tag{3.5}$$

where

$$\mathcal{N}_1(\alpha)=\mathcal{N}_1\cos\alpha+\mathcal{N}_2\sin\alpha,$$

$$\lambda_2(\alpha)=\lambda_2\cos\alpha+\lambda_1\sin\alpha\,,$$

$$r(\beta)=r\cos\beta+s\sin\beta\,,$$

$$s(\beta)=s\cos\beta-r\sin\beta\,.$$

The remaining (neutral) debris are put into (0,0) singlets:

$$\mathcal{P}_3^L,\ \mathcal{N}_2(\alpha)^L,\ \lambda_1(\alpha)^L,\ \mathcal{P}_3^R,\ \mathcal{N}_1^R,\ \lambda_2^R\,. \tag{3.6}$$

Before going into detailed discussion of the interactions implied by such an assignment we note that in principle we could have let $\mathcal{P}_1$ and $\mathcal{P}_2$ mix in Q_1^L and Q_2^L and r and s mix in Q_1^R and Q_2^R, but as it turns out no significantly different physical effects (at this early unsophisticated state of the model) will result from such mixings. Consequently for simplicity we leave them unmixed. On the other hand at $\alpha=0$ M_1 takes on its maximal value of 18 GeV and the lowest-order nonleptonic $\Delta S=1$ and SU(3)$'$ singlet transition will be forbidden in this limit. A nonvanishing value for β is needed in order that $\Delta S=2$ effects (for example the K_1K_2 mass difference) will result from an exchange of two intermediate bosons.

Given (3.2)–(3.5) we have the following hadronic weak currents:

$$\begin{aligned}J^{(1)h}&=-ie\{[\overline{\mathcal{P}}_1\mathcal{N}_1(\alpha)+\overline{\mathcal{N}}_1(\alpha)\mathcal{N}_3+\overline{\mathcal{P}}_2s(\beta)+\overline{s}(\beta)\lambda_3]_L\\&\qquad+[\overline{\mathcal{P}}_1\mathcal{N}_2+\overline{\mathcal{N}}_2\mathcal{N}_3+\overline{\mathcal{P}}_2s+\overline{s}\lambda_3]_R\}\\&=-ie\frac{\cos\alpha}{3}\sum_{i=1}^{3}(\overline{\mathcal{P}}_i\mathcal{N}_i)_L+\text{SU(3)}'\text{ nonsinglets}\,,\end{aligned} \tag{3.7}$$

$$\begin{aligned}J^{(2)h}&=-ie\{[\overline{\mathcal{P}}_2\lambda_2(\alpha)+\overline{\lambda}_2(\alpha)\lambda_3+\overline{\mathcal{P}}_1r(\beta)+\overline{r}(\beta)\mathcal{N}_3]_L\\&\qquad+[\overline{\mathcal{P}}_2\lambda_1+\overline{\lambda}_1\lambda_3+\overline{\mathcal{P}}_1r+\overline{r}\mathcal{N}_3]_R\}\\&=-ie\frac{\cos\alpha}{3}\sum_{i=1}^{3}(\overline{\mathcal{P}}_i\lambda_i)_L+\text{SU(3)}'\text{ nonsinglets}\,,\end{aligned} \tag{3.8}$$

$$\begin{aligned}J^{(0)h}&=-ie\{[\overline{\mathcal{N}}_1(\alpha)r(\beta)+\overline{\lambda}_2(\alpha)s(\beta)]_L\\&\qquad+[\overline{\mathcal{N}}_2r+\overline{\lambda}_1s]_R+\text{H.c.}\}\,.\end{aligned} \tag{3.9}$$

It is then straightforward to verify that in the lowest order only through the exchange of neutral intermediate boson Z can we get an $\Delta S=1$ *and* SU(3)$'$ singlet effective interaction for the nonleptonic decays:

$$\mathcal{L}_{\Delta S=1}=\frac{e^2}{M_0^{\,2}}\frac{\sin 2\alpha}{3}\left\{\sum_{i=1}^{3}(\overline{\mathcal{N}}_i\lambda_i)_L[\overline{s}(\beta)r(\beta)]_L+\text{H.c.}+\cdots\right\}, \tag{3.10}$$

where a Fierz transformation has been made. This interaction has octet transformation properties (hence $\Delta I=\frac{1}{2}$) since r and s, being SU(3)$\times$SU(3)$'$ singlets, are obviously SU(3) singlets as well. As we have already remarked upon in the Introduction, such a simple mechanism for the $\Delta I=\frac{1}{2}$ rule is directly related to the central feature of the model: The $\Delta S=1$ and $\Delta S=0$ transitions are mediated separately by two sets of charged intermediate vector bosons.[9a]

From (3.7)–(3.9) it is clear that our $\mathcal{N}_i$ can turn into a λ_i via r or s by emitting W^1 and W^2 when $i=3$ and two Z's when $i=1,2$. Thus the SU(3)$'$-singlet parts of the $\Delta S=2$ transitions $\overline{\mathcal{N}}_i\lambda_i\to\overline{\lambda}_j\mathcal{N}_j$ have amplitudes of the order of $(\sin^2 2\beta)G_F\alpha[m(s)-m(r)]^2/M_W^{\,2}$, which is physically quite acceptable.[24]

IV. THE LEPTON SECTOR

The hadron assignment in Sec. III implies that for the left-handed $(\frac{1}{2},\frac{1}{2})$ leptons three neutral members (one of them being the neutrino) will be needed in order to preserve universality. This extra heavy lepton allows us to place the right-handed leptons in $(\frac{1}{2},\frac{1}{2})$ quartets as well. In other ways we are left ample freedom as to the choices of relative phases among the neutral leptons (and also the mixing angles in right-handed multiplets.) In the following we shall present one assignment where $(\frac{1}{2},\frac{1}{2})_L$'s are the same as the final choice made by Pais in his O(4)$\times$U(1) model. (Basically it was guided by the requirement that absolute values of the amplitudes for μ decay and $\mathcal{N}$ and λ β decays should be in the ratio of $1:\cos\theta_C:\sin\theta_C$, and by maximal μ-e universality, which has bearings on CP-violation parameters.) However, we replace everywhere the neutrinos by

$$\begin{aligned}\nu_e(\alpha)&\equiv\tfrac{1}{3}[\sqrt{2}\cos\alpha\,\nu_e+(9-2\cos^2\alpha)^{1/2}x_2]\,,\\\nu_\mu(\alpha)&\equiv\tfrac{1}{3}[\sqrt{2}\cos\alpha\,\nu_\mu+(9-2\cos^2\alpha)^{1/2}y_2]\,,\end{aligned} \tag{4.1}$$

where x_2 and y_2 are the extra neutral heavy leptons. As for $(\frac{1}{2},\frac{1}{2})_R$, we have made such a choice that the mixing problem among the neutral leptons will be minimized. Clearly since right-handed couplings will always involve heavy leptons any particular choice made here will not affect the phenomenological predictions that concern only known leptons. Our assignment is as follows [again in the notation of (2.3)]:

$$E^L=\left[x^+,\ \frac{x_1+\nu_e(\alpha)}{\sqrt{2}},\ \epsilon^*\frac{x_1-\nu_e(\alpha)}{\sqrt{2}},\ e\right]_L,$$

$$M^L=\left[y^+,\ \frac{y_1+\nu_\mu(\alpha)}{\sqrt{2}},\ \epsilon\frac{y_1-\nu_\mu(\alpha)}{\sqrt{2}},\ \mu\right]_L,$$

$$E^R=\left[x^+,\ \frac{x_1+x_2}{\sqrt{2}},\ \epsilon^*\frac{x_1-x_2}{\sqrt{2}},\ e\right]_R, \tag{4.2}$$

$$M^R=\left[y^+,\ \frac{y_1+y_2}{\sqrt{2}},\ \epsilon\frac{y_1-y_2}{\sqrt{2}},\ \mu\right]_R,$$

where ϵ is a phase factor $\epsilon=\exp(i\pi/4)$. $x_2(\alpha)_L$ and $y_2(\alpha)_L$, which are orthogonal to $\nu_e(\alpha)_L$ and $\nu_\mu(\alpha)_L$, respectively, are the (0, 0) singlets. The leptonic weak currents are [see Eq. (2.4)–(2.6)]

$$J^{(1)l}=-i\frac{e}{\sqrt{2}}\{[\bar{x}^+(x_1+\nu_e(\alpha))+(\bar{x}_1+\bar{\nu}_e(\alpha))e+\bar{y}^+(y_1+\nu_\mu(\alpha))+(\bar{y}_1+\bar{\nu}_\mu(\alpha))\mu]_L$$
$$+[\bar{x}^+(x_1+x_2)+(\bar{x}_1+\bar{x}_2)e+\bar{y}^+(y_1+y_2)+(\bar{y}_1+\bar{y}_2)\mu]_R\}$$
$$=-ie\frac{\cos\alpha}{3}(\bar{\nu}_e e+\bar{\nu}_\mu\mu)_L+\cdots, \tag{4.3}$$

$$J^{(2)l}=-i\frac{e}{\sqrt{2}}\{[\epsilon^*\bar{x}^+(x_1-\nu_e(\alpha))-\epsilon(\bar{x}_1-\bar{\nu}_e(\alpha))e+\epsilon\bar{y}^+(y_1-\nu_\mu(\alpha))-\epsilon^*(\bar{y}_1-\bar{\nu}_\mu(\alpha))\mu]_L$$
$$+[\epsilon^*\bar{x}^+(x_1-x_2)-\epsilon(\bar{x}_1-\bar{x}_2)e+\epsilon\bar{y}^+(y_1-y_2)-\epsilon^*(\bar{y}_1-\bar{y}_2)\mu]_R\}$$
$$=-ie\frac{\cos\alpha}{3}(\epsilon\bar{\nu}_e e+\epsilon^*\bar{\nu}_\mu\mu)_L+\cdots, \tag{4.4}$$

$$J^{(0)l}=\frac{ie}{\sqrt{2}}\{[\bar{x}_1x_1+\bar{y}_1y_1-\bar{\nu}_e(\alpha)\nu_e(\alpha)-\bar{\nu}_\mu(\alpha)\nu_\mu(\alpha)+i(\bar{\nu}_e(\alpha)x_1-\bar{x}_1\nu_e(\alpha)-\bar{\nu}_\mu(\alpha)y_1+\bar{y}_1\nu_\mu(\alpha))]_L$$
$$+[\bar{x}_1x_1+\bar{y}_1y_1-\bar{x}_2x_2-\bar{y}_2y_2+i(\bar{x}_2x_1-\bar{x}_1x_2-\bar{y}_2y_1+\bar{y}_1y_2)]_R\}$$
$$=-ie\frac{\sqrt{2}\cos^2\alpha}{9}(\bar{\nu}_e\nu_e+\bar{\nu}_\mu\nu_\mu)_L+\cdots, \tag{4.5}$$

where $\cdots$ stand for terms involving heavy leptons.

By (4.3)–(4.5) and (3.7)–(3.9) we immediately have (θ_C is the Cabibbo angle)

$$\frac{G}{\sqrt{2}}\cos\theta_C=\frac{e^2\cos^2\alpha}{36M_1^{\,2}} \tag{4.6}$$

and

$$\tan\theta_C=\frac{M_1^{\,2}}{M_2^{\,2}} \tag{4.7}$$

which give for the masses for the intermediate bosons[25]

$$M_1=\cos\alpha\times 18\text{ GeV}, \tag{4.8}$$

$$M_2=\cos\alpha\times 37\text{ GeV}, \tag{4.9}$$

and by the inequality in (2.13)

$$M_0<\cos\alpha\times 41\text{ GeV}. \tag{4.10}$$

One interesting consequence of (4.7) is that in such a model the pure leptonic processes will also depend on the Cabibbo angle. For example, the cross sections for the elastic reactions $\nu_e e\to\nu_e e$ and $\bar{\nu}_e e\to\bar{\nu}_e e$ with both W^1 and W^2 contributing are $1+\sin 2\theta_C$ ($\approx$1.4) (see Ref. 25) times the corresponding values of the conventional $V-A$ theory. From the structure of $J^{(0)l}$ it is also clear that there will be no $\bar{\nu}_\mu e\to\bar{\nu}_\mu e$ reaction in the lowest order. Also even though there are $\bar{\nu}\nu$ terms in $J^{(0)l}$, the structure of $J^{(0)h}$ in (3.9) is such that $\nu+h_i\to\nu+h_f$ will not go in the lowest order if h_i and h_f are ordinary SU(3)′-singlet hadrons. (In other words, the neutrino elastic scattering on proton will be a "pure" charmed hadron production mechanism at high energies.)

We shall in the following briefly comment on the lepton mass problem. The bare mass term $m_0(\bar{E}^LE^R+\text{H.c.})$ will give equal masses to x^+, x_1, and e and there is a mixing term between ν_e and x_2 as well. To split the e and x^+ masses we must have a set of (1, 0) + (0, 1) Higgs scalars [see Eq. (2.7)] which couples invariantly with the E quartets: $(\bar{E}^LXE^R+\text{H.c.})$ After the neutral member develops vacuum expectation value there will be a mass term of the form $\delta m[\bar{E}^L(\eta_3+\zeta_3)E^R+\text{H.c.}]$. Scalars which transform as $(\frac{1}{2},\frac{1}{2})$ and couple to $x_2(\alpha)^L$ and E^R will also be needed to cancel the $\nu_e x_2$ mixing term. We can however dispense with the remaining possible (1, 1) scalars. With this scheme we have in zeroth order basically two independent mass parameters one for e and one for the x's. More precisely, among the four massive leptons we have the relations

$$m(x^+) + m(e) = m(x_1)\,, \quad (4.11)$$

$$(1 - \tfrac{2}{9}\cos^2\alpha)^{1/2} m(x_2) = m(x_1)\,. \quad (4.12)$$

Exactly the same scheme may be worked out for the M leptons.[26]

Given the phase assignment of (4.2) the situation with respect to CP violation is basically the same as the $O(4) \times U(1)$ model of Ref. 9, where details are given to show that even with the maximal CP violation phases the effects on physical processes are superweak. This comes about since nontrivial CP violation phases can crop up only in certain types of interference terms among the lepton couplings to different intermediate bosons. In particular mixings of W^1 and W^2 through lepton loops can introduce a CP-violating phase factor in the off-diagonal part of the $\overline{K}^0K^0$ mass matrix. The only aspect of this problem we shall briefly touch upon concerns the degree of divergence, and hence the need of renormalization counterterms, of such lepton bubble diagrams. In Ref. 9 an exhaustive discussion has been given on this question and its connection to various possible discrete symmetries of the theory. We shall further illustrate some of these points in our model. A straightforward calculation [using the full currents in Eqs. (4.3) and (4.4)] shows that such one-loop diagrams at small momentum transfer give the following effective interaction:

$$H_{12} = \rho(W_\mu^{1\dagger} W_\mu^2 + W_\mu^{2\dagger} W_\mu^1) + i\sigma(W_\mu^{1\dagger} W_\mu^2 - W_\mu^{2\dagger} W_\mu^1)\,, \quad (4.13)$$

with

$$\rho = (e/2\pi)^2 [m(x_1) - (1 - \tfrac{2}{9}\cos^2\alpha)^{1/2} m(x_2)][m(x^+) - m(e)]\ln\Lambda^2 + \text{finite terms}\,,$$

$$\sigma = \left(\frac{e}{2\pi}\right)^2 \{[(2 - \tfrac{2}{9}\cos^2\alpha) m^2(x_2) - m^2(x_1)] - [m(x_1) - (1 - \tfrac{2}{9}\cos^2\alpha)^{1/2} m(x_2)][m(x^+) + m(e)]\} \ln\Lambda^2 + \text{finite terms}\,,$$

where Λ is some Euclidean cutoff and we have only displayed contributions from E leptons. We note: (i) While the divergences in H_{12} in a $SU(2) \times SU(2)$ theory are generally quadratic, they are logarithmic in a model based on $SU(2) \times SU(2)$ with reflection symmetry R between the two SU(2)'s. H_{12} must be proportional to neutral lepton mass to reflect the fact that $H_{12} = 0$ in the R-invariant limit. (ii) Unlike the $O(4) \times U(1)$ model, we must have a $(1,0) + (0,1)$ Higgs scalar in order to have $m(x^+) \neq m(e)$. This means that ρ as well as σ can diverge, counterterms being available in the form of (2.11) and (2.12). (iii) The zeroth-order mass relation (4.12) reduces (4.13) to (now also putting in the M-lepton contribution)

$$\rho = \text{finite terms}\,,$$
$$\sigma = \left(\frac{e}{2\pi}\right)^2 [m^2(x_2) - m^2(y_2)] \ln\Lambda^2 + \text{finite terms}\,, \quad (4.14)$$

which to this order is basically the same as the situation in $O(4) \times U(1)$. It also shows that a finite result can emerge only if some symmetry exists between the E and M multiplets. However even with such a discrete symmetry it will be difficult in our model to enforce this finiteness of H_{12} to all orders in a natural way. [Namely, unlike the $O(4) \times U(1)$ case, the situation is complicated by availability of potential counterterms for ρ as well as for σ.] In principle we can have a Higgs system such that the vacuum expectation values of the scalar fields will take such a form, order by order, that $\mathcal{L}_{12}^{(\pm)}$ of (2.11) and (2.12) will always be zero (hence not available as counterterms). In other words these counterterms are of a type that their presence or absence can be affected by the scalar interactions of the theory. Clearly this way of putting all the difficult questions into the unknown scalar sector is not a very satisfactory approach; we shall not pursue this any further and assume that in such a theory there will be logarithmic divergences (but nothing worse) in the CP-violation parameters and one renormalization will be needed.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor A. Pais for discussions and a critical reading of the manuscript. I am also grateful to A. Zee for continual encouragement and helpful conversations.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2232.

†Address after September 1, 1973: Department of Physics, University of Missouri-St. Louis, St. Louis, Mo. 63121.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, *Elementary Particle Theory,* edited by N. Svartholm (Almqvist, Stockholm, 1968). For a recent review

on the whole subject of spontaneously broken gauge symmetry, see B. W. Lee, in *Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill.*, 1972 (NAL, Batavia, Ill., 1973), Vol. 4, p. 249.

[2]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972).

[3]B. W. Lee, Phys. Rev. D 6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972).

[4]S. Weinberg, Phys. Rev. D 5, 1962 (1972).

[5]I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. D 7, 1233 (1973).

[6]J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973).

[7]M. A. B. Bég and A. Zee, Phys. Rev. Lett. 30, 675 (1973).

[8]A. Pais, Phys. Rev. Lett. 29, 1712 (1972); 30, 114(E) (1973).

[9]A. Pais, this issue, Phys. Rev. D 8, 625 (1973).

[9a]G. Segrè, Phys. Rev. 173, 1730 (1968).

[10]"Strong PCAC (partial conservation of axial-vector current)" is assumed here. For alternative views see R. Brandt and G. Preparata, Ann. Phys. (N.Y.) 61, 119 (1970), and S. D. Drell, Phys. Rev. D 7, 2190 (1973).

[11]See, for example, J. Kuti and V. Weisskopf, Phys. Rev D 4, 3418 (1971).

[12]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970). Alternative ways have been suggested by H. Georgi and S. Glashow, Harvard report, 1973 (unpublished); see also Ref. 5.

[13]B. W. Lee, J. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973).

[14]B. W. Lee and S. B. Treiman, Phys. Rev. D 7, 1211 (1973).

[15]Lee and Treiman (Ref. 14) proposed a baryon model which will give the correct $\pi^0 \to 2\gamma$ rate but leaves open the question concerning 56 of SU(6). Within the eight-quark scheme the latter problem was discussed by Pais (Ref. 9, Appendix C) where a "nuclear model" for hadrons was described.

[16]It includes both the fractionally charged three-triplet model [the so-called "red, white, and blue quark" (RWB) model] and the integrally charged three-triplet models (Ref. 18). We shall only be interested in integrally charged ones.

[17]Using the three-triplet quarks as fundamental hadrons in gauge theories has been discussed by H. Lipkin, Phys. Rev. D 7, 1850 (1973), by H. Georgi and S. L. Glashow, Phys. Rev. D 7, 561 (1973), and by Bég and Zee (Ref. 7).

[18]M. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965). An excellent discussion on this topic may also be found in J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971). For definiteness, we shall work with the version as discussed by N. Cabibbo, L. Maiani, and G. Preparata, Phys. Lett. 25B, 132 (1967), although the original version by Han and Nambu is equal to the task.

[19]The $\Delta I = \frac{1}{2}$ rule in the standard $V-A$ theory with Han-Nambu quarks has been discussed (but from a dynamical point of view) by J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971).

[20]The $\Delta I = \frac{1}{2}$ rule in a gauge model with RWB quarks (although entirely different from the one presented here) has recently been proposed by M. A. B. Bég, this issue, Phys. Rev. D 8, 664 (1973).

[21]In the Georgi-Glashow model the mass of W must be less that 53 GeV. $\sigma(\nu_e e)$ are predicted to give the same values as those from standard $V-A$ theory.

[22]For $(\frac{1}{2}, \frac{1}{2})$ representation, $\vec{\eta} = \frac{1}{2}\vec{\tau} \otimes 1$ and $\vec{\zeta} = \frac{1}{2} 1 \otimes \vec{\tau}$ ($\vec{\tau}$ are Pauli matrices) and

$$f = \begin{pmatrix} f^+ & \frac{-f_1 + f_2}{\sqrt{2}} \\ \frac{f_1 + f_2}{\sqrt{2}} & f^- \end{pmatrix} .$$

The notations used in this paper differ from those in Refs. 8 and 9.

[23]In this type of models more than one set of $(\frac{1}{2}, \frac{1}{2})$ scalars will be needed. Here H stands collectively for all the scalar quartets.

[24]Lee, Primack, and Treiman (Ref. 13).

[25]In calculating numerically the upper bounds for $M_{0,1,2}$ and $\sigma(\nu_e e)$ we have assumed that the "bare" Cabibbo angle θ_C is not significantly different from the observed value.

[26]A similar scheme may be worked out for the hadron as well.