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Connection Between Equal-Time and Light-Cone Commutators

A-M. M. Abdel-Rahman and M. O. Taha
Department of Physics, University of Khartoum, Khartoum, Sudan
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Integral representations are given for matrix elements of equal-t (equal-time) and equal-τ (light-cone) commutators ($t=x_0$, $\tau=x_0+x_3$) of two vector currents. A plausible assumption is made connecting these commutators. Its consequences are corresponding pairs of general variable-mass sum rules. Each pair reduces to a single sum rule in the fixed-mass limit. Electroproduction is discussed as a special case.

I. INTRODUCTION

Spurred by experimental developments[1] on electroproduction and the discovery of electroproduction scaling laws,[2] as well as the theoretical possibility of operator-product expansions,[3-6] several authors[7,8] have proposed light-cone algebra as a relevant tool for investigations in particle physics. Since many of the possible fields for the application of this technique have already been probed using Gell-Mann's equal-time algebra it seems to us to be of relevance and interest to enquire as to the relationship between these two approaches and the extent to which the sets of results obtained coincide or differ. It is our object in this paper to investigate the consequences of a definite assumption that we make connecting the two algebras. This assumption is suggested by certain integral representations for both equal-time and light-cone commutators. These representations[9] constitute the formal basis for our investigation.

Denoting the Fourier transforms of the matrix elements, between equal-momenta spinless states, of the light-cone and equal-time commutators of two vector currents J_μ^i and J_ν^i by $\xi_{\mu\nu}^{ij}$ and $E_{\mu\nu}^{ij}$ one has

$$\xi_{+\nu}^{ij}=\alpha^{ij}p_\nu+\beta^{ij}q_\nu+\gamma^{ij}(\delta_{\nu 0}-\delta_{\nu 3}), \tag{1.1}$$

$$E_{0\nu}^{ij}=A^{ij}p_\nu+B^{ij}q_\nu+C^{ij}\delta_{\nu 0}. \tag{1.2}$$

We write integral representations for A^{ij}, B^{ij}, C^{ij}, α^{ij}, β^{ij}, and γ^{ij} in terms of the invariant components of the absorptive function $F_{\mu\nu}^{ij}$ where

$$F_{\mu\nu}^{ij}=\int e^{iqx}\langle p|[J_\mu^i(x),J_\nu^j(0)]|p\rangle\, d^4x. \tag{1.3}$$

These representations and the model-commutators for light-cone and equal-time algebras suggest our basic assumption for which we demand

$$\begin{aligned}\alpha^{ij}&=A^{ij},\\ \beta^{ij}&=B^{ij}.\end{aligned} \tag{1.4}$$

We show that the functions in (1.4) must be constant and obtain general sum rules involving integrals over the invariant absorptive functions. The equal-time integrals are over a parabolic path, whereas the light-cone integrals are over a linear path, in the space of the invariants. In the fixed-mass limits ($p_0=\infty$ for equal-time and $q_+=0$, or $p_+=\infty$, for light-cone) corresponding sum rules coincide. In other words the same set of fixed-mass sum rules is obtained from either equal-time or light-cone algebra.[10] The two sets of sum rules that one obtains are, however, not generally equivalent.

These sum rules are explicitly written for the case of electroproduction. In this case the fixed-mass limit of two sum rules is trivially satisfied, whereas the fixed-mass limit of another pair of sum rules gives[6,11]

$$-\frac{1}{2\pi m^2 q^2}\int_{-\infty}^{\infty}\nu W_2(\nu,q^2)d\nu=K, \tag{1.5}$$

where K is the Schwinger term in the equal-time commutator. In the scaling limit (1.5) gives

$$\frac{1}{4\pi}\int_{-1}^{1}\frac{F_2(\omega)}{\omega^2}d\omega=K. \tag{1.6}$$

Under our basic assumption (1.4), $\xi_{+\nu}^{ij}$ and $E_{0\nu}^{ij}$ are related by

$$\xi_{+\nu}^{ij}=E_{0\nu}^{ij}+(\gamma^{ij}-C^{ij})\delta_{\nu 0}-\gamma^{ij}\delta_{\nu 3}, \tag{1.7}$$

where γ^{ij} is the Fourier transform of a bilocal operator.

The material presented in this paper is organized as follows: In Sec. II we give the integral representations for the commutators. The general consequences for our basic assumption are investigated in Sec. III. Section IV deals with the special case of electroproduction, and Sec. V includes a number of remarks and points of discussion.

II. REPRESENTATION OF EQUAL-TIME AND LIGHT-CONE COMMUTATORS

We consider the Fourier transform $E_{\mu\nu}^{ij}$ of the matrix element of the equal-time commutator of two vector currents J_μ^i, J_ν^j between spinless single-particle states of equal momenta p defined by

$$E_{\mu\nu}^{ij} = \int e^{iqx}\delta(x_0)\langle p|[J_\mu^i(x), J_\nu^j(0)]|p\rangle d^4x. \quad (2.1)$$

The function $E_{\mu\nu}^{ij}$ has, under certain assumptions, the following integral representation[9]:

$$E_{\mu\nu}^{ij} = \frac{1}{2\pi}\int F_{\mu\nu}^{ij}(p, q')d\lambda, \quad (2.2)$$

where the absorptive function $F_{\mu\nu}^{ij}$ is given by

$$F_{\mu\nu}^{ij}(p, q) = \int e^{iqx}\langle p|[J_\mu^i(x), J_\nu^i(0)]|p\rangle d^4x \quad (2.3)$$

and

$$q' = (q_0 + \lambda, \vec{q}). \quad (2.4)$$

A straightforward generalization of the procedure leading to the representation (2.2) gives a similar representation for the Fourier transform $\xi_{\mu\nu}^{ij}$ of the equal-τ or light-cone commutator:

$$\xi_{\mu\nu}^{ij} = \int e^{iqx}\delta(x_0 + x_3)\langle p|[J_\mu^i(x), J_\nu^i(0)]|p\rangle d^4x. \quad (2.5)$$

The representation one obtains is expressed in terms of the same function $F_{\mu\nu}^{ij}$ with a different path of integration. One has

$$\xi_{\mu\nu}^{ij} = \frac{1}{2\pi}\int F_{\mu\nu}^{ij}(p, q'')d\lambda, \quad (2.6)$$

where

$$q'' = (q_0 + \lambda, \vec{q}_\perp, q_3 - \lambda). \quad (2.7)$$

The tensor $F_{\mu\nu}^{ij}(p, q)$ has the covariant decomposition

$$F_{\mu\nu}^{ij}(p, q) = \sum_{r=1}^{5} a_r^{ij}(\nu, q^2)\epsilon_{\mu\nu}^r(p, q), \quad (2.8)$$

where for $\{\epsilon_{\mu\nu}^r : r = 1, 2, \ldots, 5\}$ we take the set

$$\{p_\mu p_\nu, p_\mu q_\nu, q_\mu p_\nu, q_\mu q_\nu, \delta_{\mu\nu}\}. \quad (2.9)$$

When (2.8) is inserted into (2.2) one obtains

$$E_{0\nu}^{ij} = A^{ij}p_\nu + B^{ij}q_\nu + C^{ij}\delta_{\nu 0}, \quad (2.10)$$

with

$$A^{ij} = \frac{1}{2\pi}\int [p_0 a_1^{ij} + (q_0 + \lambda)a_3^{ij}]d\lambda, \quad (2.11)$$

$$B^{ij} = \frac{1}{2\pi}\int [p_0 a_2^{ij} + (q_0 + \lambda)a_4^{ij}]d\lambda, \quad (2.12)$$

$$C^{ij} = \frac{1}{2\pi}\int [\lambda p_0 a_2^{ij} + \lambda(q_0 + \lambda)a_4^{ij} + a_5^{ij}]d\lambda, \quad (2.13)$$

where in these integrals the argument of a_r^{ij} varies according to

$$a_r^{ij} = a_r^{ij}(\nu + p_0\lambda, q^2 + 2q_0\lambda + \lambda^2). \quad (2.14)$$

Similarly from (2.8) and (2.5) one gets for the light-cone commutator $\xi_{+\nu}^{ij} = \xi_{0\nu}^{ij} + \xi_{3\nu}^{ij}$,

$$\xi_{+\nu}^{ij} = \alpha^{ij}p_\nu + \beta^{ij}q_\nu + \gamma^{ij}(\delta_{\nu 0} - \delta_{\nu 3}), \quad (2.15)$$

with

$$\alpha^{ij} = \frac{1}{2\pi}\int [p_+ a_1^{ij} + q_+ a_3^{ij}]d\lambda, \quad (2.16)$$

$$\beta^{ij} = \frac{1}{2\pi}\int [p_+ a_2^{ij} + q_+ a_4^{ij}]d\lambda, \quad (2.17)$$

$$\gamma^{ij} = \frac{1}{2\pi}\int [\lambda p_+ a_2^{ij} + \lambda q_+ a_4^{ij} + a_5^{ij}]d\lambda, \quad (2.18)$$

where $p_+ = p_0 + p_3$, $q_+ = q_0 + q_3$, and, in these integrals, the functions a_r^{ij} are given by

$$a_r^{ij} = a_r^{ij}(\nu + p_+\lambda, q^2 + 2q_+\lambda). \quad (2.19)$$

Equations (2.14) and (2.19) show that the basic difference between equal-t and equal-τ commutators lies in the contour of integration followed; a parabola for equal-t and a straight line for equal-τ.
In the following section we apply these representations to deduce the consequences of the assumption (1.4).

III. CONSEQUENCES OF BASIC ASSUMPTION

Light-cone algebra, derived from the quark model[7] or the quark-parton model,[8] gives for two vector currents

$$[V_+^i(x), V_+^j(0)]\delta(x_+/\sqrt{2}) = i\sqrt{2}f_{ijk}V_+^k(x)\delta(x_-/\sqrt{2})\delta(x_+/\sqrt{2})\delta^2(\vec{x}_\perp) + \text{bilocal operator contribution}, \quad (3.1)$$

where $x_- = x_0 - x_3$. From equal-time current algebra one has

$$[V_0^i(x), V_+^j(0)]\delta(x_0) = if_{ijk}V_+^k(x)\delta^4(x) + \text{Schwinger terms}. \quad (3.2)$$

Now Eq. (2.15) gives

$$\xi_{++}^{ij} = \alpha^{ij}p_+ + \beta^{ij}q_+, \quad (3.3)$$

whereas from Eq. (2.10) one obtains

$$E_{0+}^{ij} = A^{ij}p_+ + B^{ij}q_+ + C^{ij}. \quad (3.4)$$

The V_+ terms in (3.1) and (3.2) contribute equally to the Fourier transforms ξ_{++}^{ij} and E_{0+}^{ij}. This vector contribution is proportional to p_+. The extra terms on the right-hand sides of (3.1) and (3.2) contribute to the coefficient of q_+ and not to the coefficient of p_+. It would then follow, in the context of the above models, that if these noncanonical terms contribute equally to the coefficient of q_+, one would have

$$\alpha^{ij} = A^{ij}, \tag{3.5a}$$

$$\beta^{ij} = B^{ij}. \tag{3.5b}$$

In the model commutators (3.1) and (3.2) both sides of (3.5a) are specified. The terms in (3.5b) would be known, in these models, if gradient terms in (3.1) and (3.2) were specified. In this paper, we shall not commit ourselves to any definite value for ξ_{++}^{ij} or E_{0+}^{ij} – or to any detailed internal symmetry structure – but shall investigate the consequences of the general assumption (3.5) using the representations introduced in Sec. II.

From Eq. (2.11) one obtains

$$A^{ij} = \frac{1}{2\pi}\int_{-\infty}^{\infty}[a_1^{ij}(\nu', q'^2) + p_0^{-2}(\nu' + \vec{p}\cdot\vec{q})a_3^{ij}(\nu', q'^2)]d\nu', \tag{3.6}$$

where

$$q'^2 = -\vec{q}^2 + \frac{1}{p_0^2}(\vec{p}\cdot\vec{q} + \nu')^2. \tag{3.7}$$

This shows that

$$A^{ij} = A^{ij}(p_0^{-2}, -\vec{q}^2 + p_0^{-2}(\vec{p}\cdot\vec{q})^2, p_0^{-2}\vec{p}\cdot\vec{q}). \tag{3.8}$$

Similarly Eq. (2.16) gives

$$\alpha^{ij} = \frac{1}{2\pi}\int_{-\infty}^{\infty}[a_1^{ij}(\nu', q''^2) - \eta a_3^{ij}(\nu', q''^2)]d\nu', \tag{3.9}$$

where

$$q''^2 = -\vec{q}_\perp^{\,2} + 2\eta(\tfrac{1}{2}q_+p_- - \vec{p}_\perp\cdot\vec{q}_\perp) - 2\eta\nu', \qquad \eta = -q_+/p_+, \tag{3.10}$$

so that

$$\alpha^{ij} = \alpha^{ij}(\eta, -\vec{q}_\perp^{\,2} + 2\eta(\tfrac{1}{2}q_+p_- - \vec{p}_\perp\cdot\vec{q}_\perp)). \tag{3.11}$$

Equations (3.5a), (3.8), and (3.11) clearly imply that

$$A^{ij} = K_1^{ij}, \tag{3.12}$$

$$\alpha^{ij} = K_1^{ij}, \tag{3.13}$$

where K_1^{ij} is constant. These equations are generalized noninvariant sum rules depending on several parameters whose content is further elucidated on taking special limits for these parameters. The $p_0 \to \infty$ fixed $\vec{q}^2, \vec{p}\cdot\vec{q}$, for example, gives from Eq. (3.12) the sum rule

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} a_1^{ij}(\nu', -\vec{q}^2)d\nu' = K_1^{ij} \tag{3.14}$$

provided that the integral

$$\int_{-\infty}^{\infty}\nu' a_3^{ij}(\nu', -\vec{q}^2)d\nu'$$

converges. The same sum rule (3.14) is obtained on taking the limit $\eta \to 0$ in (3.9), requiring the weaker condition that

$$\int_{-\infty}^{\infty} a_3^{ij}(\nu', -\vec{q}_\perp^{\,2})d\nu'$$

converges. It must, however, be remarked that the sum rules (3.12) and (3.13) are not generally equivalent. Apart from the obvious difference in the integrands, the function A^{ij} obtained on integrating over the parabola in (3.6) is restricted by (3.12) to a two-dimensional surface. In contrast α^{ij}, obtained by integrating along the linear path in (3.9), is restricted to a curve by (3.13). It is also clear from the above example that conditions for taking closely related limits to evaluate these sum rules may be satisfied in one case but not in the other.

Now from Eq. (3.5b) one similarly obtains the sum rules

$$B^{ij} = K_2^{ij}, \tag{3.15}$$

$$\beta^{ij} = K_2^{ij}, \tag{3.16}$$

where K_2^{ij} is constant. These sum rules may be explicitly written as

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[a_2^{ij}(\nu', q'^2) + p_0^{-2}(\nu' + \vec{p}\cdot\vec{q})a_4^{ij}(\nu', q'^2)]d\nu' = K_2^{ij}, \tag{3.17}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[a_2^{ij}(\nu', q''^2) - \eta a_4^{ij}(\nu', q''^2)]d\nu' = K_2^{ij}, \tag{3.18}$$

where q'^2, q''^2, and η are defined as in Eqs. (3.7) and (3.10).

Taken together these sum rules imply that the general light-cone and equal-time commutators $\xi_{+\nu}^{ij}$ and $E_{0\nu}^{ij}$ are related according to

$$\xi_{+\nu}^{ij} = E_{0\nu}^{ij} + (\gamma^{ij} - C^{ij})\delta_{\nu 0} - \gamma^{ij}\delta_{\nu 3}. \tag{3.19}$$

If, further, one assumes that the equal-time commutator E_{00}^{ij} is completely given by

$$E_{00}^{ij} = A^{ij}p_0, \tag{3.20}$$

then one has the condition

$$B^{ij}q_0 + C^{ij} = 0 \tag{3.21}$$

which, according to (3.15) and (3.16), implies

$$C^{ij} = -q_0\beta^{ij} = -q_0K_2^{ij}. \tag{3.22}$$

Using the representation (2.13) for C^{ij} we see that C^{ij} is of the form

$$C^{ij}=C_1^{ij}+q_0C_2^{ij}, \tag{3.23}$$

where C_1^{ij} and C_2^{ij} are independent of q_0. From the integral representation for C_2^{ij} and Eq. (3.17) one in fact finds that

$$C_2^{ij}=-K_2^{ij} \tag{3.24}$$

so that we must have

$$C_1^{ij}=0 \tag{3.25}$$

to maintain the assumption (3.20). When explicitly written, this sum rule reads

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}(z\,p_0a_2^{ij}+z^2a_4^{ij}+a_5^{ij})dz=0, \tag{3.26}$$

where, in this integral,

$$a_r^{ij}=a_r^{ij}(-\vec{p}\cdot\vec{q}+p_0z,\,-\vec{q}^{\,2}+z^2). \tag{3.27}$$

We finally remark that Eq. (3.19) gives, in particular,

$$\xi_{+\nu}^{ij}\,|_{q_+=0}=E_{0\nu}^{ij}\,|_{q_3=0}+(\delta_{\nu 0}-\delta_{\nu 3})\Lambda^{ij}, \tag{3.28}$$

where

$$\begin{aligned}\Lambda^{ij}&=\gamma^{ij}(q_0=q_3=0)\\&=\frac{1}{2\pi p_+}\int[(\nu'+\vec{p}_\perp\cdot\vec{q}_\perp)a_2^{ij}(\nu',-\vec{q}_\perp^{\,2})\\&\qquad+a_5^{ij}(\nu',-\vec{q}_\perp^{\,2})]d\nu'.\end{aligned} \tag{3.29}$$

In the next section we consider the implication of these general sum rules for the special case of electroproduction.

IV. ELECTROPRODUCTION

For electroproduction

$$\begin{aligned}F_{\mu\nu}=&\left(\frac{q_\mu q_\nu}{q^2}-\delta_{\mu\nu}\right)W_1(\nu,q^2)\\&+\frac{1}{m^2}\left(p_\mu-\frac{\nu q_\mu}{q^2}\right)\left(p_\nu-\frac{\nu q_\nu}{q^2}\right)W_2(\nu,q^2).\end{aligned} \tag{4.1}$$

Comparison with Eq. (2.8) gives

$$\begin{aligned}a_1&=\frac{1}{m^2}W_2,\\a_2&=a_3=\frac{-\nu}{m^2q^2}W_2,\\a_4&=\frac{1}{q^2}W_1+\frac{\nu^2}{m^2q^4}W_2,\\a_5&=-W_1.\end{aligned} \tag{4.2}$$

Thus the sum rules (3.12) and (3.13) read

$$\frac{1}{2\pi m^2}\int_{-\infty}^{\infty}\left(1-p_0^{-2}(\nu'+\vec{p}\cdot\vec{q})\frac{\nu'}{q'^2}\right)W_2(\nu',q'^2)d\nu'=K_1 \tag{4.3}$$

and

$$\frac{1}{2\pi m^2}\int_{-\infty}^{\infty}\left(1+\frac{\eta\nu'}{q''^2}\right)W_2(\nu',q''^2)d\nu'=K_1. \tag{4.4}$$

The value of the constant K_1 is obtained on, for example, taking the limit $\eta\to 0$ in (4.4). Since $W_2(\nu,q^2)=-W_2(-\nu,q^2)$ we see that $K_1=0$, in agreement with usually postulated equal-time and light-cone algebras. It should, however, be observed that the content of the sum rules

$$\int_{-\infty}^{\infty}\left(1-p_0^{-2}(\nu'+\vec{p}\cdot\vec{q})\frac{\nu'}{q'^2}\right)W_2(\nu',q'^2)d\nu'=0, \tag{4.5a}$$

$$\int_{-\infty}^{\infty}\left(1+\frac{\eta\nu'}{q''^2}\right)W_2(\nu',q''^2)d\nu'=0 \tag{4.5b}$$

is nontrivial, since the general absorptive function W_2 receives [for $p_0^{-2}\neq 0$ in (4.5a) and $\eta\neq 0$ in (4.5b)] contributions from various physical regions and not only from electroproduction. For electroproduction these sum rules are trivially satisfied.

The sum rules (3.15) and (3.16) become

$$\frac{1}{2\pi}\int\left[\left(\frac{-\nu'}{m^2q'^2}+p_0^{-2}(\nu'+\vec{p}\cdot\vec{q})\frac{\nu'^2}{m^2q'^4}\right)W_2(\nu',q'^2)+p_0^{-2}\,\frac{\nu'+\vec{p}\cdot\vec{q}}{q'^2}\,W_1(\nu',q'^2)\right]d\nu'=K_2, \tag{4.6}$$

$$\frac{1}{2\pi}\int\left[\left(\frac{-\nu'}{m^2q''^2}-\frac{\eta\nu'^2}{m^2q''^4}\right)W_2(\nu',q''^2)-\frac{\eta}{q''^2}W_1(\nu',q''^2)\right]d\nu'=K_2. \tag{4.7}$$

On taking the limits $p_0\to\infty$, or $\eta\to 0$, both (4.6) and (4.7) reduce to the sum rule[11]

$$-\frac{1}{2\pi m^2}\int_{-\infty}^{\infty}\frac{\nu'W_2(\nu',q^2)}{q^2}\,d\nu'=K_2. \tag{4.8}$$

A change of variable $\nu'\to\omega'=-q^2/2\nu'$ gives

$$\frac{1}{4\pi m^2}\int_{-1}^{1}\frac{\nu'W_2(\omega',q^2)d\omega'}{\omega'^2}=K_2. \tag{4.9}$$

If we now take $q^2\to-\infty$ in (4.9) and use Bjorken scaling behavior,[2] we obtain

$$\frac{1}{4\pi}\int_{-1}^{1}\frac{F_2(\omega)}{\omega^2}\,d\omega=K_2, \tag{4.10}$$

the Schwinger term sum rule.[6,11]

Finally we observe that Λ^{ij}, which occurs in the relation (3.29), will now be given by

$$\Lambda = \frac{\vec{p}_\perp \cdot \vec{q}_\perp}{2\pi m^2 p_+ \vec{q}_\perp^{\,2}} \int_{-\infty}^{\infty} \nu' W_2(\nu', -\vec{q}_\perp^{\,2}) d\nu' , \tag{4.11}$$

i.e.,

$$\Lambda = \frac{\vec{p}_\perp \cdot \vec{q}_\perp}{4\pi p_+} \int_{-1}^{1} \frac{F_2(\omega)}{\omega^2} d\omega . \tag{4.12}$$

V. CONCLUDING REMARKS

(1) We first remark on a fundamental difference between equal-time and light-cone commutators that has previously been observed[12] and arises naturally from our integral representations. One notes that, for $\vec{q} = \vec{0}$, the equal-time commutator E^{ij} has no q dependence, since it is independent of q_0. The equal-τ commutator, on the other hand, depends on q_+ even when $\vec{q} = \vec{0}$. In particular, the term γ^{ij} in the equal-τ commutator may depend on q_+ in any arbitrary manner, as is evident from Eq. (2.18). In x-space language γ^{ij} is the Fourier transform of a bilocal object.

(2) For the case of electroproduction the bilocal contribution takes the form

$$\gamma = \frac{-1}{2\pi p_+} \int \left[\frac{(\nu' - \nu)\nu'}{m^2 q'^2} \left(1 + \frac{\eta\nu'}{q'^2} \right) W_2(\nu', q'^2) + \left(1 + \frac{\eta(\nu' - \nu)}{q'^2} \right) W_1(\nu', q'^2) \right] d\nu' . \tag{5.1}$$

Using the sum rule (4.7), this equation gives

$$\gamma = \gamma_0 - \frac{\nu}{p_+} K_2 , \tag{5.2}$$

where

$$\gamma_0 = \frac{-1}{2\pi p_+} \int \left(1 + \frac{\eta\nu'}{q'^2} \right) \times \left(\frac{\nu'^2}{m^2 q'^2} W_2(\nu', q'^2) + W_1(\nu', q'^2) \right) d\nu' . \tag{5.3}$$

One observes that this bilocal contribution has a simple form in the fixed-mass limit $q_+ = 0$, since $\gamma_0(q_+ = 0)$ vanishes so that (5.2) gives

$$\gamma(q_+ = 0) = \left(\frac{\vec{p}_\perp \cdot \vec{q}_\perp}{p_+} - \tfrac{1}{2} q_- \right) K_2 . \tag{5.4}$$

(3) From the integral representations for β and γ one can see that the light-cone commutators are in general independent of q_-. Equations (2.17) and (2.18) in fact show that

$$\gamma^{ij} = \gamma_0^{ij} - \frac{\nu}{p_+} \beta^{ij} , \tag{5.5}$$

where

$$\gamma_0^{ij} = \frac{1}{2\pi p_+} \int (\nu' a_2^{ij} - \eta\nu' a_4^{ij} + a_5^{ij}) d\nu' ,$$

i.e., β^{ij} and γ_0^{ij} are independent of q_-. Thus, for example,

$$\begin{aligned} \xi_{+-}^{ij} &= \alpha^{ij} p_- + \beta^{ij} q_- + 2\gamma^{ij} \\ &= (\alpha^{ij} + \eta\beta^{ij}) p_- + 2\gamma_0^{ij} + 2\beta^{ij} \frac{\vec{p}_\perp \cdot \vec{q}_\perp}{p_+} . \end{aligned} \tag{5.6}$$

In particular, one obtains for $q_+ = 0$ on using the sum rules (3.13) and (3.16)

$$\xi_{+-}^{ij} |_{q_+=0} = K_1^{ij} p_- + 2\gamma_0^{ij}(q_+ = 0) + \frac{2\vec{p}_\perp \cdot \vec{q}_\perp}{p_+} K_2^{ij} . \tag{5.7}$$

The last two terms in (5.7) constitute the bilocal contribution when $q_+ = 0$, which for electroproduction takes the form 2Λ [see Eq. (4.12)], in agreement with Ref.13.

(4) If one requires that both the equal-time and light-cone commutators admit only single derivative terms on the right-hand side, then the Fourier transforms B^{ij} and β^{ij} of these gradient terms are such that B^{ij} is a constant given by

$$B^{ij} = \beta^{ij}(q_+ = 0) . \tag{5.8}$$

This follows from our integral representations for B^{ij} and β^{ij}. Thus, when these commutators are correctly calculated from the same dynamics (which is assumed to admit only first-order gradient terms) it is not possible to have $B^{ij} \neq 0$ unless the condition $\beta^{ij}(q_+ = 0) \neq 0$ is also satisfied. Since

$$\int \frac{F_2(\omega)}{\omega^2} d\omega \neq 0 , \tag{5.9}$$

these gradient terms must exist in both commutators of the electromagnetic current. They also satisfy (5.8).

Note added in proof. It is assumed that the asymptotic behavior of $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$ is such as to allow the appropriate limits in (4.6) and (4.7) to be taken resulting in the sum rule (4.8).

[1]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970.

[2]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3]K. G. Wilson, Phys. Rev. 129, 1499 (1969).

[4]R. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

[5]H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970).

[6]R. Jackiw, R. Van Royen, and G. West, Phys. Rev. D 2, 2473 (1970).

[7]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1.

[8]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[9]M. O. Taha, Nuovo Cimento 60A, 651 (1969); Phys. Rev. 162, 1694 (1967).

[10]M. O. Taha and A-M. M. Abdel-Rahman, Univ. of Khartoum report, 1972 (unpublished).

[11]M. O. Taha, Univ. of Khartoum report, 1972 (unpublished).

[12]R. Jackiw, invited talk at the 1972 Annual Meeting of the American Physical Society, San Francisco, 1972 (unpublished).

[13]Duane A. Dicus, Roman Jackiw, and Vigdor L. Teplitz, Phys. Rev. D 4, 1733 (1971).

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O(4) Gauge Theory with Three-Triplet Quarks*

T. P. Cheng

Rockefeller University, New York, New York 10021†

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A theory of electromagnetic and weak interactions based on the O(4) gauge group is formulated along the general program put forward recently by Pais. In our scheme the hadron symmetry $SU(3)\times SU(3)'$ is incorporated in such a way that the model has a (kinematical) $\Delta I = 1/2$ rule for nonleptonic weak decays. The lepton assignment may be made to produce CP-violation effects which are more "convergent" without enlarging the gauge group. There is no lower bound to the masses of the intermediate vector bosons, while the upper limit for one of the charged W's is only 18 GeV. The elastic reaction cross sections $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ are about 1.4 times the usual $V-A$ values. Other physical features of this model are also discussed in some detail.

I. INTRODUCTION

Following the pioneering works of Weinberg and Salam, a number of renormalizable theories of of weak and electromagnetic interactions have been constructed in the framework of spontaneously broken gauge symmetries.[1-9] Looking over the totality of the physically viable models proposed so far, we may broadly distinguish the following two classes: (i) Most of the existing models are based on the gauge group $SU(2)\times U(1)$. They differ from each other in their lepton and hadron multiplet assignments. [In this class we include those theories based on larger groups but in which some of the intermediate vector bosons are endowed with superheavy masses so that the group is effectively reduced, in the first approximation, down to the subgroup $SU(2)\times U(1)$ (Ref. 4); we also include here the Georgi-Glashow model[2] which is based on SO(3) or SU(2)—namely it may be viewed as an $SU(2)\times U(1)$ model with all multiplets transforming trivially under the U(1) group.] (ii) Recently Pais has advocated the exploration of a class of theories based on the gauge groups $O(4)\times \mathcal{G}$.[8,9] [They include $SU(2)\times SU(2)$, $O(4)\times U(1)$, $O(4)\times O(4)$, etc.] The central idea, as proposed first by Segrè,[9a] is that *two* sets of charged intermediate vector bosons of *comparable* masses are made to mediate separately the $\Delta S=0$ and $\Delta S=1$ weak decays. This also opens up the possibility of giving neutral leptons maximal CP-violating phases and yet, Pais shows, the effects on physical processes will be superweak. In this paper we shall present a model built around this central conception but based on a strict O(4) gauge group, namely the group $SU(2)\times SU(2)$ plus R, "parity" of the group O(4). (The relevance of this R symmetry will be discussed in Sec. II.) Compared with the O(4) theory of Refs. 8 and 9, it involves different lepton and hadron multiplet assignments: A larger number of neutral particles are needed here but all multiplets in this theory are R-symmetric.

Another motivation for our model originates from questions concerning hadron structures. The attractive simple picture of all hadrons made out of three fractionally charged $\mathcal{P}$, $\mathcal{N}$, and λ quarks may