and h^2 and therefore are neglecting terms of order $g^2 h\langle\phi\rangle$ and $h^3\langle\phi\rangle$. Notice, however, that the latter terms may be taken into account (e.g., by further assuming the validity of a certain "Bjorken type" expansion) at the expense of some complication and are seen not to affect our general conclusion (see Appendix).

[7]Notice that for a gauge group such as $SU(2)_L \times SU(2)_R \times U(1)$, for which the scalar densities u_0 and u_3 belong to different group representations, if the allowed mass term in $\mathcal{L}_{\text{weak}}$ is of u_0 type then all divergent radiative corrections will also be of u_0 type *without implying any special constraint between gauge bosons masses.* This will not be true for those groups, such as $SU(3)_L \times SU(3)_R \times U(1)$, for which u_0 and u_3 belong to the same group representation.

[8]As defined by retaining the leading light-cone singularity of the propagator matrix.

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Unified Weak-Electromagnetic Gauge Schemes Based on the Three-Dimensional Unitary Group*

J. Schechter and Y. Ueda†
Physics Department, Syracuse University, Syracuse, New York 13210
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$SU(3)\times U(1)$ and chiral $SU(3)\times SU(3)\times U(1)\times U(1)$ models for the weak and electromagnetic interactions of leptons and hadrons are proposed. The usual quark model for hadrons as well as the usual Cabibbo rotation scheme is assumed. Symmetry-breaking effects due to inequalities in the masses of the gauge bosons are considered. By making a simple choice of the gauge boson masses based on the U-spin subgroup of SU(3), the undesired $|\Delta S|=2$ weak nonleptonic transitions can be completely eliminated and the undesired $|\Delta S|=1$ neutral current semileptonic transitions drastically suppressed. These models can reproduce the ordinary phenomenological weak interaction but are sufficiently flexible to give the possibility of interesting new effects for neutral leptonic and semileptonic processes, as well as to allow the possibility of an over-all arbitrariness in normalization for the effective nonleptonic interaction. Another consequence of the present scheme is the existence of very heavy leptons. A brief discussion of including the strong interactions in a grand unification is also presented.

I. INTRODUCTION

Unified weak-electromagnetic gauge schemes[1-4] have recently attracted some attention because of the possibility of testing their consequences at the new National Accelerator Laboratory machine and because, when combined with the "Higgs" mechanism[5] of spontaneous symmetry breakdown, they may be renormalizable.[6] The minimal symmetry group involved in these considerations is $SU(2) \times U(1)$, the SU(2) being generated by the usual left-handed charged weak currents and the U(1) being a weak "hypercharge." If one considers pure leptonic weak interactions by themselves there is no apparent contradiction with allowed processes but, of course, improved high-energy neutrino experiments may modify this situation. For the cases of semileptonic and nonleptonic interactions one runs into nonallowed weak transitions arising from the $|\Delta S|=1$ electrically neutral hadronic currents which enter in the Cabibbo scheme. Several models[3,7,8] have been proposed to solve these difficulties but all seem to contain some unesthetic features. For example, one model[7] involves the introduction of a four-dimensional, rather than the successful three-dimensional, unitary symmetry for the hadrons.

In the present paper we would like to explore gauge schemes based on the three-dimensional unitary group.[9] Both the $SU(3)\times U(1)$ and the chiral $SU(3)\times SU(3)$ groups will be considered. We will focus our attention on the current-gauge boson part of the theory, assuming that symmetry-breaking results from inequalities in the masses of the gauge bosons. The other assumptions[10] are

(a) CP invariance,
(b) Cabibbo form for the weak currents,
(c) Analogous electron and muon interactions.

The basic problem facing us in these models is the suppression of the unobserved reactions due to the neutral hadronic currents. Our method of solving it will be the straightforward one of considering those particular intermediate bosons which couple to the "bad" currents to have arbitrarily large masses. While this method may at first seem crude it is actually very much in the spirit of the present way of thinking, in which the difference between the electromagnetic and usual weak interaction arises essentially from the enormous difference between the photon and intermediate boson masses. It is interesting to note, from a group-theoretical standpoint, that since our models single out the electric-charge direction in the three-dimensional unitary space it is convenient to classify the gauge particles according to their transformation properties with respect to the so-called "U-spin" subgroup of SU(3).

Another interesting aspect of the present models concerns the existence of heavy leptons. Since we are postulating the three-dimensional unitary group to be fundamental this means that a new (very) heavy electron and muon are predicted. There is actually some experimental evidence[11] that heavy leptons may exist. An alternative scheme due to Konopinski and Mahmoud[12] treats the electron and muon asyymetrically and gets away with just one lepton triplet. However this requires the existence of doubly charged intermediate bosons and prevents us from including hadrons in a universal way.

If the point of view of this paper is correct it implies the fascinating situation that the presently observed lepton spectrum is just an initial glimmering of the over-all pattern.

The SU(3)×U(1) model is discussed in Sec. II of this paper. It is shown how undesirable weak transitions may be eliminated or suppressed and possible consequences for pure leptonic, semileptonic, and nonleptonic interactions are noted.

The SU(3)×SU(3) model is discussed in Sec. III. An amusing point of this model is that it opens the way to the construction of a theory where the total Lagrangian of strong, electromagnetic, and weak interactions has an *exact* SU(3)×SU(3) symmetry (as well as possessing charge conjugation and parity invariance).

II. SU(3) × U(1) MODEL

We first set down our notation for the lepton part of the theory. We combine the electron and muon fields into two column vectors:

$$\psi=\begin{pmatrix}\nu_e\\ e\\ e'\end{pmatrix},\quad \chi=\begin{pmatrix}\nu_\mu\\ \mu\\ \mu'\end{pmatrix}, \tag{2.1}$$

where e' and μ' are the new heavy electron type and muon type leptons, respectively. They both have negative charge. Then the objects which transform as triplets under our left-handed SU(3) are

$$\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\\ e'\end{pmatrix},\quad \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_\mu\\ \mu\\ \mu'\end{pmatrix}. \tag{2.2}$$

[Note that $\frac{1}{2}(1+\gamma_5)\nu_e=\nu_e$, etc.] The lepton singlets are assigned to be the right-handed objects:

$$\begin{aligned}&\tfrac{1}{2}(1-\gamma_5)e,\quad \tfrac{1}{2}(1-\gamma_5)e',\\ &\tfrac{1}{2}(1-\gamma_5)\mu,\quad \tfrac{1}{2}(1-\gamma_5)\mu'.\end{aligned} \tag{2.3}$$

It is convenient to use the left- and right-handed currents, defined as

$$\begin{aligned}l^b_{a\alpha}&=i\bar\psi_b\gamma_\alpha(1+\gamma_5)\psi_a+i\bar\chi_b\gamma_\alpha(1+\gamma_5)\chi_a,\\ r^b_{a\alpha}&=i\bar\psi_b\gamma_\alpha(1-\gamma_5)\psi_a+i\bar\chi_b\gamma_\alpha(1-\gamma_5)\chi_a.\end{aligned} \tag{2.4}$$

The integrated currents of (2.4) generate a $U_L(3)\times U_R(2)$ algebra. (Note that $r^b_{1\alpha}=r^1_{b\alpha}=0$.) However, in this section we are only assuming the theory to be invariant under an $SU_L(3)\times U(1)$ subalgebra of this algebra.

Since our aim is to construct a Yang-Mills-type gauge theory we introduce nine vector gauge fields, one for each generator of the SU(3)×U(1) group. The notation is as follows:

$$\begin{aligned}&\text{octet: } (W^b_a)_\alpha \text{ with } (W^c_c)_\alpha=0,\\ &\text{singlet: } D_\alpha.\end{aligned} \tag{2.5}$$

We note that the gauge bosons can be classified according to the U-spin subgroup of SU(3) which singles out the 1 direction in unitary space. Thus we introduce the abbreviations:

$$\begin{aligned}F_\alpha&=-(3/2)^{1/2}(W^1_1)_\alpha,\\ H_\alpha&=(1/\sqrt{2})[(W^2_2)_\alpha-(W^3_3)_\alpha].\end{aligned} \tag{2.6}$$

F_α is a U-spin singlet while H_α is a member of a U-spin triplet. The photon field $\mathfrak{A}_\mu$ cannot be identified as a pure member of the octet since the octet fields only couple to the left-handed currents while the photon must couple to left plus right. Since D_α *can* couple to the right-handed currents, the simplest possibility, which we shall adopt here, is to let the fields D_α and F_α mix to give the photon and a massive neutral vector field Z_α:

$$\begin{pmatrix}F_\alpha\\ D_\alpha\end{pmatrix}=\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\mathfrak{A}_\alpha\\ Z_\alpha\end{pmatrix}. \tag{2.7}$$

In (2.7) ϕ is a mixing angle (which should not be confused with the Cabibbo angle θ). Note that in the present mixing scheme the photon is a mixture of an SU(3) singlet and a U-spin singlet member of an octet. Compare this with the SU(2) model[2,3] where the photon is a mixture of an SU(2) singlet and the $I_3=0$ member of an SU(2) triplet. We shall see that H_α is the field which couples to the undesirable neutral currents so its mass must be extremely large.

To introduce the hadrons into the theory it is convenient to assume that they can all be made out of the three quarks q_1, q_2, q_3. The objects which transform as a triplet under the left-handed SU(3) are the rotated quarks:

$$\tfrac{1}{2}(1+\gamma_5)U\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}, \tag{2.8}$$

where U is the Cabibbo matrix

$$U=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \tag{2.9}$$

with $\sin\theta\simeq\frac{1}{4}$. The hadron singlets in this model are the right-handed objects:

$$\tfrac{1}{2}(1-\gamma_5)q_1,\quad \tfrac{1}{2}(1-\gamma_5)q_2,\quad \tfrac{1}{2}(1-\gamma_5)q_3. \tag{2.10}$$

Furthermore, the hadronic currents which generate the usual chiral U(3)×U(3) are

$$\begin{aligned}(j^l)^b_{a\alpha}&=i\bar{q}_b\gamma_\alpha(1+\gamma_5)q_a,\\ (j^r)^b_{a\alpha}&=i\bar{q}_b\gamma_\alpha(1-\gamma_5)q_a.\end{aligned} \tag{2.11}$$

In terms of the currents just introduced the electromagnetic current of hadrons and leptons is

$$\begin{aligned}J^{\rm em}_\alpha &\equiv -i\bar{e}\gamma_\alpha e-i\bar{e}'\gamma_\alpha e'-i\bar{\mu}\gamma_\alpha\mu-i\bar{\mu}'\gamma_\alpha\mu'+\tfrac{2}{3}i\bar{q}_1\gamma_\alpha q_1-\tfrac{1}{3}i\bar{q}_2\gamma_\alpha q_2-\tfrac{1}{3}i\bar{q}_3\gamma_\alpha q_3\\ &=-\tfrac{1}{2}(l^2_{2\alpha}+r^2_{2\alpha}+l^3_{3\alpha}+r^3_{3\alpha})+\tfrac{1}{3}[(j^l)^1_{1\alpha}+(j^r)^1_{1\alpha}]-\tfrac{1}{6}[(j^l)^2_{2\alpha}+(j^r)^2_{2\alpha}+(j^l)^3_{3\alpha}+(j^r)^3_{3\alpha}].\end{aligned} \tag{2.12}$$

(In addition the total electromagnetic current contains a W-meson part.) Of course we have the electromagnetic interaction:

$$\mathcal{L}^{\rm em}=|e|J^{\rm em}_\alpha\mathcal{A}_\alpha, \tag{2.13}$$

with $|e|^2/4\pi\simeq1/137$. The ususal phenomenological weak interaction is

$$\mathcal{L}^w=\frac{G}{\sqrt{2}}J^{(+)}_\alpha J^{(-)}_\alpha, \tag{2.14}$$

where $|G|\simeq1.03\times10^{-5}M_p^{-2}$ and

$$J^{(+)}_\alpha=l^2_{1\alpha}+\cos\theta\,(j^l)^2_{1\alpha}+\sin\theta\,(j^l)^3_{1\alpha},\quad J^{(-)}_\alpha=l^1_{2\alpha}+\cos\theta(j^l)^1_{2\alpha}+\sin\theta\,(j^l)^1_{3\alpha}. \tag{2.15}$$

Having specified our notation we proceed to write down the SU(3)×U(1)-invariant unified weak-electromagnetic interaction of currents and gauge fields. With the mixing scheme of (2.7), and using matrix notation for the unitary octets and nonets [i.e., we make the replacement $W^b_a\to(W)_{ab}$, etc.], the unique interaction Lagrangian is

$$\mathcal{L}=g\,{\rm Tr}(l_\alpha W_\alpha)-g'D_\alpha[{\rm Tr}(l_\alpha)+\tfrac{3}{2}(r^2_{2\alpha}+r^3_{3\alpha})]+g\,{\rm Tr}(Uj^l_\alpha U^{-1}W_\alpha)+g'D_\alpha[(j^r)^1_{1\alpha}-\tfrac{1}{2}(j^r)^2_{2\alpha}-\tfrac{1}{2}(j^r)^3_{3\alpha}], \tag{2.16}$$

where g and g' are real coupling constants related to the electric charge and the mixing angle ϕ by

$$g=\frac{-|e|}{\sqrt{6}\cos\phi},\quad g'=(2/3)^{1/2}g\cot\phi. \tag{2.17}$$

Equations (2.16) and (2.17) follow from requiring the electromagnetic part of the interaction that emerges to be the usual one. Note that we have not written the characteristic Yang-Mills interaction terms of the W bosons. The kinetic-type terms should also be present in the total Lagrangian and it is important to remember that we are assuming the gauge-field masses to be split in such a way that undesirable reactions are suppressed. The exact mechanism of this symmetry breakdown is not being specified at the present stage. Of course, one of the interesting possibilities would be to try to introduce some auxiliary scalar mesons and use the Higgs mechanism to make the symmetry breakdown spontaneous. However the present approach is more general and would include such a model as a special case.

Now we take up, in turn, the pure leptonic, semileptonic, and nonleptonic effective weak interactions which result from (2.16). We shall also discuss the choices of gauge-field masses needed to achieve consistency with experiment.

The current-current pure leptonic interaction is in second order:

$$\begin{aligned}\mathcal{L}^w(\text{leptonic})=&\frac{g^2}{m^2(W^2_1)}l^2_{1\alpha}l^1_{2\alpha}+\frac{g^2}{m^2(W^3_1)}l^3_{1\alpha}l^1_{3\alpha}+\frac{g^2}{m^2(W^3_2)}l^3_{2\alpha}l^2_{3\alpha}+\frac{1}{4}\frac{g^2}{m^2(H)}(l^2_{2\alpha}-l^3_{3\alpha})^2\\ &+\frac{1}{3\sin^2\phi}\frac{g^2}{m^2(Z)}[3\sin^2\phi\,l^{\rm em}_\alpha+{\rm Tr}(l_\alpha)+\tfrac{3}{2}(r^2_{2\alpha}+r^3_{3\alpha})]^2,\end{aligned} \tag{2.18}$$

where l_α^{em} is the leptonic part of J_α^{em} given in (2.12) and $m(W_1^2)$, for example, is the mass of the W_1^2 boson. The first term in (2.18) represents the usual pure leptonic interaction so comparison with (2.14) gives

$$\frac{G}{\sqrt{2}}=\frac{g^2}{m^2(W_1^2)}. \tag{2.19}$$

Combining (2.17) and (2.19) gives $m^2(W_1^2)=\sqrt{2}\,e^2/6G\cos^2\phi>\sqrt{2}\,e^2/6G$, so that we must have $m(W_1^2)>43.1$ GeV. This differs from the lower bound of 37.3 GeV in the SU(2)×U(1) model. The second and third terms of (2.18) are responsible for the leptonic decays of the new heavy leptons. The quantities $m(W_2^3)$ and $m(H)$ will be taken to be large compared to $m(W_1^2)$ so the third and fourth terms are actually small. On the other hand there is no reason why $m(Z)$ cannot be comparable to $m(W_1^2)$, so the fifth term may give additional contributions to usual lepton scattering. For example, the effective Lagrangians for $e\nu_e$ and $e\nu_\mu$ scattering from (2.18) are

$$\mathcal{L}_{\text{eff}}(e\nu_e)=-\frac{G}{\sqrt{2}}\bar{\nu}_e\gamma_\alpha(1+\gamma_5)\nu_e\bar{e}\gamma_\alpha(g_V+g_A\gamma_5)e\,,\quad \mathcal{L}_{\text{eff}}(e\nu_\mu)=-\frac{G}{\sqrt{2}}\bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\nu_\mu\bar{e}\gamma_\alpha(g'_V+g'_A\gamma_5)e\,, \tag{2.20}$$

with

$$g_V=1+\frac{m^2(W_1^2)}{3\sin^2\phi\, m^2(Z)}(5-6\sin^2\phi)\,,\quad g_A=1-\frac{m^2(W_1^2)}{3\sin^2\phi\, m^2(Z)}\,,$$

$$g'_V=g_V-1\,,\quad g'_A=g_A-1\,.$$

These expressions may be testable in the future.

Equation (2.16) yields the following semileptonic weak interaction in second order:

$$\begin{aligned}\mathcal{L}^w(\text{semileptonic})=&\frac{g^2}{m^2(W_1^2)}\{[(j^l)^2_{1\alpha}\cos\theta+(j^l)^3_{1\alpha}\sin\theta]l^1_{2\alpha}+\text{H.c.}\}\\&+\frac{1}{2}\frac{g^2}{m^2(H)}\{[(j^l)^3_{2\alpha}+(j^l)^2_{3\alpha}]\sin2\theta+[(j^l)^2_{2\alpha}-(j^l)^3_{3\alpha}]\cos2\theta\}(l^2_{2\alpha}-l^3_{3\alpha})\\&+\frac{g^2}{m^2(W_1^3)}\{[-(j^l)^2_{1\alpha}\sin\theta+(j^l)^3_{1\alpha}\cos\theta]l^1_{3\alpha}+\text{H.c.}\}\\&+\frac{g^2}{m^2(W_2^3)}\{([(j^l)^3_{3\alpha}-(j^l)^2_{2\alpha}]\sin\theta\cos\theta+(j^l)^3_{2\alpha}\cos^2\theta-(j^l)^2_{3\alpha}\sin^2\theta)l^2_{3\alpha}+\text{H.c.}\}\\&+\frac{2}{3\sin^2\phi}\frac{g^2}{m^2(Z)}[3\sin^2\phi\, h^{\text{em}}_\alpha-(j^r)^1_{1\alpha}+\tfrac{1}{2}(j^r)^2_{2\alpha}+\tfrac{1}{2}(j^r)^3_{3\alpha}][3\sin^2\phi\, l^{\text{em}}_\alpha+\text{Tr}(l_\alpha)+\tfrac{3}{2}(r^2_{2\alpha}+r^3_{3\alpha})]\,,\end{aligned} \tag{2.21}$$

where h^{em}_α is the hadron part of J^{em}_α given in (2.12).

The first term in (2.21) represents the usual Cabibbo theory of semileptonic interactions. The second term is the real culprit. It contains neutral $|\Delta S|=1$ hadron pieces and a neutral lepton piece, giving rise to decays like $K\to\mu\bar{\mu}$, $K\to\pi\mu\bar{\mu}$, $\Sigma^+\to pe\bar{e}$, etc. Note, however, that it does not give decays into (hadrons) + ($\nu\bar{\nu}$) pairs. In any case we must require $m(H)\gg m(W_1^2)$ to suppress these unobserved processes. The third and fourth terms contribute to the decays (heavy lepton) → (light lepton) + hadrons. Some of these decays will also be suppressed by taking $m(W_2^3)$ large. Finally, the last term in (2.21) may give observable corrections to neutrino hadron scattering reactions.

The nonleptonic weak terms which result from (2.16) present some interesting features. We write the $|\Delta S|=1$ and $|\Delta S|=2$ terms below, omitting the $\Delta S=0$ ones since they would be masked by the strong interaction:

$$\begin{aligned}\mathcal{L}^w(\text{nonleptonic})=&g^2\cos\theta\sin\theta\left[\frac{1}{m^2(W_1^2)}-\frac{1}{m^2(W_1^3)}\right][(j^l)^2_{1\alpha}(j^l)^1_{3\alpha}+(j^l)^1_{2\alpha}(j^l)^3_{1\alpha}]\\&+\frac{g^2}{4}\left[\frac{1}{m^2(W_2^3)}-\frac{1}{m^2(H)}\right]\{\sin4\theta[(j^l)^2_{3\alpha}+(j^l)^3_{2\alpha}][(j^l)^3_{3\alpha}-(j^l)^2_{2\alpha}]\\&-\sin^22\theta[(j^l)^3_{2\alpha}(j^l)^3_{2\alpha}+(j^l)^2_{3\alpha}(j^l)^2_{3\alpha}]\}\,.\end{aligned} \tag{2.22}$$

The first thing to notice about (2.22) is that in the limit $m(W^2_1)=m(W^3_1)=m(W^3_2)=m(H)$, all the terms vanish. This is actually to be expected since in the exact SU(3) limit the Cabibbo rotation matrix U in (2.16) is irrelevant and the nonleptonic interaction becomes invariant under the usual (strong) SU(3), for which $\Delta S=0$ is required. The last term in (2.22) gives direct $|\Delta S|=2$ transitions and must be very severely suppressed to avoid contradicting the extremely small value of the K_L-K_S mass difference, among other things. It is gratifying that this term can be eliminated completely merely by setting $m(H)=m(W^3_2)$, without even requiring it to be particularly large. [However $m(H)$ must be large as discussed after (2.21).] Thus this model enables us to eliminate $|\Delta S|=2$ hadronic terms naturally by choosing the masses of the U-spin triplet gauge bosons to be equal and to arbitrarily suppress $|\Delta S|=1$ neutral current semileptonic terms by allowing this common mass to be arbitrarily large. The first term in (2.22) gives the usual $|\Delta S|=1$ charged-current nonleptonic interaction if we take $m(W^3_1)\gg m(W^2_1)$. However this choice for $m(W^3_1)$ is not necessary and in fact by proper adjustment of $m(W^3_1)$ we may get the nonleptonic interaction:

$$\mathcal{L}^w(\text{nonleptonic})=\frac{G}{\sqrt{2}}X\,[(j^l)^2_{1\alpha}(j^l)^1_{3\alpha}+\text{H.c.}], \tag{2.23}$$

where $X=\frac{1}{2}\sin 2\theta[1-m^2(W^2_1)/m^2(W^3_1)]$ is either a positive number less than $\frac{1}{2}\sin 2\theta$ or a negative number of arbitrary magnitude. Thus the present model *does not* fix the nonleptonic decay rates in terms of the semileptonic ones. The value of $|X|$ one needs actually depends on the notoriously difficult zero-parameter calculation of nonleptonic decay amplitudes. There is some indication[13] that the canonical value $X=\frac{1}{2}\sin 2\theta\simeq\frac{1}{4}$ is reasonable. On the other hand,[14] if one assumes octet dominance and furthermore assumes one particle dominance for the weak currents a value $X\simeq\frac{3}{5}$ fits the $K_S\to 2\pi$ decay rate. [Note that $\frac{3}{5}$ is the SU(3) Clebsch-Gordan coefficient which arises when one takes the octet part of the charged-current–charged-current interaction.] Thus the added flexibility present in (2.23) may be useful.

Finally we summarize the discussion of the gauge boson masses. The photon naturally has zero mass. In order to make the basic SU(3)×U(1)-invariant unified weak electromagnetic interaction of (2.16) consistent with experiment it is sufficient that the masses of the U-spin triplet bosons (W^3_2, W^2_3, and H) be equal and made much larger than $m(W^2_1)$. W^2_1 and W^1_2 are the "ordinary" intermediate bosons and their mass is related to the mixing angle ϕ by (2.19) and (2.17). Now if the masses of the remaining bosons – W^3_1, W^1_3, and Z – are considered to be indefinitely large we will recover the usual Cabibbo theory summarized in (2.14). On the other hand if $m(W^3_1)$ and $m(Z)$ are taken to be comparable in magnitude to $m(W^2_1)$ the interaction (2.16) yields, as noted in the preceding discussion, some new features which may soon be testable.

III. SU(3) × SU(3) MODEL

The motivation for using the chiral SU(3)×SU(3) group to describe the weak and electromagnetic interactions is that it is the *same* group which seems to be underlying the strong interactions. Thus it may lead to an extremely unified total structure.

All the fermions in this model belong to triplets. There are the left-handed lepton and quark triplets,

$$\begin{pmatrix}\nu_{eL}\\ e_L\\ e'_L\end{pmatrix},\quad \begin{pmatrix}\nu_{\mu L}\\ \mu_L\\ \mu'_L\end{pmatrix},\quad U\begin{pmatrix}q_{1L}\\ q_{2L}\\ q_{3L}\end{pmatrix}, \tag{3.1}$$

and also the right-handed lepton and quark triplets,

$$\begin{pmatrix}\nu_{eR}\\ e_R\\ e'_R\end{pmatrix},\quad \begin{pmatrix}\nu_{\mu R}\\ \mu_R\\ \mu'_R\end{pmatrix},\quad U\begin{pmatrix}q_{1R}\\ q_{2R}\\ q_{3R}\end{pmatrix}. \tag{3.2}$$

In the equations above $e_L=\frac{1}{2}(1+\gamma_5)e$ and $e_R=\frac{1}{2}(1-\gamma_5)e$, for example. The other notation is the same as in Sec. II. Note that in addition to the heavy leptons e' and μ', the SU(3)×SU(3) model contains two more neutrinos ν_{eR} and $\nu_{\mu R}$.

The SU(3)×SU(3) algebra is generated by the integrated left- and right-handed total (lepton + hadron) traceless currents

$$\begin{aligned}&l^b_{a\alpha}+(Uj^l_\alpha U^{-1})^b_a-\tfrac{1}{3}\delta^b_a(l^c_{c\alpha}+(j^l)^c_{c\alpha}),\\ &r^b_{a\alpha}+(Uj^r_\alpha U^{-1})^b_a-\tfrac{1}{3}\delta^b_a(r^c_{c\alpha}+(j^r)^c_{c\alpha}),\end{aligned} \tag{3.3}$$

where the individual terms are defined in (2.4) and (2.11) except that in the present case $r^b_{1\alpha}\neq 0$ and $r^1_{a\alpha}\neq 0$ since the neutrino fields $\nu_{eR}=\frac{1}{2}(1-\gamma_5)\psi_1$ and $\nu_{\mu R}=\frac{1}{2}(1-\gamma_5)\chi_1$ no longer vanish. In addition to the 16 currents above it is necessary to use the left- and right-handed lepton number currents

$$l^c_{c\alpha}\quad\text{and}\quad r^c_{c\alpha}. \tag{3.4}$$

The integrated currents of (3.4) give us an additional U(1)×U(1) algebra under which we also will demand invariance of the interaction terms.[15] Corresponding to the 18 currents above we need left- and right-handed octets and singlets of gauge bosons

$$\begin{aligned}&\text{octets:}\ (W^l)^b_{a\alpha},\quad (W^r)^b_{a\alpha},\\ &\text{singlets:}\ (D^l)_\alpha,\quad (D^r)_\alpha.\end{aligned} \tag{3.5}$$

These objects are assumed to have the following parity-reversal and charge-conjugation properties:

$$\text{parity: } (\vec{W}^l)^b_a \leftrightarrow -(\vec{W}^r)^b_a, \quad \vec{D}^l \leftrightarrow -\vec{D}^r,$$
$$\text{charge conjugation: } (W^l)^b_{a\alpha} \leftrightarrow -(W^r)^a_{b\alpha}, \quad (D^l)_\alpha \leftrightarrow -(D^r)_\alpha, \tag{3.6}$$

It is also convenient to use the abbreviations

$$F^l_\alpha = -(\tfrac{3}{2})^{1/2}(W^l)^1_{1\alpha}, \quad F^r_\alpha = -(\tfrac{3}{2})^{1/2}(W^r)^1_{1\alpha},$$
$$H^l_\alpha = (1/\sqrt{2})[(W^l)^2_{2\alpha} - (W^l)^3_{3\alpha}], \tag{3.7}$$
$$H^r_\alpha = (1/\sqrt{2})[(W^r)^2_{2\alpha} - (W^r)^3_{3\alpha}].$$

As in Sec. II the physical photon will correspond to a mixture of various fields. We take it to be a mixture of F^l_α, F^r_α, D^l_α, and D^r_α. Then if the photon is to have its usual parity and charge conjugation properties we must have

$$\mathfrak{A}_\alpha = \frac{1}{\sqrt{2}}\cos\omega\,(F^l_\alpha + F^r_\alpha) - \frac{1}{\sqrt{2}}\sin\omega\,(D^l_\alpha + D^r_\alpha), \tag{3.8}$$

where ω is a mixing angle. Denoting the three other physical field mixtures by Z_α, Z'_α, and Z''_α we see from (3.8) that the transformation must have the form

$$\begin{pmatrix} F^l_\alpha \\ F^r_\alpha \\ D^l_\alpha \\ D^r_\alpha \end{pmatrix} = \begin{pmatrix} (1/\sqrt{2})\cos\omega & \cdots \\ (1/\sqrt{2})\cos\omega & \cdots \\ -(1/\sqrt{2})\sin\omega & \cdots \\ -(1/\sqrt{2})\sin\omega & \cdots \end{pmatrix} \begin{pmatrix} \mathfrak{A}_\alpha \\ Z_\alpha \\ Z'_\alpha \\ Z''_\alpha \end{pmatrix}. \tag{3.9}$$

The terms outside the first column of the orthogonal matrix in (3.9) are not unique but are however not needed for our present purpose.

With the mixture in (3.9) we now are in a position to write down the unique SU(3)×SU(3)×U(1)×U(1) interaction which has the correct photon couplings:

$$\mathfrak{L} = g\,\mathrm{Tr}(l_\alpha W^l_\alpha + r_\alpha W^r_\alpha) - g'[D^l_\alpha\,\mathrm{Tr}(l_\alpha) + D^r_\alpha\,\mathrm{Tr}(r_\alpha)]$$
$$+ g\,\mathrm{Tr}(Uj^l_\alpha U^{-1}W^l_\alpha + Uj^r_\alpha U^{-1}W^r_\alpha), \tag{3.10}$$

with

$$g = \frac{-|e|}{\sqrt{3}\cos\omega}, \quad g' = (\tfrac{2}{3})^{1/2} g\cot\omega. \tag{3.11}$$

Equation (3.10) is the basic one for this model. The suppression of unwanted transitions can be achieved in a similar way to the previous model – by adjustment of the masses of the gauge bosons. Thus here we will just describe the main new features of (3.10). Note that (3.10) is *parity-invariant* and *charge-conjugation-invariant*. This means that the violation of C and P observed in the usual weak interaction arises because the masses of the right-handed gauge bosons are very much larger than the masses of the left-handed ones. The usual charged intermediate bosons in this model are identified with $(W^l)^2_{1\alpha}$ and $(W^l)^1_{2\alpha}$. Their mass satisfies the relation

$$\frac{G}{\sqrt{2}} = \frac{g^2}{m^2[(W^l)^2_1]}. \tag{3.12}$$

Combining (3.11) and (3.12) gives $m^2((W^l)^2_1) = \sqrt{2}\,e^2/3G\cos^2\omega > \sqrt{2}\,e^2/3G$. Thus $m((W^l)^2_1) > 60.9$ GeV here. This lower bound is different in both the SU(2)×U(1) and SU(3)×U(1) models.

The other gauge masses (except for the photon mass) can be taken indefinitely large so as to reproduce the usual weak-interaction theory of (2.14). Alternatively some deviations from the usual theory – which may possibly turn out to be desirable – can be achieved as in Sec. II. The additional neutrinos ν_{eR} and $\nu_{\mu R}$ would be difficult to observe since they couple to other fields through the gauge bosons with extremely large mass.

A point of interest in the interaction (3.10) is that the singlet fields D^l_α and D^r_α couple to the leptons but not to the hadrons. This is to be expected since it is well known that when a fundamental triplet has the quark charge assignment $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ the electric current belongs to an octet.

Finally we mention some properties the present model would have if it is assumed that all the symmetry breaking can be made to arise from a spontaneous breakdown mechanism of the "Higgs" type. Then the symmetry breaking would be implemented, practically speaking, by using a set of auxiliary scalar mesons with nonzero vacuum expectation value. A hitch arises because the photon is the only allowed zero-mass gauge field. In particular no zero-mass gauge field couples to the sum of the two currents in (3.4). This sum $(l^c_{c\alpha} + r^c_{c\alpha})$ is just the fermion part of the total lepton number current. The *total* lepton number is spontaneously broken.[16] However, by the structure of the Lagrangian (3.10) the lepton number for the usual fermion reactions is conserved. A similar situation was noted by Freund[17] in a different model.

A nice feature of an SU(3)×SU(3)-invariant weak-electromagnetic interaction with spontaneous breakdown would be that it opens the way for an additional unification with the strong interactions. We previously pointed out[18] that if the strong interactions are *exactly* chiral SU(3)×SU(3)-invariant they are necessarily invariant under the SU(2)×U(1) weak-electromagnetic subgroup and hence the total Lagrangian (strong + electromagnetic + weak) would have this SU(2)×U(1)-invariance. Furthermore we showed that the usual "(3, 3*)" strong-symmetry-violating terms could arise by spontaneous breakdown from an SU(2)×U(1)-invariant interaction of quarks and auxiliary scalar mesons. Now if we were to use instead a weak-electromagnetic interaction which was itself SU(3)×SU(3)-invariant (as the present model plus "Higgs" mechanism) we could generate the "(3, 3*)" strong-symmetry-violating terms from an SU(3)×SU(3)-invariant interaction of quarks and auxiliary scalar mesons. In such an event the total Lagrangian would have an *exact* SU(3)×SU(3)-invariance and *all* symmetry breaking would be spontaneous.

We are presently working with A. P. Balachandran on the possibility of "Higgs" mechanisms for models of this type.

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†Present address: Institute of Theoretical Physics, Chalmers Tekniska Hogskola, Fack, S-402 20 Göteborg 5, Sweden.

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[15] Thus our symmetry group of leptons and hadrons is actually SU(3)×SU(3)×U(1)×U(1). However, for compactness we refer to this model as the SU(3)×SU(3) model. Note that we are *not* including the left- and right-handed hadron numbers as members of our gauge group.

[16] In effect, this means that an auxiliary *scalar* field in the theory which carries lepton number will have nonzero vacuum expectation value. Thus, if this scalar field has large mass there may well be no observable consequences at present. This depends somewhat on how we let this field couple to leptons. Note that the present difficulty does not arise in the model of Sec. II. We mention that one way out of the difficulty is to assume that the usual leptons are composites of fractionally charged leptonic quarks since the additional U(1)×U(1) is then no longer necessary.

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