PHYSICAL REVIEW D VOLUME 8, NUMBER 2 15 JULY 1973

Generalized Nonlinear Lagrangian for Pseudoscalar and Scalar Mesons

P. H. Dondi and Shalom Eliezer
Department of Applied Mathematics and Theoretical Physics, Cambridge University, Cambridge, England
(Received 24 July 1972)

We construct a nonlinear Lagrangian to describe the scalar and pseudoscalar mesons such that the chiral SU(3) × SU(3) symmetry is realized by an octet of Goldstone pseudoscalar mesons while the scalar particles behave neither as parity partners of the pseudoscalar mesons nor as Goldstone bosons in the SU(3) × SU(3)-symmetry limit. The symmetry-breaking Lagrangian is assumed to transform as the $(3,\overline{3}) + (\overline{3},3)$ representation of the SU(3) × SU(3) group and contain explicit SU(3)- and SU(2)-violating terms. The transformation properties of the scalar fields together with these breaking terms in the Lagrangian enable our model to have an SU(3)-broken vacuum. We exhibit the masses of the scalar and pseudoscalar particles as well as the decay constants defining PCAC (partial conservation of axial-vector current) and PCVC (partial conservation of vector current) relations in terms of the parameters of the model, and obtain various well-known relationships (including the Glashow-Weinberg sum rules) between the physical quantities. The smoothness assumption is shown to imply approximate SU(2) × SU(2) symmetry of the Lagrangian with a small SU(3) violation in the vacuum (Gell-Mann–Oakes–Renner model), while an appropriate change in the smoothness assumption leads to approximate SU(3) symmetry of the Lagrangian with an almost SU(2) × SU(2)-invariant vacuum (Brandt-Preparata model). We calculate the symmetry-breaking parameters of the Lagrangian and vacuum and predict the mass and decay constant of the κ meson. Furthermore, from the width of the decay $\pi_N \rightarrow \eta\pi$, we obtain the decay widths of all the scalar mesons. Finally, we investigate the nonelectromagnetic SU(2) breaking and find that it gives a major contribution to the kaon mass difference but only a 5% correction to the pion mass difference. The fact that the scalar mesons are needed to get $f_K \neq f_\pi$ and different wave-function renormalization constants for the fields, which enables us to have solutions other than that of Gell-Mann, Oakes, and Renner, demonstrates the importance of the scalar mesons in SU(3)-symmetry-breaking effects.

I. INTRODUCTION

Theoretically, the necessity for the scalar mesons has been discussed by many authors.[1-24] Various studies based on current algebra,[3-8] broken SU(3) symmetry,[9] partial conservation of the vector current[6-8] (PCVC), and the violation of conformal[10-13] and chiral invariance have led to a more fundamental significance being attributed to these particles.

The existence of scalar particles is well established, although experimentally their properties (mass, width, etc.) are not decisively known.[25] The $I=1$ scalar mesons have a mass in the range 960 to 1020 MeV and width about 60 MeV, and it seems that the three candidates $\delta(962)$, $\pi_N(975)$, and $\pi_N(1016)$ are all manifestations of one physical state.[25] For the $I=\frac{1}{2}$ (κ) mesons, the mass and width is even more difficult to gauge. It seems reasonable to assume that the mass of the κ meson is about 1200 MeV with a width of 300 MeV, but the possibility of a narrow resonance (width 30 MeV) at 890 MeV is not excluded.[25-27] The isoscalar candidates are ϵ (800–1000 MeV) (Ref. 25) and the $S^*(1060)$ with a width between 150 to 300 MeV,[28] although recent evidence suggests an S^* with a mass 1250 MeV and a width about 300 MeV decaying strongly into $\pi\pi$.[29]

This evidence suggests an SU(3) octet (or nonet) of scalar particles around 1 GeV. The high mass, together with the success of SU(3) in classifying particle multiplets[30] and partial conservation of the axial-vector current[31,32] (PCAC) indicates that when the group is enlarged to SU(3)×SU(3),[33,34] the octet of pseudoscalar mesons behave as Goldstone particles[35] in the symmetry limit while the scalar mesons remain massive in this case.

Guided by the ideas described above we construct an effective nonlinear Lagrangian[36-44] containing the scalar and pseudoscalar mesons, such that the SU(3)×SU(3) symmetry is realized only by eight pseudoscalar Goldstone bosons[34] [i.e., the vacuum is SU(3)-invariant in this limit]. In this paper, we demonstrate that the scalar particles, which in our model are neither parity partners of the pseudoscalar mesons, nor Goldstone bosons, play a very important role in symmetry-breaking effects (e.g., $f_K \neq f_\pi$ due to the existence of these particles).[44] Previously, Lagrangians of this type constructed to include the $J^P = 1^-, 1^+$ mesons,[42,43] as well as the baryons,[40,41] all suffered from the defect of having an SU(3)-invariant vacuum, even when the symmetry was broken, which implies the same decay constants for all the pseudoscalar mesons ($f_K = f_\pi = f_\eta$).

In Sec. II, we construct the phenomenological Lagrangian which can be divided into two parts:

(1) The symmetric Lagrangian containing all the SU(3)×SU(3)-invariant terms formed from the covariant derivatives of the pseudoscalar and scalar fields, and from the scalar fields themselves.

(2) The symmetry-breaking part which is assumed to transform as the $(3,\bar{3})+(\bar{3},3)$ representation of the full chiral group.[15,34] The symmetry is violated only by nonderivative interactions, which ensures that the current algebra is preserved.

We exhibit, in Sec. III, the masses of the scalar and pseudoscalar mesons, as well as the decay constants defining PCAC and PCVC relations, in terms of the parameters of the model. This allows us to obtain various well-known relationships between the physical quantities.

In Sec. IV we invoke the smoothness assumption considered by many authors[16,17,20,21,34,44,45] and demonstrate how this increases the information we can obtain from our theory. We show the relation between the "smoothness coefficients" and different approximate symmetries of the Lagrangian.

The decay widths of the scalar particles, which have momentum dependence in our model, are calculated in Sec. V.

SU(2)-symmetry breaking of the Lagrangian due to nonelectromagnetic effects[9,21,45-48] is considered in Sec. VI.

Finally, in the last section we summarize and discuss the results obtained from the suggested model.

II. THE PHENOMENOLOGICAL LAGRANGIAN

The basic form of the Lagrangian density that we wish to consider can be written as

$$\mathfrak{L} = \mathfrak{L}_0 + \mathfrak{L}_{SB} = \mathfrak{L}_0 + \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3 \,, \tag{2.1}$$

where $\mathfrak{L}_0$ is invariant under the full chiral SU(3) ×SU(3) group and u_i (v_i) are the parity-even (-odd) members of the $(3,\bar{3})+(\bar{3},3)$ representation, which satisfy the equal-time commutation relations[33]

$$[Q_i^V, u_j] = if_{ijk}u_k \,, \quad [Q_i^V, u_0] = 0 \,, \tag{2.2}$$

$$[Q_i^V, v_j] = if_{ijk}v_k \,, \quad [Q_i^V, v_0] = 0 \,, \tag{2.3}$$

$$\begin{aligned}[Q_i^A, u_j] &= -i\{(\tfrac{2}{3})^{1/2}\delta_{ij}v_0 + d_{ijk}v_k\} \,, \\ [Q_i^A, u_0] &= -i(\tfrac{2}{3})^{1/2}v_i \,,\end{aligned} \tag{2.4}$$

$$\begin{aligned}[Q_i^A, v_j] &= i\{(\tfrac{2}{3})^{1/2}\delta_{ij}u_0 + d_{ijk}u_k\} \,, \\ [Q_i^A, v_0] &= i(\tfrac{2}{3})^{1/2}u_i \,.\end{aligned} \tag{2.5}$$

$\frac{1}{2}(Q_i^V \pm Q_i^A)$ are the generators of the SU(3)×SU(3) group, $i,j,k = 1,2,\ldots,8$, and summation over repeated indices is implied unless otherwise stated. The term $\epsilon_0 u_0$ breaks the chiral symmetry but conserves SU(3); $\epsilon_8 u_8$ describes the violation of SU(3) whilst conserving the SU(2) isospin symmetry. Finally, it is possible that the isospin symmetry is broken by a nonelectromagnetic interaction of the form $\epsilon_3 u_3$. Thus the symmetry-breaking Lagrangian of Eq. (2.1) is the most general that can be constructed from the $(3,\bar{3})+(\bar{3},3)$ representation which conserves the electric charge, parity, and strangeness. Obviously, the choice of the $(3,\bar{3})+(\bar{3},3)$ representation is neither new nor completely general, since other representations could be used, however, apart from the simplicity, there are strong arguments in favor of this kind of symmetry breaking as has been pointed out by many authors.[49-53]

The fields involved in our Lagrangian are those of the scalar and pseudoscalar particles, and we consider that these form octet representations of the vector SU(3) group. Thus the transformation properties under SU(3) for these fields are

$$[Q_i^V, \phi_j] = if_{ijk}\phi_k \,, \tag{2.6}$$

$$[Q_i^V, S_j] = if_{ijk}S_k \,, \tag{2.7}$$

where ϕ_i and S_i are the pseudoscalar and scalar fields, respectively, whilst their commutation relations with the chiral generators are

$$[Q_i^A, \phi_j] = iF_{ij}(\phi) \,, \tag{2.8}$$

$$[Q_i^A, S_j] = iG_{ip}(\phi)f_{pjk}S_k \,. \tag{2.9}$$

The transformation functions $F_{ij}(\phi)$ and $G_{ij}(\phi)$ are completely determined in a general framework,[37-39] and to lowest order are given by

$$F_{ij}(\phi) = f\delta_{ij} + O(\phi^2) \,, \tag{2.10}$$

$$G_{ij}(\phi) = \frac{1}{2f}f_{ijk}\phi_k + O(\phi^3) \,. \tag{2.11}$$

It is important to note that the structure of the axial transformations of the fields enable us to form an SU(3)×SU(3)-invariant mass term for the scalar particles while it is impossible to construct such terms for the pseudoscalar mesons. This mechanism ensures that the Goldstone nature of the symmetry is realized only by the pseudoscalar mesons. Finally to complete the invariant part of the Lagrangian we must use the covariant derivatives[37-39] $D_\mu\phi_i$ and $D_\mu S_i$ which to lowest order are

$$D_\mu\phi_i = \partial_\mu\phi_i + O(\phi^3) \,, \tag{2.12}$$

$$D_\mu S_i = \partial_\mu S_i + O(\phi^2 S) \,. \tag{2.13}$$

Using Eqs. (2.12) and (2.13), together with the fact that S_iS_i and $d_{ijk}S_iS_jS_k$ are SU(3)×SU(3) scalars, we can write $\mathfrak{L}_0$ up to third order in the fields as

$$\mathcal{L}_0=\tfrac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i+\tfrac{1}{2}\partial_\mu S_i\partial^\mu S_i+\alpha_S d_{ijk}\partial^\mu S_i\partial_\mu S_j S_k+\alpha_P d_{ijk}\partial_\mu\phi_i\partial^\mu\phi_j S_k+\beta_1 S_i S_i+\beta_2 d_{ijk}S_iS_jS_k\ . \tag{2.14}$$

To know the form of the symmetry-breaking terms, the $(3,\bar{3})+(\bar{3},3)$ representation must be constructed from the pseudoscalar and scalar fields. We demonstrate how this may be achieved in the Appendix. To third order in fields, we find the following expression for the symmetry-breaking Lagrangian:

$$\begin{aligned}\mathcal{L}_{SB}=&\,\epsilon_0\Big[-\sqrt{6}\,a_6 f^2+(\tfrac{2}{3})^{1/2}a_6\phi_i\phi_i+a_9S_iS_i+a_{10}d_{ijk}S_iS_jS_k-(\tfrac{1}{6})^{1/2}\frac{a_1}{f^2}d_{ijk}\phi_i\phi_jS_k\Big]\\&+\epsilon_8\Big[a_1S_8+a_3S_8(S_iS_i)-\frac{a_1}{3f^2}\phi_8(\phi_iS_i)+a_5d_{8ij}S_iS_j+a_6d_{8ij}\phi_i\phi_j-\frac{a_1}{2f^2}d_{8ij}d_{jlm}\phi_i\phi_lS_m\Big]\\&+\epsilon_3\Big[a_1S_3+a_3S_3(S_iS_i)-\frac{a_1}{3f^2}\phi_3(\phi_iS_i)+a_5d_{3ij}S_iS_j+a_6d_{3ij}\phi_i\phi_j-\frac{a_1}{2f^2}d_{3ij}d_{jlm}\phi_i\phi_lS_m\Big]\ ,\end{aligned} \tag{2.15}$$

where the a's are the remaining arbitrary coefficients describing the $(3,\bar{3})+(\bar{3},3)$ representation in terms of the available fields, and can be chosen as order unity to fix the scale of the ϵ's in $\mathcal{L}$.

The new features of our nonlinear Lagrangian are the linear terms S_8 and S_3 which must be removed by making the transformation

$$\begin{aligned}&S_i=S_i'+\delta_{i3}\langle S_3\rangle_0+\delta_{i8}\langle S_8\rangle_0\ ,\\&\phi_i=\phi_i'\ ,\end{aligned} \tag{2.16}$$

and requiring

$$2(\beta_1+a_9\epsilon_0-a_5\epsilon_8/\sqrt{3})\langle S_8\rangle_0+a_1\epsilon_8+(2/\sqrt{3})a_5\epsilon_3\langle S_3\rangle_0=0 \tag{2.17}$$

to remove the linear term in S_8', with

$$2(\beta_1+a_9\epsilon_0+a_5\epsilon_8/\sqrt{3})\langle S_3\rangle_0+a_1\epsilon_3+(2/\sqrt{3})a_5\epsilon_3\langle S_8\rangle_0=0 \tag{2.18}$$

as the corresponding equation to eliminate the linear S_3' term. Equations (2.17) and (2.18) indicate that ϵ_8 and ϵ_3 are SU(3)-symmetry-breaking parameters of the same order as $\langle S_8\rangle_0$ and $\langle S_3\rangle_0$ but say nothing about the size of ϵ_0.

At this stage we should like to point out that using the transformation (2.16), terms in $\mathcal{L}$ containing higher powers of the scalar fields than we have considered can change the quantities given by our Lagrangian. However, these contributions are of higher order in $\langle S_8\rangle_0$ and $\langle S_3\rangle_0$. Therefore a crucial assumption in this model is that an expansion in $\langle S_8\rangle_0$ and $\langle S_3\rangle_0$ makes sense such that the first-order terms are a good approximation. Further, in this model with processes described in the tree-graph approximation, the ϵ_0, ϵ_8, and ϵ_3 are not treated as perturbation parameters whilst $\langle S_8\rangle_0$ and $\langle S_3\rangle_0$ are. We can therefore summarize the philosophy regarding the parameters in our theory by stating that, generally, all the first-order contributions ϵ_i and $\langle S_i\rangle_0$ together with $\epsilon_i\langle S_j\rangle_0$ are retained but $\{\langle S_i\rangle_0\}^n$, $n>1$, are neglected.

Having removed the linear term from $\mathcal{L}$, we find that $\mathcal{L}_0$ no longer contains the desired kinetic terms $(\tfrac{1}{2}\partial_\mu\phi_i'\partial^\mu\phi_i'+\tfrac{1}{2}\partial_\mu S_i'\partial^\mu S_i')$ but has changed to

$$\begin{aligned}\mathcal{L}_0=&\tfrac{1}{2}\partial_\mu\phi_i'\partial^\mu\phi_j'\{\delta_{ij}+2\alpha_P\langle S_8\rangle_0 d_{8ij}+2\alpha_P\langle S_3\rangle_0 d_{3ij}\}+\tfrac{1}{2}\partial_\mu S_i'\partial^\mu S_j'\{\delta_{ij}+2\alpha_S\langle S_8\rangle_0 d_{8ij}+2\alpha_S\langle S_3\rangle_0 d_{3ij}\}\\&+\beta_1S_i'S_i'+3\beta_2S_i'S_j'\{\langle S_8\rangle_0 d_{8ij}+\langle S_3\rangle_0 d_{3ij}\}\ ,\end{aligned} \tag{2.19}$$

where we are obliged to keep only second order in fields. To obtain the necessary form for the kinetic Lagrangian we must first diagonalize $\mathcal{L}$ to remove the ϕ_3'-ϕ_8' and S_3'-S_8' components, and then renormalize the fields, thus

$$\begin{pmatrix}\phi^R_{\pi^0}\\ \phi^R_\eta\end{pmatrix}=\begin{pmatrix}Z_{\pi^0}^{-1/2}&0\\0&Z_\eta^{-1/2}\end{pmatrix}\begin{pmatrix}\cos\theta_P&\sin\theta_P\\-\sin\theta_P&\cos\theta_P\end{pmatrix}\begin{pmatrix}\phi_3'\\ \phi_8'\end{pmatrix}, \tag{2.20}$$

$$\phi_i^R=Z_{\phi_i}^{-1/2}\phi_i'\qquad(i=1,2,4,5,6,7)\ , \tag{2.21}$$

$$\begin{pmatrix}S^R_{\pi_N^0}\\ S^R_{S^*}\end{pmatrix}=\begin{pmatrix}Z_{\pi_N^0}^{-1/2}&0\\0&Z_{S^*}^{-1/2}\end{pmatrix}\begin{pmatrix}\cos\theta_S&\sin\theta_S\\-\sin\theta_S&\cos\theta_S\end{pmatrix}\begin{pmatrix}S_3'\\ S_8'\end{pmatrix}, \tag{2.22}$$

$$S_i^R=Z_{S_i}^{-1/2}S_i'\qquad(i=1,2,4,5,6,7)\ , \tag{2.23}$$

where R labels the physical fields.

By eliminating the $\partial_\mu \phi_3' \partial^\mu \phi_8'$ and $\partial_\mu S_3' \partial^\mu S_8'$ we get

$$\tan 2\theta_P = \tan 2\theta_S = \frac{\langle S_3\rangle_0}{\langle S_8\rangle_0} \equiv \tan 2\theta \ . \tag{2.24}$$

From Eqs. (2.19)–(2.24), it is easily seen that the renormalization constants have to be defined by

$$Z_{\pi^+}{}^{-1} = 1 + \frac{2}{\sqrt{3}} \alpha_P \langle S_8\rangle_0 \ , \tag{2.25}$$

$$Z_{\pi^0}{}^{-1} = 1 + \frac{2}{\sqrt{3}} \alpha_P \langle S_8\rangle_0 \cos 2\theta \left[1 + \left(\frac{\langle S_3\rangle_0}{\langle S_8\rangle_0}\right)^2\right] , \tag{2.26}$$

$$Z_{K^+}{}^{-1} = 1 - \frac{1}{\sqrt{3}} \alpha_P \langle S_8\rangle_0 + \alpha_P \langle S_3\rangle_0 \ , \tag{2.27}$$

$$Z_{K^0}{}^{-1} = 1 - \frac{1}{\sqrt{3}} \alpha_P \langle S_8\rangle_0 - \alpha_P \langle S_3\rangle_0 \ , \tag{2.28}$$

$$Z_\eta{}^{-1} = 1 - \frac{2}{\sqrt{3}} \alpha_P \langle S_8\rangle_0 \cos 2\theta \left[1 + \left(\frac{\langle S_3\rangle_0}{\langle S_8\rangle_0}\right)^2\right] . \tag{2.29}$$

The renormalization constants for the scalar fields have exactly the same structure, as may be expected from a glance at Eq. (2.19), with α_P replaced by α_S, i.e.,

$$Z_S = Z_\phi(\alpha_P \rightarrow \alpha_S) \ . \tag{2.30}$$

Note that in the absence of SU(2) symmetry breaking, these constants satisfy Gell-Mann–Okubo type relationships of the form

$$4 Z_K{}^{-1} = 3 Z_\eta{}^{-1} + Z_\pi{}^{-1} \ , \tag{2.31}$$

$$4 Z_\kappa{}^{-1} = 3 Z_{S^*}{}^{-1} + Z_{\pi_N}{}^{-1} \ . \tag{2.32}$$

Thus, with a similar diagonalization in the nonderivative terms (which we discuss in Sec. VI) we may rewrite our Lagrangian as a function of the physical fields

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi_i^R \partial^\mu \phi_i^R + \tfrac{1}{2}\partial_\mu S_i^R \partial^\mu S_i^R - \tfrac{1}{2} m_{\phi_i}{}^2 \phi_i^R \phi_i^R - \tfrac{1}{2} m_{S_i}{}^2 S_i^R S_i^R \ , \tag{2.33}$$

where the summation on i is over the physical states (or in other words $\phi_3^R \equiv \phi_{\pi^0}^R$ and $\phi_8^R \equiv \phi_\eta^R$ and similarly for S_i^R, and $i = 1, 2, \ldots, 8$) and the coefficients of the quadratic terms are identified as the physical masses. The appropriate expressions for these quantities are given in the following sections.

III. MESON MASSES, PCAC, AND PCVC

In this section we discuss the case with no SU(2) violation, i.e., $\epsilon_3 = \langle S_3\rangle_0 = 0$.

1. Pseudoscalar-Meson Masses

Neglecting the SU(2) breaking effects, the physical masses of the pseudoscalar mesons in the Lagrangian of Eq. (2.33) are

$$m_\pi{}^2 = \frac{2}{3} \frac{\epsilon_0 \langle u_0\rangle_0}{f^2} Z_\pi (1 + a + b + ab) \ , \tag{3.1}$$

$$m_K{}^2 = \frac{2}{3} \frac{\epsilon_0 \langle u_0\rangle_0}{f^2} Z_K (1 - \tfrac{1}{2}a - \tfrac{1}{2}b + \tfrac{1}{4}ab) \ , \tag{3.2}$$

$$m_\eta{}^2 = \frac{2}{3} \frac{\epsilon_0 \langle u_0\rangle_0}{f^2} Z_\eta (1 - a - b + 3ab) \ , \tag{3.3}$$

where the scalar densities have vacuum expectation values, measuring the symmetry of the vacuum, given by Eqs. (2.15) and (2.16) as

$$\langle u_0\rangle_0 = -\sqrt{6}\, a_6 f^2, \quad \langle u_8\rangle_0 = a_1 \langle S_8\rangle_0 \ , \tag{3.4}$$

and a and b are defined by

$$a = \frac{\epsilon_8}{\sqrt{2}\,\epsilon_0}, \quad b = \frac{\langle u_8\rangle_0}{\sqrt{2}\langle u_0\rangle_0} \ . \tag{3.5}$$

The renormalization constants, Z_{ϕ_i}, can be found in Eqs. (2.25)–(2.29) when $\langle S_3\rangle_0$ is put equal to zero.

From the expressions for the mesons given in Eqs. (3.1)–(3.3) it is interesting to consider the various limiting cases that can occur by treating a and b as variable parameters, and assuming $\alpha_P \neq 0$:

(A) SU(3)× SU(3) symmetry of $\mathcal{L}$: $\epsilon_0 = \epsilon_8 = 0$.

$$m_\pi = m_K = m_\eta = 0 \tag{3.6}$$

as is to be expected for a symmetry realized by an octet of pseudoscalar Goldstone bosons; in other words, this is an example of the Goldstone theorem.[35]

(B) SU(3) symmetry of $\mathcal{L}$: $\epsilon_8 = 0$ ($a = 0$).

(i) $b = 0$, $a_1 \neq 0$ [$\langle S_8\rangle_0 = 0$, i.e., SU(3)-invariant vacuum].

$$Z_\pi = Z_K = Z_\eta = 1 \ , \tag{3.7}$$

$$m_\pi = m_K = m_\eta \neq 0 \ . \tag{3.8}$$

(ii) $b = 0$, $a_1 = 0$ ($\langle S_8\rangle_0 \neq 0$) .

$$Z_\pi \neq Z_K \neq Z_\eta, \quad 4 Z_K{}^{-1} = 3 Z_\eta{}^{-1} + Z_\pi{}^{-1} \ , \tag{3.9}$$

$$m_K{}^2 Z_K{}^{-1} = m_\eta{}^2 Z_\eta{}^{-1} = m_\pi{}^2 Z_\pi{}^{-1} \ . \tag{3.10}$$

Equations (3.9) and (3.10) give

$$\frac{4}{m_K{}^2} = \frac{3}{m_\eta{}^2} + \frac{1}{m_\pi{}^2} \ . \tag{3.11}$$

(iii) $b \neq 0$. In this case we have Eq. (3.9) together with

$$4 m_K{}^2 Z_K{}^{-1} = 3 m_\eta{}^2 Z_\eta{}^{-1} + m_\pi{}^2 Z_\pi{}^{-1} \ . \tag{3.12}$$

(C) SU(2)×SU(2) symmetry of $\mathcal{L}$: $\epsilon_8 = -\sqrt{2}\,\epsilon_0$

$(a=-1)$.

(i) $b=0$, $a_1 \neq 0$ ($\langle S_8\rangle_0=0$, i.e., SU(3)-invariant vacuum). In this case we have Eq. (3.7) together with

$$m_\pi^2=0, \quad 4m_K^2=3m_\eta^2 \tag{3.13}$$

which is the Gell-Mann–Oakes–Renner model.[34]

(ii) $b=0$, $a_1=0$ ($\langle S_8\rangle_0 \neq 0$). Besides Eq. (3.9) we have

$$\begin{aligned} &m_\pi^2=0, \\ &4Z_K^{-1}m_K^2=3Z_\eta^{-1}m_\eta^2 . \end{aligned} \tag{3.14}$$

Equations (3.9) and (3.14) can be used to give

$$\frac{Z_K}{Z_\pi}=4\left(1-\frac{m_K^2}{m_\eta^2}\right) . \tag{3.15}$$

(iii) $b \neq 0$. Again we have Eq. (3.9) with

$$\begin{aligned} &m_\pi^2=0, \\ &\frac{Z_K^{-1}m_K^2}{Z_\eta^{-1}m_\eta^2}=\frac{3}{4}\left\{\frac{1-\frac{1}{2}b}{1-2b}\right\} \end{aligned} \tag{3.16}$$

and instead of Eq. (3.15) we have

$$\frac{Z_K}{Z_\pi}=4\left[1-\frac{m_K^2}{m_\eta^2}\left(\frac{1-2b}{1-\frac{1}{2}b}\right)\right] . \tag{3.17}$$

(D) SU(3)×SU(3) symmetry of the vacuum: $\langle S_8\rangle_0=0$ (i.e., $b=0$) and $\langle u_0\rangle_0=0$ (i.e., $a_6=0$). In this case we have simultaneously Eqs. (3.6) and (3.7).

(E) SU(3) symmetry of the vacuum: $\langle S_8\rangle_0=0$ ($b=0$).

(i) $a=0$ has been considered in (B)(i)

(ii) $a \neq 0$. Equation (3.7) holds together with

$$4m_K^2=3m_\eta^2+m_\pi^2 \tag{3.18}$$

which is the usual Gell-Mann–Okubo mass formula.

(F) SU(2)×SU(2) symmetry of the vacuum: $\langle u_8\rangle_0 = -\sqrt{2}\langle u_0\rangle_0$ ($b=-1$).

(i) $a=0$. This has Eqs. (3.9), (3.14), and (3.15).

(ii) $a \neq 0$. This implies Eq. (3.9) as well as

$$\begin{aligned} &m_\pi^2=0, \\ &\frac{Z_K^{-1}m_K^2}{Z_\eta^{-1}m_\eta^2}=\frac{3}{4}\left(\frac{1-\frac{1}{2}a}{1-2a}\right) . \end{aligned} \tag{3.19}$$

Finally, using Eq. (2.17) we deduce that ϵ_8 and $\langle S_8\rangle_0$ are SU(3)-symmetry-violating parameters of the same order and therefore, our masses given in Eqs. (3.1)–(3.3) satisfy the Gell-Mann–Okubo (GMO) mass formula identically to first order in SU(3)-symmetry breaking.

2. Scalar-Meson Masses

The expressions we obtain for the masses of the scalar particles in the Lagrangian of Eq. (2.33) are

$$m_{\pi_N}^2=Z_{\pi_N}\left[-2\beta_1-2\epsilon_0(a_9+\sqrt{3}\,a_{10}\langle S_8\rangle_0)-2\epsilon_8\left(\frac{a_5}{\sqrt{3}}+a_3\langle S_8\rangle_0\right)-2\sqrt{3}\,\beta_2\langle S_8\rangle_0\right], \tag{3.20}$$

$$m_K^2=Z_K\left[-2\beta_1-2\epsilon_0\left(a_9-\frac{\sqrt{3}}{2}a_{10}\langle S_8\rangle_0\right)+\epsilon_8\left(\frac{a_5}{\sqrt{3}}-2a_3\langle S_8\rangle_0\right)+\sqrt{3}\,\beta_2\langle S_8\rangle_0\right], \tag{3.21}$$

$$m_{S^*}^2=Z_{S^*}\left[-2\beta_1-2\epsilon_0(a_9-\sqrt{3}\,a_{10}\langle S_8\rangle_0)+2\epsilon_8\left(\frac{a_5}{\sqrt{3}}-3a_3\langle S_8\rangle_0\right)+2\sqrt{3}\,\beta_2\langle S_8\rangle_0\right]. \tag{3.22}$$

From these equations for the scalar masses, it is interesting to consider only the following limiting symmetries where we assume that α_S, β_1, and β_2 do not vanish simultaneously:

(A) SU(3)×SU(3) symmetry of $\mathcal{L}$: $\epsilon_0=\epsilon_8=0$.

(i) $\langle S_8\rangle_0=0$.

$$Z_{\pi_N}=Z_K=Z_{S^*}=1 , \tag{3.23}$$

$$m_{\pi_N}=m_K=m_{S^*} . \tag{3.24}$$

(ii) $\langle S_8\rangle_0 \neq 0$. This case implies $\beta_1=0$ [see Eq. (2.17)] and

$$Z_{\pi_N} \neq Z_K \neq Z_{S^*}, \quad 4Z_K^{-1}=3Z_{S^*}^{-1}+Z_{\pi_N}^{-1} , \tag{3.25}$$

$$m_{\pi_N}^2=-2m_K^2=-m_{S^*}^2 \tag{3.26}$$

to first order in $\langle S_8\rangle_0$. Obviously Eq. (3.26) cannot be satisfied unless β_2 is also zero. Therefore, the SU(3)×SU(3) symmetry of $\mathcal{L}$ and massive scalar particles in this limit imply SU(3) symmetry of the vacuum ($\langle S_8\rangle_0=0$).

(B) SU(3) symmetry of $\mathcal{L}$: $\epsilon_8=0$.

(i) $\langle S_8\rangle_0=0$. We have Eq. (3.23) and

$$m_{\pi_N}^2=m_K^2=m_{S^*}^2 . \tag{3.27}$$

(ii) $\langle S_8\rangle_0 \neq 0$. This implies $\beta_1+a_9\epsilon_0=0$ from Eq. (2.17), and *again* we have Eqs. (3.25) and (3.26). Therefore, it is important to realize that SU(3) symmetry of $\mathcal{L}$ and massive scalar mesons in this limit requires SU(3) symmetry of the vacuum ($\langle S_8\rangle_0=0$).

(C) SU(3)×SU(3) symmetry of the vacuum. In this case Eq. (2.17) leads to $\epsilon_8=0$ while ϵ_0 remains arbitrary, and we have Eqs. (3.23) and (3.24).

(D) SU(3) symmetry of the vacuum. This leads to exactly the same results as part (C), which demonstrates the independence of the scalar mass on the parameter $\langle u_0\rangle_0$.

We conclude this subsection by noting that to first order in SU(3) breaking, Eqs. (3.20)–(3.22) give the GMO mass formula for the scalar-meson masses.

3. PCAC

All effective Lagrangian theories without explicit derivative symmetry breaking have axial-vector divergences given by

$$\partial_\mu A_i^\mu = i[Q_i^A, \mathcal{L}_{SB}] , \qquad (3.28)$$

where A_i^μ are the octet of axial-vector currents. From the form of $\mathcal{L}_{SB}$ given in Eq. (2.1), together with the commutation relations (2.4), we find that

$$\partial_\mu A_i^\mu = [(\tfrac{2}{3})^{1/2}\epsilon_0\delta_{ik} + \epsilon_8 d_{8ik}]v_k + (\tfrac{2}{3})^{1/2}(\epsilon_8\delta_{i8})v_0 \qquad (3.29)$$

neglecting the SU(2)-violating term which leads to experimentally undetermined contributions.

The usual PCAC assumption, that the axial-vector divergences are equally good as interpolating fields for the pseudoscalar mesons as those appearing in the Lagrangian, leads us to the expressions:

$$\partial_\mu A_\pi^\mu = m_\pi^2 f_\pi \phi_\pi^R, \quad \pi = 1, 2, 3 , \qquad (3.30)$$

$$\partial_\mu A_K^\mu = m_K^2 f_K \phi_K^R, \quad K = 4, 5, 6, 7 , \qquad (3.31)$$

$$\partial_\mu A_\eta^\mu = m_\eta^2 f_\eta \phi_\eta^R, \quad \eta = 8 , \qquad (3.32)$$

to lowest order in the physical fields. To exploit these relationships, we have to expand the pseudoscalar densities v_0 and v_i in Eq. (3.29) to second order in the original fields (see Appendix),

$$v_0 = (\tfrac{2}{3})^{1/2}\frac{a_1}{f} S_i\phi_i , \qquad (3.33)$$

$$v_k = -(2a_6 f)\phi_k + \frac{a_1}{f} d_{kij}\phi_i S_j , \qquad (3.34)$$

and then use the transformation (2.16) and the renormalization (2.20) and (2.21) to obtain

$$v_0 = (\tfrac{2}{3})^{1/2}\frac{\langle u_8\rangle_0}{f} Z_\eta^{1/2}\phi_\eta^R \equiv c_0\phi_\eta^R , \qquad (3.35)$$

$$v_\pi = (\tfrac{2}{3})^{1/2}\frac{\langle u_0\rangle_0}{f} Z_\pi^{1/2}(1+b)\phi_\pi^R \equiv c_\pi\phi_\pi^R , \qquad (3.36)$$

$$v_K = (\tfrac{2}{3})^{1/2}\frac{\langle u_0\rangle_0}{f} Z_K^{1/2}(1-\tfrac{1}{2}b)\phi_K^R \equiv c_K\phi_K^R , \qquad (3.37)$$

$$v_8 = (\tfrac{2}{3})^{1/2}\frac{\langle u_0\rangle_0}{f} Z_\eta^{1/2}(1-b)\phi_\eta^R \equiv c_8\phi_\eta^R \qquad (3.38)$$

to first order in the fields. Substituting Eqs. (3.35)–(3.38) into Eq. (3.29) gives

$$\partial_\mu A_\pi^\mu = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f} Z_\pi^{1/2}(1+a)(1+b)\phi_\pi^R , \qquad (3.39)$$

$$\partial_\mu A_K^\mu = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f} Z_K^{1/2}(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b)\phi_K^R , \qquad (3.40)$$

$$\partial_\mu A_\eta^\mu = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f} Z_\eta^{1/2}[(1-a)(1-b)+2ab]\,\phi_\eta^R . \qquad (3.41)$$

Further substitution of Eq. (3.1)–(3.3) in Eqs. (3.39)–(3.41) and comparison with Eqs. (3.30)–(3.32) leads us to conclude

$$f_\pi = f Z_\pi^{-1/2} , \qquad (3.42)$$

$$f_K = f Z_K^{-1/2} , \qquad (3.43)$$

$$f_\eta = f Z_\eta^{-1/2} , \qquad (3.44)$$

Thus Eqs. (3.42)–(3.44) allow us to rewrite Eqs. (3.1)–(3.3) in the form

$$m_\pi^2 f_\pi^2 = \tfrac{2}{3}\epsilon_0\langle u_0\rangle_0(1+a)(1+b) , \qquad (3.45)$$

$$m_K^2 f_K^2 = \tfrac{2}{3}\epsilon_0\langle u_0\rangle_0(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b) , \qquad (3.46)$$

$$m_\eta^2 f_\eta^2 = \tfrac{2}{3}\epsilon_0\langle u_0\rangle_0[(1-a)(1-b)+2ab] , \qquad (3.47)$$

where the Eqs. (3.45) and (3.46) were first derived by Gell-Mann in 1961.[33] Furthermore, the ratio f_K/f_π can be written using Eqs. (3.36) and (3.37) as

$$\frac{f_K}{f_\pi} = \frac{c_\pi(1-\tfrac{1}{2}b)}{c_K(1+b)} . \qquad (3.48)$$

Finally using Eq. (2.31) together with Eqs. (3.42)–(3.44) we get

$$4f_K^2 = f_\pi^2 + 3f_\eta^2 \qquad (3.49)$$

which may be used to calculate f_η.

4. PCVC

We may extend the hypothesis described in subsection 3 above to include the strangeness-changing vector currents. Thus PCVC allows us to define a κ-meson decay constant in analogy to those of the pseudoscalar mesons. The divergences of the vector current, $V_i^\mu(x)$, are given by

$$\partial_\mu V_i^\mu = i[Q_i^V, \mathcal{L}_{SB}] , \qquad (3.50)$$

and using Eq. (2.1), with $\epsilon_3 = 0$, and Eq. (2.2), we find

$$\partial_\mu V_i^\mu = \epsilon_8 f_{8ik} u_k . \qquad (3.51)$$

With similar techniques to those used in the previous subsection, we write u_k to second order in the original scalar fields

$$u_k = a_1 S_k + a_5 d_{kpq} S_p S_q \tag{3.52}$$

and use the transformations (2.16) and (2.23) to get

$$u_\kappa = \left(a_1 - \frac{a_5\langle S_8\rangle_0}{\sqrt{3}}\right) Z_\kappa^{\ 1/2} S_\kappa^R \equiv c_\kappa S_\kappa^R, \quad \kappa = 4, 5, 6, 7 \tag{3.53}$$

to first order in the physical fields. The strangeness-changing vector-current divergence is related to the physical κ-meson field by

$$\partial_\mu V^\mu_{\kappa^+} = i\sqrt{2}\, m_\kappa^{\ 2} f_\kappa S^R_{\kappa^+} , \tag{3.54}$$

where

$$V^\mu_{\kappa^+} = V_4^\mu - iV_5^\mu , \tag{3.55}$$

and

$$S^R_{\kappa^+} = \frac{S_4 - iS_5}{\sqrt{2}} . \tag{3.56}$$

Substituting u_k from Eq. (3.53) in Eq. (3.51) and comparing with Eq. (3.54) we obtain

$$m_\kappa^{\ 2} f_\kappa = \epsilon_8 \frac{\sqrt{3}}{2} c_\kappa , \tag{3.57}$$

showing that the conservation of the strangeness-changing vector currents in the SU(3)-symmetric limit is due to the vanishing of f_κ and not $m_\kappa^{\ 2}$ as would be the case if the κ meson was a Goldstone boson.

Now we are in a position to relate the various quantities so far defined by PCAC and PCVC. By substituting Eqs. (3.42)–(3.43) into (3.45) and (3.46), and using the definitions of c_π and c_K in (3.36) and (3.37) we get

$$f_\pi m_\pi^{\ 2} = (\tfrac{2}{3})^{1/2}(\epsilon_0 + \epsilon_8/\sqrt{2})c_\pi , \tag{3.58}$$

$$f_K m_K^{\ 2} = (\tfrac{2}{3})^{1/2}(\epsilon_0 - \epsilon_8/2\sqrt{2})c_K , \tag{3.59}$$

which can be compared with Eq. (3.57) to give

$$f_\kappa m_\kappa^{\ 2} c_\kappa^{\ -1} + f_K m_K^{\ 2} c_K^{\ -1} = f_\pi m_\pi^{\ 2} c_\pi^{\ -1} . \tag{3.60}$$

This sum rule was derived by Glashow and Weinberg[15] [their Eq. (19), where our c is identical to their $Z^{1/2}$]. Finally, neglecting terms which we may consider to be of second order in SU(3) breaking, namely, $\epsilon_8\langle S_8\rangle_0$, we find from Eqs. (3.36), (3.37), (3.42), and (3.43) that

$$f_\pi c_\pi - f_K c_K = \frac{\sqrt{3}}{2} a_1\langle S_8\rangle_0 , \tag{3.61}$$

while Eqs. (3.21), (3.53), and (2.30) give

$$f_\kappa c_\kappa = \frac{-a_1^{\ 2}\epsilon_8\sqrt{3}}{4(\beta_1 + a_9\epsilon_0)} . \tag{3.62}$$

Using Eq. (2.17), which in this approximation implies

$$2(\beta_1 + a_9\epsilon_0)\langle S_8\rangle_0 = -a_1\epsilon_8 , \tag{3.63}$$

we obtain

$$f_\kappa = \frac{\sqrt{3}}{2}\langle S_8\rangle_0 , \tag{3.64}$$

and the second sum rule of Glashow and Weinberg,[15]

$$f_\pi c_\pi = f_K c_K + f_\kappa c_\kappa . \tag{3.65}$$

We emphasize that this second relationship can only be obtained from our model to first order in SU(3)-symmetry breaking.

IV. RELATIONSHIP BETWEEN SMOOTHNESS AND APPROXIMATE SYMMETRY OF THE LAGRANGIAN

In our model, we have seen in the previous section that the scalar and pseudoscalar densities of the $(3,\bar{3})+(\bar{3},3)$ representation are related to the physical fields by

$$v_i = c_{\phi_i}\phi_i^R + \text{higher orders in fields} , \tag{4.1}$$

$$u_i = c_{S_i} S_i^R + \text{higher orders in fields} , \tag{4.2}$$

where c_{ϕ_i} and c_{S_i} are defined by Eqs. (3.36), (3.37), and (3.53). The equality $c_{\phi_i} = c_{S_i} = c$ corresponds to the smoothness assumption made by other authors.[16,17,20,21,34,44,45]

Different schemes which use the $(3,\bar{3})+(\bar{3},3)$ representation to describe the chiral SU(3)×SU(3) symmetry breaking rely on contradictory values for the parameters c_{ϕ_i} and c_{S_i}. For example, the Gell-Mann, Oakes, and Renner (GMOR) model[34] assumes $c_\pi = c_K$ whilst the Brandt and Preparata (BP) model[52] has instead $c_K/c_\pi \simeq m_K^{\ 2}/m_\pi^{\ 2}$.

Previous nonlinear Lagrangians have c_π identically equal to c_K, and only the introduction of the scalar mesons, as we demonstrate in this paper, allows us to have different values for c_π and c_K.

For the reader's convenience, we collect together the equations containing c_{ϕ_i} and c_{S_i} [Eqs. (3.36), (3.37), (3.42), (3.43), (3.45), (3.46), (3.57), (3.60), and (3.65)] to show explicitly the dependence of our model on these parameters:

$$f_\pi m_\pi^{\ 2} = (\tfrac{2}{3})^{1/2}\epsilon_0 c_\pi(1+a) , \tag{4.3}$$

$$f_K m_K^{\ 2} = (\tfrac{2}{3})^{1/2}\epsilon_0 c_K(1-\tfrac{1}{2}a) , \tag{4.4}$$

$$f_\kappa m_\kappa^{\ 2} = (\tfrac{3}{2})^{1/2}\epsilon_0 c_\kappa a , \tag{4.5}$$

$$f_\pi = (\tfrac{2}{3})^{1/2}\langle u_0\rangle_0 c_\pi^{\ -1}(1+b) , \tag{4.6}$$

$$f_K = (\tfrac{2}{3})^{1/2}\langle u_0\rangle_0 c_K^{\ -1}(1-\tfrac{1}{2}b) , \tag{4.7}$$

$$f_\kappa = (\tfrac{3}{2})^{1/2}\langle u_0\rangle_0 c_\kappa^{\ -1} b , \tag{4.8}$$

which imply the Glashow-Weinberg sum rules[15]

$$f_\kappa m_\kappa^2 c_\kappa^{-1} + f_K m_K^2 c_K^{-1} = f_\pi m_\pi^2 c_\pi^{-1} , \tag{4.9}$$

$$f_K c_K + f_\kappa c_\kappa = f_\pi c_\pi , \tag{4.10}$$

where Eqs. (4.8) and (4.10) are true neglecting $\epsilon_8\langle S_8\rangle_0$.

It is now easy to see, from the above equations, that the smoothness assumption, namely $c_\pi = c_K$, gives

$$\frac{f_K}{f_\pi} = \frac{1-\frac{1}{2}b}{1+b} , \tag{4.11}$$

$$\frac{f_K m_K^2}{f_\pi m_\pi^2} = \frac{1-\frac{1}{2}a}{1+a} , \tag{4.12}$$

while $c_\pi = c_K = c_\kappa$ leads to

$$\frac{f_\kappa}{f_\pi} = \frac{3}{2}\frac{b}{1+b} , \tag{4.13}$$

$$\frac{f_\kappa m_\kappa^2}{f_\pi m_\pi^2} = \frac{3}{2}\frac{a}{1+a} . \tag{4.14}$$

Using the experimentally suggested value $f_K/f_\pi = 1.2$ we get from Eqs. (4.11) and (4.12)

$$a = -0.90, \quad b = -0.12 \tag{4.15}$$

and together with Eqs. (4.13) and (4.14) we predict

$$f_\kappa/f_\pi = -0.20, \quad m_\kappa = 1.22 \text{ GeV} \tag{4.16}$$

in good agreement with the experimental value of m_κ.

It is important to note that the mass of the κ meson is quite sensitive to small changes in f_K/f_π. For example, the value $f_K/f_\pi = 1.26$ gives

$$m_\kappa = 1.04 \text{ GeV}, \quad f_\kappa/f_\pi = -0.26 . \tag{4.17}$$

It is interesting to point out that the BP suggestion[52]

$$\frac{c_\pi}{m_\pi^2} = \frac{c_K}{m_K^2} = \frac{c_\kappa}{m_\kappa^2} , \tag{4.18}$$

is symmetric to the smoothness assumption

$$c_\pi = c_K = c_\kappa \tag{4.19}$$

under the interchange of $a \leftrightarrow b$ in our model. Then, if we assume Eq. (4.18) to hold instead of Eq. (4.19) we have

$$b = -0.90, \quad a = -0.12 , \tag{4.20}$$

while the predicted values of f_κ and m_κ in Eqs. (4.16) and (4.17) are not changed.

Thus we can conclude that the smoothness assumption (4.19) *implies* approximate SU(2)×SU(2) symmetry of the Lagrangian and almost SU(3)-invariant physical states,[44] while the assumption (4.18) leads to an approximately SU(3)-symmetric Lagrangian and physical states which are near to SU(2)×SU(2) invariance.[24]

V. THE DECAY WIDTHS OF THE SCALAR MESONS

The Lagrangian that we have constructed can obviously be enlarged to calculate all n-particle processes involving the scalar and pseudoscalar mesons. However, as the experimental data are so poor, the extension to many-particle interactions is in general both tedious and uninteresting. The one possibility that does remain after a discussion of the scalar masses is to try to estimate the various widths for the scalar mesons to decay into two pseudoscalars. In order to obtain predictions from the Lagrangian rather than a many-parameter fit (and since the data are poor, this is evidently very easy to do) we make several simplifying assumptions which reduce the accuracy of our predictions by an amount of the order of SU(3)-breaking effects.

Firstly, we neglect all SU(3)-symmetry-breaking contributions to the scalar-pseudoscalar-pseudoscalar (SPP) interactions which are given from Eqs. (2.14) and (2.15) by

$$\mathcal{L}_{SPP} = \alpha_P d_{ijk} S_i^R \partial_\mu \phi_j^R \partial^\mu \phi_k^R - \epsilon_0 (\tfrac{1}{6})^{1/2} \frac{a_1}{f^2} d_{ijk} \phi_i^R \phi_j^R S_k^R , \tag{5.1}$$

which in the tree-graph approximation gives an invariant amplitude

$$g_{S_i P_j P_k} = 2\left[\alpha_P d_{ijk}(k_1 \cdot k_2) - \epsilon_0(\tfrac{1}{6})^{1/2}\frac{a_1}{f^2} d_{ijk}\right] , \tag{5.2}$$

where k_1 and k_2 are the four-momenta of the pseudoscalars (with mass m_1 and m_2) involved. In the usual manner, the decay rate is defined as

$$\Gamma_{S_i \to P_j P_k} = \frac{1}{2m_{S_i}} \int |g_{S_i P_j P_k}|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3} \times (2\pi)^4 \delta(m_{S_i} - \omega_1 - \omega_2)\delta^3(\vec{k}_1 + \vec{k}_2) S ,$$

where m_{S_i} is the mass of the scalar meson, ω_1 and ω_2 are the energies of the corresponding pseudoscalar mesons and $S = \prod_i 1/n_i!$ for n_i identical particles in the final state. Completing the integrations, we are led finally to the decay rate

$$\Gamma_{S_i \to P_j P_k} = \frac{(d_{ijk})^2 S}{2\pi m_{S_i}^2} k_0 F(k_0) , \tag{5.3}$$

where

$$k_0 = \frac{m_{S_i}}{2}\left[1 - \frac{(m_1+m_2)^2}{m_{S_i}^2}\right]^{1/2}\left[1 - \frac{(m_1-m_2)^2}{m_{S_i}^2}\right]^{1/2} \tag{5.4}$$

and

$$F(k_0) \equiv F(k_1 \cdot k_2)$$
$$= \left[\alpha_P (k_1 \cdot k_2) - \frac{\epsilon_0 a_1}{\sqrt{6} f^2} \right]^2 , \quad (5.5)$$

with

$$k_1 \cdot k_2 = \omega_1 \omega_2 + k_0^2$$
$$= (m_1^2 + k_0^2)^{1/2} (m_2^2 + k_0^2)^{1/2} + k_0^2 . \quad (5.6)$$

Obviously, we do not neglect the SU(3) breaking in the masses as this would lead to a trivial relationship between the decay rates of the scalar mesons and the symmetric tensor d_{ijk}.

Simple manipulations on Eq. (5.3) allow us to exhibit the decay widths for the physical particles as

$$\Gamma_{\pi_N^+ \to \eta\pi^+} = \frac{1}{6\pi m_{\pi_N}^2} k_0 F(k_0) ,$$
$$\Gamma_{\pi_N^+ \to K^+ \bar{K}^0} = \frac{1}{4\pi m_{\pi_N}^2} k_0 F(k_0) ,$$
$$\Gamma_{S^* \to \pi\pi} = \frac{1}{4\pi m_{S^*}^2} k_0 F(k_0),$$
$$\Gamma_{S^* \to KK} = \frac{1}{8\pi m_{S^*}^2} k_0 F(k_0) , \quad (5.7)$$
$$\Gamma_{\kappa^+ \to K\pi} = \frac{3}{8\pi m_\kappa^2} k_0 F(k_0) ,$$
$$\Gamma_{\kappa^+ \to K^+\eta} = \frac{1}{24\pi m_\kappa^2} k_0 F(k_0) ,$$

where the appropriate masses must be inserted into the $k_0 F(k_0)$ factor in each case, and the decay of S^* into two pions and two kaons includes both charged and neutral modes, while the

$$\Gamma_{\kappa^+ \to K\pi} = \Gamma_{\kappa^+ \to K^+\pi^0} + \Gamma_{\kappa^+ \to K^0\pi^+} .$$

Thus we have six decay widths in Eq. (5.7) and only two parameters as given in Eq. (5.5).

Secondly, we introduce the smoothness assumption, $c_\pi = c_K$, and use the expressions for m_π^2 and m_K^2 [Eqs. (3.1) and (3.2)] to first order in SU(3) breaking to obtain

$$\frac{\epsilon_0 a_1}{\sqrt{6} f^2} = \alpha_P \left(\frac{m_\pi^2 + 2m_K^2}{6} \right) . \quad (5.8)$$

Thus, substituting this equation into (5.5) gives

$$F(k_0) = \alpha_P^2 \left[(k_1 \cdot k_2) + \frac{m_\pi^2 + 2m_K^2}{6} \right]^2 , \quad (5.9)$$

which enables us to fit the data with just the single parameter α_P^2.

The decay widths for the scalar particles are given in Table I for two cases:

(1) $m_{\pi_N} = 1.01$ GeV, $\Gamma(\pi_N \to \eta\pi) = 50$ MeV, $m_\kappa = 1.22$ GeV, and $m_{S^*} = 1.27$ GeV.

(2) $m_{\pi_N} = 0.98$ GeV, $\Gamma(\pi_N \to \eta\pi) = 50$ MeV, $m_\kappa = 1.04$ GeV, and $m_{S^*} = 1.06$ GeV.

The mass and width of π_N are taken as input, the mass m_κ is as calculated in our model [Eqs. (4.16) and (4.17)] and m_{S^*} is given by the GMO mass formula.

VI. SU(2) BREAKING

In this section, we calculate the contribution of the nonelectromagnetic SU(2)-symmetry breaking, described by the u_3 term in our original Lagrangian, Eq. (2.1), up to first order in the parameters describing the symmetry violations.

From Eqs. (2.17) and (2.18) which eliminate the linear terms in the Lagrangian we find to first order that

$$\frac{\langle S_3 \rangle_0}{\langle S_8 \rangle_0} = \frac{\langle u_3 \rangle_0}{\langle u_8 \rangle_0} = \frac{\epsilon_3}{\epsilon_8} . \quad (6.1)$$

We have also shown in Sec. II that diagonalization of the kinetic terms leads to

$$\theta_S = \theta_P = \theta = \frac{\langle S_3 \rangle_0}{2\langle S_8 \rangle_0} . \quad (6.2)$$

Further, diagonalization of the pseudoscalar as well as the scalar mass terms give

$$\theta = \frac{\epsilon_3}{2\epsilon_8} \quad (6.3)$$

which is consistent with Eq. (6.1) and (6.2).

The pseudoscalar-meson masses are now given by

$$m_{\pi^+}^2 = \frac{2}{3} \frac{\epsilon_0 \langle u_0 \rangle_0}{f^2} (1 + x) , \quad (6.4)$$

TABLE I. Decay widths of the scalar particles. Case I: $m_{\pi_N} = 1.01$ GeV, $m_\kappa = 1.22$ GeV, $m_{S^*} = 1.27$ GeV. Case II: $m_{\pi_N} = 0.98$ GeV, $m_\kappa = 1.04$ GeV, $m_{S^*} = 1.06$ GeV.

	$S \to PP$	$\pi_N^+ \to \eta\pi^+$	$\pi_N^+ \to K^+\bar{K}^0$	$S^* \to \pi\pi$	$S^* \to K\bar{K}$	$S^* \to \eta\eta$	$\kappa \to K\pi$	$\kappa \to K\eta$
(I)	k_0 (GeV)	0.33	0.04	0.61	0.38	0.30	0.48	0.30
(I)	Γ (MeV)	50 (input)	6	340	40	25	260	13
(II)	k_0 (GeV)	0.32	···	0.51	0.19	···	0.39	···
(II)	Γ (MeV)	50 (input)	···	250	12	···	180	···

$$m_{\pi^0}^2 = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f^2}(1+x+2y\theta)\ , \tag{6.5}$$

$$m_{K^+}^2 = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f^2}(1-\tfrac{1}{2}x+\tfrac{1}{2}\sqrt{3}\,y)\ , \tag{6.6}$$

$$m_{K^0}^2 = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f^2}(1-\tfrac{1}{2}x-\tfrac{1}{2}\sqrt{3}\,y)\ , \tag{6.7}$$

$$m_\eta^2 = \frac{2}{3}\frac{\epsilon_0\langle u_0\rangle_0}{f^2}(1-x-2y\theta)\ , \tag{6.8}$$

where

$$x = a+b-\frac{2\alpha_P}{\sqrt{3}}\langle S_8\rangle_0\ , \tag{6.9}$$

$$y = \frac{\epsilon_3}{\sqrt{2}\,\epsilon_0}+\frac{\langle u_3\rangle_0}{\sqrt{2}\langle u_0\rangle_0}-\frac{2\alpha_P}{\sqrt{3}}\langle S_3\rangle_0\ . \tag{6.10}$$

Neglecting the $y\theta$ terms, Eqs. (6.4) to (6.10) together with (6.1) give

$$\frac{\epsilon_3}{\epsilon_8} = \frac{\sqrt{3}}{2}\frac{(m_{K^0}^2-m_{K^+}^2)}{(m_K^2-m_\pi^2)}\ . \tag{6.11}$$

It is important to mention at this stage that when discussing mass differences within SU(2) multiplets one has to consider also the electromagnetic (em) contribution, thus assuming only these two types of SU(2)-symmetry breaking (i.e., em and u_3), we have

$$(m_{K^0}-m_{K^+})_{\rm expt} = (m_{K^0}-m_{K^+})_{u_3}+(m_{K^0}-m_{K^+})_{\rm em} \tag{6.12}$$

with a similar equation for the pions. It has been shown by Dashen[32] that the em contribution gives the sum rule

$$(m_{K^0}^2-m_{K^+}^2)_{\rm em} = (m_{\pi^0}^2-m_{\pi^+}^2)_{\rm em} \tag{6.13}$$

which can be expressed as

$$(m_{K^0}-m_{K^+})_{\rm em} = \frac{m_\pi}{m_K}(m_{\pi^0}-m_{\pi^+})_{\rm em}\ . \tag{6.14}$$

The electromagnetic mass difference of the pions has been calculated, using current-algebra techniques,[54] to give

$$(m_{\pi^0}-m_{\pi^+})_{\rm em} = -5.0\ {\rm MeV} \tag{6.15}$$

which leads to

$$(m_{K^0}-m_{K^+})_{\rm em} = -1.4\ {\rm MeV}\ . \tag{6.16}$$

Inserting Eq. (6.16) into (6.12) and using the experimental result $m_{K^0}-m_{K^+}=4.0$ MeV, we find

$$(m_{K^0}-m_{K^+})_{u_3} = 5.4\ {\rm MeV}\ . \tag{6.17}$$

To calculate the value of ϵ_3/ϵ_8 we insert Eq. (6.17) into (6.11) for the K^0-K^+ mass difference, to get

$$\epsilon_3/\epsilon_8 = 2.2\times10^{-2} \tag{6.18}$$

in agreement with many calculations given in the literature.[21,46,48,55] If we replace the denominator in Eq. (6.11) by using the GMO mass formula we find using Eq. (6.3)

$$\theta = \frac{1}{\sqrt{3}}\frac{(m_{K^0}^2-m_{K^+}^2)}{(m_\eta^2-m_\pi^2)} \tag{6.19}$$

which may be compared with the form

$$\theta = \frac{1}{\sqrt{3}}\frac{(m_{K^0}^2-m_{K^+}^2)+(m_{\pi^+}^2-m_{\pi^0}^2)}{(m_\eta^2-m_\pi^2)} \tag{6.20}$$

obtained by Okubo and Sakita,[56] as in this approximation (neglecting $y\theta$ terms) $m_{\pi^+}=m_{\pi^0}$ (which is to be expected since u_3 is an isotriplet and the pion mass difference is pure $I=2$).

Including now the $y\theta$ terms in the expressions (6.4)–(6.8) we can get the u_3 contribution to the π^+-π^0 mass difference. Using Eqs. (6.11) and (6.19) we have

$$(m_{\pi^0}-m_{\pi^+})_{u_3} = (-)\frac{(m_{K^0}-m_{K^+})_{u_3}^2}{m_\pi}\frac{1}{(1-m_\pi^2/m_K^2)}\ . \tag{6.21}$$

The numerical value for $(m_{K^0}-m_{K^+})_{u_3}$ is given in Eq. (6.17) and leads to

$$(m_{\pi^0}-m_{\pi^+})_{u_3} = -0.23\ {\rm MeV} \tag{6.22}$$

which, as has been pointed out by other authors,[48] is in the wrong direction but within the probable 10% error of the current-algebra calculations of Das *et al.*[54] We note however that although $y\theta$ is of order ϵ_3^2/ϵ_8, this is still a second-order SU(2)-breaking effect, and since we have not taken into account the full contribution to this order, our result in Eq. (6.22) may be considerably altered.

Finally, again neglecting the $y\theta$ terms, we see that the masses of the pseudoscalars given in Eqs. (6.4)–(6.8) satisfy the GMO formula with SU(2) corrections.[57]

$$2(m_{K^+}^2+m_{K^0}^2) = 3m_\eta^2+m_\pi^2\ . \tag{6.23}$$

To the same approximation we find that

$$2(m_{\kappa^+}^2+m_{\kappa^0}^2) = 3m_{S^*}+m_{\pi_N}^2 \tag{6.24}$$

for the scalar particles.

VII. SUMMARY AND DISCUSSION

The present paper is concerned with the spontaneous breakdown of chiral SU(3)×SU(3) symmetry realized by an octet of Goldstone pseudoscalar mesons. Thus, in the limit that symmetry breaking is neglected, SU(3)×SU(3) does not appear as a symmetry of the particle states as SU(3) does. Guided by this idea, and the existence of an octet of scalar mesons with masses around 1 GeV, we have constructed a nonlinear Lagrangian such that the scalar particles behave neither as parity partners of the pseudoscalar mesons nor like Goldstone bosons in the chiral SU(3)×SU(3) limit. Although these scalar mesons have no particular significance in the symmetry limit, they play a special role in symmetry-breaking effects.

We have considered a symmetry-breaking Lagrangian which transforms as the $(3,\bar{3})+(\bar{3},3)$ representation of the group SU(3)×SU(3) and contains explicitly nonderivative SU(3) and SU(2) violations which conserve the current algebra. The transformation properties of the scalar fields enable us to have linear terms in S_3 and S_8 in the Lagrangian which cause the vacuum to violate SU(3) and SU(2) symmetry. However, it turns out that SU(3) symmetry of the Lagrangian implies SU(3) symmetry of the vacuum if the scalar mesons are to remain massive in this limit.

The symmetric Lagrangian is given in Eq. (2.14) while the symmetry-breaking part is written in Eq. (2.15). The new couplings of major importance are:

(a) α_P and α_S which produce renormalization effects for the wave functions [Eqs. (2.25)–(2.30)] and symmetric, derivative-dependent scalar-pseudoscalar-pseudoscalar (*SPP*) and scalar-scalar-scalar (*SSS*) couplings. As has been demonstrated in Sec. V, the decay widths given from this *SPP* term are evidently consistent with the present experimental data (see Table I).

(b) β_1 produces the scalar particle mass in the limit of chiral symmetry.

(c) a_1 is the coefficient of the linear terms in the scalar fields and gives the nonderivative SPP couplings. The presence of a_1 forces us to make a transformation of the scalar fields to get a nonvanishing vacuum expectation value which breaks SU(3) symmetry.

(d) As usual, the coefficient a_6 contributes the nonzero vacuum expectation value of the SU(3) singlet, u_0 [Eq. (3.4)].

The main conclusions that can be drawn from our Lagrangian model after it has been rewritten in terms of physical fields are as follows:

(1) The pseudoscalar-meson masses, exhibited in Eqs. (3.1)–(3.3) show the Goldstone behavior of these particles. To first order in SU(3) breaking, these particles satisfy the GMO mass formula.

(2) The scalar particle masses are given in Eqs. (3.20)–(3.22), and satisfy the GMO mass formula to first order in SU(3). These expressions together with the pseudoscalar masses demonstrate the possible symmetry limits in our model as has been discussed in Sec. III. As expected, our model contains the physically relevant limit with an SU(3) ×SU(3)-invariant Lagrangian and SU(3)-symmetric vacuum.

(3) To first order in the fields our Lagrangian implies partial conservation of the axial-vector current (PCAC). The decay constants (f_π, f_K, and f_η) defined by the PCAC Eqs. (3.30)–(3.32) are shown to be related to the wave-function renormalization constants [Eqs. (3.42)–(3.44)] which allows us to have $f_K \neq f_\pi$. The fact that the scalar mesons are needed to reproduce this physical result, exhibits the importance of these mesons in SU(3)-symmetry-breaking effects. Furthermore, we see that the first-order SU(3)-breaking corrections to the ratio f_K/f_π are nonzero in direct contradiction to the assumption of Ref. 58 based on a nongeneral proof.[59,44]

(4) Extension to include partial conservation of the vector current (PCVC) for the strangeness-changing vector current, together with the expressions obtained using PCAC, gives the Glashow-Weinberg sum rules[15] [Eqs. (3.60) and (3.65)] in our model. It is important to mention that we have these relationships although the κ meson is not a Goldstone particle in the chiral SU(3)×SU(3) limit.

(5) The smoothness assumption [Eq. (4.19)] which is discussed in Sec. IV, is shown to imply approximate SU(2)×SU(2) symmetry of the Lagrangian with a small SU(3) violation in the vacuum.

(6) An appropriate change in the smoothness assumption [see Eqs. (4.18)] as has been suggested by Brandt and Preparata[52] leads to approximate SU(3) symmetry of the Lagrangian, while the vacuum is almost SU(2)×SU(2)-invariant.

(7) Both assumptions [Eqs. (4.18) and (4.19)] mentioned above give the same results for the pseudoscalar masses, f_K/f_π, f_κ/f_π, and m_κ [Eqs. (4.3)–(4.8)].

Using the known pseudoscalar masses and f_K/f_π as input, we calculate the symmetry-breaking parameters [Eqs. (4.15) and (4.20)] of the model and predict f_κ/f_π and m_κ [Eqs. (4.16 and (4.17)] in agreement with experimental data.

(8) The widths of the scalar particles are obtained in Sec. V neglecting the SU(3)-breaking effects in the *SPP* couplings and assuming smoothness. Making these assumptions we are able to calculate the decay widths in agreement with experiment taking as input only m_{π_N} and $\Gamma(\pi_N \to \eta\pi)$. (See Table I.) A feature of our numerical results is the existence of a broad S^* decaying strongly into two pions, as suggested in Ref. 29.

(9) In our model, we found that the nonelectromagnetic SU(2)-breaking term gives the major contribution to the mass difference of the kaons [Eq. (6.17)]. Finally, we have a modified GMO mass formula and expression for the $\pi\eta$ mixing angle as given in earlier literature [Eqs. (6.19), (6.23), and (6.24)].

Thus this model can be considered as a generalization of the usual nonlinear Lagrangians to include many more interesting symmetry-breaking phenomena as well as describing the scalar mesons.

ACKNOWLEDGMENTS

We should like to thank B. Renner for bringing Refs. 58 and 59 to our attention and H.D.I. Abarbanel for an interesting discussion on this point.

P.H.D. wishes to acknowledge financial support from the Science Research Council, and S.E. is grateful to the Royal Society Postdoctoral Fellowship exchange program with the Israel Academy of Science.

APPENDIX

The $(3,\bar{3})+(\bar{3},3)$ representation can be constructed as a function of fields with known chiral SU(3) $\times$SU(3) transformation properties in the manner given by Coleman *et al.*[37] Thus

$$\begin{pmatrix} u_\alpha \\ v_\alpha \end{pmatrix} = \exp\begin{bmatrix} 0 & -(\xi_i d_{i\alpha\beta}) \\ (\xi_i d_{i\alpha\beta}) & 0 \end{bmatrix}\begin{pmatrix} X_\beta \\ 0 \end{pmatrix} , \qquad (A1)$$

where $\alpha, \beta = 0, 1, 2, \ldots, 8$ and $i = 1, 2, \ldots, 8$ and ξ_i is a 0^- octet constructed from the pseudoscalar-meson fields. Since we are interested only in the pseudoscalar- and scalar-meson fields, the general form of X_β is a function of the scalar fields given by

$$X_0 = c_1 + c_2 S_i S_i + c_3 d_{ijk} S_i S_j S_k + \cdots , \qquad (A2)$$

$$X_i = c_4 S_i + c_5 d_{ijk} S_j S_k + c_6 S_i S_j S_j + \cdots . \qquad (A3)$$

Then as has been shown in Ref. 37 we may choose a particular coordinate system $\xi_i = \phi_i / f$ in Eq. (A1) to get u_α and v_α as a series expansion in ϕ_i and S_i.

However, it is interesting to note that the $(3,\bar{3})+(\bar{3},3)$ representation can be constructed in a general coordinate system using only the equal-time commutation relations for the fields with the group generators. Using our theorem given in Ref. 44, the representation is given by

$$u_i, u_0, v_i = i\tfrac{3}{5} d_{ijk} [Q_i^A, u_j]^{(8^S+1)}$$

and (A4)

$$(\tfrac{2}{3})^{1/2} v_0 = i\tfrac{1}{8}\delta_{ij}[Q_i^A, u_j]^{(8^S+1)} ,$$

where the notation indicates that the commutator is restricted to contain only the singlet and the symmetric octet under SU(3), i.e., in general, having formed an arbitrary octet vector u_i from the scalar and pseudoscalar fields, this commutator contains the 27, 10, $\overline{10}$, 8^S, 8^A and singlet representations of SU(3), and we require the coefficients of the 27, 10, $\overline{10}$, and 8^A to vanish. Taking the general form for u_k to third order in the fields as

$$\begin{aligned} u_k = {} & S_k(a_1 + a_2\phi_i\phi_i + a_3 S_i S_i) \\ & + a_4\phi_k(\phi_i S_i) + a_5 d_{kij} S_i S_j \\ & + a_6 d_{kij}\phi_i\phi_j + a_7 d_{kij} d_{jlm}\phi_i\phi_l S_m \\ & + a_8 d_{kij} d_{jlm}\phi_l\phi_m S_i , \end{aligned} \qquad (A5)$$

we find that constraining $[Q_i^A, u_j]$ to have only singlet and symmetric octet gives

$$\begin{aligned} & a_2 = a_8 = 0 , \\ & a_4 = -a_1/3f^2, \\ & a_7 = -a_1/2f^2 . \end{aligned} \qquad (A6)$$

Then using Eq. (A4), we get

$$v_i = -2a_6 f\phi_i + \frac{a_1}{f} d_{ijk}\phi_j S_k , \qquad (A7)$$

$$v_0 = (\tfrac{2}{3})^{1/2}\frac{a_1}{f} S_p\phi_p , \qquad (A8)$$

while the commutator

$$[Q_i^A, u_0] = -i(\tfrac{2}{3})^{1/2} v_i \qquad (A9)$$

allows us to deduce u_0 in the form

$$\begin{aligned} u_0 = {} & -\sqrt{6}\, a_6 f^2 + (\tfrac{2}{3})^{1/2} a_6\phi_i\phi_i - (\tfrac{1}{6})^{1/2}\frac{a_1}{f^2} d_{ijk}\phi_i\phi_j S_k \\ & + a_9 S_i S_i + a_{10} d_{ijk} S_i S_j S_k . \end{aligned} \qquad (A10)$$

[1]L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962); Phys. Rev. 133, B812 (1964).

[2]M. Gell-Mann, Physics 1, 63 (1964).

[3]S. L. Adler, Phys. Rev. 140, B736 (1965).

[4]F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968).

[5]B. Dutta-Roy and I. Lapidus, Phys. Rev. 169, 1357 (1968).

[6]J. E. Mackey, J. M. McKessic, D. M. Scott, and W. W. Wada, Phys. Rev. 172, 1590 (1969).

[7]H. D. Dahmen, K. D. Rothe, and L. Schülke, Nucl. Phys. B7, 472 (1968); B8, 150 (1968).

[8]S. Eliezer and P. Singer, Nucl. Phys. B11, 514 (1969); Nuovo Cimento 4A, 638 (1971); Lett. Nuovo Cimento 4, 443 (1970).

[9]S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964).

[10]P. Carruthers, Phys. Reports 1, 1 (1970), and references therein.

[11]P. Carruthers, Phys. Rev. D 3, 959 (1971).

[12]E. Golowich, E. Lasley, and V. Kapila, Phys. Rev. D 4, 2070 (1971).
[13]S. Eliezer and R. Dutt, Phys. Letters 40B, 250 (1972); S. Eliezer, DAMTP Report No. 72/27 (unpublished).
[14]M. Lévy, Nuovo Cimento 52A, 23 (1967).
[15]S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).
[16]S. L. Glashow, in *Hadrons and their Interactions*, edited by A. Zichichi (Academic, New York, 1968), pp. 83–140.
[17]P. R. Auvil and N. G. Deshpande, Phys. Rev. 183, 1463 (1969).
[18]L. K. Pande, Phys. Rev. Letters 25, 777 (1970); Lett. Nuovo Cimento 4, 401 (1970).
[19]S. Okubo and V. S. Mathur, Phys. Rev. D 1, 2046 (1970).
[20]R. Dutt, S. Eliezer, and P. Nanda, Phys. Rev. D 4, 3759 (1971).
[21]R. Dutt and S. Eliezer, Particles and Nuclei 2, 61 (1971); Nuovo Cimento 8A, 741 (1972).
[22]P. Carruthers and R. W. Haymaker, Phys. Rev. Letters 27, 455 (1971); Phys. Rev. D 4, 1808 (1971).
[23]R. Olshansky, Phys. Rev. D 4, 2440 (1971).
[24]L. Bessler, T. Muta, H. Umezawa, and D. J. Welling, Phys. Rev. D 2, 349 (1970); L. Bessler and D. J. Welling, *ibid.* 6, 1092 (1972).
[25]Particle Data Group, Phys. Letters 39B, 1 (1972).
[26]T. G. Trippe *et al.*, Phys. Letters 28B, 203 (1968).
[27]H. Yuta *et al.*, Phys. Rev. Letters 26, 1502 (1971).
[28]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).
[29]J. T. Carroll *et al.*, Phys. Rev. Letters 28, 318 (1972).
[30]M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).
[31]S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).
[32]R. F. Dashen, Phys. Rev. 183, 1245 (1969).
[33]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[34]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).
[35]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).
[36]S. Weinberg, Phys. Rev. 166, 1568 (1968).
[37]C. J. Isham, Nuovo Cimento 59A, 356 (1969); S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969); C. G. Callan, Jr., S. Coleman, J. Wess, and B. Zumino, *ibid.* 177, 2247 (1969).
[38]A. J. Macfarlane, A. Sudbery, and P. Weisz, Proc. Roy. Soc. (London) A314, 217 (1970).
[39]K. J. Barnes, P. H. Dondi, and S. C. Sarkar, Proc. Roy. Soc. (London) A330, 389 (1972).
[40]S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969), and references therein.
[41]A. J. Macfarlane and P. H. Weisz, Lett. Nuovo Cimento 2, 25 (1969).
[42]K. J. Barnes and P. Dittner, Nucl. Phys. B31, 183 (1971).
[43]J. Wess and B. Zumino, Phys. Letters 37B, 95 (1971).
[44]P. H. Dondi and S. Eliezer, Nucl. Phys. B47, 390 (1972).
[45]R. Dutt and S. Eliezer, Lett. Nuovo Cimento 1, 918 (1971); Phys. Rev. D 4, 180 (1971).
[46]N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968); Phys. Rev. D 1, 707 (1970).
[47]R. J. Oakes, Phys. Letters 29B, 683 (1969).
[48]A. Goyal, L.-F. Li, and G. Segrè, Phys. Rev. D 2, 2373 (1970).
[49]Our Refs. 10–24, 33, 34, 40–42, 44–48, and 50–53 all consider this type of symmetry breaking, and this is by no means an exhaustive list.
[50]R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters 26, 104 (1971).
[51]K. J. Barnes, P. Dittner, and P. H. Dondi, Nucl. Phys. B31, 195 (1971).
[52]R. A. Brandt and G. Preparata, Phys. Rev. Letters 26, 1605 (1971); Ann. Phys. (N.Y.) 61, 119 (1970).
[53]B. Renner, G.I.F.T. seminar on broken chiral symmetry (unpublished).
[54]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).
[55]H. Osborn and D. J. Wallace, Nucl. Phys. B20, 23 (1970).
[56]S. Okubo and B. Sakita, Phys. Rev. Letters 11, 50 (1963).
[57]R. H. Dalitz and D. G. Sutherland, Nuovo Cimento 37, 1777 (1965); 38, 1945(E) (1965).
[58]S. C. Chhajlany, L. K. Pandit, and G. Rajasekaran, Phys. Letters 35B, 166 (1971).
[59]L. K. Pandit and G. Rajasekaran, Nucl. Phys. B9, 531 (1969).