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[5] This assumes that the factors $(r'+\vec{n}\cdot\vec{r}')^{-1}$ are finite over the forward radiation cone. If the angle between $\vec{n}$ and $-\vec{r}'$ is smaller than γ^{-1}, then the analysis no longer applies. In fact, one must then consider both retarded time terms, which leads to a cancellation and reduction in power over that obtained from a single retarded time term.

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Comment on U(6) × O(2)-Symmetry Breaking*

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It is shown that if U(6)×O(2) symmetry is "l-broken," then there must exist Carlitz-Kislinger (CK) branch points in the complex angular momentum plane. In view of experimental and theoretical indications against their existence, modifications of "l breaking" are considered that do not lead to CK cuts. The approach to U(6)×O(2) breaking based on the Melosh transformation and on the partial conservation of axial-vector currents appears to be free of this difficulty.

Recently considerable attention has been devoted to the problem of $\mathrm{U}(6)_W\times\mathrm{O}(2)_{L_z}$-symmetry breaking. Two approaches have been discussed. The more fundamental approach[1] makes use of the Melosh transformation and of the partial conservation of axial-vector currents (PCAC) to find the pattern of pionic couplings in broken U(6)×O(2). The more heuristic approach[2] of "l-broken" U(6)×O(2) assumes the extra quark-antiquark pair that appears upon the decay of a nonexotic meson (baryon) into two nonexotic mesons (a nonexotic meson and a nonexotic baryon) to be created in the SU(3)-singlet $^3P_0$ state. The patterns of U(6) breaking obtained in the two approaches coincide to a large extent. This has led to speculation on their possible equivalence. It is the purpose of this note to show that the heuristic approach requires for its consistency the existence of Carlitz-Kislinger[3] (CK) type branch points in the complex angular momentum plane. Present experimental evidence points against the existence of such branch points. Moreover no dynamical mechanism (dual or otherwise) has been found so far to generate them. Were one therefore to deny the possibility of CK cuts, one would also have to abandon the heuristic l breaking of U(6)×O(2), except possibly as a phenomenological ansatz for the coupling of a few *low-lying* hadrons. Remarkably, the fundamental approach does *not* require CK cuts and is thus not affected. Finally, we note that a modification of l-broken U(6)×O(2) that overcomes the difficulty is possible, but at the price of lowered predictivity.

Consider the Regge family of mesons corresponding to the multiplets $(6,\bar{6};l)$ $(l=0,1,2,\dots)$ of $\mathrm{U}(6)\times\mathrm{U}(6)\times\mathrm{O}(3)_L$, of which the $l=0$ multiplet contains the familiar 0^- and 1^- mesons (π, ρ, etc.). As usual we represent the $L=l$ multiplet by $M^{(l)}_{\mu_1\cdots\mu_l}{}^{\beta}_{\alpha}(p)$, where $\alpha=aA$, $\beta=bB$, a and b are spinor indices and range from 1 to 4, A and B are SU(3) indices and range from 1 to 3, μ_1 through μ_l are Lorentz tensor indices and run from 0 to 3, and p is the four-momentum of the multiplet. $M^{(l)}(p)$ obeys the mesonic Bargmann-Wigner equations, is symmetric in the l indices $\mu_1\cdots\mu_l$ and traceless in any pair of indices $\mu_i\mu_j$, and obeys $p^{\mu_i}M^{(l)}_{\mu_1\cdots\mu_l}(p)=0$.

The $M^{(l)}M^{(0)}M^{(0)}$ coupling according to l-broken U(6)×O(2) has the form

$$(m_0)^{2-l}P^{\mu_2}\cdots P^{\mu_l}\,\mathrm{Tr}(M^{(l)}_{\mu_1\cdots\mu_l}(p_3)\{(m_0)^{-1}g_l P^{\mu_1}[M^{(0)}(p_1)M^{(0)}(p_2)]_l+h_l[M^{(0)}(p_1)\gamma^{\mu_1}M^{(0)}(p_2)]_{l-1}$$
$$+h_l'\gamma^{\mu_1}[M^{(0)}(p_1)M^{(0)}(p_2)]_{l-1}+h_l''[M^{(0)}(p_1)M^{(0)}(p_2)]_{l-1}\gamma^{\mu_1}\}). \qquad (1a)$$

Here the trace is to be taken with respect to the Dirac and SU(3)-spinor indices, and the generic notation

$$[f(p_1, p_2)]_l = f(p_1, p_2) + (-1)^l f(p_2, p_1) \tag{1b}$$

for any $f(p_1, p_2)$ has been introduced. Further,

$$\begin{aligned} &P = p_1 - p_2\,, \\ &p_1 + p_2 + p_3 = 0\,, \\ &p_1^2 = p_2^2 = m_0^2\,, \\ &p_3^2 = m_l^2\,, \end{aligned} \tag{1c}$$

and the coupling constants g_l correspond to U(6) $\times$O(2)-invariant couplings while the h_l, h_l', and h_l'' correspond to l-breaking couplings each containing the ${}^3P_0\,\gamma_\mu$ spurion *once*. The expression (1a) is unambiguous for $l \geq 1$. For $l=0$, however, there is no orbital index on $M^{(l)}$, so that the h, h', and h'' couplings become impossible,

$$h_0 = h_0' = h_0'' = 0\,, \tag{1d}$$

and the vertex takes the form

$$g_0 m_0 \mathrm{Tr}(M^{(0)}(p_3)[M^{(0)}(p_1)M^0(p_2)]_0)\,. \tag{1e}$$

Consider now the odd-signature Regge trajectory of the ${}^3l_{J=l+1}$ mesons (l = even) on which the ρ meson lies. Let me call ρ^J ($J=1, 3, 5, \ldots$) the (odd) spin-J recurrence of the ρ meson ($\rho = \rho^1$). The most general $\rho^{0J}\rho^-\rho^+$ coupling has the form

$$\begin{aligned} \sqrt{2}\, m_0^{1-J} P^{\mu_3}\cdots P^{\mu_J}\rho^{0J}_{\mu_1\cdots\mu_J}\rho^-_\alpha\rho^+_\beta \\ \times[\, a_1^J(2m_0^2)^{-1}P^{\mu_1}P^{\mu_2}P^\alpha P^\beta \\ +4a_2^J m_0 g^{\alpha\mu_1}g^{\beta\mu_2} \\ +a_3^J P^{\mu_2}(P^\alpha g^{\beta\mu_1} + P^\beta g^{\alpha\mu_1}) \\ +a_4^J P^{\mu_1}P^{\mu_2}g^{\alpha\beta}]\,. \end{aligned} \tag{2a}$$

According to (1), one can express for each J the four coupling constants $a_1^J, \ldots, a_4^J$ in terms of the l-broken U(6)$\times$O(2) coupling constants $g_l, \ldots, h_l''$. It is easy to show that all couplings of the ρ trajectory (in particular $a_1^J, \ldots, a_4^J$) are independent of h_l' and h_l''. For each J, l-broken U(6)$\times$O(2) thus allows one to express the four couplings $a_1^J, \ldots, a_4^J$ in terms of the two couplings g_l and h_l. We find

$$\begin{aligned} &a_1^J = g_{J-1}/\eta_J\,, \\ &a_2^J = h_{J-1}\eta_J(1+\eta_J)\,, \\ &a_3^J = h_{J-1}(1+\eta_J - \tfrac{1}{2})/\eta_J - g_{J-1}(1+\eta_J)/\eta_J\,, \\ &a_4^J = g_{J-1}(1+\eta_J) - \tfrac{1}{2}h_{J-1}\,, \end{aligned} \tag{2b}$$

where

$$\eta_J = m_{J-1}/2m_0\,. \tag{2c}$$

The Carlson interpolations $a_i(t) \equiv a_i^{\alpha_\rho(t)}$ of the a_i^J, if they exist, would represent the "ρ-side" factors of the residues of the ρ^0 Regge pole in various invariant hadron-ρ^+ scattering amplitudes. Without any loss of generality we can assume these Carlson interpolations to exist.[4] In terms of the Carlson interpolations $g(t) \equiv g_{\alpha_\rho(t)-1}$ and $h(t) = h_{\alpha_\rho(t)-1}$ of the $g_{J=1}$'s and h_{J-1}'s and of the interpolation $\sqrt{t}/2m_0$ of η_J we have

$$\begin{aligned} \frac{a_2(t)}{a_1(t)} &= \frac{h(t)}{g(t)}\,\frac{t}{4m_0^2}\left(1+\frac{\sqrt{t}}{2m_0}\right), \\ \frac{a_3(t)}{a_1(t)} &= \frac{h(t)}{g(t)}\left(1+\frac{\sqrt{t}}{2m_0}-\frac{1}{2}\right) - \left(1+\frac{\sqrt{t}}{2m_0}\right), \\ \frac{a_4(t)}{a_1(t)} &= \frac{\sqrt{t}}{2m_0}\left(1+\frac{\sqrt{t}}{2m_0}-\frac{1}{2}\frac{h(t)}{g(t)}\right). \end{aligned} \tag{3}$$

All residues of the ρ Regge pole must be analytic in the neighborhood of $t=0$. Hence the ratios $a_i(t)/a_1(t)$ can have no branch point at $t=0$. According to (3) this means

$$\frac{h(t)}{2g(t)(1-\sqrt{t}/2m_0)} = 1 \tag{4}$$

in the whole complex t plane. In particular, at $t = m_0^2$, $h(m_0^2) = h_0 = 0$ [Eq. (1d)], so that (4) implies

$$g(m_0^2) = g_0 = 0\,. \tag{5a}$$

Equations (1d) and (5a) imply

$$g_{\rho\pi\pi} = 0 \tag{5b}$$

in blatant contradiction with experiment (via exchange degeneracy one gets into the same trouble also for the couplings of the even-signature Regge trajectories).

How does one avoid the catastrophic result (5)? *A priori* there are four possibilities:

(1) doubling of the meson spectrum,
(2) the existence of CK cuts,
(3) modification of the l-breaking mechanism, and
(4) renunciation (partial or complete) of the l-breaking mechanism in favor of the fundamental approach (mentioned in the Introduction).

We now discuss each of these alternatives in detail.

(1) If in addition to and degenerate with the $(6, \overline{6};\, l)$ Regge family one would also find a $(\overline{6}, 6;\, l)$ family, he could arrange the couplings so that the $\sqrt{t}$ terms all cancel in Eqs. (3). The ratios a_i/a_1 are then automatically analytic at $t=0$ without the further condition (4), and thus the bad relations (5) do not follow. In the absence of U(6)$\times$O(2) breaking this possibility has been explored by Delbourgo and Salam.[5] Unfortunately no such doubling is observed in nature.

(2) In the presence of CK cuts the Regge residues

can have a branch point at $t=0$; however, the branch point contributed to the scattering amplitude by the Regge pole cancels against that contributed by the CK cut. Unfortunately, again experimental evidence indicates the absence of CK cuts. Moreover no dynamical model that leads to Regge poles accompanied by CK cuts has ever been found. Indeed it is questionable whether their existence is compatible with duality.[6]

(3) One could contemplate a modification of the heuristic approach in order to avoid the catastrophic Eqs. (5). We shall briefly sketch how this is done and point out the attendant loss in predictivity.

To be specific consider again the $M^{(l)}M^{(0)}M^{(0)}$ couplings. The decay of a nonexotic meson into two nonexotic mesons is described by quark diagrams with three quark lines. The appearance of *one* $^3P_0$ spurion γ_μ on either of these three quark lines corresponds to the three coupling constants h, h', and h'' in (1a). One can consider the "higher-order" terms in which two or more spurions are activated. If two spurions were to appear on the same quark line one would have a quark propagator in between them. Additional U(6)×O(2) breaking would then occur, for instance on account of the projection operator of the quark propagator. The precise form of this breaking would depend on details of quark dynamics. Our considerations are thus necessarily limited to higher-order terms with *at most one* γ_μ *spurion per quark line*. One can rationalize this possibility by postulating that on each quark line one spurion γ_μ has a certain probability of appearing but that this is the whole extent of U(6)×O(2) breaking. The existence of any dynamical model implementing this postulate is highly doubtful.

Whether or not terms with two or more γ_μ spurions per quark line are important, we want to show now that already terms with at most one γ_μ spurion per quark line are sufficient to remove the prediction (5). Terms with at most one spurion per quark line fall into four categories: those in which (a) zero, (b) one, (c) two, or (d) all three quark lines support a spurion. The terms (a) and (b) have been included in the vertex (1a). In order not to obtain equation (5a) we need new vertices such that the corresponding couplings do not vanish for $l=0$. Then instead of (5a) we would constrain to zero not $g(m_0^2)$ but only a linear combination of $g(m_0^2)$ and the new couplings and thus not necessarily the $\rho\pi\pi$-coupling constant. The most general term of type (c) or (d) such that the corresponding couplings do not vanish for $l=0$ is

$$k_l(m_0)^{1-l}P^{\mu_1}\cdots P^{\mu_l}\,\mathrm{Tr}(M^{(l)}_{\mu_1\cdots\mu_l}(p_3)\{[\gamma_\mu M^{(0)}(p_1)\gamma^\mu M^{(0)}(p_2)]_l+b_l[M^{(0)}(p_1)\gamma_\mu M^{(0)}(p_2)\gamma^\mu]_l+c_l[\gamma_\mu M^{(0)}(p_1)M^{(0)}(p_2)\gamma^\mu]_l\}). \quad (6)$$

Bose statistics requires

$$b_0=c_0=1. \quad (7)$$

For simplicity (without thereby losing any essential features) we impose the quark-antiquark symmetry

$$b_l=1 \text{ for all } l. \quad (8)$$

We introduce the Carlson interpolations $k(t)$ and $c(t)$ of the k_{l-1}'s and c_{l-1}'s. We further make the ansatz

$$c(t)=r(t)+\sqrt{t}\,s(t), \quad (9)$$

where both r and s are analytic functions regular at $t=0$. An argument similar to that which led to Eq. (4) now leads to

$$\frac{h(t)}{2g(t)(1-\sqrt{t}/2m_0)}+\frac{2[r(t)+1]k(t)}{g(t)}=1. \quad (10)$$

If $k(m_0^2)\neq 0$, then using Eqs. (1), (6), and (7) we find

$$\left|\frac{g_{\rho^0\pi^-\pi^+}}{g_{\rho^0\rho^-\rho^+}}-1\right|=\left|\frac{4k}{3g-8k}\right| \quad (11)$$

in violation of ρ universality. With some additional assumptions[7] one can derive from experiment that the left-hand side of Eq. (11) is $\lesssim 0.2$, so that k/g must be small. In fact one can impose exact ρ universality:

$$k(m_0^2)=0. \quad (12)$$

For $t\to m_0^2$, $h(t)\to 0$ and $g(t)\to g(m_0^2)=$ finite, so that near $t=m_0^2$ Eq. (10) takes the form

$$2[r(t)+1]k(t)\approx g(m_0^2). \quad (13)$$

The remarkable feature is that Eqs. (12) and (13) do *not* imply Eq. (5a). Indeed $c(m_0^2)=c_0=1$ according to Eq. (7) but $r(m_0^2)$ and $s(m_0^2)$ need not be finite. $r(t)$ and $s(t)$ can each have a pole at $t=m_0^2$ provided only the pole cancels from $c(t)$. But *if* $r(t)$ *is infinite at* $t=m_0^2$, *then even upon imposing Eq. (13) we can have* $g(m_0^2)\neq 0$. As a simple example

$$c(t)\approx\frac{a}{t-m_0^2}\left(1-\frac{\sqrt{t}}{m_0}\right) \quad (14a)$$

and

$$k(t)\approx\frac{g(m_0^2)}{2a}(t-m_0^2) \quad (14b)$$

obey both Eqs. (12) and (13) [in Eqs. (14) a is a constant].

If in addition to the residues of the ρ Regge pole in the processes $A+\rho^+ \to A+\rho^+$ we also consider the processes $A+\pi^0 \to A+\omega$ and $A+\pi^+ \to A+\pi^+$, then in the limit $m_\pi = m_\rho = m_\omega$ we can *derive* Eq. (12), and consequently $r(m_0^2)$ must be infinite if $g_{\rho\pi\pi}$ is not to vanish.

Briefly, what we have shown is that *if terms with two spurions are included, the catastrophic relations (5) disappear.*

Of course there are many terms of the types (c) and (d) in addition to those of Eq. (6), and this leads to a considerable reduction in predictivity. It would be nevertheless interesting to see whether simple physical principles (such as the ρ universality used above) could be used to constrain these terms.

We come now to the last alternative.

(4) If on the basis of the prediction (5) we reject l-broken $U(6)\times O(2)$, we must find an alternative way of breaking this symmetry. One could say that l-broken $U(6)\times O(2)$ is a good approximation for small l values but is in need of significant corrections for larger l. This would deny us the right to Reggeization which was at the basis of the catastrophe (5). Still, this would mean that we are far from understanding how U(6) is broken and that we have only a phenomenological rule of thumb that happens to work for small l.

Maybe all this means is that one should stick to the fundamental approach and steer clear of heuristic arguments. Nevertheless one has to make sure that no similar catastrophes occur in the fundamental approach. As the fundamental approach uses PCAC to work its way to hadronic couplings, it obviously says nothing about couplings like $\rho^J\rho\rho$ which involve no pion. With vector-meson dominance (VMD) $\rho^J\rho\rho$ couplings could be analyzed even within the fundamental approach, but then VMD is certainly less reliable than PCAC. Thus the fundamental approach by itself does not lead to Eqs. (5). If one invokes only PCAC the fundamental approach can be used to correlate $\rho^J\pi\omega$ and $\rho^J\pi\pi$ couplings. In the unphysical limit $m_\omega = m_\pi = m_0$ one can then show that the ratio of the $\rho^J\pi\omega$ and $\rho^J\pi\pi$ Regge factors is $\sqrt{t}/2m_0$, so that the fundamental approach also leads to a difficulty (this result obtains of course also for the heuristic approach). However, when the degeneracy condition $m_\omega = m_\pi$ is lifted the situation is changed and no difficulty seems to arise. It is thus fair to suggest as a hopeful avenue the replacement of the heuristic by the fundamental approach.

Finally, another feature that emerges at this point is that both the heuristic and fundamental approaches concern themselves with hadronic couplings and have so far very little to say about the effects of $U(6)\times O(2)$ breaking on the hadronic mass spectrum. It may be that a more comprehensive theory that incorporates symmetry breaking in masses will also overcome the difficulties pointed out in this paper.

To sum up, in the absence of CK cuts l-broken $U(6)\times O(2)$ can be at best a phenomenological description of the couplings of low-lying hadrons. It would be therefore very useful if the problem of the existence of CK cuts would be unambiguously settled one way or the other. If one were to take the experimental and theoretical indications against CK cuts seriously, one would either have to renounce l-broken $U(6)\times O(2)$ in favor of the fundamental approach or else modify the symmetry-breaking mechanism along the lines discussed in this paper.

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[3]R. Carlitz and M. Kislinger, Phys. Rev. Lett. 24, 186 (1970).

[4]If the couplings grow too fast for large J, one need only multiply in a suitable exponential. As below we are concerned only with ratios of Regge couplings, such exponentials have no effect. For that matter, one could deal directly with ratios of couplings provided they have only a finite number of zeros.

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[6]R. Carlitz, S. D. Ellis, P. G. O. Freund, and S. Matsuda, Caltech Report No. CALT-68-260, 1970 (unpublished); S. D. Ellis, Phys. Rev. D 5, 2366 (1972).

[7]Assuming f-ρ exchange degeneracy, f dominance of the Pomeranchuk, and smoothness of Regge residue ratios near $t=0$, one can deduce from the relations $d\sigma_{\rho N}/dt \simeq d\sigma_{\pi N}/dt$ and $\sigma_{\rho N} \simeq \sigma_{\pi N}$ (obtained from ρ-photoproduction experiments via VMD) that $g_{\rho\pi\pi}/g_{\rho\rho\rho}$ equals unity to within 20%.