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Reggeization of Elementary Particles in Renormalizable Gauge Theories: Vectors and Spinors

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The question of the Reggeization of the elementary particles in renormalizable gauge theories with the Higgs-Kibble mechanism is examined. It is concluded that the massive non-Abelian gauge vector mesons and $J=\frac{1}{2}$ fermions of these theories lie on Regge trajectories in every case in which a counting criterion due to Mandelstam is satisfied. Several explicit examples are presented which verify that the necessary factorization condition of the Born helicity matrices is satisfied at $J=1$ and $J=\frac{1}{2}$, supporting this conclusion.

I. INTRODUCTION

The idea that the elementary particles of a Lagrangian field theory, which at first sight give rise to nonanalytic Kronecker δ terms in the complex angular momentum plane, in fact lie on Regge trajectories dates back more than ten years to a series of papers of Gell-Mann *et al.*[1] Although their ideas initiated a sizable industry which specialized in summing Feynman diagrams,[2] with the objectives of exhibiting possible Regge asymptotic behavior of the S-matrix elements and demonstrating that the elementary particles of the theory lie on these Regge trajectories, their program was to a large extent unsuccessful. The only positive result[1] was the Reggeization of the fermion in a particular model with a spin-$\frac{1}{2}$ particle coupled to vector mesons by means of a conserved vector current. Subsequently, Mandelstam[3] presented criteria to establish when an elementary particle *must* Reggeize, which indeed agreed with the conclusion that the fermion of the vector-spinor theory should lie on a Regge trajectory. His arguments also indicated that the elementary particles of other Lagrangian theories need not Reggeize, and explained the results of the theoretical experiments carried out by computing Feynman diagrams. The Mandelstam criteria are to a large extent kinematical and suggest where one should look for Reggeization. These criteria involve comparing, for the particular model being studied, the number of arbitrary parameters appearing in the scattering amplitudes with the number of constraints satisfied by these amplitudes. The dynamical criteria of the program are very general ones requiring that the theory lead to amplitudes which satisfy analyticity and unitarity; they appear to be related to the renormalizability of the Lagrangian field theory. Indeed Teplitz and collaborators[4-6] have exhibited a number of models which satisfy Mandelstam's counting criteria but fail to Reggeize because of the absence of renormalizability, which implies a violation of unitarity bounds in each order of perturbation theory. A particularly interesting example[5] is the failure to Reggeize the gauge vector mesons of a (nonrenormalizable) massive Yang-Mills theory.

Since renormalizable theories of massive Yang-Mills fields are now known,[7] we have reexamined the question of the Reggeization of gauge vector mesons. Our study was further motivated by the fact that the zero-slope limit of certain dual models[8] leads to amplitudes which are identical to those constructed from particular renormalizable Yang-Mills Lagrangians. Although our work has not given us any special insight into this aspect of

Regge theory, we do believe that there are many physically interesting Lagrangian theories which have better behavior in the angular momentum plane than one might expect.

Our calculations have been limited to low orders of perturbation theory,[9] and in particular to the first nontrivial condition that a theory must satisfy[1,3] in order to Reggeize, the so-called factorization condition of the Born approximation (see Appendix A). If indeed the Mandelstam arguments[3] apply and the theory Reggeizes, the factorization condition must hold. Even though the factorization condition follows from general principles, we feel that an explicit check is worthwhile, because of possible subtleties in crypto-renormalizable theories[7] and in the derivation of the Mandelstam criteria. Furthermore, explicit calculations provide information as to the actual number and behavior of the Regge trajectories in the theory, as contrasted with the Mandelstam argument, which only indicates the *maximum* number of trajectories in any given channel.

In this paper we discuss in some detail the Reggeization of the gauge vector mesons in a renormalizable SU(2) Yang-Mills theory.[9] We check the factorization of the Born-approximation helicity amplitudes for vector-vector scattering at $J=1$, $I=1$, and deduce, to order g^2, the form of the vector-meson Regge trajectory and residues. (It turns out that it is not possible to demonstrate that Abelian vector mesons Reggeize by our methods because of the absence of two-body "nonsense" states at $J=1$ in this case, and of trilinear vector couplings. A discussion of the Abelian case may require consideration of many-body scattering, a task beyond the scope of this paper.[10]) The non-Abelian vector mesons appear as elementary-particle poles in other $I=1$ two-body channels that couple to the vector-vector channel: vector-scalar states as well as fermion-antifermion states if fermions are present in the theory. We verify that the factorization condition is in fact satisfied by all relevant two-body amplitudes at $J=1$, $I=1$.

To see if *all* elementary particles of a renormalizable massive Yang-Mills theory Reggeize, one should also investigate the situation at $J=0$. This has been done with mixed results, which will be described in a sequel to this paper.[11] We merely note here that the Mandelstam argument indicates that the theory need not Reggeize at $J=0$.[3]

If fermions are present, one can also study Reggeization at $J=\frac{1}{2}$. It is already known that factorization holds at $J=\frac{1}{2}$ as a result of the work of Gell-Mann *et al.*,[1] as extended by Abers *et al.*[6] to the Yang-Mills case. (However, the Lagrangian considered by Abers *et al.* was not renormalizable; ours is, so we expect the fermion to Reggeize.)

Our work is easily extended to other groups. For U(n) models of the type discussed by Bardakçi and Halpern,[12] the Reggeization of the SU(n) gauge mesons follows easily. We have no results for the Abelian gauge meson present in the U(n) model. Another example of physical interest is provided by a theory of chiral $SU(2)\times SU(2)$ with ρ and A_1 gauge mesons. (No identification with physical vector mesons is implied; we use the labeling for identification purposes only.) One can then consider a spontaneously broken chiral symmetry along the lines of Bardakçi and Halpern.[12] In this example the ρ and A_1 mesons need not have equal masses; nevertheless both the ρ and A_1 mesons Reggeize, as in the previous cases. One may also add fermions to this particular model. However, in contradistinction to the other examples studied, the presence of fermions in a chiral $SU(2)\times SU(2)$ theory raises some questions, since the interaction of the axial-vector meson with the spinor field can lead to anomalies[13] in diagrams involving closed spinor loops. If such anomalies are present, the theory is no longer renormalizable, so that unitarity bounds need not be satisfied order by order in perturbation theory, and the Reggeization program along the lines we are considering may ultimately fail. We illustrate these problems by considering a model discussed by Gross and Jackiw[14] and show that in lowest order factorization still persists; anomalies do not play a role in the Reggeization program until one reaches the one-loop approximation.[13]

II. GAUGE VECTOR MESONS AS REGGEONS

A. Vector-Vector Scattering

In this section we discuss the simplest model known to us in which elementary gauge vector mesons Reggeize, the so-called SU(2) restricted spectrum model of Gervais and Neveu.[8] The model is constructed from a renormalizable Yang-Mills Lagrangian for massless SU(2) gauge mesons in interaction with a complex doublet of scalar mesons. The self-interaction of the scalars can be arranged so that the $I=0$ member of this scalar quartet has a nonvanishing vacuum expectation value. The Yang-Mills mesons thus acquire a mass by means of the Higgs-Kibble mechanism.[15] The theory in the U gauge then describes a triplet of $I=1$ massive vector mesons in interaction with a massive $I=0$ scalar meson, which are the *physical elementary particles of the model*. (If the masses of the vector and scalar are arbitrarily set equal in lowest order, one arrives at the "restricted spectrum" model considered by Gervais and Neveu,[8] in the context of the zero-slope limit of a dual resonance model. We will not set these masses equal in our work, since this restriction is not necessary for the Reggeization of the vector mesons to succeed and since the general case is easy to analyze.) For the purpose of this paper the relevant part of the U-gauge interaction Lagrangian is

$$\mathcal{L}_I = -g\epsilon^{abc}\partial_\mu V^a_\nu V^b_\mu V^c_\nu - \tfrac{1}{4} g^2 \epsilon^{abe}\epsilon^{cde} V^a_\mu V^b_\nu V^c_\mu V^d_\nu + \tfrac{1}{2} gm V^a_\mu V^a_\mu \sigma + \tfrac{1}{8} g^2 V^a_\mu V^a_\mu \sigma^2 - \frac{g\mu^2}{4m}\sigma^3 - \tfrac{1}{32} g^2 \frac{\mu^2}{m^2}\sigma^4 . \tag{2.1}$$

Here V^a_μ is an $I=1$ gauge field with mass m, while σ is a scalar field for a *physical* meson with $I=0$ and mass μ.

Let us now concentrate on vector-vector scattering in this model, as it contains an elementary vector-meson pole which is a candidate for Reggeization and may have Kronecker δ terms at $J=1$. The Born approximation for vector-vector scattering in the U gauge is described by the diagrams of Fig. 1. For $I=1$, the s-channel partial-wave helicity amplitudes, computed in the Born approximation, contain terms of the form $\delta_{J,1}(s-m^2)^{-1}$ coming from the amplitude described by Fig. 1(a). We now argue that this nonanalytic term in complex angular momentum is converted into a Regge pole by the procedure of Gell-Mann *et al.*[1] The helicity amplitudes for Figs. 1(a) were computed by Dicus, Freedman, and Teplitz,[5] but they did not consider Fig. 1(b) since there was no elementary scalar meson in their model. (There are some misprints in their helicity amplitudes.) The theory considered by Dicus *et al.*[5] was nonrenormalizable, and as a result scattering amplitudes exceeded unitarity bounds in the Born approximation. Furthermore, the Born-approximation partial-wave amplitudes at $J=1$, evaluated from Fig. 1(a) alone, did not factorize.[5]

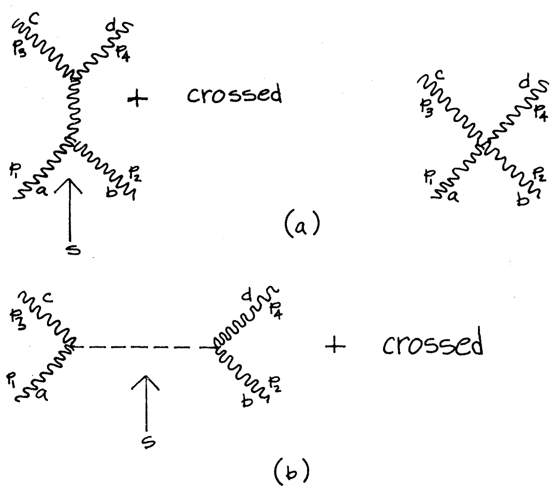

FIG. 1. The Feynman diagrams for vector-vector scattering in the Born approximation. The wiggly lines represent vector mesons and dashed lines represent scalar mesons. (a) The vector-exchange graph and contact graph; (b) the scalar-exchange graph.

Our finding[9] is that when *all* the diagrams of Fig. 1 are added together with couplings given by the Higgs-Kibble mechanism as required by renormalizability, the resulting partial-wave helicity amplitudes *do* factorize at $J=1$. The general argument of Mandelstam[3] implies that the vector meson lies on a Regge trajectory, to all orders in the coupling constant and with no restrictions on the value of the coupling constant or the vector meson mass.[16]

Let us describe some of the details leading to these results. Given the scattering amplitude computed from the Feynman diagrams of Fig. 1, one reads off the J-plane behavior near $J=1$ from the large-z behavior of the s-channel helicity amplitudes.[17] (The cosine of the s-channel scattering angle is $\cos\theta_s \equiv z$.) The $I=1$ amplitudes increase linearly with z while the $I=0$ and $I=2$ amplitudes approach constants. Therefore, only the $I=1$ amplitudes have nonanalytic behavior at $J=1$, with Kronecker δ's for the sense-sense transitions and singular sense-nonsense and nonsense-nonsense transitions. The actual partial-wave projections of the helicity amplitudes near $J=1$ are carried out using the prescriptions, formulas, and tables of Ref. 1. Unless otherwise specified, we adhere to the notation and conventions of Ref. 1 (see also Appendix B).

The methods of Appendix A as adapted to vector-vector scattering require that one calculate the normal-parity partial-wave helicity amplitudes $T^{(+)}_{\lambda_3\lambda_4;\lambda_1\lambda_2}(s,J)$ for vector-vector scattering in the Born approximation. These amplitudes, near $J=1$, fall into three sets, sense-sense ($|\lambda| = |\lambda_1-\lambda_2| \le 1$, $|\mu| = |\lambda_3-\lambda_4| \le 1$), sense-nonsense ($|\mu|\le 1$, $|\lambda|=2$), and nonsense-nonsense ($|\lambda| = |\mu| = 2$), and their singular parts have the following behavior[9]:

$$\tilde{B}_{\lambda_3\lambda_4;\lambda_1\lambda_2} = -g^2 b_{\lambda_3\lambda_4} b_{\lambda_1\lambda_2}(s)\delta_{J,1} \quad \text{(sense-sense)}, \tag{2.2a}$$

$$\tilde{B}_{1-1;\lambda_1\lambda_2} = g^2 b_{1,-1} b_{\lambda_1\lambda_2}(s)(J-1)^{-1/2} \quad \text{(nonsense-sense)}, \tag{2.2b}$$

$$\tilde{B}_{1-1;1-1} = g^2[b_{1,-1}(s)]^2(J-1)^{-1} \quad \text{(nonsense-nonsense)}. \tag{2.2c}$$

As we observed in Ref. 9, the Born-approximation helicity matrix factorizes and is of rank 1 near $J=1$. According to our discussion in Appendix A, the Reggeized amplitude near $J=1$ takes the form

$$T_{\lambda_3\lambda_4;\lambda_1\lambda_1}=\beta_{\lambda_3\lambda_4}(s)\beta_{\lambda_1\lambda_2}(s)[J-\alpha(s)]^{-1} \quad \text{(sense-sense)}, \tag{2.3a}$$

$$\frac{T_{1-1;\lambda_1\lambda_2}}{(J-1)^{1/2}}=\beta_{1,-1}(s)\beta_{\lambda_1\lambda_2}(s)[J-\alpha(s)]^{-1} \quad \text{(nonsense-sense)}, \tag{2.3b}$$

$$T_{1-1;1-1}=[\beta_{1-1}(s)]^2[\alpha(s)-1][J-\alpha(s)]^{-1} \quad \text{(nonsense-nonsense)}, \tag{2.3c}$$

with $\beta_{\lambda_1\lambda_2}(s)$, computed in lowest order, given by

$$\begin{aligned}
\beta_{11}&=g\left[\frac{3m^4}{k^2(s-m)^2}\right]^{1/2}(\alpha-1)^{1/2},\\
\beta_{01}&=g\left[\frac{3m^2 s}{k^2(s-m^2)}\right]^{1/2}(\alpha-1)^{1/2},\\
\beta_{00}&=g\left[\frac{4}{3}\,\frac{1}{k^2(s-m^2)}\right]^{1/2}(\tfrac{1}{4}s+2m^2)(\alpha-1)^{1/2},\\
\beta_{1-1}&=g\left[\frac{2(s-m^2)}{k^2}\right]^{1/2}(\alpha-1)^{-1/2},
\end{aligned} \tag{2.4}$$

and

$$\operatorname{Im}\alpha=8g^2\frac{s-m^2}{s-4m^2}\rho(s).$$

This is consistent with

$$\alpha(s)=1+\frac{8g^2}{\pi}(s-m^2)\int_{4m^2}^{\infty}\frac{ds'}{(s'-s)}\,\frac{\rho(s')}{(s'-4m^2)}, \tag{2.5}$$

as discussed in Appendix A. This representation should, of course, be checked by higher-order calculations. If indeed Eq. (2.5) is the correct form of α, we shall have a single Regge pole passing through $J=1$ at $s=m^2$. The elementary massive gauge vector meson of a renormalizable SU(2) Yang-Mills theory thus lies on a Regge trajectory.

In examining the detailed dynamical features required for this result, two factors seem to be crucial:

(1) The non-Abelian structure of the gauge group introduces a *trilinear* self-coupling of the vector mesons. Thus Abelian gauge mesons do not Reggeize by means of the mechanism studied in this paper,[10] since there is no vector-meson pole in Abelian vector-vector scattering, and no nonsense amplitude in vector-scalar scattering (which does have a vector-meson pole).

(2) The presence of a scalar meson, with couplings as given by the Higgs-Kibble mechanism,[15] is required in order to improve the high-energy behavior so that the Born approximation satisfies unitarity bounds.[18] It is remarkable that factorization is then achieved simultaneously with the improved high-energy behavior.

B. Vector-Meson Pole in Other Processes

At $J=1$, $I=1$ there exist, in addition to the two-vector-meson state, a vector-scalar state and a fermion-antifermion state. Fermions are introduced into the theory in a gauge-invariant, renormalizable manner by adding the interaction term

$$\mathcal{L}_I=g\bar{\psi}\gamma_\mu T_a\psi V_\mu^a \tag{2.6}$$

to Eq. (2.1), where T is the appropriate isotopic-spin matrix. (Notice that there is no direct fermion-scalar coupling.) Transition amplitudes between these states all contain a vector s-channel pole and Kronecker δ terms. They must Reggeize if the program is to be consistent. In Born approximation the relevant diagrams are shown in Fig. 2. The only nonsense state is still the helicity-2 vector-vector state. We have computed the helicity amplitudes for all the processes described by

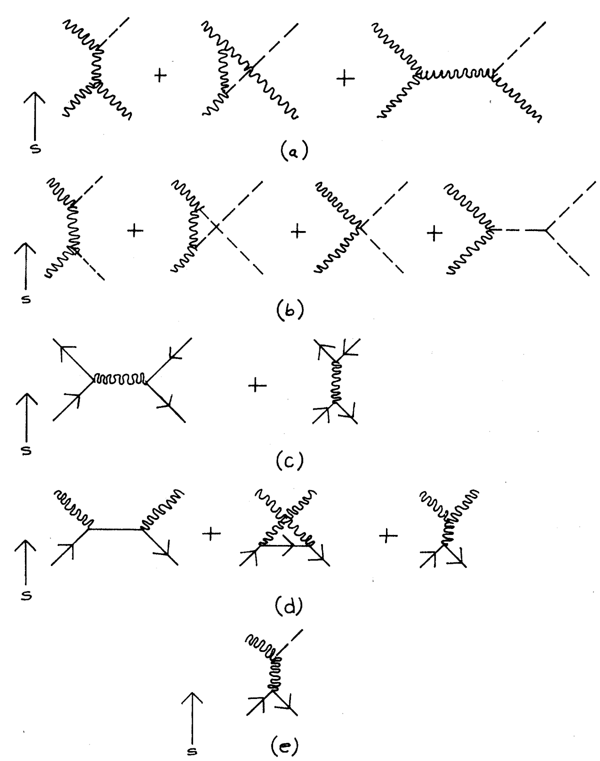

FIG. 2. The Feynman diagrams for two-body processes which in Born approximation couple to vector-vector scattering at $J=1$. The solid lines represent fermions. (a) The graph for vector-vector to vector-scalar transitions; (b) the graphs for vector-scalar scattering; (c) the graphs for fermion-antifermion scattering; (d) the graphs for fermion-antifermion to vector-vector transitions; (e) the graph for fermion-antifermion to vector-scalar transitions.

Fig. 2, and have found that the factorization condition is maintained for this enlarged system. The 8×8 matrix of Born terms has rank one, and can be written (as in Appendix B) in the form $v_{ij} = g^2 b_i b_j$, where the b's are given in Table I. Hence the coupled-channel system *maintains* the Regge behavior exhibited by the vector-vector scattering states. The Reggeized form of the amplitudes for all of these processes is $T_{ij} = \gamma_i\gamma_j[J-\alpha(s)]^{-1}$ near $J=1$, with the γ's being obtained trivially from the b's given in (2.4) and Table I. The Regge pole is still generated by the vector-vector nonsense state, and is coupled to all other states through the sense-nonsense terms in the various scattering amplitudes.

III. THE $J=\frac{1}{2}$ FERMION AS A REGGEON

It is known from the classic work of Gell-Mann *et al.* that the fermion Reggeizes in a theory of fermions coupled to a neutral vector meson by means of a conserved current. This result was extended to the case of a fermion interacting with a massive Yang-Mills vector meson by Abers *et al.*,[6] who showed that the Born approximation for the vector-spinor scattering amplitude factorized. One may doubt that the work of Abers *et al.* demonstrates that the fermion Reggeizes in Yang-Mills theory, since they studied a nonrenormalizable theory. However, the renormalizable SU(2) gauge theory described in Sec. II will lead to the *identical* Born approximation found by Abers *et al.*, since the model described by Eqs. (2.1) and (2.6) does not have a direct coupling of the scalars to the fermions. Thus, if one uses this renormalizable SU(2) gauge theory to calculate higher-order corrections, Mandelstam's arguments[3] should be applicable in implying that the fermion *must* Reggeize. Hence in fact the conclusion of Abers *et al.*[6] that the fermion Reggeizes is confirmed.

One can also consider the question of coupled channels containing a $J=\frac{1}{2}$ pole, as shown in Fig. 3. Although there is no fermion-scalar scattering, there is a fermion-vector to fermion-scalar transition in lowest order, made possible by vector-meson exchange, as illustrated in Fig. 3(b).

The implications of Appendix A are that the nonsense-nonsense part of the fermion–vector-meson amplitude, the nonsense-sense part of the fermion-vector to fermion-scalar transition, and the sense-sense fermion-scalar elastic amplitude (which is zero in the Born approximation) must factorize near $J=\frac{1}{2}$, as prescribed by Eq. (A10). It turns out that Eq. (A10) is satisfied in the SU(2) model of (2.1) and (2.6), because the singular part of the nonsense-sense fermion-vector to fermion-scalar transition of Fig. 3(b) vanishes for purely kinematical reasons. However, we can easily introduce additional scalar and/or pseudoscalar mesons, with gauge-invariant couplings to the vector mesons and fermions, into the SU(2) Lagrangian. In these cases there are nontrivial singular parts in the scalar-fermion and pseudoscalar-fermion chan-

TABLE I. The Born-approximation normal-parity helicity matrix expressed at $J=1$ and $I=1$, as $v_{\mu\lambda} = g^2 b_\mu \times b_\lambda$ for the SU(2) model with the "restricted spectrum." In all the tables, we denote the vector-meson, scalar-meson, and fermion masses by m, μ, and M, respectively. For a given two-particle channel we denote the center-of-mass energies for the vector meson and fermion by ω and E, respectively, the center-of-mass momentum by k, and the (total energy)^2 by s. (a) The vector-vector states; (b) the vector-scalar states; (c) the fermion-antifermion states for the special case of the $I=\frac{1}{2}$ fermion.

	(a) (VV)
b_{11}	$m^2\left[\dfrac{3}{k^2(s-m^2)}\right]^{1/2}$
b_{01}	$m\left[\dfrac{3s}{k^2(s-m^2)}\right]^{1/2}$
b_{00}	$(\frac{1}{2}s+4m^2)\left[\dfrac{1}{3k^2(s-m^2)}\right]^{1/2}$
b_{1-1}	$\left[\dfrac{2(s-m^2)}{k^2}\right]^{1/2}$
	(b) $(V\sigma)$
b_1	$m\left[\dfrac{2}{3(s-m^2)}\right]^{1/2}$
b_0	$\omega\left[\dfrac{2}{3(s-m^2)}\right]^{1/2}$
	(c) $(F\bar{F})$ for $I=\frac{1}{2}$ fermions
$b_{1/2,1/2}$	$\left[\dfrac{1}{3(s-m^2)}\right]^{1/2}$
$b_{1/2,-1/2}$	$\dfrac{1}{M}\left[\dfrac{2s}{3(s-m^2)}\right]^{1/2}$

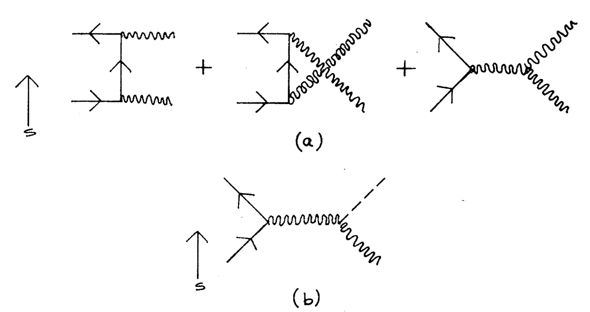

FIG. 3. The Feynman diagrams for two-body processes which contribute in the Born approximation to the Reggeization of the elementary fermions. (a) The vector-fermion scattering graphs; (b) the graph for vector-fermion to scalar-fermion transitions.

nels. We find that even with these additional channels, the Reggeization of the fermions still persists.[1] We illustrate this by introducing an isospin-one pseudoscalar meson $\vec{\pi}$ into the SU(2) model, with the relevant interactions:

$$\mathcal{L}_I = -g\vec{V}_\mu \times \vec{\pi}\cdot \partial_\mu \vec{\pi} - \tfrac{1}{2}g^2(\vec{V}_\mu\times\vec{\pi})\cdot(\vec{V}_\mu\times\vec{\pi}) + iG\bar{\psi}\vec{\tau}\cdot\vec{\pi}\gamma_5\psi . \quad (3.1)$$

We have one additional channel: $\vec{\pi}\vec{\pi}$ for the vector mesons and $\vec{\pi}F$ for the fermions. The normal-parity Born matrices, which are 9×9 for $J=1$, $I=1$ and 4×4 for $J=\frac{1}{2}$, $I=\frac{1}{2}$, both factorize as prescribed by (A10), and both are of rank one. We list the result for the $\pi\pi$ channel in Table II and the result for the FV and πF channels in Table III.

IV. OTHER GROUPS

In this section we consider Reggeization in other renormalizable Yang-Mills theories. These examples lead us to believe that non-Abelian gauge mesons will Reggeize in any *renormalizable* massive Yang-Mills theory when Mandelstam's counting criterion[3] is satisfied for vector-vector scattering.

A. U(n) and the Extended Spectrum

Consider U(n) as a local gauge group, and construct a theory of massive gauge vector mesons using the scheme of Bardakçi and Halpern.[12] Let $\frac{1}{2}\lambda_a$ $(a=0,1,\ldots,n^2-1)$ be the generators of U(n) and let λ_0 be the generator of the U(1) subgroup. The λ_a can be represented by the $n\times n$ matrices which satisfy

$$[\lambda_a,\lambda_b]=2if_{abc}\lambda_c ,$$
$$\{\lambda_a,\lambda_b\}=2d_{abc}\lambda_c ,$$
$$\lambda_0=\sqrt{2/n}\,I , \quad (4.1)$$

and

$$d_{0ab}=(2/n)^{1/2}\delta_{ab} ,$$

with f_{abc} the structure constant of the group and d_{abc} defined by the anticommutation relation, and with repeated indices summed.

Let

$$V_\mu=\sum_a \frac{1}{\sqrt{2}}\lambda_a V_\mu^a \quad (4.2)$$

and

$$M=\frac{1}{\sqrt{2}}\sum_a \lambda_a(\sigma^a+i\pi^a) , \quad (4.3)$$

where V_μ^a is the gauge vector field, and σ^a and π^a are Hermitian scalar fields. Accordingly, the covariant derivative appropriate to (4.3) is

$$D_\mu M=\partial_\mu M-\frac{ig}{\sqrt{2}}V_\mu M . \quad (4.4)$$

Assume that the self-interaction of the scalar mesons can be arranged so that $\langle\sigma^0\rangle=v$, which allows one to write $\sigma^a=v\delta_{a0}+s^a$, with $\langle s^a\rangle=0$. In the unitary gauge one finds the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}(\partial_\mu V_\nu^a-\partial_\nu V_\mu^a+gf_{abc}V_\mu^bV_\nu^c)^2+\tfrac{1}{2}m^2V_\mu^aV_\mu^a + \tfrac{1}{2}gmd_{abc}V_\mu^aV_\mu^bs^c+\tfrac{1}{8}g^2d_{aed}d_{bce}V_\mu^bV_\mu^cs^as^d + \tfrac{1}{2}(\partial_\mu s^a)^2+\tfrac{1}{2}gf_{abc}(s^a\partial_\mu s^b)V_\mu^c+P(s) , \quad (4.5)$$

where $m^2=g^2v^2/2n$ is the common vector-meson mass squared.

Note that it is only the SU(n) non-Abelian subgroup that has trilinear self-couplings for the vector mesons, so that our discussion of Reggeization applies only to these n^2-1 non-Abelian gauge mesons. On restriction to the SU(n) subgroup, one finds that the Born approximation for vector-vector scattering is identical to that described by Fig. 1, with the couplings suitably modified in accord with the Lagrangian given by (4.5). One can transcribe our results directly from the SU(2) model considered in Sec. II, with the replacement $\delta_{ab}\to d_{abc}$ for the vector-vector-scalar coupling, and $\epsilon_{abc}\to f_{abc}$ for the trilinear vector coupling. One then observes that the non-Abelian vector meson Reggeizes, and there is no other pole at $J=1$ (neither sense- nor nonsense-choosing). The crucial computational ingredients for this conclusion are the relations

TABLE II. The contribution to the Born-approximation $J=1$ normal-parity helicity matrix of the $I=1$ $\pi\pi$ state in the SU(2) model.

	$(\pi\pi)$
b	$2\left[\frac{2k^2}{3(s-m^2)}\right]^{1/2}$

TABLE III. The contribution to the Born-approximation $J=\frac{1}{2}$ normal-parity helicity matrix of the $I=\frac{1}{2}$. We denote the total energy by W. (a) VF states; (b) πF states.

	(a) (FV)
b_0	$\left[\frac{3m^2}{8Mk^2}\frac{(E+M)}{(W-M)}\right]^{1/2}$
b_1	$\left[\frac{3(E-\omega-M)^2}{16Mk^2}\frac{(E+M)}{(W-M)}\right]^{1/2}$
b_{-1}	$\left[\frac{3(E+M)(W-M)}{8Mk^2}\right]^{1/2}$
	(b) $(F\pi)$
b	$\left[\frac{3}{2M}\frac{(E-M)}{(W-M)}\right]^{1/2}$

$$f_{aeb}f_{cde} = f_{cae}f_{deb} - f_{dae}f_{ceb} = d_{cae}d_{deb} - d_{dae}d_{ceb}\,, \tag{4.6}$$

with repeated indices summed.

As explained in Sec. II B, one must also consider the vector-meson pole in other processes coupled to vector-vector scattering in order to establish the consistency of the identification of the vector meson as a Regge pole. A few additional remarks must be added to the discussion of Sec. II B, since Eq. (4.5) allows a greater variety of two-body processes than discussed earlier. For simplicity, we describe details for the group U(2). [The model of Sec. II is the SU(2) "restricted spectrum" model of Gervais and Neveu,[8] while the U(n) model of Eq. (4.5) is their "extended spectrum" model.] For purposes of notation we denote the ($I=1$, $I=0$) vector-meson and scalar-meson multiplets as $(\vec{V}_\mu, \omega_\mu)$ and $(\vec{s}, \sigma)$, respectively. One may also add a fermion to the theory exactly as in Eq. (2.6). The coupled two-body processes to be considered are

$$\begin{pmatrix} \vec{V}\vec{V} \\ \vec{V}\sigma \\ \vec{s}\vec{s} \\ \omega\vec{s} \\ F\bar{F} \end{pmatrix} \longleftrightarrow \begin{pmatrix} \vec{V}\vec{V} \\ \vec{V}\sigma \\ \vec{s}\vec{s} \\ \omega\vec{s} \\ F\bar{F} \end{pmatrix} \tag{4.7}$$

(note that the only nonsense state at $J=1$ is in the $\vec{V}\vec{V}$ channel).

Our finding is that the 11×11 Born matrix describing the processes (4.7) factorizes at $J=1$ in accord with Eq. (A8), and is of rank one. The results are represented as in Eqs. (B5) and (B6). They are essentially the same as those of Tables I and II, and need not be listed separately.

B. Broken Chiral SU(2)

This example is very similar to the one just discussed. The major difference is that, as a result of the spontaneous breakdown of chiral symmetry, multiplets of gauge mesons of opposite parity have different mass. Our purpose in presenting this example is to illustrate the fact that mass splittings arising from spontaneous symmetry breakdown play no role in determining whether gauge mesons Reggeize or not. For the moment, we do not introduce fermions into this model, in order to avoid a discussion of anomalies,[13] which is delayed until Sec. IV C.

Let us consider the gauge group $SU(2)_L \times SU(2)_R$, with gauge mesons

$$R_\mu = \frac{1}{\sqrt{2}}\vec{\tau}\cdot\vec{R}_\mu$$

and

$$L_\mu = \frac{1}{\sqrt{2}}\vec{\tau}\cdot\vec{L}_\mu\,. \tag{4.8}$$

Consider a set of scalar fields[12] ϕ_1, ϕ_2, and M transforming according to the $(0,\frac{1}{2})$, $(\frac{1}{2},0)$, and $(\frac{1}{2},\frac{1}{2})$ representations of the group, respectively. The appropriate covariant derivatives are

$$D_\mu\phi_1 = \partial_\mu\phi_1 - igR_\mu\phi_1\,,$$

$$D_\mu\phi_2 = \partial_\mu\phi_2 - igL_\mu\phi_2\,, \tag{4.9}$$

and

$$D_\mu M = \partial_\mu M - igR_\mu M + igML_\mu\,.$$

One arranges the self-interaction of the scalars so that

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \frac{1}{\sqrt{2}}v\begin{pmatrix}0\\1\end{pmatrix}$$

and

$$\langle M\rangle = \tfrac{1}{2}u\,. \tag{4.10}$$

It is convenient to decompose the scalar multiplets as follows:

$$\phi_1 = \frac{1}{\sqrt{2}}(\sigma_1 + v + i\vec{\tau}\cdot\vec{\pi}_1)\begin{pmatrix}0\\1\end{pmatrix}\,,$$

$$\phi_2 = \frac{1}{\sqrt{2}}(\sigma_2 + v + i\vec{\tau}\cdot\vec{\pi}_2)\begin{pmatrix}0\\1\end{pmatrix}\,, \tag{4.11}$$

and

$$M = \frac{1}{\sqrt{2}}\left(\sigma_3 + \frac{u}{\sqrt{2}} + i\vec{\tau}\cdot\vec{\pi}_3\right)\,.$$

The interaction Lagrangian for the theory in the U gauge is

$$\begin{aligned}\mathcal{L}_I = {}& \tfrac{1}{2}gm_V\vec{V}_\mu\cdot\vec{V}_\mu\epsilon + \tfrac{1}{2}g\vec{A}_\mu\cdot\vec{A}_\mu(m_V\epsilon + 2m_A\sin\theta\sigma) + gm_V\eta\vec{V}_\mu\cdot\vec{A}_\mu - gm_A\sin\theta\cos\theta\vec{A}_\mu\cdot\vec{V}_\mu\times\vec{\pi} + g\vec{V}_\mu\cdot\vec{\pi}\times\partial_\mu\vec{\pi} \\ & + g\cos\theta\vec{A}_\mu\cdot(\vec{\pi}\partial_\mu\sigma - \sigma\partial_\mu\vec{\pi}) + \tfrac{1}{2}g\sin\theta\vec{V}_\mu\cdot(\eta\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\eta) + \tfrac{1}{2}g\sin\theta\vec{A}_\mu\cdot(\epsilon\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\epsilon) \\ & + \tfrac{1}{8}g^2\sin^2\theta(\vec{V}_\mu\cdot\vec{V}_\mu + \vec{A}_\mu\cdot\vec{A}_\mu)\vec{\pi}\cdot\vec{\pi} + \tfrac{1}{2}g^2\cos^2\theta[(\vec{V}_\mu\times\vec{\pi})\cdot(\vec{V}_\mu\times\vec{\pi}) + (\vec{\pi}\cdot\vec{A}_\mu)(\vec{\pi}\cdot\vec{A}_\mu)] - g^2\cos\theta\sigma\vec{A}_\mu\cdot\vec{V}_\mu\times\vec{\pi} \\ & + \tfrac{1}{8}g^2[(\eta^2+\epsilon^2)(\vec{V}_\mu\cdot\vec{V}_\mu + \vec{A}_\mu\cdot\vec{A}_\mu) + 4\vec{V}_\mu\cdot\vec{A}_\mu\eta\epsilon] + P(\vec{\pi},\epsilon,\eta,\sigma)\,,\end{aligned} \tag{4.12}$$

where $m_V=(1/\sqrt{2})gv$ and $m_A=(1/\sqrt{2})g(v^2+u^2)^{1/2}$ are the masses of the vector mesons and axial-vector mesons, respectively. We have introduced the scalar fields $\epsilon=(1/\sqrt{2})(\sigma_1+\sigma_2)$ and $\sigma=\sigma_3$. The pseudoscalar fields are $\eta=(1/\sqrt{2})(\sigma_1-\sigma_2)$ and $\vec{\pi}$ such that in the U gauge

$$\begin{aligned} \vec{\pi}_1 &\xrightarrow[U\text{ gauge}]{} -\frac{1}{\sqrt{2}}\sin\theta\vec{\pi}\,, \\ \vec{\pi}_2 &\longrightarrow \frac{1}{\sqrt{2}}\sin\theta\vec{\pi}\,, \\ \vec{\pi}_3 &\longrightarrow \cos\theta\vec{\pi}\,, \end{aligned} \tag{4.13}$$

where

$$\cos^2\theta=m_V^2/m_A^2\,.$$

The coupled two-body channels to be considered are VV, AA, $V\epsilon$, $A\pi$, $\pi\pi$, and $\pi\eta$ for the vector mesons and VA, $V\pi$, $V\eta$, $A\sigma$, and $\sigma\pi$ for the axial-vector mesons. Note that only the VV, AA, and VA channels have nonsense states at $J=1$. We find that the Born matrices, which are 16×16 for the vector mesons and 13×13 for the axial-vector mesons, factorize at $J=1$ in accord with Eq. (A8) and are both of rank one. The results are presented in Table IV according to the representation (B6), with the conclusion that both vector and axial-vector mesons Reggeize even though chiral symmetry is broken spontaneously. The reader should appreciate that, as a consequence of the broken chiral symmetry, the nonsense matrix for the Reggeization of the vector meson is *nontrivially* of dimension 2; nevertheless, the Born matrix is of rank one in this case. Thus although two Regge trajectories were possible in principle, only one was found [see Appendix A, expecially Eq. (A11)]. [Note that, aside from the obvious changes of kinematics and coupling constants, which can be read off from Eq. (4.12), the entries for $V\epsilon$, $A\pi$, $V\pi$, $V\eta$, and $A\sigma$ are the same as that of $V\sigma$ in Table I, and $\pi\pi$, $\pi\eta$, $\sigma\pi$, and $\pi\epsilon$ are the same as that of $\vec{\pi}\vec{\pi}$ in Table II. Therefore, we do not list these results separately.]

TABLE IV. The Born-approximation normal-parity helicity matrix at $J=1$ and $I=1$ for the $SU(2)_L\times SU(2)_R$ model. The VV states are identical to Table I(a). (a) AA states; (b) VA states.

	(a) (AA)
b_{11}	$(4m_A^2-m_V^2)\left[\dfrac{1}{3k^2(s-m_V^2)}\right]^{1/2}$
b_{01}	$\dfrac{(4m_A^2-m_V^2)}{m_A}\left[\dfrac{s}{3k^2(s-m_V^2)}\right]^{1/2}$
b_{00}	$\dfrac{[s(2m_A^2-m_V^2)+8m_A^4]}{2m_A^2}\left[\dfrac{1}{3k^2(s-m_V^2)}\right]^{1/2}$
b_{1-1}	$\left[\dfrac{2(s-m_V^2)}{k^2}\right]^{1/2}$
	(b) (VA)
b_{11}	$\dfrac{(2m_V^2+m_A^2)s-(m_A^2-m_V^2)^2}{s}\left[\dfrac{1}{3k^2(s-m_A^2)}\right]^{1/2}$
b_{01}	$m_V[3s+(m_A^2-m_V^2)]\left[\dfrac{1}{3sk^2(s-m_A^2)}\right]^{1/2}$
b_{10}	$\dfrac{[(2m_A^2+m_V^2)s-(m_A^2-m_V^2)(2m_A^2-m_V^2)}{m_A}\left[\dfrac{1}{3sk^2(s-m_A^2)}\right]^{1/2}$
b_{00}	$\dfrac{[s^2-(m_A^2-m_V^2)^2]m_V+8m_A^2m_Vs}{2sm_A}\left[\dfrac{1}{3k^2(s-m_A^2)}\right]^{1/2}$
b_{1-1}	$\left[\dfrac{2(s-m_A^2)}{k^2}\right]^{1/2}$

C. Fermions and Anomalies

In general, if fermions are present in a gauge theory with axial-vector mesons, the question of anomalies must be considered,[13] since anomalies destroy the renormalizability of the theory, and the gauge mesons would not Reggeize. The problems of anomalies seem to be well understood,[13] so that a full-scale investigation on our part does not seem to be called for. Anomalies first appear in calculations involving at least one fermion closed loop, so that no difficulties are expected in the Born approximation. We have examined some simplified examples to convince ourselves that the Born matrices factorize as expected.

We have studied the Reggeization of the fermion in the Abelian gauge group $U(1)_{\gamma_5}$, where the axial-vector gauge meson and fermion acquire a mass by means of the Higgs-Kibble mechanism. The Born-approximation amplitude for axial-vector–fermion scattering in the U gauge is described by Fig. 4. This process has already been considered by Dicus and Teplitz,[17] who only computed the contribution of Fig. 4(a) and consequently failed to obtain factorization at $J=\frac{1}{2}$. Figure 4(b), which occurs because of the symmetry-breaking mechanism, ensures that the Born matrix (i) satisfies unitarity bounds and (ii) factorizes at $J=\frac{1}{2}$ as is required if the fermion is to Reggeize. In addition to the AF channel, we have the σF channel. We find that once again the whole 4×4 normal-parity Born matrix at $J=\frac{1}{2}$ factorizes as prescribed by (A8) and is of rank one. We present the results in Table V, according to the representation (B6). Although the question of anomalies must be considered in higher orders, this problem does not

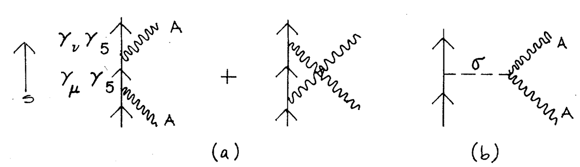

FIG. 4. The Feynman graph in the Born approximation for axial-vector–spinor scattering in $U(1)_{\gamma_5}$ gauge theory. (a) The fermion-pole graphs; (b) the scalar-exchange graph.

TABLE V. The Born-approximation normal-parity helicity matrix at $J=\frac{1}{2}$ for the $U(1)\gamma_5$ model. (a) AF states; (b) σF states.

	(a) (FA)
b_0	$\left[\frac{(m^2-2\omega M)^2}{2Mm^2k^2}\frac{(E-M)}{(W-M)}\right]^{1/2}$
b_1	$\left[\frac{(E-\omega-3M)^2}{4Mk^2}\frac{(E-M)}{(W-M)}\right]^{1/2}$
b_{-1}	$\left[\frac{(E-M)(W-M)}{2Mk^2}\right]^{1/2}$
	(b) $(F\sigma)$
b	$\left[\frac{(E+M)}{2M(W-M)}\right]^{1/2}$

make its presence felt in the Born approximation.

The other model which we have studied in detail is a version[14] of $U(1)_L\times U(1)_R$ with spontaneous symmetry breakdown, which provides a mechanism for the gauge mesons and fermion to acquire a mass. The conclusions are identical to that found in the $U(1)_{\gamma_5}$ model.

Note added in proof. We have found models, with special choice of parameters, in which certain $J=0$ amplitudes factorize.

APPENDIX A

In this appendix we wish to review and summarize certain technical aspects of Reggeization relevant to our work. We hope that the interested reader will find here a useful introduction to the subject, to be supplemented of course by a study of the papers of Gell-Mann *et al.*,[1] Mandelstam,[3] and Abers and Teplitz.[6]

The particles of a conventional Lagrangian field theory are said to Reggeize if the scattering amplitudes of the theory exhibit Regge asymptotic behavior $\sim\sum s^{\alpha_i(t)}$ and if the elementary particles of the theory lie on Regge trajectories, i.e., for a particle of mass m and spin σ, $\alpha_i(m^2)=\sigma$ for some α_i.

An alternative and essentially equivalent definition of Reggeization is the following: In a Lagrangian theory it may be possible to define Regge interpolations $T(s,J)$ of the physical partial-wave amplitudes $T_J(s)$, for sufficiently large J. At small physical values J_0 of J, the analytic continuation of $T(s,J)$ (if it exists) need not agree with the physical amplitudes. The latter contain, in general, contributions from polynomial functions of the scattering angle. The former, defined by a Froissart-Gribov continuation, do not contain such polynomial contributions. The difference is a "Kronecker δ term" $T(s,J)-T_J(s)\sim\delta_{JJ_0}$. Polynomial contributions *are* present, and Kronecker δ terms may *appear* at J_0 if there exists an elementary particle of spin J_0 in the s channel under consideration, or if the s channel can communicate with states having sufficiently large total spin ($\sigma_1+\sigma_2\geq J_0+1$ for two-particle states). If, however, the Kronecker δ's are absent, that is, if the Regge continuation and the physical amplitudes agree at J_0, the theory is said to Reggeize at this value of angular momentum. In particular, an elementary particle of spin J_0 originally present in the Lagrangian has been turned into a Reggeon by higher-order corrections.

In our work we consider conventional *renormalizable* Lagrangian theories, so that our elementary particles' spins do not exceed unity. Therefore, we are concerned with Reggeization at $J_0=0$, $\frac{1}{2}$, and 1. We believe that, to any finite order of perturbation theory, $T(s,J)$ and $T_J(s)$ agree for $J>1$. A necessary condition for Reggeization according to the mechanism of Gell-Mann *et al.*[1] is the existence of nonsense states. One examines, for instance, two-particle helicity states $|\lambda_1\lambda_2\rangle$ at a given J, and labels the states "nonsense" or "sense" if the total helicity $\lambda=|\lambda_1-\lambda_2|$ does or does not exceed J_0. Helicity amplitudes are referred to as sense-sense, sense-nonsense, and nonsense-nonsense, and denoted T_{ss}, T_{sn}, and T_{nn}. Note that at J_0 only the sense-sense amplitude is physical.

In the Born approximation the helicity amplitudes near J_0 have the characteristic behavior

$$B_{ss}\sim-v_{ss}\delta_{JJ_0}+\text{analytic terms},$$

$$B_{sn}\sim v_{sn}(J-J_0)^{-1/2}+\text{analytic terms},$$

and

$$B_{nn}\sim v_{nn}(J-J_0)^{-1}+\text{analytic terms}.$$

A second necessary condition for Reggeization at J is the factorization condition $v_{ss}=v_{sn}v_{nn}^{-1}v_{ns}$. We shall "derive" this later on. If this condition holds, Gell-Mann *et al.*[1] argue that the theory will Reggeize if higher-order terms have good high-s behavior. In principle an investigation, order by order in perturbation theory, is required to verify the high-energy behavior.

Following the work of Gell-Mann *et al.*,[1] Mandelstam[3] devised an *alternate* criterion for Reggeization, which is simple and essentially kinematical in character. He pointed out that at J_0 the *physical* amplitudes T_{ss}(physical) satisfy a set of dynamical equations, whose solution involves a certain number of arbitrary parameters: subtraction constants and the mass and coupling constants that describe elementary particles in a Lagrangian. The *Regge* amplitudes satisfy similar equations, and in particular the solution T_{ss} contains other arbitrary parameters brought about by the coupling of T_{ss} to T_{sn} and T_{nn}. Both T_{ss} and T_{ss}(physical) satisfy at J_0 certain kinematical constraints having to do with their behavior at thresholds and "conspiracy" points. Because of the presence of arbitrary parameters, T_{ss}(physical) and T_{ss} need not agree, but if the number of constraints exceeds the number of parameters T_{ss}(physical) and T_{ss} *must* be equal, and the theory will Reggeize at J_0. The Mandelstam argument therefore sets up a counting procedure which can be carried out to establish if a given theory *must* Reggeize at a given J_0. If the counting criteria are satisfied, Reggeization will take place for any values of masses and coupling constants. If the criteria are not satisfied, Reggeization may still take place, but only for special values of these parameters, and one must fall back on detailed, dynamical calculations.

The essential ingredients for the validity of the Mandelstam argument appear to be analyticity, unitarity, and the existence of nonsense channels. It is reasonable to assume that the first two hold in any renormalizable Lagrangian theory. Indeed in the examples studied by Teplitz and collaborators,[5,17] where Mandelstam counting suggested Reggeization should take place, and yet the Born approximation did not factorize, lack of renormalizability of the Lagrangian seems to be the only possible culprit.

In our work we have been guided by a heuristic N/D approach to the solution of the equations satisfied by the Regge amplitudes. A direct attack of the field theory means essentially summing diagrams and going to the high-energy limit, or examining the J-plane behavior of sums of diagrams, a task made very difficult by the intricate complications of non-Abelian gauge theories. The use of partial-wave dispersion relations and their N/D solution suffers from uncertainties having to do with subtraction constants. In principle the Mandelstam argument states that these subtraction constants can be determined, but in practice this is difficult and we can only claim a certain plausibility for our results, to be checked in principle by higher-order calculations.

We consider calculating the Regge amplitudes from the dispersion relations

$$T_{\mu\lambda}(s,J)=V_{\mu\lambda}(s,J)+\frac{1}{2\pi i}\int\frac{ds'}{s'-s}\operatorname{disc}[T_{\mu\lambda}(s,J)]_{\rm el} \qquad \text{(A1)}$$

and "elastic unitarity"

$$\operatorname{disc}[T_{\mu\lambda}]_{\rm el}=2i\sum_{\nu}\rho_\nu T^*_{\nu\mu}T_{\nu\lambda}. \qquad \text{(A2)}$$

Here T is defined by means of a Froissart-Gribov continuation. The "potential" V contains left-hand cut and inelastic contributions (three-or-more-particle intermediate states; we include all two-body states in our elastic unitarity terms). The "potential" is calculated to a given order of perturbation theory from the t- and u-channel absorptive parts, and does not contain s-channel elementary-particle poles or other contributions from terms analytic in t or u.

The above equations (or their solution) can be continued to a value of J near J_0 where one divides them into sense and nonsense pieces. We write in block matrix form

$$T_{\mu\lambda}=\begin{pmatrix}T_{ss} & T_{sn}\\ T_{ns} & T_{nn}\end{pmatrix}, \qquad \text{(A3)}$$

where T_{ns} is the transpose of T_{sn}. The potential in the Born approximation has the form

$$V_{\mu\lambda}=\begin{pmatrix}0 & v_{sn}(J-J_0)^{-1/2}\\ v_{ns}(J-J_0)^{-1/2} & v_{nn}(J-J_0)^{-1}\end{pmatrix}+\text{analytic terms}. \qquad \text{(A4)}$$

Near J_0 the nonsense-channel potential $v_{nn}(J-J_0)^{-1}$ dominates and provides the main force that "drives" the system. The sense-nonsense potential $v_{sn}(J-J_0)^{-1/2}$ provides the principal coupling of the nonsense to the sense channels, and the analytic terms can be ignored.

A crude N/D calculation, with $N=V$, gives disc$D$ $=-2iV\rho$. We note that disc$D$ is bounded as $s\to\infty$. This reflects our choice of normalization of the amplitude, and the fact that *in a renormalizable theory the elements of V do not exceed unitarity bounds*. We also note that aside from kinematical singularities (factors of momentum in the denominator) which can be removed by redefining the amplitudes, the elements of V are polynomials in s (or $\sqrt{s}$ for fermion-boson scattering). We therefore choose as our solution for D the form

$$D=1-VK, \qquad \text{(A5)}$$

where K is an appropriate integral involving the phase-space factor ρ, and suitably defined so as to cancel singular factors of V. (Essentially we are taking the polynomial factors of V outside the dispersion integral.) Equation (A5) implies a particular subtraction philosophy, which in principle can be checked by higher-order calculations or by verifying that the solutions satisfy all the constraints. Our Eq. (2.5) for the trajectory $\alpha(s)$ illustrates the meaning of our remarks.

We find from (A3)–(A5)

$$T=\begin{pmatrix}0 & \dfrac{v_{sn}}{(J-J_0)^{1/2}}\\ \dfrac{v_{ns}}{(J-J_0)^{1/2}} & \dfrac{v_{nn}}{J-J_0}\end{pmatrix}\times\begin{pmatrix}1 & \dfrac{-v_{sn}K}{(J-J_0)^{1/2}}\\ \dfrac{-v_{ns}K}{(J-J_0)^{1/2}} & 1-\dfrac{v_{nn}K}{J-J_0}\end{pmatrix}^{-1} \tag{A6}$$

and in particular

$$T_{ss}=v_{sn}\frac{K}{J-J_0-v_{nn}K}v_{ns}. \tag{A7}$$

We have dropped a fourth-order term $v_{sn}Kv_{ns}$ in the denominator, since other fourth-order contributions to the denominator have not been taken into account. At $J=J_0$

$$T_{ss}=-v_{sn}v_{nn}{}^{-1}v_{ns}. \tag{A8}$$

On the other hand, the physical amplitude has the form

$$T_{ss}(\text{physical})=-v_{ss}+\cdots, \tag{A9}$$

where $\cdots$ are terms expected to agree with terms in the Regge continuation that we have neglected, and which are analytic near J. In our approximation the requirement that T_{ss}(physical) and T_{ss} agree is the factorization condition

$$v_{ss}=v_{sn}v_{nn}{}^{-1}v_{ns}. \tag{A10}$$

The matrix $\alpha=J_0+v_{nn}K$ is symmetric, and it will have in general as many eigenvalues $\alpha_\nu(s)$ as there are nonsense channels. We can write a spectral decomposition, $\alpha=\sum P_\nu\alpha_\nu(s)$, and represent T_{ss} as a sum of Regge poles passing near J_0 (to lowest order) with factorized residues since the P's are projection operators:

$$T_{ss}=\sum_\nu v_{sn}P_\nu v_{ns}K\frac{1}{J-\alpha_\nu(s)}=\sum_\nu\frac{\gamma_s^\nu\gamma_s^\nu}{J-\alpha_\nu(s)}. \tag{A11}$$

For the special case of a single nonsense state, we observe that $\mathrm{Im}\,\alpha=\rho v_{nn}$.

We note that if there is a pole in the s channel corresponding to an elementary particle of spin J_0 so that $v_{ss}\sim(s-m^2)^{-1}$, the factorization condition will require $\det v_{nn}$ to have a simple zero at $s=m^2$. In general at least one of the eigenvalues $\alpha_\nu(s)-J_0$ will have a zero at $s=m^2$, and the particle will lie on the corresponding trajectory. If some eigenvalue $\alpha_\nu(s)-J_0$ has a zero at $s=s_0$, but v_{ss} has no pole at this point, the factorization condition will ensure that the corresponding Regge residue also has a zero at $s=s_0$, in which case the trajectory has chosen nonsense at $J=J_0$.

APPENDIX B

We devote this appendix to a summary of conventions and definitions used in this paper. For convenience, we present these in schematic fashion.

Conventions.

(i) Feynman rules are derived as in Ref. 19.

(ii) Helicity amplitudes are defined as in Ref. 20.

(iii) Polarization vectors are defined as in Ref. 4 and as by Gell-Mann *et al.*[1]

(iv) Partial-wave helicity amplitudes are derived as by Gell-Mann *et al.*[1]

Definitions.

$T^{(+)}_{\lambda_3,\lambda_4;\lambda_1,\lambda_2}(s,J)$

$\equiv$ normal-parity, partial-wave helicity amplitude for a two-body process, with $\lambda_1,\ldots,\lambda_4$ the helicity, $\lambda\equiv\lambda_1-\lambda_2$, $\mu=\lambda_3-\lambda_4$, (B1)

$B_{\lambda_3\lambda_4;\lambda_1\lambda_2}(s,J)\equiv$ Born approximation, (B2)

$B_{\mu\lambda}(s,J)=\tilde{B}_{\mu\lambda}(s,J)+$analytic terms at $J=J_0$, (B3)

with

$\tilde{B}_{\mu\lambda}(s,J)\equiv$ the singular part (in J) of the Born approximation near $J\simeq J_0$, (B4)

where

$$\tilde{B}_{ss}(s,J)=-v_{ss}\delta_{J,J_0}, \tag{B5a}$$

$$\tilde{B}_{sn}(s,J)=v_{sn}(J-J_0)^{-1/2}, \tag{B5b}$$

and

$$\tilde{B}_{nn}(s,J)=v_{nn}(J-J_0)^{-1}. \tag{B5c}$$

If the amplitudes factorize near $J\sim J_0$ and the matrix $v_{\mu\lambda}$ is of rank one, then for $J_0=1$ we have

$$v_{\mu\lambda}=g^2b_\mu\times b_\lambda, \tag{B6}$$

where g is the gauge coupling constant of the gauge group; for $J=\frac{1}{2}$ we have in general

$$v_{\mu\lambda}=g(\mu)b_\mu\times g(\lambda)b_\lambda, \tag{B7}$$

where $g(\mu)$ is the relevant coupling constant for the state "μ"; for instance, $g(\mu)=g$ for the fermion–vector-meson states, $g(\mu)=G$ for the fermion–scalar-meson states.

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