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Electromagnetic Theory of Strong Interaction

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From dielectric-diamagnetic properties shown to be inherent in the electromagnetic interaction itself by its very nature, and the fact that the dielectric attraction effect in a charged dielectric medium may dominate over the pure Coulomb repulsion, a hypothetical mechanism, which under special circumstances seems to be able to hold a charged object together, is presented, the diamagnetic property making the mechanism independent of the velocity of the object. This approach is shown to give a Yukawa-type equation for the electromagnetic field within a charged medium, and leads to a tentative electromagnetic interpretation of strong interaction analogous to the theory of plasmons. In addition to the prediction of the pion mass from the nuclear interaction range as in the Yukawa theory, the electromagnetic approach also predicts the existence of a lighter stable structure with a mass which agrees in order of magnitude with the mass of the electron, and suggests the existence of an excited state of this structure with a mass which agrees with the mass of the muon. On a macroscopic scale, the hypothetical charge-confinement mechanism presented gives energy contents for ball lightning which are of the same order of magnitude as the extremely high values ($\approx 10^7$ J) reported for this phenomenon.

I. INTRODUCTION

The subject of this paper is the mysterious fact that although physical experience seems to tell us that charged substances tend to fly apart as a result of the Coulomb repulsion, still in nature there are objects like ball lightning,[1] atomic nuclei, and elementary particles, which seemingly contradict this experience. Ingenious theoretical models have been proposed to explain the forces necessary to counterbalance the repulsive forces in these cases, such as the Yukawa theory[2] for the strong nuclear interaction, through which the existence of the pion was also predicted. Ascribing the mechanisms holding the objects together to forces outside electromagnetism, however, introduces a logical inadequacy[3] in the electromagnetic theory: on one hand, electromagnetism is a relativistically invariant theory, and on the other hand, one then has to resort to forces outside electromagnetism in order to construct a relativistically correct energy-mass relationship for a finite charged object. In view of this, an attempt will be made here to find a mechanism for the strong interaction within the framework of electromagnetism.

The principle upon which the theory of this paper is built is a phenomenon which has been known ever since ancient times and which even has given electricity its name: the fact that, e.g., a piece of amber ($\eta\lambda\epsilon\kappa\tau\rho o\nu$) after rubbing will attract uncharged objects. This implies that even if the objects are charged with the same kind of charge as the piece of amber itself, the attraction due to the polarization in the objects may dominate over the Coulomb repulsion. This principle, extended to the conditions inside a medium with density of charge ρ_e and electric permittivity ϵ, will describe[4] the force $\vec{f}$ per volume element, caused by an electric field $\vec{E}$, as consisting of a Coulomb contribution, together with a dielectric effect which may make $\vec{f}$ directed opposite to the electric field,

$$\vec{f}=\rho_e\vec{E}-\tfrac{1}{2}\vec{E}^2\nabla\epsilon+\tfrac{1}{2}\nabla\left(\vec{E}^2\rho\frac{\partial\epsilon}{\partial\rho}\right), \tag{1}$$

where ρ is a measure of the density of the medium (e.g., charge or mass density). An analogous expression[5] is valid for the volume force $\vec{f}$ due to a magnetic field $\vec{B}$ in a medium with a current density $\vec{j}=\rho_e\vec{v}$ and magnetic permeability μ,

$$\vec{f}=\rho_e\vec{v}\times\vec{B}+\tfrac{1}{2}\vec{B}^2\nabla\left(\frac{1}{\mu}\right)-\tfrac{1}{2}\nabla\left(\vec{B}^2\rho\frac{\partial(1/\mu)}{\partial\rho}\right), \tag{2}$$

expressing the fact that a diamagnetic substance will be repelled by a magnetic field.

Now, electromagnetic interaction has an intrinsic dielectric-diamagnetic quality in the sense that in line with LeChatelier's principle, the effect of the interaction of the electromagnetic field on a charge, or an ensemble of charges, will tend to weaken the field itself, just like the polarization and magnetization in a dielectric or diamagnetic substance would do. And, for instance, so will a charge subjected to an acceleration due to an electric field create, by electromagnetic induction, an electric-field contribution opposite to the original field. Similarly, the Thomas rotation[6] of a charge in a magnetic field will create a magnetic-field contribution, weakening the original magnetic field. These effects will be studied in more detail in Sec. II, and will be shown to lead to a Yukawa-type equation for the electromagnetic field within a charged medium. Under common laboratory conditions with comparatively weak fields and quasineutral substances, these effects will be negligible compared to the ordinary polarization and magnetization of the atomic constituents of the substances.

The idea that dielectric effects might be responsible for the strong interaction is not new. In his pioneering paper[7] on the quantum-mechanical treatment of α decay, electric polarization was one of the effects Gamow proposed as a possible cause for the nuclear forces. What is new in the present approach, however, is that it is shown that dielectric effects can occur not only as a result of charge separation in a quasineutral medium, but also as a result of electromagnetic induction when an ensemble of, say, solely positive charges is being accelerated.

In contrast to the confinement mechanisms of magnetic nature commonly discussed in plasma physics, the attraction mechanism studied in this paper is electric in nature. However, in addition to the electric field, the magnetic field is also derived in the following in order to demonstrate the relativistic invariance of the attraction mechanism and the symmetry of the wave equations for the $\vec{E}$ and $\vec{B}$ fields.

II. THE YUKAWA EQUATION

We consider a cloud of charge with inertial mass density ρ_m, density of (unipolar) electric charge ρ_e, electric permittivity ϵ_0 (as in vacuum), magnetic permeability μ_0 (as in vacuum), and moving with a constant translational velocity $\vec{v}$ ($v\ll c$) with respect to a stationary reference system K. We assume that, seen from a system K' moving with the cloud, there are *at a particular instant* no internal velocities and hence no internal magnetic field in the cloud. (Later[14] some observational aspects will be discussed.) If the electric field in K of the cloud is $\vec{E}$, then the magnetic field in K of the cloud will be

$$\vec{B}=\vec{v}\times\vec{E}/c^2.$$

The acceleration of a volume element of the cloud caused by the electric field can be written as

$$\vec{a}=(\rho_e/\rho_m)\vec{E}-\vec{a}_0, \tag{3}$$

where $\vec{a}_0$ denotes possible dielectric reaction effects due to the acceleration caused by $\vec{E}$. We first study the case $\vec{a}_0=0$, i.e., assuming that there are no constraints on the accelerations of the constituents of the cloud. Owing to the acceleration $\vec{a}$, as seen from K there will then be a Thomas rotation $\vec{\omega}=-\vec{v}\times\vec{a}/(2c^2)$ of each infinitesimal part of the cloud, i.e.,

$$\vec{\omega}=-(\rho_e/2\rho_m)\vec{B}. \tag{4}$$

Thus, even if there are no internal velocities in the cloud as seen from the system K' moving with the cloud, this will not be the case as seen from the stationary system K, where superimposed on the translational velocity of the cloud there will also be an infinitesimal internal rotational velocity so that $\vec{v}_{tot}=\vec{v}+\vec{\omega}\times\delta\vec{r}$, where $\delta\vec{r}$ is the radius vector from the center of rotation ($\delta\vec{r}$ must be infinitesimal, otherwise velocities would not necessarily be Galilean-invariant in the nonrelativistic limit between K and K'). Although the Thomas rotation has no effect on the total velocity itself since $\delta\vec{r}$ is arbitrarily close to 0, it does have the effect that, e.g.,

$$\begin{aligned}\nabla\times\vec{v}_{tot}&=\nabla\times\vec{v}+[\nabla\times(\vec{\omega}\times\delta\vec{r})]_{\delta\vec{r}\to 0}\\&=[(\delta\vec{r}\cdot\nabla)\vec{\omega}-(\vec{\omega}\cdot\nabla)\delta\vec{r}+\vec{\omega}\,\nabla\cdot\delta\vec{r}\\&\quad-\delta\vec{r}\,\nabla\cdot\vec{\omega}]_{\delta\vec{r}\to 0}\\&=-(\vec{\omega}\cdot\nabla)\delta\vec{r}+\vec{\omega}\nabla\cdot\delta\vec{r}=2\vec{\omega},\end{aligned}$$

i.e., from Eq. (4)

$$\nabla\times\vec{v}_{tot}=-(\rho_e/\rho_m)\vec{B}. \tag{5}$$

This equation can be regarded[6] as a reformulation of Larmor's theorem.

From the Maxwell equations,

$$\nabla\cdot\vec{E}=\rho_e/\epsilon_0, \tag{6}$$

$$\nabla\times\vec{E}=-\partial\vec{B}/\partial t, \tag{7}$$

$$\nabla\cdot\vec{B}=0, \tag{8}$$

$$\nabla\times\vec{B}=\mu_0\vec{j}+\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}, \tag{9}$$

we can now derive wave equations for the electromagnetic field inside the charged cloud.

Operating on Eq. (9) with $\nabla\times$ gives

$$\nabla\times(\nabla\times\vec{B})=\mu_0\nabla\times\vec{j}+\frac{1}{c^2}\frac{\partial}{\partial t}(\nabla\times\vec{E}).$$

Using Eq. (7) and the formula $\nabla\times(\nabla\times\vec{B})=\nabla(\nabla\cdot\vec{B})-\Delta\vec{B}$, where $\nabla\cdot\vec{B}=0$ according to (8), we obtain

$$\Delta\vec{B}-\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}=-\mu_0\nabla\times\vec{j}.$$

Since the current density is $\vec{j}=\rho_e\vec{v}_{tot}$, where $\vec{v}_{tot}=\vec{v}+(\vec{\omega}\times\delta\vec{r})_{\delta\vec{r}\to 0}$ as above, we get $\nabla\times\vec{j}=\rho_e\nabla\times\vec{v}_{tot}-\vec{v}\times\nabla\rho_e$. Inserting the expression for $\nabla\times\vec{v}_{tot}$ from Eq. (5), and regarding $\vec{v}\times\nabla\rho_e$ as a source term $\vec{S}_B$, we obtain a Yukawa-type equation for the $\vec{B}$ field:

$$\Box^2\vec{B}-\frac{\rho_e^2}{\rho_m}\mu_0\vec{B}=\mu_0\vec{S}_B. \tag{10}$$

This equation will be valid also in the limit $v=0$.

Similarly, we can obtain an equation for the $\vec{E}$ field from Eqs. (7), (9), and (6):

$$\Box^2\vec{E}-\mu_0\frac{\partial\vec{j}}{\partial t}=\frac{1}{\epsilon_0}\nabla\rho_e.$$

Regarding $\nabla\rho_e$ as a source term $\vec{S}_E$ and evaluating (for $v=0$)

$$\frac{\partial\vec{j}}{\partial t}=\frac{\partial}{\partial t}(\rho_e\vec{v})=\rho_e\vec{a}=\left(\frac{\rho_e^2}{\rho_m}\right)\vec{E},$$

we get

$$\Box^2\vec{E}-\frac{\rho_e^2}{\rho_m}\mu_0\vec{E}=\frac{1}{\epsilon_0}\vec{S}_E. \tag{11}$$

Thus, inside the charged cloud the usual electromagnetic wave equations $\Box^2\vec{E}=0$, $\Box^2\vec{B}=0$ for vacuum are replaced by the two Yukawa-type Eqs. (10) and (11). Making the free-wave ansatz constant $\times e^{i(\omega t-kz)}$ in these equations we obtain the dispersion relation

$$\omega^2/c^2-k^2-\rho_e^2\mu_0/\rho_m=0.$$

After relating frequency and wave number to energy and momentum we get

$$E^2/c^2-p^2=\rho_e^2\mu_0\hbar^2/\rho_m,$$

i.e., the photons corresponding to the $\vec{E}$ and $\vec{B}$ fields within the cloud can be interpreted to have the rest mass

$$m_Y=(\rho_e^2\mu_0\hbar^2/\rho_m c^2)^{1/2}. \tag{12}$$

Just as the electromagnetic interaction in vacuum takes place[8] via the exchange of virtual, massless photons, so—provided it is at all possible to physically realize such a charged cloud as we have assumed here—would the electromagnetic interaction inside the cloud take place via the exchange of virtual Yukawa photons with mass m_Y as in Eq. (12). Using the relations[9] $\vec{B}=\nabla\times\vec{A}$ and $\vec{E}=-\nabla V-\partial\vec{A}/\partial t$, we may, from the wave equations in $\vec{B}$ and $\vec{E}$ [Eqs. (10) and (11)], deduce similar wave equations for the scalar and vector potentials V and $\vec{A}$. Furthermore, as will be mentioned in Sec. IV there may be, due to the "mass" term in these equations, no coupling between the scalar and vector fields, i.e., in contrast to the usual electromagnetic photons in vacuum, the Yukawa photons discussed here may not necessarily be vector particles. For the "diffusion length"

$$L=(m_Y c/\hbar)^{-1} =(\rho_e^2\mu_0/\rho_m)^{-1/2} \tag{13}$$

in Eq. (11) we may, for a wide range of applications, and in line with techniques[10] used in the treatment of the diffusion equation, use a constant value obtained by "homogenizing" $(\rho_e^2\mu_0/\rho_m)^{-1/2}$ over the region of space in question. Also, the relativistic changes in densities and inertial mass will cancel each other and m_Y will be independent of the velocity of the cloud, as is required if it is to be interpreted as a rest mass.

The static $l=0$, zero-leakage[11] solutions to Eqs. (10) and (11) are

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\iiint\frac{\vec{S}_E e^{-r_{12}/L}}{r_{12}}dV, \tag{14}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\iiint\frac{\vec{S}_B e^{-r_{12}/L}}{r_{12}}dV, \tag{15}$$

where $\vec{r}_{12}$ is the vector from the source point to the field point.

The Yukawa photons appearing in the above description of the electromagnetic field inside the assumed charged cloud are formally identical to the plasmons[12] in a quasineutral medium. Just as the plasmons in a quasineutral medium are involved in an effort of the electric field

$$\vec{E}=-\nabla V-\partial\vec{A}/\partial t$$

to eliminate, through the term $-\nabla V$, deviations from charge neutrality, so the above Yukawa photons are involved in a similar effort of the electric field to eliminate, through the term $-\partial\vec{A}/\partial t$, deviations from the assumed initial situation with no internal velocities in the cloud (when $v\ll c$, $\vec{A}$ is a linear functional of $\vec{v}$, i.e., $\partial\vec{A}/\partial t$ is a linear functional of $\partial\vec{v}/\partial t$; $\partial\vec{A}/\partial t$ may have a nonvanishing value even though $\vec{A}$ is zero). Equations (14) and (15) correspond to evanescent waves[13] in an overdense plasma. Owing to the effect of the exponential factor, the range of the fields in Eqs. (14) and (15) is small compared to the usual Coulomb field. In Sec. III, the attraction effect caused by Yukawa photons in a charged medium will be studied.

III. THE DIELECTRIC-ATTRACTION EFFECT

The electric and magnetic fields in a charged medium allowing unconstrained accelerations are, according to Eqs. (14) and (15),

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\iiint\frac{\vec{S}_E e^{-r_{12}/L}}{r_{12}}dV,$$

$$\vec{B}=\frac{\mu_0}{4\pi}\iiint\frac{\vec{S}_B e^{-r_{12}/L}}{r_{12}}dV.$$

Since $\vec{S}_E=\nabla\rho_e$, $\vec{S}_B=\vec{v}\times\vec{S}_E$, and

$$\iiint\frac{\nabla\rho_e e^{-r_{12}/L}}{r_{12}}dV - \iiint(1+r_{12}/L)\frac{\rho_e e^{-r_{12}/L}\vec{r}_{12}}{r_{12}^3}dV = \iiint\nabla\left(\frac{\rho_e e^{-r_{12}/L}}{r_{12}}\right)dV = \iint\vec{n}\frac{\rho_e e^{-r_{12}/L}}{r_{12}}dS=0,$$

$\vec{n}$ being the normal to an element dS of an enclosing surface outside the source region, we may rewrite the above fields as

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\iiint(1+r_{12}/L)\frac{\rho_e e^{-r_{12}/L}\vec{r}_{12}}{r_{12}^3}dV, \tag{16}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\iiint(1+r_{12}/L)\frac{\rho_e e^{-r_{12}/L}\vec{v}\times\vec{r}_{12}}{r_{12}^3}dV. \tag{17}$$

The true nature of these fields is in the form of quanta, such that for sufficiently small time intervals there will be a contribution from only one source point at a time.[14] Since $\rho_e dV$ describes the rate of emission of virtual photons from a particular volume element dV of the cloud, then the total charge of the cloud, $q=\iiint\rho_e dV$, describes the rate of emission from any one point in the cloud, which then for each small time interval will be located stochastically in the cloud with a probability proportional to ρ_e. Analogously to, e.g., the γ-dose rate from a radioactive sample, the average instantaneous electric and magnetic fields from the cloud itself, corresponding to Eqs. (16) and (17), will thus be

$$\vec{E}=\frac{q}{4\pi\epsilon_0}\left(1+\frac{r_{12}}{L}\right)\frac{e^{-r_{12}/L}\vec{r}_{12}}{r_{12}^3}, \tag{18}$$

$$\vec{B}=\frac{\mu_0 q}{4\pi}\left(1+\frac{r_{12}}{L}\right)\frac{e^{-r_{12}/L}\vec{v}\times\vec{r}_{12}}{r_{12}^3}, \tag{19}$$

where $\vec{r}_{12}$ is the vector from the momentary source point.

Thus the solutions to the Yukawa equations for the $\vec{E}$ and $\vec{B}$ fields in a charged medium allowing unconstrained accelerations can formally be described as consisting of Coulomb contributions in a dielectric-diamagnetic medium, with

$$\epsilon=\epsilon_0\frac{e^{r_{12}/L}}{1+r_{12}/L} \tag{20}$$

and

$$\mu=\mu_0(1+r_{12}/L)e^{-r_{12}/L}. \tag{21}$$

The dielectric characteristic thus inherent in the electromagnetic interaction itself will, as discussed in connection with Eq. (1) in Sec. I, give rise to an attraction effect in a charged medium, which in the case of a strong, divergent field may dominate over the repulsive effect. (The quantum property[14] introduced above is essential to ensure a divergent field and hence attraction; classically the field, e.g., in a uniformly charged sphere, is convergent and the dielectric-attraction effect discussed here would then not occur.) Since $\epsilon=1/\mu$ the repulsive diamagnetic effect compensates,[15] through the force in Eq. (2), the relativistic change in $\vec{E}$ and makes the attraction mechanism in a charged object valid regardless of its velocity.

Due to the dielectric effect, the acceleration $\vec{a}$ in Eq. (3) will contain a nonvanishing dielectric attraction term $\vec{a}_0$. In particular, if the volume force could be made to vanish, then the dielectric-attraction term from Eq. (1),

$$\vec{a}_0=\left[\tfrac{1}{2}E^2\nabla\epsilon-\tfrac{1}{2}\nabla\left(E^2\rho\frac{\partial\epsilon}{\partial\rho}\right)\right]\Big/\rho_m,$$

would be equal to $(\rho_e/\rho_m)\vec{E}$. This may formally be interpreted as an unconstrained Coulomb acceleration $(\rho_e/\rho_m)\vec{E}$ of a volume element in the medium when it absorbs a virtual Yukawa photon, immediately followed by an unconstrained dielectric retardation $\vec{a}_0$ of equal magnitude when space reacts on the electromagnetic-field change caused by the acceleration. This reaction effect is more immediate and more universal than the reaction effect which an accelerating charge encounters in a quasineutral medium and which, after a short transient phase, leads to a field-dependent equilibrium velocity as described by Ohm's law $\vec{j}\propto\vec{E}$.

A trivial stability condition of the system is approached by an unlimited expansion of the cloud; however, if the charged medium is sufficiently dense, there also exist nontrivial stability conditions. Assuming a charged cloud with total charge q and uniform charge and mass densities ρ_e and ρ_m, we may calculate the dielectric-attraction effects from Eq. (1). Using Eqs. (20), (13), and with $\vec{r}$ being the vector from the momentary source point, we obtain

$$\begin{aligned}\rho\frac{\partial\epsilon}{\partial\rho}&=\frac{\partial\epsilon}{\partial(1/L)}\rho\frac{\partial(1/L)}{\partial\rho}\\&=\left(r\epsilon-\frac{r\epsilon}{1+r/L}\right)\frac{1}{2}\frac{1}{L}\\&=\frac{1}{2}\frac{(r/L)^2}{1+r/L}\epsilon\,.\end{aligned}$$

Since the third term in Eq. (1) is then

$$\begin{aligned}\frac{1}{2}\frac{\partial}{\partial r}\left(E^2\rho\frac{\partial\epsilon}{\partial\rho}\right)&=\tfrac{1}{2}\frac{\partial}{\partial r}\left(E^2\frac{1}{2}\frac{(r/L)^2}{1+r/L}\epsilon\right)\\&=\tfrac{1}{2}E^2\frac{r/L^2}{1+r/L}\epsilon\\&\quad+\frac{1}{4}\left(\frac{r}{L}\right)^2\frac{\partial}{\partial r}\left(E^2\frac{\epsilon}{1+r/L}\right),\end{aligned}$$

the second term in Eq. (1),

$$-\tfrac{1}{2}E^2\frac{\partial}{\partial r}\epsilon=-\tfrac{1}{2}E^2\frac{r/L^2}{1+r/L}\epsilon\,,$$

is canceled (just like, e.g., in an ordinary gas[16]), and the momentary volume force becomes

$$f=\rho_eE+\frac{1}{4}\left(\frac{r}{L}\right)^2\frac{\partial}{\partial r}\left(E^2\frac{\epsilon}{1+r/L}\right). \tag{22}$$

Inserting for E the field from Eq. (18) and assuming the cloud to be in the form of a sphere with radius r_0 so that $\rho_e=q/\frac{4}{3}\pi r_0^3$, we obtain

$$\begin{aligned}f&=\frac{q^2}{(4\pi)^2\epsilon_0}3(1+r/L)\frac{e^{-r/L}}{r_0^3r^2}\\&\quad+\frac{1}{4}\left(\frac{r}{L}\right)^2\frac{q^2}{(4\pi)^2\epsilon_0}\left(-\frac{4}{r}-\frac{1}{L}\right)\frac{e^{-r/L}}{r^4}\\&=\frac{q^2}{(4\pi)^2\epsilon_0}\frac{e^{-r/L}}{4r_0^3r^2}\left(12+\frac{12r}{L}-\frac{4r_0^3}{L^2r}-\frac{r_0^3}{L^3}\right).\end{aligned} \tag{23}$$

For small values of r, the volume force will obviously be negative, i.e., despite the highly repulsive nature of the Coulomb force the net force will still be nonrepulsive. It should thus be possible to realize a stable charged structure, e.g., in the form of a spherical cloud, provided that the radius of the cloud does not exceed a certain value of the order of the attractive range. With $r=r_0$ in Eq. (23) we can obtain an estimate of the radius of the cloud from the equation

$$12+12r_0/L-4(r_0/L)^2-(r_0/L)^3=0\,, \tag{24}$$

or (the other two roots are negative)

$$r_0=2.55L=\frac{2.55\hbar}{m_Yc}\,, \tag{25}$$

where we have also used the relation of Eq. (13). Now if m_c is the mass of the cloud, we obtain from Eq. (13)

$$L^2=\frac{\rho_m}{\mu_0\rho_e^{\,2}}=\frac{m_c/\frac{4}{3}\pi r_0^{\,3}}{\mu_0 q^2/(\frac{4}{3}\pi r_0^{\,3})^2}\,,$$

i.e., after using Eq. (25)

$$L=\frac{\mu_0 q^2}{m_c\frac{4}{3}\pi(2.55)^3}, \tag{26}$$

or by using Eq. (13) and the relation $\mu_0=(\epsilon_0 c^2)^{-1}$,

$$m_Y=\frac{(2.55)^3}{3}\frac{4\pi\epsilon_0\hbar c}{q^2}m_c\,,$$

i.e.,

$$m_Y=5.5\,\alpha^{-1}\left(\frac{e}{q}\right)^2 m_c\,, \tag{27}$$

where e is the electron charge and α the fine-structure constant $\alpha=e^2/(4\pi\epsilon_0\hbar c)=1/137.04$. The mass of the charged cloud and the mass of the corresponding Yukawa photons of the electromagnetic field in the cloud thus occur intrinsically connected by the fine-structure constant as in Eq. (27). Owing to the approximations made above (zero-leakage solution[11] and $r=r_0$), the numerical factor in Eq. (27) may be expected to be accurate only within a factor of about 2 or 3.

Within the attractive range $\approx r_0$, the force in Eq. (23) on a volume element of the cloud is directed opposite to the momentary electric field, and any acceleration due to the electric field tending to change the assumed initial situation with no internal velocities in the cloud will be countered by a dielectric reaction of equal magnitude [a nonvanishing net acceleration *opposite* to $\vec{E}$ due to the force in Eq. (23) would again create a repulsive contribution in Eq. (11) opposing this acceleration]. For the volume element Newton's second law thus gives

$$\rho_m\frac{d^2\vec{r}}{dt^2}=0\,. \tag{28}$$

After scalar multiplication by $\vec{r}$ and integration, we obtain the virial[17,18] expression

$$\tfrac{1}{2}\int\int\int\rho_m\frac{d^2}{dt^2}(r^2)dV-\int\int\int\rho_m\left(\frac{dr}{dt}\right)^2dV=0\,,$$

or, expressed in moment of inertia I and kinetic energy E_K,

$$\frac{1}{2}\frac{d^2I}{dt^2}=2E_K\,. \tag{29}$$

Using coordinates such that there is no collective translational motion of the cloud, Eq. (29) states that the rate of expansion of the cloud is determined by its internal velocities. Since we assume that initially there are to be no internal velocities, and since accelerations due to the electric field are canceled within the attractive range $\approx r_0$ by dielectric effects as discussed above, the constituents of the cloud would be expected to remain "frozen" in their initial positions, and the cloud to behave much like a solid—although, again, the mechanism involved is due to reactions to changes in the velocity distribution, not the spatial distribution, of the charges. Equation (29) is identical to the case with a cloud of a neutral substance; however, the charge of the cloud will have an effect on its stability to other disturbances, such as an initial stochastic motion in the cloud. Such disturbances would be expected to play a role in diminishing the stability only if—analogously to the case of a solid, or a gas in a gravitational field[17]—they are not negligible compared to the energies involved in the attraction mechanism, energies which, in the case of the charged cloud being studied, can be very high.

As is evidenced, e.g., in ionic crystals, the virial theorem of plasma physics, stating that a self-confined plasma is impossible, is not applicable[18] to an ensemble of discrete charges when the thermal energies are much smaller than the Coulomb energies. The above deduction shows that the quantum nature[14] of the electromagnetic field makes the conditions in the charged cloud analogous to this situation by introducing a (non-local) potential energy due to the dielectric attraction.

IV. POSSIBLE PHYSICAL INTERPRETATIONS

Although the identification with strong interaction must be regarded as tentative, here we will study the above attraction mechanism in the nuclear case, assuming the pion ($m_\pi\approx 135$ MeV/c^2) to be the Yukawa photon involved in the mechanism. The theory proposed then predicts the existence of a stable structure with a radius, according to Eq. (25), of the order of 3.7 F and a mass, according to Eq. (27), of the order of 0.2 MeV/c^2, values which are of the same order of magnitude as the classical radius (2.82 F) and mass (0.511 MeV/c^2) of the electron. The pion, in this picture, would be described as an electron in an excited state analogous to a plasma oscillation with the plasma frequency. (N.B. that the oscillation in this case may be stationary and without coupling between the $\vec{E}$ and $\vec{B}$ fields.[19]) In analogy with the theory of plasmons, one would, in addition to this volume excitation, then also expect another excitation mode confined to the surface and corresponding to a surface plasmon which, using results[20] for a plane vacuum-plasma interface, would be expected to have an energy of about $(\frac{1}{2})^{1/2}$ of the volume plasmon, i.e., a mass of approximately $m_\pi/\sqrt{2} \approx 100$ MeV/c^2. This excited state which, since the excitation is confined to the surface only, is otherwise to be expected to have properties very similar to the unperturbed electron, would thus be interpreted as the muon (mass 106 MeV/c^2). Again, the idea that the muon might correspond to a surface excitation of the electron is not new, and has been suggested, e.g., by Dirac.[21]

As a dual description it should be pointed out that the above deduction will also be valid if what is described above as a cloud would be the probability distribution of a quantum-mechanical *Zitterbewegung*[22] of a particle instead, i.e., if the electron, instead of as a charged cloud, would be pictured as a point pion trapped by its own field. Owing to the quantum nature[14] of the electromagnetic interaction discussed above it does not seem possible to decide by observations which picture is right, and this question may thus be regarded as lying outside the realm of physics.

The mechanism presented, which under special circumstances seems to be able to hold charged objects together, may not—if valid—necessarily be restricted to the microscopical scale but may also possibly shed some light on the macroscopic phenomenon of ball lightning, which so far has defied satisfactory explanation. A crucial point is the extremely high energy contents of the order of 10^7 J reported[23-25] for these objects, ranging in size from a walnut to a football. Since they are reported to seem to float around in air and their mass thus must be of the order of grams only, their energy contents must be more than a thousand times that of the corresponding amount[26] of TNT, and a nonclassical explanation seems necessary. If we assume the extreme field and current conditions present during a thunderstorm to be able to deplete an ionized region of air from electrons (since they have greater mobility than the ions), and thus for a short time form a dense, positively charged plasma, we may have conditions which, according to the hypothetical interaction mechanism discussed above, might permit a stable solution in the form of a spherical cloud, i.e., we speculatively conceive of a ball lightning as a macroscopic electron-type structure. Assuming a fraction κ of the electrons in each air molecule which takes part in the process to be ionized ($0 \leq \kappa \leq 1$), the charge-to-mass ratio of the accelerating particles will be $\kappa/(2\times 1800)$ of that of the electron. From Eq. (11) it follows that the dielectric reaction effect to the instantaneous acceleration will attain its maximum for $\kappa = 1$, and the energetically most-favored motion in the extremely high static field in the electron-depleted plasma thus is when the oxygen and nitrogen nuclei which constitute the excess charge momentarily behave as completely free particles. The charge-to-mass ratio of the particles involved in the mechanism is then $1/(2\times 1800)$ that of the electron. Using for the radius of an average[24] ball lightning the value 5 cm, i.e., 1.8×10^{13} times the classical electron radius, the proportionality $r_0 \propto L \propto q^2/m$ from Eqs. (25) and (26) gives for the total excess charge of the ball lightning a value $1.8\times 10^{13}\times 2\times 1800 = 6.5\times 10^{16}$ times the electron charge, i.e., 0.01 As. With a radius of 5 cm as above, this would correspond to a total electrostatic energy of the ball lightning of the order of $(6.5\times 10^{16})^2/1.8\times 10^{13} = 2.3\times 10^{20}$ times the self-energy of the electron, or 2×10^7 J, i.e., the conjecture of ball lightning as a macroscopic manifestation of the strong-interaction mechanism proposed in this paper gives for the energy content of an average ball lightning an estimate which is of the same order of magnitude as reported. If the surrounding air also has a high degree of electron depletion, only a fraction of the energy would be liberated explosively at the breakdown of the ball, which would explain the comparatively harmless disappearance of some ball lightnings.

Assuming normal density of the electron-depleted air forming the cloud above, the energy of the order of 10^7 J involved in the mechanism corresponds to a potential energy of the order of several keV per atom of the cloud. Normal thermal and ionization energies are negligible compared to this binding energy, and the cloud will thus be expected to exhibit a high degree of stability against ordinary thermal disturbances. Needless to say, the conditions inside and around the cloud assumed above with a positive excess charge of 0.01 As would be very extreme and hard to predict. In a normal surrounding, such a cloud would certainly tend to polarize the air strongly and surround itself with a negative space charge. Whether this would lead to an immediate neutralization of the excess charge of the cloud, as one would expect, is a more difficult question, e.g., since the mechanism presented above requires the nuclei constituting the excess charge to behave momentarily as if completely stripped, which might delay the neutralization process considerably. Whatever the relevance of the mechanism discussed in this paper may be to the Kugelblitz phenomenon, it should again be pointed out that an energy of the order of several keV per atom, which is what observations of ball lightning imply,[23–25] is something which is several orders of magnitude greater than what can be achieved by any chemical reaction or conventional recombination process.

If the mechanism presented were correct, it would suggest that artificial production of ball lightning might be accomplished by depleting electrons from ionized air inside a large metallic tank, in which case it might be possible to contain the electron-depleted air during the critical stage before sufficient excess charge is formed. It is interesting to note several reports[27] of spontaneous creations of ball lightning under similar conditions in aircraft. The mechanism presented is also compatible with results reported from experiments with circuit breakers, indicating that a threshold energy output of the order of 10^6 J is necessary for the formation of the type of ball lightning which occasionally and accidentally occurs in battery-powered submarines.[25] This suggests an alternative method for artificial production of ball lightning, namely, using a low-voltage, extremely high-current discharge by which a momentary charge density of the magnitude required by the mechanism proposed might be attained.

V. CONCLUDING REMARKS

Above we have, within the framework of electrodynamics, found a hypothetical interaction mechanism which in the nuclear case exhibits a considerable likeness to the Yukawa theory of strong interaction, and which—if the theory has a bearing on physical reality—might suggest a possible electromagnetic origin of the Poincaré stresses[28] necessary for relativistically consistent properties of finite charge distributions. Also, just as from Eq. (12) the electron might formally be described as a Yukawa photon in atomic matter, and the pion as a Yukawa photon in electronic matter, the theory speculatively predicts—as the first in a sequence of interactions of unknown (and possibly infinite) length beyond the strong interaction—the existence of a new type of interaction, the agent of which being an entirely new elementary object with a mass from Eq. (27) of the order of $(m_\pi/m_e)m_\pi \approx 37\ \mathrm{GeV}/c^2$, and which would be the Yukawa photon in pionic matter. It is interesting to note that this mass value agrees with the value ($37.29\ \mathrm{GeV}/c^2$) obtained by Lee[29] for the intermediate boson of the weak interaction, assuming the weak interaction to be of electromagnetic origin.

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[1]In this paper it is assumed that the observations of ball lightning [see, e.g., the compilation in S. Singer, *The Nature of Ball Lightning* (Plenum, New York, 1971)] can be regarded as essentially trustworthy and relating to one and the same type of object. If ball lightning exists at all in the sense of a uniform phenomenon, then several circumstances in connection with reports on their behavior and decay seem to point to a deviation from quasineutrality: They may move against the wind and be strongly influenced by the presence of conductors (*ibid.* p. 31, p. 44); their decay is often accompanied by strong electric effects (*ibid.* p. 35, p. 38, p. 42); when decaying they may, in a manner resembling ordinary lightning, kill or temporarily lame persons (*ibid.* p. 9, p. 28, p. 45).

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[5]J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), p. 154.

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[7]G. Gamow, Z. Phys. 51, 204 (1928).

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[9]R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Ref. 3), part II, p. 21–5.

[10]J. H. Ferziger and P. F. Zweifel, *The Theory of Neutron Slowing Down in Nuclear Reactors* (MIT Press, Cambridge, Mass., 1966), Chaps. II and III.

[11]Cf. K. H. Beckurts and K. Wirtz, *Neutron Physics* (Springer, Berlin, 1964), p. 110. For a medium with extension $\lesssim 3L$, the actual solution will fall off more rapidly than the nonleakage solution, i.e., with an effective diffusion length $< L$. When using the nonleakage solution, the dielectric-attraction effect discussed in the following thus will be, for source points near the boundary, somewhat underestimated.

[12]D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1964), p. 95 ff.

[13]P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading,

Mass., 1969), p. 147.

[14]This quantum nature of the electromagnetic field—necessary to avoid the "ultraviolet catastrophe" of classical black-body radiation [see, e.g., R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Ref. 3), part I, p. 41–6]—obviously also makes it impossible to *observe* the simultaneous momentary position and velocity of the cloud with infinite accuracy in line with the uncertainty relation. For if P_1 and P_2 are two successive momentary source points in the cloud, then it will not be possible from observations of P_1 and P_2 (which are the only type of information we can get about the cloud) to distinguish unambiguously between a possible motion of the whole cloud and the stochastic change of position of the momentary source point within the cloud. Owing to the unpredictability of the locations of the points in the cloud between which the exchange of virtual photons will take place, there will also be, for sufficiently small time intervals, a similar unpredictability of the potential energy of the cloud. Thus, although the cloud momentarily will be observed as a point charge, its self-energy will not be infinite since there is, for a cloud with finite extension, a vanishing probability that the emission and absorption of the photon will take place in the same point.

As will be discussed later in the paper, this discrete quality of the electromagnetic field is essential also for the virial of a system to vanish and, hence, for the existence of stable, localized charge configurations. Thus, just as the stability of, e.g., a proton-electron system against collapse due to the Coulomb force is a quantum-mechanical effect (cf. R. P. Feynmann *et al.*, *The Feynman Lectures on Physics*, part III, p. 2-5ff), so will a stability of the charged cloud against explosion or implosion be a result of the quantum nature of the electromagnetic field.

[15]Using four-vector formalism, see, e.g., R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Ref. 3), part II, Sec. 26–4.

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