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Bjorken Scaling in Quantum Field Theory*

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We discuss the conditions under which renormalization-group methods can imply Bjorken scaling for deep-inelastic structure functions. We prove and refine a conjecture of Parisi's concerning the high-spin behavior of anomalous dimensions in order to show that in most theories Bjorken scaling is possible only if the renormalization group has a fixed point at the *origin* of coupling-constant space.

I. INTRODUCTION

In recent years many people have explored the attractive possibility that renormalization-group techniques might allow one to understand Bjorken scaling within the framework of standard renormalizable field theory. The general idea is this: The familiar logarithms of perturbation theory can be shown to sum to a power provided that a certain universal function of coupling constant for the theory in question has a zero. The power with which a given amplitude behaves is itself a linear combination of the "anomalous dimensions" of the external fields entering in the amplitude. In the case of forward Compton scattering, the growth with external photon mass of the nth moment of a structure function with respect to the scaling variable $x=-q^2/2\nu$ turns out to be governed by the anomalous dimension γ_n of the angular momentum n member of the series of operators of "twist" two (twist equals canonical dimension minus spin). Bjorken scaling means that γ_n is zero for *all n* since if the structure function has a limit, then so do all its moments. The anomalous dimensions γ_n are *a priori* independent functions of the coupling constant and the problem for the renormalization-group approach is to explain why an infinite set of them should simultaneously vanish.

Typically, the twist-two operators are bilinear in the fundamental fields of the theory. For example, in scalar field theory one is concerned with the series $A^{(n)}_{\mu_1\cdots\mu_n}=\phi\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi$. If one could ignore the subtractions needed to define $A^{(n)}$, its anomalous dimension would be just 2γ, where γ is the anomalous dimension of the fundamental field ϕ. This would be doubly interesting, for, first, one would be able to assign a phenomenological significance to the otherwise unmeasurable but fundamentally important anomalous dimension γ, and second, one would need only to control, via the renormalization group, one anomalous dimension in order to recover electroproduction scaling.

Of course, the subtractions needed to define $A^{(n)}$ are important and, in general, destroy the naive connection between γ_n and γ. Parisi, however, has suggested that the naive connection is reestablished in the limit of large angular momentum for $A^{(n)}$, i.e., that $\gamma_n\to 2\gamma$ as $n\to\infty$ (Ref. 1). The nonrigorous argument for this would be that in the limit of infinite n, the infinite number of derivatives in $\phi\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi$ effectively separate the space-time arguments of the two fields so that no subtractions are needed to define $A^{(\infty)}$ beyond those needed to define ϕ itself. Our purpose in this paper is to show directly that in selected renormalized field theories Parisi's conjecture is correct and to explore the consequences of this result for the renormalization-group approach to scaling. Most importantly we show that for such theories Bjorken scaling implies the existence of an ultraviolet-stable fixed point at the origin of coupling-constant space.

In Sec. II we will review the salient features of the renormalization-group approach. In Sec. III we will carry out the proof of Parisi's conjecture for scalar field theories (discussion of more complicated theories will, in the interests of readability, be relegated to appendixes). In Sec. IV we will discuss in detail the rather surprising consequences of this theorem.

II. THE OPERATOR-PRODUCT EXPANSION AND THE RENORMALIZATION GROUP

Deep-inelastic lepton-hadron scattering is most naturally analyzed with the aid of Wilson's operator-product expansion.[2] This expansion expresses a product of local operators in terms of a series of local operators of decreasing importance at short distances. Indeed to any finite order in perturbation theory the product of two currents (for simplicity we shall discuss scalar currents) is given by

$$J(x)J(-x)\approx\sum_n C_n(x,g)O_n(0)\,, \tag{2.1}$$

where O_n are a complete set of local operators and

C_n are functions of x_μ and the coupling constants (represented here by g). The utility of this expansion is based on the fact that, to any finite order in perturbation theory, the small-distance behavior of $O_n(x)$ is determined, up to logarithms, by the canonical (naive) dimensions of the operators J and O_n; i.e.,

$$C_n(x, g) \underset{x_\mu \to 0}{\simeq} |x|^{2d_J - d_n} f(\ln x^2, g) . \qquad (2.2)$$

The above result is of little use unless one can say something about the logarithmic corrections to $C_n(x)$. When one sums to all orders in perturbation theory, these logarithms could change the naive power behavior of $C_n(x)$. Wilson[2] has argued that the net effect of the logarithms is merely to change the value of the dimensions of most operators, but that the singularity of $C_n(x)$ is still determined by the dimensions of J and O_n, which however are not given by their canonical values but are "anomalous."

That this can occur can be seen by analyzing the operator-product expansion with the aid of renormalization-group techniques. The renormalization-group equations are most simply derived from the Callan-Symanzik equations.[3] Consider (for simplicity) a $g\phi^4$ field theory. The one-particle irreducible n-point Green's functions $\Gamma^{(n)}(p_1, \ldots, p_n)$ satisfy

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + n\gamma(g)\right] \Gamma^{(n)}(p_1, \ldots, p_n; g) = \Gamma_\theta^{(n)}(0; p_1, \ldots, p_n; g), \qquad (2.3)$$

where $\beta(g)$ and $\gamma(g)$ are finite functions of g, μ is the scalar mass, and $\Gamma_\theta^{(n)}(0; \ldots)$ is the n-point Green's function with the insertion of the operator $\theta = \mu^2\phi^2$ with zero momentum.

These equations are useful in analyzing the large- (Euclidean) momentum behavior of Green's functions. This is because, to any finite order in perturbation theory, the right-hand side of (2.3) can be neglected if one uses Weinberg's theorem. Specifically, if $p_i = \lambda q_i$, $q_i^2 < 0$, then, if one defines $\Gamma_{\text{ASY}}^{(n)}(p_i)$ to be the sum of all leading terms in $\Gamma^{(n)}(p_i)$ as $\lambda \to \infty$ [neglecting terms which decrease by powers of λ relative to $\Gamma_{\text{ASY}}^{(n)}(p_i)$], we have

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + n\gamma(g)\right] \Gamma_{\text{ASY}}^{(n)}(p_i; g) = 0 . \qquad (2.4)$$

These are the renormalization-group equations, first derived for quantum electrodynamics (QED) by Gell-Mann and Low.[4] If we define the effective coupling constant $\bar{g}(g, t)$ by

$$\frac{d\bar{g}}{dt} = \beta(\bar{g}), \quad \bar{g}(g, 0) = g, \quad t = \ln\lambda , \qquad (2.5)$$

then the explicit solution of (2.4) is

$$\Gamma_{\text{ASY}}^{(n)}(\lambda q_i, g) = \lambda^{4-n} \Gamma_{\text{ASY}}^{(n)}(q_i; \bar{g}(g, t)) \times \exp\left[-n \int_0^t dt' \, \gamma(\bar{g}(g, t'))\right] , \qquad (2.6)$$

where we have used dimensional analysis:

$$\Gamma_{\text{ASY}}^{(n)}(p_i, g) = \mu^{4-n} f^{(n)}\left(\frac{\lambda q_i}{\mu}, g\right). \qquad (2.7)$$

As Gell-Mann and Low[4] discovered and Wilson has emphasized recently,[5] (2.6) has a very simple asymptotic form if β has a zero [a fixed point of the renormalization-group equation (2.5)]:

$$\beta(g_f) = 0 \text{ and } \beta'(g) < 0 . \qquad (2.8)$$

In that case if g is sufficiently close to g_f then

$$\bar{g}(g, t) \underset{t = \ln\lambda \to \infty}{\longrightarrow} g_f , \qquad \int_0^t dt' \, \gamma(\bar{g}(g, t')) \underset{t \to \infty}{\simeq} t\gamma(g_f) , \qquad (2.9)$$

so that

$$\Gamma_{\text{ASY}}^{(n)}(\lambda q_i; g) \underset{\lambda \to \infty}{\simeq} \lambda^{4-n-n\gamma(g_f)} \Gamma_{\text{ASY}}^{(n)}(q_i; g_f) \times \exp\left\{-n \int_0^\infty dt [\gamma(\bar{g}(g, t)) - \gamma(g_f)]\right\} . \qquad (2.10)$$

This is the asymptotic behavior we would expect if the scalar field ϕ had dimension $d = 1 + \gamma(g_f)$, so that $\gamma(g_f)$ is the anomalous dimension of ϕ. Thus the asymptotic behavior of the theory is controlled by the fixed points of the renormalization group. If $\beta'(g_f) < 0$, then the fixed point is relevant only when $t \to -\infty$ and is therefore called an infrared- (IR) stable fixed point, whereas if $\beta'(g_f) < 0$ then g_f is an ultraviolet- (UV) stable fixed point.

The above analysis is easily generalized to discuss the ultraviolet behavior of more complicated renormalizable field theories involving many fields and coupling constants. The equations then involve in place of $\beta(g)\partial/\partial g$ a sum of derivatives $\beta_i(g_1, \ldots, g_n)\partial/\partial g_i$ for each independent coupling constant g_i. The fixed points of the renormalization group, which determine the asymptotic behavior of the theory, will be determined by the simultaneous vanishing of all the β_i.

The renormalization group can be applied to the operator-product expansion.[6] The functions $C_n(x)$ will satisfy an equation similar to (2.4):

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_n(g) - 2\gamma_J(g)\right] C_n(x, g) = 0 , \qquad (2.11)$$

where γ_J (γ_n) is the anomalous dimension of the operator J (O_n). The solution of this equation, as previously, determines *small-x* behavior of $C_n(x,g)$ to be

$$C_n(x, g) \underset{x_\mu \to 0}{\simeq} |x|^{d_n(g_f) - 2d_J(g_f)} , \qquad (2.12)$$

where $d_J(g_f)$ $[d_n(g_f)]$ is the dimension of J $[O_n]$ at the UV-stable fixed point g_f; i.e., the canonical dimension of the operator plus $\gamma_J(g_f)$ $[\gamma_n(g_f)]$.

The structure functions of deep-inelastic lepton-nucleon scattering probe the product of electromagnetic currents near the light cone. When one uses the operator-product expansion[7] to analyze these structure functions, one finds that an infinite tower of operators contributes in the Bjorken scaling region. The dominant operators are those that have smallest "twist" (dimension minus spin) and thereby contribute to the leading light-cone singularity of the product of currents. Thus in a $g\phi^4$ theory the dominant operators are those with canonical twist $(2+n)-n=2$:

$$O^{(n)}_{\mu_1\cdots\mu_n}(x)=\phi^*(x)\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi(x),$$

and the operator-product expansion of electromagnetic currents has the form (to any finite order of perturbation theory)

$$J_\mu(x)J_\nu(-x)\underset{x^2\sim 0}{\sim}\sum_n C_n(x^2,g)x^{\mu_1}\cdots x^{\mu_n}O^{(n)}_{\mu_1\cdots\mu_n}(0)\times\partial_\mu\partial_\nu(x^2+i\epsilon x_0)^{-1}, \qquad (2.13)$$

where $C_n(x^2,g)$ naively is finite as $x^2\to 0$, but actually satisfies

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_n(g)\right]C_n(x^2,g)=0. \qquad (2.14)$$

Note that the anomalous dimension of the electromagnetic current J_μ is identically zero since the conserved charge $Q=\int d^3x J_0(x)$ must be dimensionless.

The Fourier transform of $C_n(x^2,g)$ is measured, up to an unknown constant determined by the matrix element of $O^{(n)}(0)$, by the moments of the current-hadron structure functions in the Bjorken scaling limit.

As we have seen, the small-x^2 behavior of $C_n(x^2)$ is determined by a UV-stable fixed point of the renormalization group (say at g_f), so that

$$C_n(x^2)\underset{x^2\to 0}{\sim}(x^2)^{\gamma_n(g_f)/2}. \qquad (2.15)$$

This implies for the now-familiar structure function $\nu W_2(q^2,\nu)=F_2(q^2,\omega=-q^2/2\nu)$ that

$$\int_0^1 d\omega\,\omega^{n-2}F_2(q^2,\omega)\underset{q^2\to-\infty}{\sim}(-q^2)^{-\gamma_n(g_f)/2}. \qquad (2.16)$$

Bjorken scaling is the hypothesis that the structure function $F_2(q^2,\omega)$ has a finite limit as $q^2\to-\infty$. This implies that all the anomalous dimensions of the twist-two operators must vanish, i.e., $\gamma_n(g_f)=0$. This would appear, at first hand, to be very unlikely. With the exception of the energy-momentum tensor, none of the operators $O^{(n)}$ is constrained to have canonical dimensions. Thus $\gamma_2\equiv 0$ but γ_n $(n>2)$ will in general be nonzero. Furthermore the requirement that all the γ_n vanish seems exceedingly unlikely since it would appear to impose an infinite set of conditions on functions of a fixed parameter g_f.

On the other hand, the anomalous dimensions are not totally unrelated. The positivity of $F_2(q^2,\omega)$ ensures that γ_n increases with n:

$$0=\gamma_2\leqslant\gamma_3\leqslant\gamma_4\leqslant\cdots\leqslant\gamma_n. \qquad (2.17)$$

In fact one can prove in addition that the γ_n satisfy the inequality

$$\frac{m}{m+2}\leqslant\frac{(\gamma_{n+m}-\gamma_n)}{(\gamma_{n+m+2}-\gamma_n)} \qquad (2.18)$$

as a consequence of the positivity of F_2.[8] Therefore if in addition to γ_2 any additional γ_k vanishes, then (2.17) implies that all γ_i vanish for $i<k$ and (2.18) implies that all γ_k vanish for $i>k$. Therefore to achieve Bjorken scaling (or to test it experimentally) it is sufficient that two operators of twist two have canonical dimensions. One of these is automatically provided by the energy-momentum tensor, the other must be a consequence of the structure of the theory since we do not have the freedom to adjust g_f.

There is, in any theory, a fixed point of the renormalization group at which all anomalous dimensions vanish. This is the fixed point where all the coupling constants vanish. In most theories this is *not* a UV-stable fixed point. In the following we shall prove, for a large class of theories, that this is the only fixed point for which $\gamma_n=0$, i.e., for which one can have Bjorken scaling.

III. LARGE-SPIN BEHAVIOR OF ANOMALOUS DIMENSIONS

A naive consideration of the composite operator $O^{(n)}=\phi^*\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi$ (in a $g\phi^4$ theory) would lead one to the conclusion that its dimension is simply given in terms of the dimension of the field ϕ, i.e., $d_n=n+2d_\phi$, so that the anomalous dimension would satisfy

$$\gamma_n(g)=2\gamma_\phi(g). \qquad (3.1)$$

This is of course false, since the composite operator involves a singular product of fields at the same point and thus requires subtractions which might (and indeed do) spoil the naive additivity of dimensions. One might expect intuitively that as n becomes very large, the increasing number of derivatives act effectively to separate the two fields, thus reinstating the additivity. Thus one might conjecture that

$$\lim_{n\to\infty}\gamma_n(g)=2\gamma_\phi(g). \qquad (3.2)$$

Indeed Parisi has recently argued that at a fixed point of the renormalization group ($g=g_f$) the above relation is true in a $g\phi^4$ theory. In the following we shall prove that (3.2) is true, to all orders in perturbation theory, for any value of g.

To illustrate how the theorem works, we shall first consider the anomalous dimensions γ_n of the series of operators of lowest twist (= dimension − spin) in scalar field theory. The lowest twist in this theory is 2 and the corresponding series of operators is

$$O^{(n)}_{\mu_1\cdots\mu_n} = \frac{i^n}{2^n}\,\phi^*\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi - (\text{trace terms}).$$

The trace terms are chosen to make $O^{(n)}$ symmetric and traceless and therefore pure spin n. To make $O^{(n)}$ well defined, one has to specify how its matrix elements are subtracted. Because $O^{(n)}$ is pure spin n, only the two-particle matrix element is divergent and it requires only one subtraction. We will perform the needed subtractions at zero four-momentum according to the usual Bogolubov-Parasiuk-Hepp (BPH) procedure.

The anomalous dimensions of operators other than the fundamental field ϕ itself are defined by the scaling Ward identities of the theory. If for a general operator A_i we denote the n-particle one-particle-irreducible (1PI) Green's function by $\Gamma^{(n)}_{A_i}(k;p_1,\ldots,p_n)$, then these Ward identities have the form

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+n\gamma(g)\right]\Gamma^{(n)}_{A_i}(k;p_1,\ldots,p_n) - \sum_j \gamma_{ij}(g)\Gamma^{(n)}_{A_j}(k;p_1,\ldots,p_n) = i\Gamma^{(n)}_{\theta A_i}(0,k;p_1,\ldots,p_n), \quad (3.3)$$

where the sum over j is in general over all operators A_j of canonical dimension less than or equal to that of A_i and of the same spin as A_i, θ is the mass insertion, γ is the anomalous dimension of the field ϕ, γ_{ij} is an anomalous dimension matrix for the operators A_i, and $\beta(g)$ is the same eigenfunction of the coupling constant as appears in the Ward identity for the particle Green's functions of the theory.

If A_i is in the lowest-twist series, then this equation simplifies considerably: An operator of the same spin as A_i cannot have lower dimension without being also of lower twist, which is impossible. Furthermore, in a simple scalar field theory, the operator of lowest twist and given spin is unique. Then the scaling equation for $O^{(n)}$ simplifies to

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-n\gamma(g)+\gamma_m(g)\right]\Gamma^{(n)}_{O^{(m)}}(k;p_1,\ldots,p_n) = -i\Gamma^{(n)}_{\theta O^{(m)}}(0,k;p_1,\ldots,p_n), \quad (3.4)$$

with γ_n having the meaning of anomalous dimension of $O^{(n)}$.

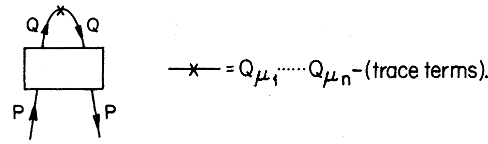

FIG. 1. Typical contribution to $\Gamma^{(2)}_{\theta O^{(n)}_{\mu_1\cdots\mu_k}}(0,0;p)$.

Let us now look at this equation for $n=2$ and $k=0$. We define "form factors" by

$$\Gamma^{(2)}_{O^{(n)}_{\mu_1\cdots\mu_n}}(0;p) = F^{(n)}(p^2)\{p_{\mu_1}\cdots p_{\mu_n} - (\text{trace terms})\}, \quad (3.5)$$

$$\Gamma^{(2)}_{\theta O^{(n)}_{\mu_1\cdots\mu_n}}(0;p) = F^{(n)}_\theta(p^2)\{p_{\mu_1}\cdots p_{\mu_n} - (\text{trace terms})\}.$$

The scaling Ward identity then reads

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_n(g)-2\gamma(g)\right]F^{(n)}(p^2) = -iF^{(n)}_\theta(p^2). \quad (3.6)$$

The subtraction procedure we have adopted is such that at $p=0$, $F^{(n)}$ takes on its free-field-theory value, $F^{(n)}(0)=1$. Therefore

$$\gamma_n - 2\gamma = -iF^{(n)}_\theta(0). \quad (3.7)$$

$F^{(n)}_\theta$ is a matrix element which needs no subtraction and we therefore have no prior knowledge of its value. We will now show that it has simple behavior for $n\to\infty$, allowing us to say something about γ_n for large n.

To compute $F^{(n)}_\theta$ one writes down all the graphs for $\Gamma^{(2)}_{\theta O^{(n)}}(00;p)$. The typical graph has the structure given in Fig. 1, where the box is a product of vertices, scalar propagators, and one mass insertion. The product of propagators can be simplified by the usual Feynman parameter techniques to the form

$$C\int\prod_{i=1}^{L}dl_i\prod_{i=1}^{N}d\alpha_i\,\delta\left(1-\sum^N\alpha_i\right)\left[\sum^N\alpha_i(q_i^2-m_i^2)\right]^{-N}\{q_{\mu_1}\cdots q_{\mu_n}-(\text{trace terms})\}, \quad (3.8)$$

where the $\{l_i\}$ are a set of independent loop momenta and q_i is the momentum of the ith internal line. This is further simplified by choosing each q_i to be an appropriate sum of loop momenta plus a finite piece p_i and requiring that the combined denominator have no terms linear in the loop momenta. It is well known that the p_i are then determined in terms of the external momenta by the Kirchoff circuit conditions, treating α_i as the resistance and p_i as the current of the ith line. Since there is only one external momentum p, one has $p_i=\phi_i(\alpha_i)p$. Consequently Eq. (3.8) can be further simplified to

$$C\int\prod_{i=1}^{L} dl_i \prod_{j=1}^{N} d\alpha_j\, \delta\Big(1-\sum^{N}\alpha_i\Big)\Big[\sum \eta_{ij} l_i\cdot l_j + A(\alpha)p^2+B(\alpha)\Big]^{-N}\Big(\sum \epsilon_j l_j+\phi(\alpha)p\Big)_{\mu_1}\cdots\Big(\sum \epsilon_j l_j+\phi(\alpha)p\Big)_{\mu_n}, \tag{3.9}$$

where $(\sum\epsilon_j l_j+\phi(\alpha)p)$ is just the momentum q chosen consistent with the Kirchoff conditions ($\epsilon_j=\pm1, 0$ according to whether and in what direction l_j flows through q).

We require the spin-n part of this amplitude and so must read off the coefficient of $p_{\mu_1}\cdots p_{\mu_n}$. It is clear from the structure of Eq. (3.9) that powers of p_μ cannot arise from the integration over loop momenta, and the coefficient of $p_{\mu_1}\cdots p_{\mu_n}$ must therefore be

$$C\int\prod_{i=1}^{N} d\alpha_i\, \delta\Big(1-\sum^{N}\alpha_i\Big)\phi^n(\alpha)\int\prod_{i=1}^{L} dl_i\Big(\sum\eta_{ij}l_i\cdot l_j+Ap^2+B\Big)^{-N}. \tag{3.10}$$

We recall that $\phi(\alpha)$ is the fraction of the external momentum p, which flows through the internal line q if the flow is computed according to Kirchoff conditions, taking the α_i as the resistances of the corresponding lines. But in a two-terminal network with positive resistances (all $\alpha_i>0$), it is intuitively obvious and not difficult to demonstrate that the current flowing in a given internal line is always less than or equal in magnitude to the external current. In other words, $|\phi(\alpha)|\le 1$. Therefore in the limit $n\to\infty$, the integrand of Eq. (3.10) vanishes except for those points in α space, where $|\phi(\alpha)|=1$. If $|\phi(\alpha)|=1$, all the external momentum flows through the line q and the circuit analogy implies that the line q must be connected to the external lines of the graph by a short circuit. This means that some subset of the α's, including the α's of the two lines which connect to $O^{(n)}$ must vanish. Therefore $|\phi(\alpha)|=1$ only in certain "corners" of the domain of integration of the α_i, and the volume in α space associated with the points where $|\phi(\alpha)|=1$ is zero. Thus as $n\to\infty$ the integrand in Eq. (3.10) will vanish.

To estimate how fast it vanishes, one notes that the integral to be done has the structure

$$\int_0^1 d\beta\,\beta^n(1-\beta)^{k-1}\sim\frac{1}{n^k},$$

where k is the minimal number of internal lines needed to get a short circuit including the insertion of $O^{(n)}$. The factor $(1-\beta)^{k-1}$ is the volume element in α space associated with the set of points such that $1-|\phi|=1-\beta\approx 0$. Clearly the minimum number of lines in the short circuit is two, so the behavior for large n must be at least $1/n^2$.

We so far have pretended that all the loop momentum integrations converge so that we may discuss just the Feynman parameter integration without worrying about the need for renormalization. However, if we regulate by the Pauli-Villars technique, it is apparent that all the extra contributions associated with regulators and subtraction terms have the same structure in Feynman parameter space, so that the combined object which has no divergent loop integrations is subject to the same argument concerning its large-n behavior. Therefore $\lim_{n\to\infty}F_\theta^{(n)}=0$ and we conclude that $2\gamma-\gamma_n\to_{n\to\infty}0$. Therefore, in the limit of large angular momentum the anomalous dimension of the lowest-twist operator becomes equal to twice the anomalous dimension of the fundamental field.

The above theorem can be generalized to theories involving renormalizable couplings of fermion and scalar fields. The analysis, although complicated by the occurrence of more than one operator of lowest twist, and nontrivial numerators in the Feynman graphs, proceeds essentially as above. The proof for a Yukawa-like theory is given in Appendix A. The only remaining renormalizable theories are (a) theories involving a massive vector meson coupled to a conserved current (vector-gluon theories); (b) gauge theories of the Abelian (QED) or Yang-Mills variety.

We conjecture that an analogous theorem holds for vector-gluon theories. In Appendix B we discuss this more fully. In the case of non-Abelian gauge theories it is clear that such a theorem cannot hold. To see this it is sufficient to consider second-order perturbation theory, in which the anomalous dimensions of all the fields are gauge-dependent, whereas the γ_n's are independent of the gauge.

IV. CONSEQUENCES OF THE THEOREM

In this section we shall prove, for the class of theories enumerated above, that Bjorken scaling implies that the origin of coupling-constant space is a UV-stable fixed point of the renormalization group. We shall then explore the consequences of such an occurrence for the phenomonology of deep-inelastic scattering.

Consider a theory in which one has Bjorken scaling. Under the assumption that the asymptotic behavior of the theory is identical with that of the leading powers of finite-order perturbation theory, one can use the renormalization group equations described previously. Bjorken scaling then means (i) the existence of a UV-stable fixed point determined by the vanishing $\beta_i(g_1, \ldots, g_N)$, $i = 1 \ldots N$, where β_i is the coefficient of $\partial/\partial g_i$ in the renormalization-group equations and g_i $(i = 1 \ldots N)$ are the independent coupling constants of the theory; (ii) the anomalous dimensions of the dominant operators (twist two) on the light cone all vanish.

We make the further assumptions that these operators are identical with those that dominate the light-cone behavior to any finite order in perturbation theory. Let us consider specifically a $g\phi^4$ theory. In that case we are assuming that the twist-two operators are given by $O^{(n)} = \phi^* \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_n} \phi$ as perturbation theory would indicate. We are excluding the possibility that "dimension crossing" occurs, that is, that an operator such as $(\phi^* \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu \phi)\phi\phi$ acquires an anomalous dimension such that its total dimension is equal to that of $\phi \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu \phi$.

With the aid of the theorem proved above we learn that Bjorken scaling implies that the anomalous dimension of the field Φ, at the fixed point $g = g_f$, vanishes. This then implies that g_f must be zero. To prove this we consider the $g\phi^4$ theory with massless scalar fields, where the coupling constant (defined to be the 4-point Green's function at some arbitrary Euclidean point) is set equal to g_r. In such a theory the scalar propagator $D(p^2, \mu^2)$ (renormalized at some Euclidean point $p^2 = -\mu^2$) satisfies the renormalization-group equation

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g_f)\frac{\partial}{\partial g} - 2\gamma(g_f)\right] D(p^2, \mu^2, g) = 0,$$

which expresses the fact that a change of the (arbitrary) renormalization point (μ) can be absorbed by a change in the renormalized coupling constant and in the scale of the field. Since $\beta(g_f) = \gamma(g_f) = 0$ and $D(p^2, \mu^2, g^2) = (i/p^2) f(p^2/\mu^2)$, we deduce that

$$D(p^2, \mu^2, g_f) = \frac{i}{p^2} . \tag{4.1}$$

We then invoke the Federbush-Johnson theorem,[9] which informs us that if the full propagator is equal to the free propagator then the theory is a free-field theory. In particular the connected 4-point function vanishes identically and therefore $g_f = 0$.

Thus we have proved, within the framework of the renormalization-group approach, that Bjorken scaling can occur in a $g\phi^4$ theory *only if* $g = 0$ is a UV-stable fixed point. It is easy however to show that although $g = 0$ is a fixed point, it is not UV stable. To investigate the UV stability of the origin it is sufficient to consider Eq. (2.5) for $\bar{g} \approx 0$. In that case we can restrict attention to the lowest-order term in β, which is $\beta(g) = 3g^2/8\pi^2$, so that for g sufficiently small Eq. (2.5) is

$$\frac{d\bar{g}}{dt} = \frac{3}{8\pi^2} \bar{g}^2 . \tag{4.2}$$

Thus $\bar{g}$ increases as t increases and we are driven away from the origin.[10] Therefore, Bjorken scaling cannot occur in a pure $g\phi^4$ scalar field theory.

The analysis of theories involving many fermion scalar fields and correspondingly many coupling constants is more complicated. One way of achieving Bjorken scaling is for the anomalous dimensions of all twist-two operators to vanish at the fixed point. Since we have proved (in Appendix A) that these anomalous dimensions are equal, for large spin, to twice the anomalous dimensions of the fields from which they are constructed, the anomalous dimensions of all the fields must vanish. The Federbush-Johnson theorem then implies that *all* renormalized coupling constants must vanish at the fixed point. In other words, Bjorken scaling implies that the relevant fixed point is the origin in coupling-constant space. To investigate whether or not this is possible, one merely has to calculate all the β functions to lowest order in perturbation theory and check whether $g_i = 0$ is an attractive (UV-stable) fixed point of the equations:

$$\frac{d\bar{g}_i}{dt} = \beta(\bar{g}_1, \ldots, \bar{g}_n) . \tag{4.3}$$

Zee has shown that in Yukawa theories invariant under a general Lie group and having a single coupling constant, the origin is UV-unstable.[11] In addition Coleman and one of us (D.G.) have investigated fermion-scalar theories involving an arbitrary number of independent coupling constants.[12] This investigation showed that no renormalizable theory of fermions and scalars can be ultraviolet stable at the origin.

This however would not definitely exclude the possibility of Bjorken scaling holding for such theories. This is because it is possible that only some of the naive twist-two operators have canonical dimension while others develop anomalous dimensions and do not contribute to the light-cone behavior. Specifically, if the fermion operators (generically $\bar{\psi}\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2} \cdots \overleftrightarrow{\partial}_{\mu_n}\psi$) have canonical dimensions at the fixed point, while the scalar operators (generically $\phi^* \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_n} \phi$) do not, then one can only conclude that the *fermion* fields have canonical dimension. This would then imply that the Yukawa coupling constants vanish at the fixed point, but not that the quartic scalar couplings vanish. This interesting possibility is not one that can be explored in perturbation theory since one would have to compute the β functions of the Yukawa coupling constants to all orders in the quartic scalar meson coupling constants.

In a theory involving a massive vector gluon coupled to a conserved Abelian current, the behavior of the anomalous dimension of the twist-two operators for large spin is much harder to analyze. In Appendix B we discuss such theories and conjecture, on the basis of a second-order calculation, that the anomalous dimension of the dominant twist-two operator approaches, for large spin, twice the anomalous dimension of the gluon field. If this is so, then once again we conclude that Bjorken scaling implies that the origin in coupling-constant space is a UV-stable fixed point. This possibility is easy to exclude by explicit calculation. Thus if our conjecture is correct, vector-gluon theories cannot exhibit Bjorken scaling.

At this point it is appropriate to recapitulate. Having realized that naive scaling of a renormalizable field theory is broken by logarithms in perturbation theory we use the renormalization-group equations to sum the logarithms. Scaling is then recovered if these equations possess a fixed point. To achieve Bjorken scaling the anomalous dimension of the dominant twist-two operators must vanish, implying for a large class of theories, that the fixed point must be at the origin of coupling-constant space. This possibility is then shown by simple calculation to be exceedingly unlikely in these theories.

Recently it has been shown that for non-Abelian gauge theories, zero coupling is UV-stable fixed point of the renormalization group.[13,14] Some of these theories can even contain fermions and scalar mesons.[14,15] The arguments of this paper suggest that *Bjorken scaling (if it persists as an experimental fact of life) might force one to non-Abelian gauge theories of the strong interactions.*

Let us now discuss the implications of an assumed fixed point at zero coupling constants for the phenomenological description of deep-inelastic scattering. Of course the detailed structure of the operator-product expansion will depend on the theory under consideration—however, certain features will be shared by all such theories.[16]

First we note that in such asymptotically free theories *naive scaling is violated by calculable logarithmic terms*. This occurs because the fixed point at zero coupling constant is a double zero of β. The generic renormalization-group equation for a trilinear coupling constant g is

$$\frac{d\bar{g}}{dt}=\beta(\bar{g})\approx-2b\bar{g}^3, \qquad (4.4)$$

where we set $b>0$ so that the origin is UV-stable. In that case we have

$$\bar{g}^2(t)=\frac{g^2}{1+bg^2t} \qquad (4.5)$$

so that as $t=\ln\lambda\to+\infty$, $\bar{g}^2(t)\to 1/bt$; i.e., approaches zero as the inverse of the logarithm of the scale of the momenta. This is too slow for the integral in Eq. (2.10) to converge, since anomalous dimensions will (in general) be of order g^2 for small g. Therefore the coefficient $C_{A,B,C}(x,g)$ of the operator C in the expansion of $A(x)\cdot B(-x)$ will behave, when $x_\mu\to 0$, like

$$C_{A,B,C}(x,g)\underset{x_\mu\to 0}{\sim}(\ln x)^{(\hat{\gamma}_C-\hat{\gamma}_A-\hat{\gamma}_B)/b}, \qquad (4.6)$$

where γ_A is defined to be equal to

$$\frac{d^2}{dg^2}\gamma_A(g^2)\bigg|_{g^2=0},$$

and $\gamma_A(g^2)$ is the anomalous dimension of $A(x)$:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+n\gamma(g)+\gamma_A(g)\right]\Gamma^{(n)}_A=-i\Gamma^{(n)}_{A\theta}\,. \qquad (4.7)$$

These logarithmic deviations from canonical scaling can therefore be determined by calculating γ and β to the lowest nontrivial order in perturbation theory. The coefficients of $\ln x$ will be independent of the physical coupling constants and determined only by the structure of the theory.

Certain naive scaling results will be preserved due to the fact that the anomalous dimensions of conserved or partially conserved currents are zero. For example, consider the vacuum expectation value of the commutator of electromagnetic currents:

$$\int d^4x\, e^{iq\cdot x}\langle 0|[J_\mu(x),J_\nu(0)]|0\rangle = \epsilon(q_0)(g_{\mu\nu}q^2-q_\mu q_\nu)\rho(q^2). \qquad (4.8)$$

The total annihilation cross section for $e^+e^-\to$ hadrons is proportional to ρ (where $\sqrt{q^2}$ = center-of-mass energy of the leptons). The renormalization group tells us that the leading powers in $\rho(q^2)$,

$$\rho(q^2)\to\rho_0\left(\ln\frac{q^2}{\mu^2}\right)+\frac{\mu^2}{q^2}\rho\left(\ln\frac{q^2}{\mu^2}\right)+\cdots,$$

sum to give

$$\rho_0\left(\ln\frac{q^2}{\mu^2},g\right)=\rho_0\left(1,\bar{g}\left(\ln\frac{q^2}{\mu^2}\right)\right) \underset{q^2\to-\infty}{\sim}\rho_0(1,0)+O\left(\frac{1}{\ln(q^2/\mu^2)}\right). \qquad (4.9)$$

This means that the commutator approaches logarithmically, for large q^2, its free-field-theory value, and that

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons})_{q^2 \to -\infty} \left(\sum Q_i^2\right)\sigma(e^+e^- \to \mu^+\mu^-)\,, \tag{4.10}$$

where Q_i is the charge of the ith fundamental field (parton).

Let us now discuss the expansion of the product of vector or axial-vector [SU(3)×SU(3)] currents near the light cone. The dominant terms in this expansion are

$$J_\mu^a(x)J_\nu^b(-x)\underset{x^2\to 0}{\longrightarrow}\sum_n C_{\mu\nu\alpha}^{n,abc}(x,g)x^{\mu_1}\cdots x^{\mu_n}\Theta_{\alpha\mu_1\cdots\mu_n}^{n,c}(0)\,, \tag{4.11}$$

where $\Theta_{\alpha\mu_1\cdots\mu_n}^{(n+1)}$ are the twist-two local operators in the theory [generically $\bar\psi\gamma_\alpha\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\lambda^a(1\pm\gamma_5)\psi$]. The naive parton-model results[7] are derived by assuming that the coefficient of $\Theta^{(n)}$, $C_{\mu\nu\alpha}^{n,abc}(x,g)$, is proportional to its free-field-theory value, i.e., $C_{\mu\nu\alpha}^{n,abc}(x,0)$, as $x^2\to 0$. In our case this is not strictly true since $\Theta^{(n)}$ has an anomalous dimension γ^n. In that case

$$C_{\mu\nu\alpha}^{n,abc}(x,g)\underset{x^2\to 0}{\sim}C_{\mu\nu\alpha}^{n,abc}(x,0)\exp\left[\int_0^{\ln(x^2\mu^2)}dt\,\gamma_n(\bar g(g,t))\right]. \tag{4.12}$$

This, however, does not modify the naive parton or light-cone model results, since the tensorial and SU(3)×SU(3) structure is not affected. The net effect of the logarithmic corrections is that the nth moments of the structure functions do not scale (except for $n=2$, where $\gamma^{(2)}=0$ since $\Theta^{(2)}$ is the energy-momentum tensor). Instead

$$\int_0^1 d\omega\,\omega^n F(\omega,q^2)_{q^2\to-\infty}\!\!\sim C_n\left(\ln\frac{q^2}{\mu^2}\right)^{-\hat\gamma_n/2b}, \tag{4.13}$$

where

$$\hat\gamma_n=\frac{d^2}{dg^2}\gamma_n(g^2)\bigg|_{g^2=0},$$

but the relations between various moments of different structure functions[7] are preserved. Thus, for example, in a theory in which the constituents of the electromagnetic current are spin-$\frac12$, the ratio of the moments of the longitudinal and transverse structure functions R_n all vanish, albeit logarithmically:

$$R_n(q^2)=\frac{\int_0^1 d\omega\,\omega^n F_L(q^2,\omega)}{\int_0^1 d\omega\,\omega^n F_T(q^2,\omega)}\ \underset{q^2\to-\infty}{\sim}\ \frac{1}{\ln(-q^2/\mu^2)}\,. \tag{4.14}$$

The end result is perhaps disappointing. Initially we demanded that Bjorken scaling not be violated by powers of q^2 and we were almost inevitably led to theories with a UV-stable fixed point at the origin. In such theories Bjorken scaling is indeed violated only by powers of $\ln q^2$. However, the approach to scaling is only logarithmic, and the naive parton-model results apply only when $\ln q^2 \gg 1$. Since it is rather hard to believe that present experiments are in this region, one still does not understand the presently observed precocious scaling. Whether or not the present data are consistent with logarithmic variation in q^2 remains to be seen. Since the precise power of $\ln q^2$ with which each moment of the structure function grows is calculable and theory-dependent, such growth, if observed, would provide a sensitive test of particular theories.

APPENDIX A: YUKAWA THEORIES

When we consider theories with fermions, the argument becomes more complicated. In the discussion of scalar-meson theory, the crucial point was that certain two-particle matrix elements of the twist-two operator of spin n vanish in the limit $n\to\infty$. If we refer to the discussion following Eq. (3.7) we see that we made essential use of the scalar nature of the internal propagators. If fermions are present, this is no longer true and the argument has to be modified.

For definiteness let us take a model of a single Fermi field and a single scalar field interacting via a Yukawa coupling. There are now two series of operators having twist two at the level of naive dimension counting:

$$A_{\mu_1\cdots\mu_n}^{(n)}=\frac{i^n}{n2^n}\{\bar\psi\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_n}\psi + \text{permutations} - \text{trace terms}\},$$

$$B_{\mu_1\cdots\mu_n}^{(n)}=\frac{i^n}{2^n}\{\phi\overleftrightarrow{\partial}_{\mu_1}\cdots\overleftrightarrow{\partial}_{\mu_n}\phi-\text{trace terms}\}\,. \tag{A1}$$

Let us use the notation $\Gamma_O^{(r,s)}$ to indicate a Green's function involving r fermions and/or antifermions, s bosons, and one insertion of the operator O. Then the scaling equation has the form

$$\begin{aligned}\left(\mu\frac{\partial}{\partial\mu}+\sum\beta_i\frac{\partial}{\partial g_i}-r\gamma_\psi-s\gamma_\phi\right)&\Gamma_{A(n)}^{(r,s)}(k;p_1,\ldots,p_{r+s})\\ &+\gamma_{AA}^n\Gamma_{A(n)}^{(r,s)}(k;p_1,\ldots,p_{r+s})\\ &+\gamma_{AB}^n\Gamma_{B(n)}^{(r,s)}(k;p_1,\ldots,p_{r+s})\\ &=-i\Gamma_{\theta A(n)}^{(r,s)}(0k;p_1,\ldots,p_{r+s})\,,\end{aligned} \tag{A2}$$

where we write $\sum\beta_i\partial/\partial g_i$ since there will be more than one coupling constant; γ_ψ and γ_ϕ are the anomalous dimensions of the fermion and boson fields, respectively, and θ is the mass insertion operator of the theory. A similar equation holds for $\Gamma_{B(n)}^{(r,s)}$. Just as in the scalar case, the operator of lowest twist only mixes with other operators of the same twist (and the same spin, of course). In this theory the operator of lowest twist and given spin is not unique, so the anomalous dimension appears as a matrix (γ_{ij}).

The BPH subtraction procedure will fix certain of the two-particle matrix elements of $A^{(n)}$ and $B^{(n)}$ at the subtraction point, which we take to be at zero four-momentum. We define scalar form factors by

$$\Gamma^{(2,0)}_{A^{(n)}_{\mu_1\cdots\mu_n}}(0;p) = A^{(n)}_{\psi}(p^2)\left\{\frac{1}{n}\gamma_{\mu_1}p_{\mu_2}\cdots p_{\mu_n} + \text{permutations} - \text{trace terms}\right\}$$

$$+\hat{A}^{(n)}_{\psi}(p^2)\{\not p\, p_{\mu_1}\cdots p_{\mu_n} - \text{trace terms}\}, \tag{A3}$$

$$\Gamma^{(0,2)}_{A^{(n)}_{\mu_1\cdots\mu_n}}(0;p) = A^{(n)}_{\phi^2}(p^2)\{p_{\mu_1}\cdots p_{\mu_n} - \text{trace terms}\},$$

with corresponding definitions for the operators $B^{(n)}$. The BPH subtraction procedure guarantees that

$$A^{(n)}_{\psi}(0)=1, \quad A^{(n)}_{\phi}(0)=0,$$
$$B^{(n)}_{\psi}(0)=0, \quad B^{(n)}_{\phi}(0)=0. \tag{A4}$$

The form factors $\hat{A}^{(n)}_{\psi}$ are not divergent and so are not determined at the subtraction point.

If we follow the by now familiar procedure of applying the scaling equation at the subtraction point, we find that

$$\begin{pmatrix} \gamma^{(n)}_{AA} - 2\gamma_\psi & \gamma^{(n)}_{AB} \\ \gamma^{(n)}_{BA} & \gamma^{(n)}_{BB} - 2\gamma_\phi \end{pmatrix} = -i\begin{pmatrix} A^{(n)}_{\psi\theta}(0) & A^{(n)}_{\phi\theta}(0) \\ B^{(n)}_{\psi\theta}(0) & B^{(n)}_{\phi\theta}(0) \end{pmatrix}, \tag{A5}$$

when $A^{(n)}_{\psi\theta}$, etc., are the form factors, as defined in Eq. (A3), of $\Gamma^{(2,0)}_{\theta A^{(n)}_{\mu_1\cdots\mu_n}}(00;\phi)$, etc. To prove the desired theorem we must show that $\lim_{n\to\infty} A^{(n)}_{\psi\theta}(0) = 0$, etc.

To show how the argument goes, we shall consider in detail the behavior of $A^{(n)}_{\psi\theta}$, the fermion matrix elements of the twist-two fermion operator. The argument differs from the pure scalar case mainly because internal fermion propagators are present, so that even when the integrand has been fully "Feynmanized," as in Eq. (3.7), the integrand depends on the external leg four-momentum (as opposed to four-momentum squared) via the Fermi propagator numerators.

Let us consider a particular graphical contribution to $\Gamma^{(2,0)}_{A^{(n)}}(0;p)$ in which there are $\bar N$ Fermi propagators present, and N internal propagators altogether. Then by arguments similar to those used after Eq. (3.6) we can reduce the integral to

$$C\int\prod^{L} dl_i \int\prod^{N} d\alpha_i\, \delta(1-\textstyle\sum\alpha_i)\Big[\sum_j \eta_{ij} l_i\cdot l_j + A(\alpha)p^2 + B(\alpha)\Big]^{-N}$$
$$\times\Big\{\gamma_{\mu_1}\Big(\sum\epsilon_j l_j + \phi(\alpha)p\Big)_{\mu_2}\cdots\Big(\sum\epsilon_j l_j+\phi(\alpha)p\Big)_{\mu_n}\Big\}$$
$$\times M_{\alpha_1\cdots\alpha_{\bar N}\beta_1\cdots\beta_{\bar N}}\Big(\sum\epsilon^{(1)}_j \not l_j + \phi^{(1)}(\alpha)\not p + m\Big)_{\alpha_1\beta_1}\cdots\Big(\sum_j\epsilon^{(\bar N)}_j\not l_j + \phi^{(\bar N)}(\alpha)\not p + m\Big)_{\alpha_{\bar N}\beta_{\bar N}},$$

where M is some numerical matrix describing how the various inverse propagators are coupled together (an important point is that it is a Lorentz scalar—since we are dealing with a Yukawa theory, it is essentially a product of Kronecker δ's), $\epsilon^{(i)}_j$ is ±1 or 0 according to whether and in which direction the loop momentum l_j flows in the inverse propagator in question, and $\phi^{(i)}(\alpha)p$ is the fraction of the external momentum flowing through the propagator as determined by the Kirchoff conditions. As in the scalar case, $|\phi^{(i)}(\alpha)| \le 1$ and $|\phi^{(i)}(\alpha)|=1$ only when the propagator in question is part of a "short circuit" connecting to the external lines.

We now wish to identify the coefficient of $\gamma_{\mu_1}p_{\mu_2}\cdots p_{\mu_n}$ in the limit of large n. A maximum of $\bar N$ of the factors of p_μ can now come from the inverse Fermi propagators so that the fact that $|\phi(\alpha)| \le 1$ again causes the leading contribution to come from that region of α space where $|\phi(\alpha)|=1$.

Therefore the leading contribution comes from a short circuit connecting the $A^{(n)}$ fermion vertex to the external lines.

For a particular choice of short circuit, let us suppose that there are a propagators in the short circuit. Let us assume that b of the factors of p_μ in the desired covariant come from the inverse fermion propagators in the short circuit ($b \leq a$) and that c factors come from internal fermion propagators not in the short circuit. These bring along with them factors $\phi^{(j)}(\alpha)$ which clearly vanish, by the circuit analogy, in the short-circuit limit. The remaining $(n-b-c-1)$ factors come from the $A^{(n)}$ fermion vertex and yield a combinatorial factor of n^{b+c-1}. Then the n dependence of the interesting form factor is the same as that of

$$I_n = n^{b+c-1}\int_0^1 d\beta(1-\beta)^{c+a-1}\beta^n$$
$$\to \frac{1}{n^{a-b+1}} \leq \frac{1}{n} \to 0\,.$$

The factor of $(1-\beta)^{a-1}$ comes from the volume element of the corner of α space associated with the short circuit, while the remaining factor of $(1-\beta)^c$ comes from the vanishing at the short-circuit point of the c factors of $\phi(\alpha)$ associated with the momenta p_μ picked from the internal, nonshort-circuited fermion propagators. Consequently $\lim_{n\to\infty} A_{\psi\theta}^{(n)}(0) = 0$, as required. A similar line of argument shows that the other matrix elements on the right-hand side of Eq. (A5) vanish as $n \to \infty$. Consequently we find that the anomalous dimension matrix has the desired limit

$$\begin{pmatrix} \gamma_{AA}^{(n)} & \gamma_{AB}^{(n)} \\ \gamma_{BA}^{(n)} & \gamma_{BB}^{(n)} \end{pmatrix} \xrightarrow[n\to\infty]{} \begin{pmatrix} 2\gamma_\psi & 0 \\ 0 & 2\gamma_\phi \end{pmatrix} \quad . \tag{A6}$$

APPENDIX B: VECTOR-MESON THEORIES

Theories involving fundamental vector mesons are of considerable theoretical importance, but do not satisfy the simple theorems we have been able to derive for scalar and Yukawa theories. The major reason for this is that the estimates of asymptotic behavior, such as those carried out in Appendix A are profoundly altered by the presence of the four-vector Dirac matrices. Matters are further complicated by the fact that the vector-meson field itself has twist zero so that the number of independent operators with twist two grows with increasing angular momentum. In spite of these complications, a reasonable conjecture suggests itself. In this appendix we shall state the conjecture and show how it is supported by low-order perturbation calculations.

We consider a theory of a massive vector meson coupled to a single massive Fermi field. We are as usual interested in twist-two operators of arbitrary angular momentum n and wish to study the behavior of their anomalous dimensions as $n \to \infty$. To render the perturbation expansion of this theory manifestly renormalizable it is customary to introduce a massive scalar field which couples to the divergence of the vector field and which, as a consequence of the equations of motion, is a free field. This can be done in the functional integral approach by adding a quadratic "gauge-fixing" term $(1/2\epsilon)(\partial\cdot A)^2$ to the Lagrangian, and one finds that ϵ is just the ratio of the scalar mass to the vector mass. Since the physically interesting quantities will not depend on the scalar mass, it is only necessary to consider operators whose matrix elements will be ϵ-independent. Since ϵ can be changed by gauge transformations on the fields, we want to consider operators of twist two which are gauge-invariant in the usual sense. As a result there are only two relevant series of operators:

$$O^{\psi}_{\mu_1\cdots\mu_n} = \sum_{r=2}^{n} \overline{\psi}\gamma_{\mu_1}(\vec{\partial}_{\mu_2} - igA_{\mu_2})\cdots(\vec{\partial}_{\mu_r} - igA_{\mu_r}) \times(\vec{\partial}_{\mu_{r+1}} - igA_{\mu_{r+1}})\cdots(\vec{\partial}_{\mu_n} - igA_{\mu_n})\psi\,, \tag{B1}$$
$$O^{A}_{\mu_1\cdots\mu_n} = F_{\mu_1\alpha}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_{n-1}}F^{\alpha}_{\ \mu_n}\,,$$

where $F_{\mu\nu}$ is the usual field tensor, and each operator is appropriately symmetrized and trace-subtracted so as to carry angular momentum n.

These operators will of course be described by a two-by-two anomalous dimension matrix since they may mix with one another. The asymptotic power behavior of Green's functions is determined by the eigenvalues of this matrix and, as far as Bjorken scaling is concerned, only the smallest of the two eigenvalues is relevant. In this theory a relation of the type $\gamma_n \to 2\gamma_\phi$ can only make sense if γ_ϕ is the anomalous dimension of the vector-meson field. The quantity γ_n is, by construction, gauge-invariant, while the dimension of the Fermi field is gauge-variant. Therefore one might conjecture that in this theory the lower of the two dimension eigenvalues has the simple asymptotic limit $\gamma_N \to 2\gamma_A$. This would lead to similar consequences as before since Bjorken scaling would still imply that the anomalous dimension of a fundamental field vanishes and would still lead to effective free-field behavior. Furthermore it is trivial to show that the origin cannot be an ultraviolet-stable fixed point of the renormalization group for an Abelian vector-gluon theory. Thus, if our conjecture is correct, such theories could not exhibit Bjorken scaling. The general analysis of the conjecture is, for the moment, beyond us due to the diagrammatic complexity of matrix elements of the operator $A^{(n)}$.

However, it is easy to see how things work in the lowest nontrivial order of perturbation theory.

We will adopt the notation $\Gamma_O^{(a,b)}$ to indicate the one-particle-irreducible Green's function of a Fermi fields, b vector fields, and one insertion of the operator O. The scaling equations which suffice to determine our 2×2 dimension matrix are

$$\left[\mu\frac{\partial}{\partial\mu}+M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}-2\gamma_\psi\right]\Gamma^{(2,0)}_{O_n^\psi}+(\gamma^{n}_{\psi\psi}\Gamma^{(2,0)}_{O_n^\psi}+\gamma^n_{\psi A}\Gamma^{(2,0)}_{O_n^A}) = -i\Gamma^{(2,0)}_{\theta O_n^\psi}\,, \quad \text{(B2)}$$

$$\left[\mu\frac{\partial}{\partial\mu}+M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}-2\gamma_\psi\right]\Gamma^{(2,0)}_{O_n^A}+(\gamma^{n}_{A\psi}\Gamma^{(2,0)}_{O_n^\psi}+\gamma^n_{AA}\Gamma^{(2,0)}_{O_n^A}) = -i\Gamma^{(2,0)}_{\theta O_n^A}\,, \quad \text{(B3)}$$

$$\left[\mu\frac{\partial}{\partial\mu}+M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}-2\gamma_A\right]\Gamma^{(0,2)}_{O_n^\psi}+(\gamma^{n}_{\psi\psi}\Gamma^{(0,2)}_{O_n^\psi}+\gamma^n_{\psi A}\Gamma^{(0,2)}_{O_n^A}) = -i\Gamma^{(0,2)}_{\theta O_n^\psi}\,, \quad \text{(B4)}$$

$$\left[\mu\frac{\partial}{\partial\mu}+M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}-2\gamma_A\right]\Gamma^{(0,2)}_{O_n^A}+(\gamma^{n}_{A\psi}\Gamma^{(0,2)}_{O_n^\psi}+\gamma^n_{AA}\Gamma^{(0,2)}_{O_n^A}) = -i\Gamma^{(0,2)}_{\theta O_n^A}\,, \quad \text{(B5)}$$

where μ and M are the vector and Fermi masses and θ is, as usual, the mass insertion. We desire to calculate the anomalous dimension matrix to $O(g^2)$ and so only need calculate the various Green's functions out to $O(g^2)$. Since $\beta(g)$ is $O(g^3)$, the β terms may all be dropped. Further, since $\Gamma^{(2,0)}_{OA}$ and $\Gamma^{(0,2)}_{O\psi}$ are both $O(g^2)$, and since all γ's are $O(g^2)$, all terms in which these Green's functions appear multiplied by any γ may be dropped.

Finally, since all the Green's functions appearing in these equations are at most logarithmically divergent, we can manipulate a generic scaling equation of the form

$$\left[\mu\frac{\partial}{\partial\mu}+\gamma\right]\Gamma_O^{(2)}=-i\Gamma^{(2)}_{\theta O} \quad \text{(B6)}$$

to conclude that γ is equal to the coefficient of the logarithmically divergent piece of the unsubtracted Feynman integral for $\Gamma_O^{(2)}$. If we denote this coefficient by $[\Gamma_O^{(a,b)}]_{\log}$ and make use of the aforementioned simplifications arising from working only to $O(g^2)$ we get the result

$$[\Gamma^{(2,0)}_{O_n^\psi}]_{\log}=\gamma^n_{\psi\psi}-2\gamma_\psi\,, \quad \text{(B7)}$$

$$[\Gamma^{(2,0)}_{O_n^A}]_{\log}=\gamma^n_{A\psi}\,, \quad \text{(B8)}$$

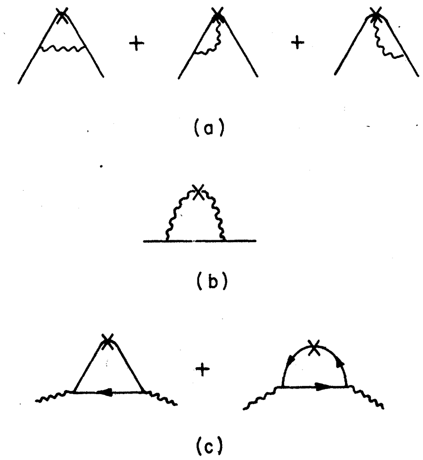

FIG. 2. Lowest-nontrivial-order graphs for two-body matrix elements of O^A and O^ψ.

$$[\Gamma^{(0,2)}_{O_n^\psi}]_{\log}=\gamma^n_{\psi A}\,, \quad \text{(B9)}$$

$$[\Gamma^{(0,2)}_{O_n^A}]_{\log}=\gamma^n_{AA}-2\gamma_A=O(g^4)\,, \quad \text{(B10)}$$

where the last result follows from the nonexistence of $O(g^2)$ corrections to $\Gamma^{(0,2)}_{O_n^A}$. The diagrams which must be considered to evaluate $[\Gamma^{(2,0)}_{O^\psi}]_{\log}$, $[\Gamma^{(2,0)}_{O^A}]_{\log}$, and $[\Gamma^{(0,2)}_{O^\psi}]_{\log}$ are displayed in Figs. 2(a), 2(b), and 2(c), respectively. The graph with a point interaction between O_n^ψ and two Fermi fields plus a vector field arises from the covariant derivatives in O_n^ψ. To evaluate the dimension matrix of the operators O_n^ψ and O_n^A, one must also compute γ_ψ and γ_A, a simple exercise with one-loop graphs.

The result of carrying out these computations is

$$\begin{pmatrix}\gamma^n_{\psi\psi} & \gamma^n_{A\psi}\\ \gamma^n_{\psi A} & \gamma^n_{AA}\end{pmatrix} = \frac{g^2}{8\pi^2}\begin{pmatrix} 1-\dfrac{2}{n(n+1)}+4\sum\limits_2^n\dfrac{1}{j} & \dfrac{8(n^2+n+2)}{n(n+1)(n+2)}\\[2ex] \dfrac{n^2+n+2}{n(n^2-1)} & \dfrac{4}{3}\end{pmatrix}. \quad \text{(B11)}$$

The lower right-hand entry is just $2\gamma_A$ as follows from our remark that $\gamma^n_{AA}-2\gamma_A=O(g^4)$. As $n\to\infty$, the upper diagonal element grows like $\log n$ and the two off-diagonal elements vanish, so that the two on-diagonal elements become the eigenvalues. Evidently, the lower of the two eigenvalues is $2\gamma_A$, as indicated. Whether this phenomenon repeats itself in higher orders is not clear, but worth investigating.

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[9]P. Federbush and K. Johnson, Phys. Rev. 120, 1926 (1960). Note that an essential ingredient in the proof of the Federbush-Johnson theorem is positivity. Thus in the case of gauge theories, even if we could prove an analogous theorem to that proved here, the absence of a positive definite metric would prevent our using this theorem.

[10]This would not be the case if one were to choose the "wrong" sign of g [namely, if $L=-(g/4!)\phi^4$, $g>0$ is the "right" sign]. Then $g=0$ would be UV-stable. However, one can then use similar renormalization-group techniques to argue that the ground-state energy is unbounded from below, as one might naively expect.

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Electromagnetic Theory of Strong Interaction

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From dielectric-diamagnetic properties shown to be inherent in the electromagnetic interaction itself by its very nature, and the fact that the dielectric attraction effect in a charged dielectric medium may dominate over the pure Coulomb repulsion, a hypothetical mechanism, which under special circumstances seems to be able to hold a charged object together, is presented, the diamagnetic property making the mechanism independent of the velocity of the object. This approach is shown to give a Yukawa-type equation for the electromagnetic field within a charged medium, and leads to a tentative electromagnetic interpretation of strong interaction analogous to the theory of plasmons. In addition to the prediction of the pion mass from the nuclear interaction range as in the Yukawa theory, the electromagnetic approach also predicts the existence of a lighter stable structure with a mass which agrees in order of magnitude with the mass of the electron, and suggests the existence of an excited state of this structure with a mass which agrees with the mass of the muon. On a macroscopic scale, the hypothetical charge-confinement mechanism presented gives energy contents for ball lightning which are of the same order of magnitude as the extremely high values ($\approx 10^7$ J) reported for this phenomenon.

I. INTRODUCTION

The subject of this paper is the mysterious fact that although physical experience seems to tell us that charged substances tend to fly apart as a result of the Coulomb repulsion, still in nature there are objects like ball lightning,[1] atomic nuclei, and elementary particles, which seemingly contradict this experience. Ingenious theoretical models have been proposed to explain the forces necessary to counterbalance the repulsive forces in these cases, such as the Yukawa theory[2] for the strong nuclear interaction, through which the existence of the pion was also predicted. Ascribing the mechanisms holding the objects together to forces outside electromagnetism, however, introduces a