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Remarks on Massive Spin-One Particles

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The construction of a Lagrangian such that its isospin current is the source of the vector-meson field, when the vector mesons are described by a tensor field, is studied. The physical significance of the contact terms of the tensor description, as far as the s-wave scattering lengths are concerned, is discussed.

The usual description of the massive spin-one particles is based on Proca's field $V'_\mu(x)$. These mesons can also be described by means of an antisymmetric second-rank tensor field $T_{\mu\nu}(x)$, which transforms under the representation $(1,0)\oplus(0,1)$ of the Lorentz group,[1-6] while the field $V'_\mu(x)$ transforms under the representation $(\frac{1}{2},\frac{1}{2})$. The tensor description has certain advantages in comparison with the usual one, related, for example, to the gauge invariance of the theory,[3,6] etc. Several aspects of the tensor description have been examined in previous papers.[1,2,4-6] In this paper we investigate the problem of constructing a Lagrangian such that the isospin current which is obtained from it is the source of the vector-meson field. We show that this condition cannot be satisfied exactly by a Lagrangian, which is a polynomial in the coupling constant g. However it can be satisfied to certain order in g, and a Lagrangian is constructed which satisfies this condition to order g^2. This Lagrangian gives an interaction Hamiltonian in the interaction representation identical to the interaction Hamiltonian of the Yang-Mills theory, to order g^2. The way of constructing the infinite series, which satisfies exactly the above condition, is indicated. Also the physical significance of the contact terms, which appear in the tensor description of spin-one mesons, as far as the s-wave scattering lengths are concerned, is discussed.

Let $\underline{L}_{\rm tot}$ be the total Lagrangian[7] of all particles involved, in which the massive vector mesons are described by the tensor field $\underline{T}_{\mu\nu}(x)$. The isospin current $\vec{\underline{I}}_\mu$ is obtained from the relation

$$\begin{aligned}\vec{\underline{I}}_\mu &= \frac{\partial \underline{L}_{\rm tot}}{\partial(\partial_\lambda \vec{\underline{T}}_{\lambda\rho})}\times \vec{\underline{T}}_{\rho\mu} + \vec{\underline{I}}'_\mu \\ &= \partial_\lambda \vec{\underline{T}}_{\lambda\rho}\times\vec{\underline{T}}_{\rho\mu} + \vec{\underline{J}}_\rho\times\vec{\underline{T}}_{\rho\mu} + \vec{\underline{I}}'_\mu\,, \end{aligned}\tag{1}$$

where $\vec{\underline{I}}'_\mu$ is the isospin current of all particles in the model except those described by the fields $\vec{\underline{T}}_{\mu\nu}$, and $\vec{\underline{J}}_\mu$ is defined in terms of the interaction Lagrangian $\underline{L}_{\rm int}$ by

$$\vec{\underline{J}}_\mu = \frac{\partial \underline{L}_{\rm int}}{\partial(\partial_\lambda \vec{\underline{T}}_{\lambda\mu})}\,. \tag{2}$$

For simplicity we have assumed in Eq. (1) that the free Lagrangian $L^T_{\rm free}$ of the fields $T_{\mu\nu}$ is given by

$$L^T_{\rm free} = \tfrac{1}{2}\partial_\nu \vec{\rm T}_{\nu\mu}\cdot\partial_\lambda\vec{\rm T}_{\lambda\mu} + \tfrac{1}{4}m^2\vec{\rm T}_{\mu\nu}\cdot\vec{\rm T}_{\mu\nu}\,. \tag{3}$$

The above Lagrangian differs from the free Lagrangian of previous papers by unimportant four-divergences. From the equations of motion of the fields $\vec{\underline{T}}_{\mu\nu}$ we get

$$\partial_\mu(\partial_\mu\vec{\underline{V}}_\nu - \partial_\nu\vec{\underline{V}}_\mu - \vec{\underline{J}}_{\mu\nu}) - m^2\vec{\underline{V}}_\nu = -m\vec{\underline{J}}_\nu\,, \tag{4}$$

where

$$\vec{\underline{V}}_\nu = \frac{1}{m}(\partial_\mu\vec{\rm T}_{\mu\nu} + \vec{\underline{J}}_\nu)\,, \tag{5}$$

$$\vec{\underline{J}}_{\mu\nu} = \frac{\partial \underline{L}_{\rm int}}{\partial\vec{\underline{T}}_{\mu\nu}}\,. \tag{6}$$

The expression $\vec{\underline{V}}_\nu$ corresponds to the spin-one field of the usual description of spin-one mesons.

Construction of a model in which the isospin current $\vec{\underline{I}}_\nu$ is the source of the vector-meson field means to find a Lagrangian which will imply

$$\vec{\underline{J}}_\nu = 2g\vec{\underline{I}}_\nu \tag{7}$$

or

$$\frac{\partial \underline{L}_{\rm int}}{\partial(\partial_\mu\vec{\underline{T}}_{\mu\nu})} = 2g\left[\frac{\partial \underline{L}_{\rm tot}}{(\partial_\mu\vec{\underline{T}}_{\mu\rho})}\times\vec{\underline{T}}_{\rho\nu} + \vec{\underline{I}}'_\nu\right]\,, \tag{8}$$

where $2g$ is the coupling strength of the fields $\partial_\lambda\vec{\underline{T}}_{\lambda\mu}$ to the isospin current $\vec{\underline{I}}'_\mu$. We easily see that the above equation can never be satisfied exactly by a Lagrangian which is a polynomial in g. Therefore when the vector mesons are described by a tensor field using Lagrangians which are polynomials in g, it is not possible to construct a model in which the source of these mesons is the isospin current.

Models in which the isospin current is approximately the source of the vector-meson field can of course be constructed. Consider the Lagrangian

$$\underline{L}_{\rm tot}=\tfrac{1}{2}(\partial_\nu\underline{\vec{T}}_{\nu\mu}+2g\underline{\vec{I}}'_\mu)\cdot(\partial_\lambda\underline{\vec{T}}_{\lambda\mu}+2g\underline{\vec{I}}'_\mu)+\tfrac{1}{4}m^2\,\underline{\vec{T}}_{\mu\nu}\cdot\underline{\vec{T}}_{\mu\nu}$$
$$+g(\partial_\rho\underline{\vec{T}}_{\rho\mu}+2g\underline{\vec{I}}'_\mu)\cdot(\partial_\lambda\underline{\vec{T}}_{\lambda\nu}+2g\underline{\vec{I}}'_\nu)\times\underline{\vec{T}}_{\nu\mu}$$
$$+2g^2(\partial_\rho\underline{\vec{T}}_{\rho\mu}\times\underline{\vec{T}}_{\mu\nu})\cdot(\partial_\lambda\underline{\vec{T}}_{\lambda\sigma}\times\underline{\vec{T}}_{\sigma\nu})+\underline{L}'_{\rm free}\,, \qquad (9)$$

where $\underline{L}'_{\rm free}$ is the free Lagrangian of all particles except those described by the fields $\underline{\vec{T}}_{\mu\nu}$. The above Lagrangian gives

$$\underline{\vec{J}}_\mu=2g(\partial_\lambda\underline{\vec{T}}_{\lambda\nu}+2g\underline{\vec{I}}'_\nu)\times\underline{\vec{T}}_{\nu\mu}+4g^2(\partial_\lambda\underline{\vec{T}}_{\lambda\sigma}\times\underline{\vec{T}}_{\sigma\nu})\times\underline{\vec{T}}_{\nu\mu}+2g\underline{\vec{I}}'_\mu$$
$$=2g(\partial_\lambda\underline{\vec{T}}_{\lambda\nu}\times\underline{\vec{T}}_{\nu\mu}+\underline{\vec{J}}_\nu\times\underline{\vec{T}}_{\nu\mu}+\underline{\vec{I}}'_\mu)+O(g^3)$$
$$=2g\underline{\vec{I}}_\mu+O(g^3)\,, \qquad (10)$$

i.e., the relation (7) is satisfied to order g^2.

The Lagrangian of Eq. (9) gives the following effective interaction Hamiltonian in the interaction representation[1,2] to order g^2:

$$(H_{\rm int})_{\rm eff}=-L_{\rm int}+\tfrac{1}{2}\vec{J}_\mu\cdot\vec{J}_\mu+\frac{1}{4m^2}\vec{J}_{\mu\nu}\cdot\vec{J}_{\mu\nu}$$
$$=\tfrac{1}{4}(\partial_\nu\vec{V}_\mu-\partial_\mu\vec{V}_\nu-2gm\vec{V}_\mu\times\vec{V}_\nu)^2$$
$$-\tfrac{1}{4}(\partial_\mu\vec{V}_\nu-\partial_\nu\vec{V}_\mu)^2-2gm\vec{V}_\mu\cdot\vec{I}'_\mu\,, \qquad (11)$$

which is the effective interaction Hamiltonian of a Yang-Mills theory in which the vector field is coupled to the isospin current $\underline{\vec{I}}'_\mu$. It is well known that in such a theory the isospin current is the source of the vector-meson field.

We argued before that Eq. (8) cannot be satisfied exactly by a Lagrangian which is a polynomial in g. It can be satisfied however by a Lagrangian which is an infinite series in g, namely, the one which is equivalent to the Lagrangian of the Yang-Mills theory. This can be obtained from the Lagrangian

$$\underline{L}'_{\rm tot}=-\tfrac{1}{2}m^2\,\underline{\vec{V}}_\mu\cdot\underline{\vec{V}}_\mu+\tfrac{1}{4}m^2\,\underline{\vec{T}}_{\mu\nu}\cdot\underline{\vec{T}}_{\mu\nu}+m\,\partial_\lambda\underline{\vec{T}}_{\lambda\mu}\cdot\underline{\vec{V}}_\mu$$
$$+2gm\underline{\vec{I}}'_\mu\cdot\underline{\vec{V}}_\mu-m^2g\,\underline{\vec{T}}_{\mu\nu}\cdot\underline{\vec{V}}_\mu\times\underline{\vec{V}}_\nu\,. \qquad (12)$$

From (12) we get the field equations

$$\partial_\mu\underline{\vec{V}}_\nu-\partial_\nu\underline{\vec{V}}_\mu-m\underline{\vec{T}}_{\mu\nu}+2mg\underline{\vec{V}}_\mu\times\underline{\vec{V}}_\nu=0\,, \qquad (13)$$

$$\partial_\lambda\underline{\vec{T}}_{\lambda\mu}-m\underline{\vec{V}}_\mu+2g\underline{\vec{I}}'_\mu-2gm\underline{\vec{V}}_\nu\times\underline{\vec{T}}_{\mu\nu}=0\,. \qquad (14)$$

Using Eq. (13) to eliminate the fields $\underline{\vec{T}}_{\mu\nu}$ from the Lagrangian (12) we obtain the Yang-Mills Lagrangian, apart from a four-divergence, which can be dropped. If however we use Eqs. (13) and (14) to eliminate the fields $\underline{\vec{V}}_\mu$ from (12), the procedure is iterative giving $\underline{L}'_{\rm tot}$ in the form of an infinite series in g, which is the Lagrangian we are looking for. This expression and the Lagrangian (9) coincide to order g^2.

We have shown before[1,2] that in our tensor formalism the effective interaction Hamiltonian in the interaction representation differs from the expression $-L_{\rm int}$ by contact terms, which are of the current × current type. The additional contact terms give rise in general to nonrenormalizable interactions and have several interesting physical consequences. Let us describe, for example, the ρ mesons by a tensor field and let us consider the interaction Lagrangian

$$\underline{L}_{\rm int}(x)=-\frac{f_{\rho\pi\pi}}{m_\rho}\,\partial_\mu\underline{\vec{T}}_{\mu\nu}(x)\cdot[\underline{\vec{\pi}}(x)\times\partial_\nu\underline{\vec{\pi}}(x)]\,. \qquad (15)$$

From the above Lagrangian we get the effective interaction Hamiltonian in the interaction representation

$$H_{\rm int}(x)_{\rm eff}=f_{\rho\pi\pi}\vec{\rho}_\nu(x)\cdot[\vec{\pi}(x)\times\partial_\nu\vec{\pi}(x)]$$
$$+\frac{1}{2}\left(\frac{f_{\rho\pi\pi}}{m_\rho}\right)^2[\vec{\pi}(x)\times\partial_\nu\vec{\pi}(x)]\cdot[\vec{\pi}(x)\times\partial_\nu\vec{\pi}(x)]\,. \qquad (16)$$

From the above Hamiltonian one can calculate the $\pi\pi$ scattering amplitudes $A_I^{(2)}(\pi\pi)$ in the isospin state I to second order in the coupling constant $f_{\rho\pi\pi}$. We easily find that these amplitudes give no contribution to the s-wave $\pi\pi$ scattering lengths $\alpha_I^0(\pi\pi)$. This result was expected since the tensor field does not carry the spin-zero component. Proceeding in an analogous fashion we can generalize the above result to the s-wave scattering lengths $\alpha_I^0(AB)$, where A, B may be π, K, ..., etc.

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[7]We use underlined letters for quantities in the Heisenberg representation, and nonunderlined letters for quantities in the interaction representation.