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Electromagnetic Waves in an Expanding Universe

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The electromagnetic perturbation equations are solved in a closed Robertson-Walker space-time; the frequency spectrum and the proper modes are found. A simple quantum theory of electromagnetic waves in the background metric is then developed using the proper modes of the static metric conformally connected with the closed Robertson-Walker metric. It is shown that the number of "photons" does not change as a result of expansion (or contraction). The application of these results to the cosmic background radiation is discussed.

I. INTRODUCTION

In an attempt to connect the quantum nature of matter and light with the curvature of space-time Schrödinger[1] pointed out an interesting phenomenon. In an expanding (or contracting) universe, matter could be created or annihilated merely by expansion (or contraction), whereas with light there would be a reflection of electromagnetic waves in space. The question of the backscattering of light by expansion (or contraction) has been investigated further by Parker[2] in a classical context. It is the purpose of this paper to present a simple quantum theory of electromagnetic waves in an expanding model universe given by the Robertson-Walker metric (with positive spatial curvature) and prove that the corresponding "photon" number operator is independent of time. This is a partial generalization of Parker's result that in Robertson-Walker space-times there is no backscattering of light due to expansion (or contraction). As also noted by Parker,[2] the basic reason for the absence of "photon" creation or annihilation is the conformal invariance of Maxwell's equations and the fact that expansion (or contraction) is isotropic in a Robertson-Walker space-time. When the relative expansion rates for different spatial directions vary with time, the number of photons is expected to change also.

The perturbations of the expanding model universe under consideration have been studied before.[3] However, we present an explicitly conformally invariant formalism which is convenient for quantization. The frequency spectrum and the eigenfunctions of the proper modes are found in the comoving coordinate frame, and a simple quantization procedure based on the usual Dirac method is proposed. An important defect of our method is its explicit dependence on the comoving coordinate frame. It is well known[4] that difficulties arise when attempts are made to give generally covariant quantization rules.

Finally, we discuss the application of our results to the cosmic background radiation and show that the frequency spectrum thus obtained is the same as the Planck spectrum for usual frequencies.

II. ELECTROMAGNETIC PERTURBATIONS

The covariant Maxwell's equations for electromagnetic waves in a gravitational field are[5]

$$F^{\mu\nu}{}_{;\nu}=0$$

and (1)

$$F_{\mu\nu;\sigma}+F_{\nu\sigma;\mu}+F_{\sigma\mu;\nu}=0\,.$$

In a given coordinate frame where the background metric is $g_{\mu\nu}$, $-ds^2=g_{\mu\nu}dx^\mu dx^\nu$, these equations can be written as

$$[(-g)^{1/2}F^{\mu\nu}]_{,\nu}=0$$

and (2)

$$F_{\mu\nu,\sigma}+F_{\nu\sigma,\mu}+F_{\sigma\mu,\nu}=0\,,$$

where $g=\mathrm{Det}(g_{\mu\nu})$.

It can be shown[6] that in a Cartesian coordinate system the substitution $F_{\mu\nu} \to (\vec{E}, \vec{B})$ and $(-g)^{1/2}F^{\mu\nu} \to (-\vec{D}, \vec{H})$ casts Eqs. (2) into the form of Maxwell's equations in a material medium. The equations of electromagnetic waves in a gravitational field (for a given metric $g_{\mu\nu}$) can then be interpreted as the Maxwell's equations in flat space-time but in a material medium[7] with the constitutive relations[8]

$$D_i = \epsilon_{ik}E_k - (\vec{G}\times\vec{H})_i$$

and (3)

$$B_i = \mu_{ik}H_k + (\vec{G}\times\vec{E})_i ,$$

where

$$\epsilon_{ik} = \mu_{ik} = -(-g)^{1/2}\frac{g^{ik}}{g_{00}}$$

and (4)

$$G_i = -\frac{g_{0i}}{g_{00}} .$$

The problem of the propagation of electromagnetic waves in a gravitational field can thus be reduced to the problem of wave propagation in a material medium in flat space-time.

The well-known duality rotation

$$\begin{aligned} \vec{E}' &= \vec{E}\cos\alpha - \vec{H}\sin\alpha ,\\ \vec{D}' &= \vec{D}\cos\alpha - \vec{B}\sin\alpha ,\\ \vec{H}' &= \vec{E}\sin\alpha + \vec{H}\cos\alpha ,\\ \vec{B}' &= \vec{D}\sin\alpha + \vec{B}\cos\alpha \end{aligned} \qquad (5)$$

leaves invariant not only the Maxwell's equations

$$\begin{aligned} \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} ,\\ \vec{\nabla}\times\vec{H} &= \frac{\partial\vec{D}}{\partial t} ,\\ \vec{\nabla}\cdot\vec{B} &= 0 ,\\ \vec{\nabla}\cdot\vec{D} &= 0 , \end{aligned} \qquad (6)$$

but also the constitutive relations (3). This latter fact is due to the equality of ϵ_{ik} and μ_{ik}.

The energy-momentum tensor of the electromagnetic field is given by

$$4\pi T_\alpha{}^\beta = -F_{\alpha\mu}F^{\beta\mu} + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\delta_\alpha{}^\beta , \qquad (7)$$

which in the formalism under discussion has components[8]

$$8\pi(-g)^{1/2}T_0{}^0 = \vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H} , \qquad (8)$$

$$\begin{aligned} 4\pi(-g)^{1/2}T_0{}^i &= (\vec{E}\times\vec{H})_i ,\\ 4\pi(-g)^{1/2}T_i{}^0 &= (\vec{D}\times\vec{B})_i , \end{aligned} \qquad (9)$$

$$4\pi(-g)^{1/2}T_i{}^j = E_iE_j + H_iB_j - \tfrac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})\delta_i{}^j . \qquad (10)$$

Let us now restrict our attention to a particular gravitational field, namely, the field of an isotropic expanding model universe. We shall consider the "closed" Robertson-Walker metric given by

$$-ds^2 = -dt^2 + a^2(t)R^2\,[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] , \qquad (11)$$

where $a(t)$ is the expansion parameter and R is the radius of the model universe at time τ, $a(\tau) = 1$. The electromagnetic perturbations of the radiation-free space-time satisfy Maxwell's equations (1) with the metric given by (11). Under the coordinate transformation

$$\begin{aligned} dt &= a(t)d\eta ,\\ x^1 &= 2R\tan(\tfrac{1}{2}\chi)\sin\theta\cos\phi ,\\ x^2 &= 2R\tan(\tfrac{1}{2}\chi)\sin\theta\sin\phi ,\\ x^3 &= 2R\tan(\tfrac{1}{2}\chi)\cos\theta , \end{aligned} \qquad (12)$$

the metric takes the form

$$-ds^2 = a^2[-d\eta^2 + f^2(\rho)(\delta_{ij}dx^i dx^j)] , \qquad (13)$$

where

$$\rho = \left[\sum_i (x^i)^2\right]^{1/2} = 2R\tan(\tfrac{1}{2}\chi)$$

and

$$f(\rho) = (1 + \rho^2/4R^2)^{-1} .$$

This isotropic form of the metric in the comoving coordinate system is particularly convenient for the discussion of the electromagnetic perturbations in this expanding model. In fact, the perturbation equations can be thought of as Maxwell's equations in flat space-time but in a material medium with

$$\epsilon_{ik} = \mu_{ik} = f(\rho)\delta_{ik} .$$

Let $\vec{F} = \vec{E} + i\vec{H}$, then one can show that

$$-i\vec{\nabla}\times\vec{F} = f(\rho)\frac{\partial\vec{F}}{\partial\eta}$$

and (14)

$$\vec{\nabla}\cdot(f\vec{F}) = 0$$

are equivalent to Maxwell's equations (6) and the constitutive relations (3). The expansion factor does not come into Eqs. (14) due to the explicit conformal invariance of the formalism we have used. The spherical symmetry of the comoving coordinate system implies that a solution can be found for a definite angular momentum J and its component M along the x^3 axis. Let[9]

$$\vec{F}_{JM\omega}(\vec{\rho})e^{-i\omega\eta} = \sum_{\sigma=e,m,0} F^{(\sigma)}_{J\omega}(\rho)\vec{Y}^{(\sigma)}_{JM}(\hat{\rho})e^{-i\omega\eta} \qquad (15)$$

be a solution of (14) and define

$$\psi^{(\sigma)}_{Jn}(\chi) \equiv R\,\rho(\chi)F^{(\sigma)}_{J\omega}(\rho(\chi)), \quad n \equiv R\omega$$

and

$$U_J(\chi) \equiv \frac{J(J+1)}{\sin^2\chi}.$$

The substitution of (15) into (14) gives

$$\frac{d^2\psi^{(m)}_{Jn}}{d\chi^2} + (n^2 - U_J)\psi^{(m)}_{Jn} = 0, \qquad (16)$$

$$\psi^{(e)}_{Jn} = -n^{-1}\frac{d\psi^{(m)}_{Jn}}{d\chi}, \qquad (17)$$

and

$$\psi^{(0)}_{Jn} = -n^{-1}[J(J+1)]^{1/2}\frac{\psi^{(m)}_{Jn}}{\sin\chi}. \qquad (18)$$

It turns out that the solution of Eq. (16) with the boundary condition that the electromagnetic perturbation be finite everywhere is

$$\psi^{(m)}_{Jn} = 2^{J+1} n\, J! \left[\frac{n'!}{(n+J)!}\right]^{1/2} (\sin\chi)^{J+1} C^{J+1}_{n'}(\cos\chi), \qquad (19)$$

where $C^{J+1}_{n'}$ is a Gegenbauer polynomial[10] and

$$n = n' + J + 1, \qquad (20)$$

with $J = 1, 2, 3, \ldots$ and $n' = 0, 1, 2, \ldots$. The Gegenbauer polynomials $C^{\lambda}_n(x)$ are orthogonal over the interval $(-1, 1)$ with the weight function $w(x) = (1-x^2)^{\lambda-1/2}$, $\lambda > -\frac{1}{2}$. The solutions (19) are normalized such that

$$\int_0^\infty F^{(m)}_{J\omega}(\rho)F^{(m)}_{J\omega'}(\rho)f(\rho)\rho^2 d\rho = 2\pi\omega\delta_{\omega\omega'}. \qquad (21)$$

Thus the frequency of the electromagnetic modes in a "closed" expanding model universe characterized by a Robertson-Walker metric can be obtained from the formula $\omega_n = n/R$ where $n = 2, 3, 4, \ldots$.

It is simple to show that a solution for the electric and magnetic fields exists which has a definite parity and angular momentum, namely,

$$\vec{E}^m_{JM\omega}(\vec{\rho}) = F^m_{J\omega}(\rho)\vec{Y}^{(m)}_{JM}(\hat{\rho}) \qquad (22)$$

and

$$\vec{H}^m_{JM\omega}(\vec{\rho}) = -i[F^e_{J\omega}(\rho)\vec{Y}^{(e)}_{JM}(\hat{\rho}) + F^0_{J\omega}(\rho)\vec{Y}^{(0)}_{JM}(\hat{\rho})]. \qquad (23)$$

In analogy with the flat-space case, we call this solution the magnetic J-pole radiation. From Eq. (5) we see that another independent solution is

$$\vec{E}^e_{JM\omega} = -\vec{H}^m_{JM\omega}$$

and (24)

$$\vec{H}^e_{JM\omega} = \vec{E}^m_{JM\omega},$$

which we shall call the electric J-pole radiation. The general solution of the perturbation equations is then a linear combination of these independent solutions. Thus

$$\vec{E}(\vec{\rho},\eta) = \sum_{\substack{JM\omega \\ \lambda=e,m}} (a^{\lambda}_{JM\omega}\vec{E}^{\lambda}_{JM\omega} + a^{\lambda*}_{JM\omega}\vec{E}^{\lambda*}_{JM\omega}) \qquad (25)$$

and

$$\vec{H}(\vec{\rho},\eta) = \sum_{\substack{JM\omega \\ \lambda=e,m}} (a^{\lambda}_{JM\omega}\vec{H}^{\lambda}_{JM\omega} + a^{\lambda*}_{JM\omega}\vec{H}^{\lambda*}_{JM\omega}), \qquad (26)$$

where

$$\frac{d}{d\eta}a^{\lambda}_{JM\omega}(\eta) = -i\omega a^{\lambda}_{JM\omega}(\eta)$$

and an asterisk denotes complex conjugation.

It should be emphasized that Eqs. (25) and (26) give the electric and magnetic fields in the expanding model universe at a time η corresponding to $t=\tau$. For any other η they give the field quantities for the static metric conformally connected to (13), namely,

$$-d\bar{s}^2 = -d\eta^2 + f^2(\rho)(\delta_{ij}dx^i dx^j), \qquad (13')$$

where $f(\rho)$ depends on the parameter R. Thus the conformal invariance of Maxwell's equations is reflected in the fact that the expanding model universe given by (13) is electromagnetically equivalent to the static space-time given by (13′) at the instant when the parameter R in (13′) is equal to the radius of the expanding universe. Then Eqs. (25) and (26) are valid for both metrics at that instant. This is a general result and holds for any instant of time. That is, for any given time $t=\tau'$, when the radius of the universe is R', $R(t) = a(t)R = a'(t)R'$ and $a'(\tau') = 1$, all the steps from Eqs. (11) to (26) can be repeated with $R \to R'$ and $\eta \to \eta'$, where $dt = a'(t)d\eta'$. It follows that the frequencies of the electromagnetic modes (of the static space-time conformally connected to the expanding universe) at τ' are given by $\omega'_n = n/R'$, $n = 2, 3, 4, \ldots$. Therefore for a given mode n, the frequency decreases as the universe expands. This is the familiar red shift of the spectral lines of the distant galaxies due to the expansion of the universe.

III. QUANTIZATION OF THE ELECTROMAGNETIC WAVES

A quantum theory of electromagnetic waves in the gravitational field under consideration can be developed along the same lines as the standard method[11] for the flat space. That is, using Eqs. (8), (25), and (26), the total electromagnetic field energy can be written as the sum of the energies of harmonic oscillators which correspond to the electromagnetic modes of the model universe. The canonical quantization of the oscillators results in the interpretation that $a^{\lambda}_{JM\omega}(\eta)$ and $a^{\lambda *}_{JM\omega}(\eta) \to a^{\lambda\dagger}_{JM\omega}(\eta)$ are the annihilation and creation operators for "photons" of total angular momentum J, x^3 component of angular momentum M, energy ω, and type $\lambda = e, m$. The time η is such that if the radius of the model universe is R at η, then $\omega = \omega_n$ for some n, where $\omega_n = n/R$ and $n = 2, 3, 4, \ldots$. These operators satisfy the equal-time commutation relations

$$\begin{aligned} &[a^{\lambda}_{JM\omega}, a^{\lambda'}_{J'M'\omega'}] = 0\,, \\ &[a^{\lambda\dagger}_{JM\omega}, a^{\lambda'\dagger}_{J'M'\omega'}] = 0\,, \\ \text{and} \\ &[a^{\lambda}_{JM\omega}, a^{\lambda'\dagger}_{J'M'\omega'}] = \delta_{JJ'}\delta_{MM'}\delta_{\omega\omega'}\delta_{\lambda\lambda'}\,. \end{aligned} \tag{27}$$

The total energy of the system can be written as

$$\begin{aligned} U &= \int (-g)^{1/2} T_0{}^0 \rho^2\, d\rho\, d\Omega \\ &= \sum_{JM\omega\lambda} \omega (a^{\lambda\dagger}_{JM\omega} a^{\lambda}_{JM\omega} + \tfrac{1}{2})\,, \end{aligned} \tag{28}$$

where the energy associated with a mode is assumed to be equal to its frequency.

The energy, total angular momentum, and the component of angular momentum along the x^3 axis should be diagonal in this particle-number representation. The angular momentum operator is

$$\vec{M} = \int (\vec{\rho} \times \vec{P})\, d^3\rho\,, \tag{29}$$

where $\vec{P}$ is the Hermitian operator corresponding to $(-g)^{1/2} T_i{}^0$. Hence

$$8\pi\vec{M} = \int \vec{\rho} \times (\vec{E} \times \vec{H} - \vec{H} \times \vec{E}) f^2(\rho) \rho^2 d\rho\, d\Omega\,.$$

An explicit calculation of M_3 using

$$8\pi M_3 = \int (E_3 \vec{H} \cdot \hat{\rho} + \vec{H} \cdot \hat{\rho} E_3 - H_3 \vec{E} \cdot \hat{\rho} - \vec{E} \cdot \hat{\rho} H_3) \times f^2(\rho) \rho^3 d\rho\, d\Omega$$

reduces, after somewhat tedious algebra, to

$$M_3 = \sum_{JM\omega\lambda} M (a^{\lambda\dagger}_{JM\omega} a^{\lambda}_{JM\omega})\,. \tag{30}$$

This result confirms the interpretation $a^{\lambda}_{JM\omega}$ and $a^{\lambda\dagger}_{JM\omega}$ as the annihilation and creation operators for "photons", i.e., the quanta of electromagnetic waves in the curved space-time under consideration.

We are now in a position to discuss the commutation relations between the field operators. Only the equal-time commutators will be considered since the explicit calculation of the unequal-time commutators appears to be difficult. Let us consider then

$$[E_i(\vec{\rho}, \eta), E_j(\vec{\rho}\,', \eta)] = 2i\, \mathrm{Im}\Phi_{ij}(\vec{\rho}, \vec{\rho}\,')\,,$$

where

$$\Phi_{ij} = \sum_{JM\omega\lambda} [\vec{E}^{\lambda}_{JM\omega}(\vec{\rho}) \cdot \hat{x}^i][\vec{E}^{\lambda}_{JM\omega}(\vec{\rho}\,') \cdot \hat{x}^j]^*\,.$$

From the fact that

$$\vec{E}^{\lambda *}_{JM\omega}(\vec{\rho}) = (-1)^{J+M+\epsilon} \vec{E}^{\lambda}_{J-M\omega}(\vec{\rho})\,,$$

where $\epsilon = 1$ for $\lambda = e$, and $\epsilon = 0$ for $\lambda = m$, one can easily show that Φ_{ij} is real, and hence

$$[E_i(\vec{\rho}, \eta), E_j(\vec{\rho}\,', \eta)] = 0\,. \tag{31}$$

Similarly, we have

$$[H_i(\vec{\rho}, \eta), H_j(\vec{\rho}\,', \eta)] = 0\,. \tag{32}$$

The results (31) and (32) are the same as in the flat-space theory. The same components of electric and magnetic fields also commute with each other at a given time. More generally, it can be shown (see the Appendix) that

$$[E_i(\vec{\rho}, \eta), H_j(\vec{\rho}\,', \eta)] = 4\pi i f^{-1}(\rho') \epsilon^{ijk} \times \frac{\partial}{\partial x^k} [f^{-1}(\rho') \delta(\vec{\rho} - \vec{\rho}\,')]\,. \tag{33}$$

The relation (33) reduces to the flat-space result when $R \to \infty$. We note that one can measure $\vec{E}(\vec{\rho}, \eta)$ throughout all of space at a fixed time η. The same statement is true for $\vec{H}(\vec{\rho}, \eta)$, too. However, the simultaneous measurement of $E_i(\vec{\rho}, \eta)$ and $H_j(\vec{\rho}, \eta)$, $i \neq j$ is not possible, in general. The uncertainty relation in such a measurement will depend on the radius of the model universe at η.

The formalism we have used above breaks down for $\omega \gtrsim \omega_p$, where $\omega_p = 1$ in the units we have used. In conventional units $\omega_p = c/L^*$, $L^* = (\hbar G/c^3)^{1/2} \simeq 1.6 \times 10^{-33}$ cm is the Planck length.[12] For such frequencies the energy of a mode is comparable to the fluctuation energy of the gravitational field ($\sim \hbar c/L^*$) and the quantum nature of this field should be taken into account. We note that with an upper bound of ω_p for the range of possible frequencies, the zero-point energy of the electromagnetic field $\sim (R/L^*)^4 (\hbar c/R)$ is comparable in magnitude to the energy of the gravitational fluctuations[12] $\sim -(R/L^*)^3 \times (\hbar c/L^*)$.

IV. DISCUSSION

We have seen that creation and annihilation operators for "photons" in the expanding model universe vary with time as $\exp(\pm i\omega\eta)$, where $\omega = n/R$ for some $n=2,3,4,\ldots$, and R is the radius of the universe at η. Let $R(t)=Ra(t)$; then these operators vary with time as $\exp[\pm in\int^t R^{-1}(t')dt']$ for some n. It is then clear that the number operator $a^{\lambda\dagger}_{JM\omega}a^{\lambda}_{JM\omega}$ for a given n is independent of time. Therefore as the model universe expands, the number of "photons" in the universe does not change.[13]

The above considerations find application in the cosmic background radiation. The thermal microwave radiation discovered by Penzias and Wilson[14] has been interpreted to be the primeval fireball radiation with a black-body spectrum and temperature $T\simeq 2.7$ °K.[15,16] The striking isotropy of the background radiation leads one to suppose that on a large scale the universe was essentially homogeneous and isotropic at about the red shift $z_0 \sim 10^3$, at which the radiation decoupled from matter. We shall thus assume that the large-scale structure of the universe can be described by the Robertson-Walker metric (11) for $0\leq z\leq z_0$. From the estimate for the present matter density ($\rho_m c^2 \simeq 10^{-9}$ erg/cm^3) and background electromagnetic-radiation density ($\rho_r c^2 \simeq 3\times 10^{-13}$ erg/cm^3), it follows that $\rho_m > \rho_r$ up to a red shift $z\sim z_0$. This apparent coincidence[17] allows one to treat the background radiation as the electromagnetic perturbation on the matter-dominated expanding universe since the moment of last scattering of the radiation.

When the radiation is at a temperature T, then the average number of "photons" with frequency ω_n is $[\exp(\omega_n/kT)-1]^{-1}$. For a given $n=R\omega_n$ the number of "photons" does not change during the expansion, so RT is a constant. The total electromagnetic energy is then

$$U=2\sum_{n=2}^{\infty}\omega_n(R^2\omega_n^{\,2}-1)[\exp(\omega_n/kT)-1]^{-1}, \qquad (34)$$

where we have used the fact that for a given n the degeneracy of the corresponding mode is $2(n^2-1)$.

It is clear from (34) that for $n\geq 2$ there is deviation from the usual black-body spectrum. For experimentally significant energies, however, $n\gg 1$, so the energy spectrum is $2R^3\omega_n^{\,3}[\exp(\omega_n/kT)-1]^{-1}$, which is the same as the Planck spectrum for a frequency ω_n and volume of the space $V=2\pi^2R^3$.

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APPENDIX

We start with

$$[E_i(\vec{\rho},\eta),H_j(\vec{\rho}\,',\eta)]=2\sum_{JM\omega\lambda}[\vec{\mathrm{E}}^{\lambda}_{JM\omega}(\vec{\rho})\cdot\hat{x}^i]\times[\vec{\mathrm{H}}^{\lambda}_{JM\omega}(\vec{\rho}\,')\cdot\hat{x}^j]^*, \qquad (\mathrm{A1})$$

and substitute in the right-hand side of this equation for $\vec{\mathrm{E}}^{\lambda}_{JM\omega}$ and $\vec{\mathrm{H}}^{\lambda}_{JM\omega}$ using (22), (23), and (24). The completeness of Gegenbauer polynomials can then be used to write Eq. (A1) in the form

$$[E_i(\vec{\rho},\eta),H_j(\vec{\rho},\eta)]=\frac{4\pi i}{R^2\rho\rho'}\left[\frac{\partial\delta(\chi-\chi')}{\partial\chi'}P_{ij}-\frac{\delta(\chi-\chi')}{\sin\chi}Q_{ij}\right], \qquad (\mathrm{A2})$$

where

$$P_{ij}=\sum_{JM}\{[\vec{\mathrm{Y}}^{(e)}_{JM}(\hat{\rho})\cdot\hat{x}^i][\vec{\mathrm{Y}}^{(m)}_{JM}(\hat{\rho}')\cdot\hat{x}^j]^* -[\vec{\mathrm{Y}}^{(m)}_{JM}(\hat{\rho})\cdot\hat{x}^i][\vec{\mathrm{Y}}^{(e)}_{JM}(\hat{\rho}')\cdot\hat{x}^j]^*\}$$

and

$$Q_{ij}=\sum_{JM}[J(J+1)]^{1/2}\{[\vec{\mathrm{Y}}^{(m)}_{JM}(\hat{\rho})\cdot\hat{x}^i][\vec{\mathrm{Y}}^{(0)}_{JM}(\hat{\rho}')\cdot\hat{x}^j]^* +[\vec{\mathrm{Y}}^{(0)}_{JM}(\hat{\rho})\cdot\hat{x}^i][\vec{\mathrm{Y}}^{(m)}_{JM}(\hat{\rho}')\cdot\hat{x}^j]^*\}.$$

From the relations between the vector spherical harmonics and their completeness, we get

$$P_{ij}=\epsilon^{ijk}\frac{x'^k}{\rho'}\delta(\hat{\rho},\hat{\rho}'), \qquad (\mathrm{A3})$$

$$Q_{ij}=\left[(\vec{\rho}\times\vec{\nabla})^i\frac{x'^j}{\rho'}+\frac{x^i}{\rho}(\vec{\rho}\,'\times\vec{\nabla}')^j\right]\delta(\hat{\rho},\hat{\rho}'). \qquad (\mathrm{A4})$$

When we put (A3) and (A4) back in (A2) we get

$$[E_i(\vec{\rho},\eta),H_j(\vec{\rho}\,',\eta)]=4\pi i\,f^{-1}(\rho')\epsilon^{ijk}\frac{\partial}{\partial x'^k}\times[\rho^{-2}f^{-1}(\rho')\delta(\rho-\rho')\delta(\hat{\rho},\hat{\rho}')].$$

From this result Eq. (33) follows immediately.

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Highly Excited Nuclear Matter*

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It is suggested that very hot and dense nuclear matter may be formed in a transient state in "head-on" collisions of very energetic heavy ions with medium and heavy nuclei. A study of the particles emitted in these collisions should give clues as to the nature of dense hot nuclear matter. Some simple models regarding the effects of meson and N^* production on the properties of dense hot nuclear matter are discussed.

I. INTRODUCTION

Heavy cosmic-ray particles impinging on nitrogen nuclei in the atmosphere have produced characteristic patterns. These patterns are the result of small momentum exchange between the collision partners accompanied by an excitation of both partners. The originally stationary partner disintegrates and forms a more or less isotropic star. The moving partner would form a similar star in its rest system which, however, appears in the lab as a narrow forward-directed jet. It is indeed highly plausible that collisions of this kind should be the most prevalent ones. Full-body collisions with big momentum exchange are less probable and could be studied with cosmic rays only to a limited extent.

The Bevalac, producing nuclei with 2 GeV per nucleon,[1] could be used to study the rare phenomena of strong momentum exchange. The ideal situation would be to shoot a fully accelerated uranium atom on another uranium atom where roughly half the energy would be available in the center-of-mass system. This would provide several hundred MeV per nucleon in the center-of-mass system. In general, for the collision of two identical nuclei the energy per baryon, ϵ, in the center-of-mass system will be

$$\epsilon = Mc^2(1 + E/2Mc^2)^{1/2}, \qquad (1)$$

where E is the lab kinetic energy per nucleon of the incident nucleus and M is the nucleon rest mass. In the near future, the uranium-on-uranium collision is out of the question. However, argon on argon may be quite feasible, and experiments of argon on uranium are also quite interesting. In the latter case we have the advantage of bigger cross sections than for argon on argon, but somewhat lesser energy per nucleon.

In the case of two equal nuclei the resulting picture which we expect to hold is that three regions may develop: a segment of the target nu-