Big-Bang Model Without Singularities*

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A uniform cosmological model filled with fluid which possesses pressure and second viscosity is developed. The Einstein equations can be integrated exactly. One solution is the steady-state cosmology, but this is unstable. Other solutions start from the steady-state oné in the infinite past but expand more and more slowly as viscosity dies out. At any finite proper time in the past the curvature is finite. Some comments on possible origins of this viscosity are given.

INTRODUCTION

It is widely believed that only cosmological models having an extremely hot, dense initial state can explain the observed features of our universe. This is due, in large part, to the discovery of the microwave background which can best be explained as the greatly red-shifted radiation from this "fireball" state.

Many attempts have been made to describe the early evolution of the universe and to deal with such problems as the fireball spectrum, helium synthesis, and so forth.[1] Some attempts have had some success but it must be remembered that the number of things known for certain in observational cosmology is distressingly small. There is considerable uncertainty in the Hubble parameter, the deacceleration parameter is not well known, and most of the energy of the universe could still be "hidden" in neutrinos, gravitons, or collapsed objects. Thus a wide range of models can fit the data.

There is also the problem of singularities. Most of the models which have a hot, dense state develop infinite space-time curvature at some finite time in the past. Wheeler has pointed out that to allow singularities in a field theory is really to allow anything at all.[2] Singularities make a theory unsatisfactory. The problem here is precisely the same as that of the final state in gravitational collapse.

Few physicists expect one formula to describe all the properties of the real universe. Any cosmological model which correctly describes some properties should be at least kept in mind for a possible future synthesis with models describing other properties. The model presented in this paper has some peculiar physical features but, as far as I can tell, none that conflict with observation if numbers are chosen properly. It is simple mathematically and has the attractive quality of avoiding a singularity, while still possessing a hot, dense state. This is made possible by inclusion of the second viscosity of the fluid filling the model.

A VISCOUS COSMOLOGY

I will assume a Robertson-Walker metric with flat space sections:

$$ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2). \tag{1}$$

(Units are chosen so that $c = G = 1$.) Thus the model possesses expansion but no shear or rotation. It will be filled with a uniform fluid with energy density ϵ and pressure p, related to ϵ by $p = \epsilon(\gamma - 1)$ with $1 \leq \gamma \leq 2$. I will take viscosity into account by adding the viscous part of the energy-momentum tensor given by Landau and Lifshitz[3] to the usual tensor for a perfect fluid. When the motion is one of pure expansion this simply has the effect of replacing the pressure by

$$p' = p - (3\zeta - 4\eta)(\dot{R}/R), \tag{2}$$

with η and ζ the first and second viscosity coefficients, respectively. (A dot denotes the time derivative.) I will neglect η in comparison with ζ and, as suggested by formulas for nonrelativistic fluids[4] or for particles interacting with radiation,[5] put $\zeta = \alpha\epsilon$ with α a constant. Then

$$p' = \epsilon(\gamma - 1 - 3\alpha\dot{R}/R). \tag{3}$$

With $H = \dot{R}/R$ as the Hubble parameter the Einstein equations give[6]

$$\begin{aligned} 8\pi\epsilon/3 &= H^2, \\ 8\pi(\gamma - 1 - 3\alpha H)\epsilon &= -(2\dot{H} + 3H^2). \end{aligned} \tag{4}$$

Elimination of ϵ yields

$$\dot{H}/H^2 + 3(\gamma - 3\alpha H)/2 = 0. \tag{5}$$

We note first that there is a solution $H =$ constant $= \gamma/3\alpha \equiv H_0$. This is the de Sitter metric which is required by the postulates of steady-state cosmology. McCrea has pointed out that it can be obtained as a solution of the Einstein equations (without cosmological term) if we allow a large negative pressure.[7] When $H=H_0$,

$$p'=(\gamma-1-\gamma)\epsilon=-\epsilon,$$

although p is always positive in the present model.

The general solution of (5) is

$$3\gamma H_0(t-t_0)/2=\ln R^{3\gamma/2}+CR^{3\gamma/2}, \tag{6}$$

with α replaced by $\gamma/3H_0$. t_0 and C are integration constants. We can adjust clocks to make $t_0=0$.

When R is sufficiently small, $|\ln R^{3\gamma/2}|\gg CR^{3\gamma/2}$ and $R=\exp(H_0t)$ so that all solutions approach the de Sitter metric. On the other hand, when $CR^{3\gamma/2}\gg|\ln R^{3\gamma/2}|$ then $R=(3\gamma H_0t/2C)^{2/3\gamma}$ and the effects of viscosity are negligible. For $\gamma=1$ this is the dust-filled Einstein–de Sitter model.

Sketches of R versus t are shown in Fig. 1. The value of C is critical. $C=0$ gives the steady-state solution which all models approach as $t\to-\infty$. This solution is unstable in the sense that any perturbation corresponding to a change of C from 0 will make the R-t curve peel away from the $C=0$ curve. When $C<0$ all observers "run out of time"—t cannot get beyond a certain finite value.

The solutions with $C>0$ are the ones of most physical interest. Here the expansion continually slows down but never reverses. The Hubble parameter is

$$H=H_0/(1+CR^{3\gamma/2}). \tag{7}$$

These models have no geometric singularity—the space-time curvature is finite for all the finite proper time of any observer.

The usual positivity condition on the energy-momentum tensor is equivalent to[8]

$$\epsilon+p'\geq 0, \quad \epsilon+3p'\geq 0.$$

In the present model the first is $1\geq H/H_0$, which

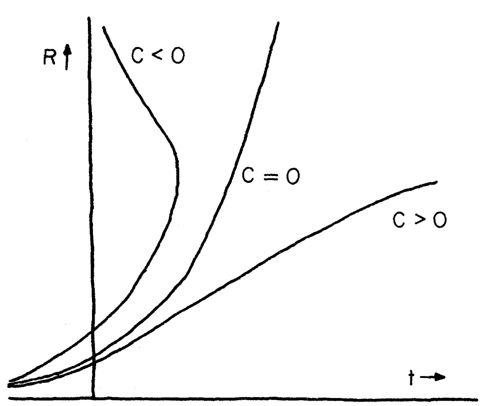

FIG. 1. R versus t for viscous cosmologies.

is always true when $C>0$. The second condition is $\gamma(1-H/H_0)\geq\frac{2}{3}$ and this fails when H becomes sufficiently close to H_0.

The "age of the universe" is infinite in these models but for many purposes the time when H begins to depart significantly from the "primordial" value H_0 can be taken as a beginning. For example, we could choose t_0 in (6) so that $t=0$ is the instant at which the positivity condition is first satisfied. This gives us the result

$$-3\gamma H_0t_0/2=a-\ln C,$$

where

$$a=2/(3\gamma-2)+\ln[2/(3\gamma-2)]$$

is always of order unity. Let us assume that at the present epoch $H\ll H_0$ (so that viscosity is now negligible). Then (6) and (7), with the value we have derived for t_0, give

$$t\simeq 2/3\gamma H-2a/3\gamma H_0\simeq 2/3\gamma H. \tag{8}$$

This is the time from the singular state in the corresponding model with $\zeta=0$. Thus, with the stated assumptions, our model cannot lengthen the evolutionary time for the universe in terms of the Hubble time $T=1/H$.

INTERPRETATION

Like all cosmological models, this one contains many oversimplifications. The dependence of the second viscosity on energy density would be more complex in reality. In addition, γ will not be constant but will change as the proportions of nonrelativistic particles, radiation, mesons, etc. change. A conservative procedure would be to put $\gamma=1$ for the present epoch and $\gamma=\frac{4}{3}$ in the intermediate stages of the fireball, and perhaps to allow $\gamma\to 2$ as strong interactions dominate.

However, it is possible that most of the energy density of the universe is in the form of neutrinos or gravitons and has so far remained unobserved. In this case it might make sense to put $\gamma=\frac{4}{3}$ throughout.

It is not easy to see how second viscosity could be the dominant feature of the universe at present and it seems more reasonable to assume, as we have done, that $H\ll H_0$ and that viscosity was important only in the distant past.

In order to discover a source of this second viscosity, we may look back to the time when the density of matter approached that of quantum gravitational field fluctuations, about 5×10^{93} g/cm^3.[9] This gives a value of about 5×10^{43} sec^{-1} for H_0, which may be compared with the present estimate of $H\simeq 50$ (km/sec)/Mpc $\simeq 1.8\times10^{-18}$ sec^{-1}.[10] Obviously the viscosity would be negligible by now. By the time the universe had expanded to nuclear density, $\rho \simeq 2\times10^{14}$ g/cm^3, the viscosity could be ignored and the model could be fitted onto one of the models which have been worked out for the "early" universe.

Under such extreme conditions one would expect graviton production in graviton-graviton scattering. This would lead to an increase in entropy which could be described crudely by a second viscosity coefficient. Of course in such a state, when the observable universe would be squeezed into a volume approaching that of a classical electron, any simple model can be considered only the crudest approximation. The gravitons of which I have spoken cannot at all be separated from the general curvature of space-time and a detailed description in terms of the single concept of second viscosity in not to be thought of. But it does seem possible that the model presented here might serve as a starting point for a more complete description.

*This paper is a slightly modified version of one which was awarded an honorable mention in the 1973 Gravity Research Foundation Essay Contest.

[1]See, e.g., S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 15.
[2]J. A. Wheeler, Rev. Mod. Phys. 33, 63 (1961).
[3]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, Reading, Mass., 1959), Chap. XV.
[4]Reference 3, p. 304.
[5]Reference 1, p. 57.
[6]Reference 1, p. 472.
[7]W. H. McCrea, Proc. R. Soc. 206, 562 (1951).
[8]See, e.g., R. Geroch, in *Relativity*, edited by M. Carmeli, S. I. Fickler, and L. Witten (Plenum, New York, 1970), Appendix D.
[9]J. A. Wheeler, *Geometrodynamics* (Academic, New York, 1962), p. 79.
[10]A. Sandage, paper presented at the Mayall Symposium, Tuscon, Arizona, 1971 (unpublished).

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Search for Correlations in the Arrival Times of Extensive Air Showers and Weber Gravitational Waves*

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We have searched for correlations in the arrival times of extensive air showers and Weber gravitational waves. Extensive air showers with energies greater than 5×10^{14} eV were detected at the Jadwin Physics Building, Princeton, New Jersey, from March to November 1971. The apparatus was not sensitive to cosmic rays coming directly from the galactic center. The shower-arrival times were compared to the arrival times of Weber's gravitational waves occurring over the same period. Undelayed- and delayed-time coincidences were checked for in the analysis, as well as possible time-delay dispersion effects.

INTRODUCTION

Several mechanisms have been proposed for generation of the gravitational waves detected by Weber.[1] Weber[2] and others have proposed that collapse of a star would give rise to gravitational waves. Misner[3] and others have conjectured that the gravitational waves are generated by synchrotron radiation of massive orbiting bodies. The most recent conjecture[4] is that they are associated with the recently discovered periodic x-ray sources.