

states are to be evaluated with  $\epsilon_3 = 0$ . Thus, in evaluating  $\partial_\mu A_\mu^i = -i[Q_5^i, \mathcal{H}]$  for terms in the perturbation expansion, we drop the  $\epsilon_3 U_3$  term.

<sup>15</sup>In unified gauge theories, one must include weak radiative corrections to isospin-violating parity-conserving amplitudes (as for  $\eta \rightarrow 3\pi$  decay), since, in general, they are of order  $\alpha$ , one may ask whether this could resolve the  $\eta \rightarrow 3\pi$  difficulty. We have examined this question as well allowing for different possible renormalizable strong interactions and find that the answer is in the *negative*. The reasons, briefly speaking, are that the relevant corrections are either like a  $U_3$  term (which is already considered), or have the same commutation relations with the chiral generators ( $Q_5^i$ ) as either  $\mathcal{H}_{em}$  or  $U_3$ .

<sup>16</sup>In a quark-model language, this amounts to saying that the weak hadron current has the form  $\mathcal{P}\gamma_\mu(1 + \gamma_5)\mathcal{H}_0$  in terms of  $(\mathcal{P}, \mathcal{H}_0, \lambda_0)$  fields, while the mass matrix in terms of the same fields has the form

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta \\ 0 & \Delta & m_{\lambda_0} \end{pmatrix}$$

which is  $U(2) \times U(2)$ -invariant if  $\Delta \rightarrow 0$ . The "physical"

fields  $(\mathcal{P}, \mathcal{H}, \lambda)$  are obtained by diagonalization (Cabibbo rotation) of the mass matrix; in terms of which the weak hadron current has the Cabibbo form  $\mathcal{P}\gamma_\mu(1 + \gamma_5) \times (\mathcal{H} \cos\theta + \lambda \sin\theta)$ . The same rotation gives a small mass  $(-\Delta^2/m_{\lambda_0})$  to the  $\mathcal{H}$  quark (if  $\Delta/m_{\lambda_0}$  is small). One obtains in this way;  $\tan 2\theta = 2\Delta/m_{\lambda_0}$  and  $\epsilon_3 \approx (\frac{2}{3})^{1/2} \sin^2\theta \approx 0.06$ . Thus, chiral  $U(2) \times U(2)$  breaking, non-electromagnetic isospin breaking, and Cabibbo angle are nonvanishing entirely because of  $\Delta \neq 0$ .

<sup>17</sup>R. J. Oakes, Phys. Lett. **30B**, 262 (1969). Similar suggestions have also been made from other considerations by R. Gatto, G. Sartori, and M. Tonin, *ibid.* **28B**, 128 (1968).

<sup>18</sup>This problem has been noted by a number of authors: S. L. Glashow, R. Jackiw, and S. S. Shei, Phys. Rev. **87**, 1916 (1969); M. Gell-Mann, Lecture Notes, Summer School of Theoretical Particle Physics, Univ. of Hawaii, 1969 (unpublished). We thank Professor B. W. Lee for bringing it to our attention.

<sup>19</sup>In principle, weak PCAC [R. A. Brandt, M. Goldhaber, C. A. Orzalesi, and G. Preparata, Phys. Rev. Lett. **24**, 1517 (1970)] could be a possible source of resolution; however with the  $H$  meson slowly disappearing from the Particle Data Tables, it is not clear whether this could resolve the dilemma either.

## $\rho$ - $A_2$ Exchange Degeneracy and the Adler Sum Rule\*

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On the basis of  $\rho$ - $A_2$  exchange degeneracy, the structure functions for virtual Compton scattering with  $I = 1$  and  $C = \pm 1$  in the  $t$  channel are assumed proportional to one another. The constant of proportionality is determined by the threshold relation of Bloom and Gilman. The Adler sum rule ( $I = 1$ ,  $C = -1$ ) is found to be consistent with the observed behavior of the structure function ( $I = 1$ ,  $C = +1$ ) in the scaling region. A simple proportionality relation between  $\nu(W_2^{\nu p} - W_2^{\nu n})$  and  $\nu(W_2^{\nu p} - W_2^{\nu n})$  is obtained in the scaling limit.

Some time ago, Harari<sup>1</sup> conjectured that, in the Adler sum rule<sup>2</sup> for charged-photon scattering, there might be a term that depends on  $q^2$  (negative square of the photon mass) in addition to the constant that arises from the current-algebra restriction<sup>3</sup> on the scattering amplitude. It was suggested that such a term could be identified with a possible fixed  $J = 1$  pole in the single-helicity-flip amplitude, besides the one in the double-helicity-flip amplitude whose residue has been identified with the current algebra constant.<sup>4</sup> The purpose of this work is to make an attempt at an evaluation of the terms in question in the Bjorken scaling limit<sup>5</sup> by relating the appropriate structure function for the charged-photon scattering to that for the neutral-photon scattering on the basis of theoretical

models. For the latter, there exist extensive data from the MIT-SLAC experiment on deep-inelastic electron-proton scattering.<sup>6</sup>

As is well known, the forward amplitude for the scattering of charged photons off a spin-averaged proton,  $\gamma^*(q) + p(p) \rightarrow \gamma^*(q) + p(p)$ , is given in terms of invariants as follows:

$$\begin{aligned} T_{\mu\nu}^{\pm} &= i \int d^4x e^{-iqx} \langle p(p) | \theta(x_0) [j_\mu^\pm(x), j_\nu^\pm(0)] | p(p) \rangle \\ &= p_\mu p_\nu A^\pm(\nu, q^2) + (p_\mu q_\nu + p_\nu q_\mu) B^\pm(\nu, q^2) \\ &\quad + q_\mu q_\nu C^\pm(\nu, q^2) + \delta_{\mu\nu} D^\pm(\nu, q^2), \end{aligned} \quad (1)$$

where  $\nu = -(p \cdot q)/M$ . The conservation of the isovector current and the current algebra<sup>3</sup> then give

$$\begin{aligned} \nu MA^+(\nu, q^2) - q^2 B^+(\nu, q^2) &= \pm(2M)^{-1}, \\ \nu MB^+(\nu, q^2) - q^2 C^+(\nu, q^2) - D^+(\nu, q^2) &= 0. \end{aligned} \quad (2)$$

Writing

$$\begin{aligned} \text{Im}[A^-(\nu, q^2) - A^+(\nu, q^2)] &\equiv \text{Im}A^{(-)}(\nu, q^2) \\ &= \frac{\pi}{2M^2} W_2^{(-)}(\nu, q^2), \end{aligned} \quad (3)$$

the Adler sum rule reads<sup>2</sup>

$$\int_0^\infty d\nu W^{(-)}(\nu, q^2) = 1. \quad (4)$$

Since  $A^{(-)}(\nu, q^2)$  is an  $s$ - $u$  crossing-antisymmetric amplitude, the  $t$  channel ( $I=1, C=-1$ ) is presumably dominated by the  $\rho$ -trajectory exchange. With two units of helicity flip involved in the  $t$  channel, the Regge asymptotic behavior of  $A^{(-)}(\nu, q^2)$  should be  $\nu^{\alpha_\rho-2}$ , where  $\alpha_\rho \approx \frac{1}{2}$ . Thus, for a purely hadronic scattering  $W_2^{(-)}(\nu, q^2)$  should satisfy a superconvergence relation. The current-algebra restriction, the unity on the right-hand side of (4), has been reconciled with a fixed  $J=1$  pole in the amplitude  $A^{(-)}(\nu, q^2)$ .<sup>4</sup> Should there be a fixed  $J=1$  pole also in  $B^{(-)}(\nu, q^2)$ , we can write for the asymptotic forms

$$\begin{aligned} A^{(-)}(\nu, q^2) &= \beta_A(q^2, \alpha_\rho) \left( i + \tan \frac{\pi \alpha_\rho}{2} \right) \nu^{\alpha_\rho-2} \\ &\quad + K_A(q^2) \nu^{-1}, \\ B^{(-)}(\nu, q^2) &= \beta_B(q^2, \alpha_\rho) \left( i + \tan \frac{\pi \alpha_\rho}{2} \right) \nu^{\alpha_\rho-1} \\ &\quad - K_B(q^2) M^{-1}, \end{aligned} \quad (5)$$

where  $K_A(q^2)$  and  $K_B(q^2)M^{-1}$  are the residues of the fixed poles in  $A^{(-)}(\nu, q^2)$  and  $B^{(-)}(\nu, q^2)$ , respectively. These residues are assumed to be real. From (2) it then follows that

$$-M^2 K_A(q^2) = q^2 K_B(q^2) + 1. \quad (6)$$

Assuming unsubtracted dispersion relation for  $A^{(-)}(\nu, q^2)$ , we then obtain

$$\int_0^\infty d\nu W^{(-)}(\nu, q^2) = 1 + q^2 K_B(q^2). \quad (7)$$

This is the form of the sum rule, modified in relation to (4), that was considered by Harari.<sup>1,7</sup>

While there is no *a priori* reason why there should not be the fixed pole in  $B^{(-)}(\nu, q^2)$ , the resulting  $q^2$ -dependent term on the right-hand side of (7) has a serious consequence with regard to the parton model,<sup>8-10</sup> in which high-energy photon-

hadron scattering is viewed as arising from the interaction of the virtual photon with pointlike constituents of the hadron. Indeed, the independence of the right-hand side of (4) upon  $q^2$  is strongly suggestive of such a model.<sup>11</sup>

Since the neutral photon does not enter into (4) and (7), only the forthcoming neutrino reactions can test these sum rules in a model-independent way.<sup>12</sup> One way of testing these sum rules is to relate the structure functions  $W_2^{(-)}(\nu, q^2)$  to the corresponding structure function  $W_2^{\gamma^p}(\nu, q^2) - W_2^{\gamma^n}(\nu, q^2)$  for the proton-neutron difference in the Compton scattering of the virtual photon.

For large  $\nu$ ,  $W_2^{(-)}(\nu, q^2)$  and  $W_2^{\gamma^p}(\nu, q^2) - W_2^{\gamma^n}(\nu, q^2)$  are presumably dominated by  $\rho$  and  $A_2$  exchanges in the  $t$  channel, respectively. It is then natural to assume that, under the  $\rho$ - $A_2$  exchange degeneracy, they are related to one another by a factor which depends only on  $q^2$ . This point can be illustrated by the following model.

Let us assume SU(3) symmetry for the  $\rho\rho\rho$  and  $K^*\bar{K}^*\rho$  couplings ( $F$  type) and U(3) symmetry for the  $\rho\omega_8 A_2$ ,  $\rho\phi_1 A_2$ , and  $K^*\bar{K}^* A_2$  couplings ( $D$  type), with the ideal mixing between the eighth component  $\omega_8$  of the SU(3) octet and the SU(3) singlet  $\phi_1$ .<sup>13</sup> Then, the assumption of  $\rho$ - $A_2$  exchange degeneracy that follows from the absence of exotic states in the  $K^*p$  channel leads to the ratios given in Fig. 1 for the absorptive parts of the vector-meson-nucleon scattering amplitudes.

The absorptive part of the forward Compton amplitude reads

$$W_{\mu\nu} = \frac{1}{2} \int d^4x e^{-iqx} \langle p(p) | [j_\mu^{\text{em}}(x), j_\nu^{\text{em}}(0)] | p(p) \rangle. \quad (8)$$

Introducing the electromagnetic current in the vector-dominance model (VDM)

$$j_\mu^{\text{em}}(x) = \frac{m_\rho^2}{f_\rho} \rho_\mu^0(x) + \frac{m_\omega^2}{f_\omega} \omega_\mu(x) + \frac{m_\phi^2}{f_\phi} \phi_\mu(x) \quad (9)$$

and the ratios given in Fig. 1, it can then be shown that

$\rho^\pm$	$\rho^\pm$	$K^{*+} K^{*+}$	$K^{*+} K^{*+}$	$\omega$	$\rho^0$	$\omega$	$\rho^0$
$\rho$	$\rho$	$\rho$	$A_2$	$A_2$	$A_2$		$A_2$
$p$	$p$	$p$	$p$	$p$	$p$	$n$	$n$
$\pm 2$	$\pm 1$	$\pm 1$	$-1$	$-2$	$-2$	$+2$	$+2$

FIG. 1. The relative magnitudes of the absorptive parts of the vector-meson-nucleon scattering amplitudes in pole approximation.

$$W_{\mu\nu}^{\gamma\rho} - W_{\mu\nu}^{\gamma n} = \frac{m_\rho^2 m_\omega^2}{f_\rho f_\omega} [(q^2 + m_\rho^2)(q^2 + m_\omega^2)]^{-1} \int d^4x e^{-iqx} \{ \langle p(p) | [j_\mu^{\rho^0}(x), j_\nu^{\omega^0}(0)] | p(p) \rangle + \langle p(p) | [j_\mu^{\omega^0}(x), j_\nu^{\rho^0}(0)] | p(p) \rangle \}, \quad (10)$$

where  $j_\mu^{\rho^0}$  and  $j_\mu^{\omega^0}$  are the source currents of the  $\rho^0$  and  $\omega$  fields, respectively. For the charged photon scattering with the proton as the target, we have

$$W_{\mu\nu}^{(-)} = \frac{1}{2} \left( \frac{m_\rho^2}{f_\rho} \right)^2 (q^2 + m_\rho^2)^{-2} \int d^4x e^{-iqx} \{ \langle p(p) | [j_\mu^{\rho^+}(x), j_\nu^{\rho^-}(0)] | p(p) \rangle - \langle p(p) | [j_\mu^{\rho^-}(x), j_\nu^{\rho^+}(0)] | p(p) \rangle \}, \quad (11)$$

where  $j_\mu^{\rho^\pm}$  are the source currents of the  $\rho^\pm$  fields. From the ratios given in Fig. 1, we then get

$$W_{\mu\nu}^{(-)} = \frac{1}{2} \left( \frac{m_\rho}{m_\omega} \right)^2 \frac{f_\omega q^2 + m_\omega^2}{f_\rho q^2 + m_\rho^2} [W_{\mu\nu}^{\gamma\rho} - W_{\mu\nu}^{\gamma n}]. \quad (12)$$

The same relation must hold between  $W_2^{(-)}$  and  $W_2^{\gamma\rho} - W_2^{\gamma n}$ . Unfortunately, VDM is known to fail for large  $q^2$ .<sup>6</sup> Thus, this relation gives only an illustrative example in support of our assumption.

The  $q^2$ -dependent proportionality between  $W_2^{(-)}(\nu, q^2)$  and  $W_2^{\gamma\rho}(\nu, q^2) - W_2^{\gamma n}(\nu, q^2)$  assumed on the basis of the  $\rho$ - $A_2$  exchange degeneracy lies within the domain of the Regge theory. By virtue of the duality hypothesis,<sup>14</sup> these amplitudes must be made up of  $N^*$  resonances in the direct channel. While resonance phenomena are coherent properties of the nucleon, in the parton model it is the incoherent scattering by pointlike constituents of the hadrons that receives the special emphasis. Nonetheless, it has been shown by Bloom and Gilman<sup>15</sup> that there exists a remarkable consistency between the scaling<sup>5</sup> for which the parton model provides a possible mechanism and the resonance phenomena, both observed in the deep-inelastic electron scattering. Although there exists at present no theoretical explanation of this consistency,<sup>15,16</sup> it does suggest strongly that the aforementioned  $q^2$ -dependent proportionality between  $W_2^{(-)}(\nu, q^2)$  and  $W_2^{\gamma\rho}(\nu, q^2) - W_2^{\gamma n}(\nu, q^2)$  transcends the Regge regime and remains valid in the region where scaling sets in.<sup>17</sup> The  $q^2$ -dependent proportionality factor is expected to become a constant in the scaling limit ( $q^2 \rightarrow \infty$ ), as is seen in (12). Assuming this to be the case, in the limit  $q^2 \rightarrow \infty$ , the assumption predicts that the structure functions  $\nu(W_2^{\gamma\rho} - W_2^{\gamma n})$  and  $\nu W_2^{(-)}$  should scale with the same functional dependence on the scale variable.<sup>18</sup>

The proportionality constant can be computed in the scaling limit in the vicinity of the elastic threshold using the relation obtained by Bloom and Gilman from their finite-energy sum rule<sup>15</sup>:

$$\lim_{\substack{q^2 \rightarrow \infty \\ \omega' \rightarrow 1}} \nu W_2(\omega') = \lim_{\substack{q^2 \rightarrow \infty \\ \omega' \rightarrow 1}} \frac{1}{\omega' - 1} \left( -q^2 \frac{d}{dq^2} [G(q^2)]^2 \right), \quad (13)$$

where

$$\omega' = (2M\nu + M^2)/q^2,$$

$$[G(q^2)]^2 = [F_1(q^2)]^2 + q^2 [F_2(q^2)/2M]^2$$

$$= \frac{[G_E(q^2)]^2 + q^2 [G_M(q^2)/2M]^2}{1 + q^2/4M^2}. \quad (14)$$

Following their steps, it can easily be shown that<sup>19</sup>

$$\begin{aligned} \lim_{\substack{q^2 \rightarrow \infty \\ \omega' \rightarrow 1}} \frac{\nu W_2^{(-)}(\omega')}{\nu [W_2^{\gamma\rho}(\omega') - W_2^{\gamma n}(\omega')]} \\ = \lim_{q^2 \rightarrow \infty} \frac{(d/dq^2) [G_M^V(q^2)]^2}{(d/dq^2) [G_M^S(q^2) G_M^V(q^2)]}, \end{aligned} \quad (15)$$

where  $G_M^S(q^2)$  and  $G_M^V(q^2)$  are the isoscalar and isovector elastic magnetic form factors of the nucleon, respectively. Introducing the relations

$$\begin{aligned} G_M^S(q^2) &= \frac{1}{2} [G_{Mp}(q^2) + G_{Mn}(q^2)], \\ G_M^V(q^2) &= \frac{1}{2} [G_{Mp}(q^2) - G_{Mn}(q^2)], \end{aligned} \quad (16)$$

and the "scaling" of the elastic form factors

$$G_{Mp}(q^2)/\mu_p = G_{Mn}(q^2)/\mu_n, \quad (17)$$

we obtain<sup>20</sup>

$$\begin{aligned} \lim_{\substack{q^2 \rightarrow \infty \\ \omega' \rightarrow 1}} \frac{\nu W_2^{(-)}(\omega')}{\nu [W_2^{\gamma\rho}(\omega') - W_2^{\gamma n}(\omega')]} &= \frac{\mu_p - \mu_n}{\mu_p + \mu_n} \\ &= 5.35. \end{aligned} \quad (18)$$

As was stated earlier, according to our assumption, the structure functions  $\nu W_2^{(-)}$  and  $\nu(W_2^{\gamma\rho} - W_2^{\gamma n})$  should scale with the same functional dependence on  $\omega'$ . Thus, the above ratio evaluated in the limit of the elastic threshold ( $\omega' \rightarrow 1$ ) is expected to hold for all values of  $\omega'$ . Changing the variable  $\omega'$  of (14) to the Bjorken scale variable  $\omega = 2M\nu/q^2$ ,<sup>5</sup> the sum rule (7) becomes

$$\frac{\mu_p - \mu_n}{\mu_p + \mu_n} \int_1^\infty \frac{d\omega}{\omega} \nu (W_2^{\gamma\rho} - W_2^{\gamma n}) = 1 + [q^2 K_B(q^2)]_{q^2 \rightarrow \infty}. \quad (19)$$

The value for the integral has been determined from the experimental data,<sup>6</sup> assuming that the

scaling persists for all values of  $\omega$  above the experimentally observed region and adopting the Regge asymptotic behavior for the scaling<sup>21</sup>:

$$\int_1^\infty \frac{d\omega}{\omega} \nu(W_2^{\gamma^p} - W_2^{\gamma^n}) = 0.19 \pm 0.08. \quad (20)$$

From (19), we then get

$$\lim_{q^2 \rightarrow \infty} q^2 K_B(q^2) = 0.02 \pm 0.43. \quad (21)$$

We thus find that, within the scheme based on the assumption of proportionality between the structure functions with  $I=1$  and  $C=\pm 1$  in the  $t$  channel, and the Bloom-Gilman duality in the scaling region, the Adler sum rule (4) for the  $C=-1$  structure function is consistent with the experimentally observed behavior of the  $C=+1$  structure function in the scaling region within the experimental error. The absence of the fixed  $J=1$  pole in the amplitude  $B^{(-)}$  is consistent with the parton model for the scaling of the  $C=-1$  structure function.

The assumption evidently can be extended to the case where both vector and axial-vector currents are present. There is no vector-axial-vector interference term in the spin-averaged forward amplitude. Thus, the structure function for the neutrino reactions  $W_2^{\bar{\nu}p} - W_2^{\nu p}$  is a linear sum of  $W_2^{(-)}$  for charged photons and the corresponding one for "axial-vector charged photons." In analogy to the former, the latter is again assumed to be proportional to  $W_2^{\gamma^p} - W_2^{\gamma^n}$ . We then expect that there exists a constant proportionality between  $\nu(W_2^{\bar{\nu}p} - W_2^{\nu p})$  and  $\nu W_2^{(-)}$  in the scaling region. The constant of proportionality can be determined this

time by referring to the Adler sum rules in the scaling limit

$$\int_1^\infty \frac{d\omega}{\omega} \nu W_2^{(-)} = 1 \quad (22)$$

and

$$\int_1^\infty \frac{d\omega}{\omega} \nu(W_2^{\bar{\nu}p} - W_2^{\nu p}) = 2. \quad (23)$$

We then expect that the relation

$$\nu(W_2^{\bar{\nu}p} - W_2^{\nu p}) = 2\nu W_2^{(-)} \quad (24)$$

should hold in the scaling region. From (18) and (24), it is then predicted that

$$\nu(W_2^{\bar{\nu}p} - W_2^{\nu p}) = 2 \frac{\mu_p - \mu_n}{\mu_p + \mu_n} \nu(W_2^{\gamma^p} - W_2^{\gamma^n}). \quad (25)$$

The validity of the scheme presented in this work depends crucially on experimental verification of this relation.

With regard to the residue  $K_B(q^2)$  of the fixed pole in  $B^{(-)}$  amplitude, it has previously been shown that  $K_B(0) \approx 0$  by saturation of the Cabibbo-Radicati sum rule.<sup>7</sup> We have obtained the result  $K_B(q^2 \rightarrow \infty) \approx 0$  in our scheme. The value for  $K_B(q^2)$  in the nonscaling region remains yet unknown.

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<sup>7</sup>Taking out explicitly the elastic contribution to the integral in (7) and differentiating the result with respect to  $q^2$ , after an isospin rotation, one obtains at

$$q^2 = 0$$

$$K_B(0) = -\frac{1}{3} \langle r^2 \rangle_E^V + (F_2^V/2M)^2 + \frac{1}{2\pi^2\alpha} \int_{\nu_0}^\infty \frac{d\nu}{\nu} [2\sigma_T^V(\nu, I=\frac{1}{2}) - \sigma_T^V(\nu, I=\frac{3}{2})].$$

The well-known Cabibbo-Radicati sum rule corresponds to  $K_B(0) = 0$  [N. Cabibbo and L. A. Radicati, Phys. Lett. **19**, 697 (1966)]. Various authors have shown that the sum rule is satisfied with  $K_B(0) \approx 0$ . See F. J. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966); S. L. Adler and F. J. Gilman, *ibid.* **152**, 1460 (1966) and **156**, 1598 (1967); F. J. Gilman, *ibid.* **167**, 1365 (1968); G. C. Fox and D. Z. Freedman, Phys. Rev. **182**, 1628 (1969); G. J. Aubrecht and W. W. Wada, Ann. Phys. (N.Y.) **78**, 376 (1973).

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paper, the structure functions in the Bjorken scaling limit are discussed in terms of the Regge asymptotic functions.

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- <sup>19</sup>The elastic Born term for  $W_2^{(-)}$  is  $[F_1^{\gamma}(q^2)]^2 + q^2[F_2^{\gamma}(q^2)/2M]^2$ , whereas that for  $W_2^{\gamma p} - W_2^{\gamma n}$  is  $F_1^{\gamma}(q^2)F_1^{\gamma}(q^2) + q^2[F_2^{\gamma}(q^2)/2M][F_2^{\gamma}(q^2)/2M]$ .
- <sup>20</sup>It is interesting to compare this ratio (5.34) with that (1.45) from (12) based on SU(6) symmetry [ $(f_\omega/f_\rho) = 3$ ]. The discrepancy may be attributed to the failure of the VDM in the scaling region.
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## Consistency of New Meson-Nucleon Elastic Scattering Data with a Previously Conjectured Universal Curve\*

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We show that important new large- $t$   $\pi^+p$  data at  $p_L = 13.8$  and  $22.6$  GeV/ $c$  are in good agreement with a universal curve suggested last year.

Recently a detailed experimental study<sup>1</sup> of elastic pion-proton scattering was made at large energies ( $p_L = 13.8$  and  $22.6$  GeV/ $c$ ) over a substantial part of the forward scattering region [ $|t| \lesssim 5$  (GeV/ $c$ )<sup>2</sup>] in order "to determine if the shapes and magnitudes of the cross sections are approaching limiting values as the momentum increases."<sup>1</sup> In this note we will show that these new data are in close agreement with an apparently universal curve for meson-nucleon scattering which was found last year in a study<sup>2</sup> of the high-energy ( $p_L \gtrsim 5$  GeV/ $c$ ) elastic data then available. At that time it was shown that existing forward and backward data fell on a simple curve if cross sections normalized to unity at  $t=0$ ,

$$f = \frac{d\sigma}{dt} / \left( \frac{d\sigma}{dt} \right)_{t=0}, \quad (1)$$

were plotted versus the dimensionless variable

$$\tau = -bt(s+t)/s, \quad (2)$$

where  $b(s)$  is the slope of the forward peak [ $|t| \lesssim 0.5$  (GeV/ $c$ )<sup>2</sup>] appropriate to the reaction and energy. The angular regions which fell on the universal curve were defined by  $|t| \lesssim 3$  (GeV/ $c$ )<sup>2</sup> in the forward region and  $|u| \lesssim 1$  (GeV/ $c$ )<sup>2</sup> in the backward region. The quadratic character of the variable  $\tau$  reflects the backward peak onto the forward peak. The universal character of the curve was expected to become even more evident at higher energies. If this curve is valid, a large body of existing data has been reduced to a determination of two quantities of obvious geometrical (optical) significance,  $b$  and  $\sigma_t$  (which fixes  $d\sigma/dt|_{t=0}$  via the optical theorem, assuming the real part of the forward amplitude is small).

The recent high-energy forward-region data of Cornillon *et al.*<sup>1</sup> give strong additional support to