bootstrap conditions for certain reaction amplitudes involving internal $P$ mesons only if the $P$ representation is $\underline{8}\oplus\underline{1}\oplus\underline{1}$. Two of the isoscalar mesons are predicted to be particular octet-singlet mixtures, while the third must be a pure singlet. It was suggested in Ref. 2 that the $E$ is the pure singlet. However, as pointed out above, this is incompatible with the $E$-decay data. In this model the $X(958)$ must be the pure singlet, and the $\eta$ and $E$ the singlet-octet mixtures.[4] We take the $\eta$ to be the mixture with the larger octet component.

If this assignment is made, the ratio of the $K^*(890)$-$K\pi$ and $K^*(890)$-$KE$ couplings may be computed from the interaction constants of Ref. 2. The phase-space factor for the $K^*\to K\pi$ decay is often taken as $p^3/M^2$, where $p$ is the decay momentum in the $K^*$ rest system, and $M$ is the $K^*$ mass. If the $VPP$ vertex is of the type $e\cdot(p_1-p_2)$, where $e$ is the $V$ polarization four-vector, and $p_1$ and $p_2$ are the four-momenta of the two $P$ mesons, the corresponding phase-space factor for the $E\to K^*\bar{K}$ decay is $p^3/E_{K^*}{}^2$, where $E_{K^*}$ is the decay energy of the $K^*$, approximately the $K^*$ mass. If these phase-space factors are used, a $K^*$ width of 50 MeV leads to a predicted $(\bar{K}K^*+\bar{K}K^*)$ partial width of the $E$ of ~9 MeV. This compares favorably with the tentative value of ~12 MeV given in Ref. 1.

Finally, we want to point out that if the $E$ is in an axial-vector meson, there remains a problem with the SU(3) classification. The $A_1(1100)$ has the appropriate $G$ parity to belong to the $E$ octet. The $A_1\to\pi\rho$ decay appears experimentally to occur predominantly in the $S$ wave. If one uses a simple phase-space factor of $p$ for the $S$-wave decays, the ~300-MeV width of the $A_1\to\pi\rho$ decay leads to a predicted $E\to(K\bar{K}^*+\bar{K}K^*)$ partial width of about 200 MeV, if the $E$ is a pure octet particle. This is compatible with the measured value of ~12 MeV only if the octet component of the $E$ is extremely small.

Part of this work was done while the author was visiting the Lawrence Berkeley Laboratory.

*Work supported in part by the National Science Foundation.

[1]Particle Data Group, Rev. Mod. Phys. Suppl. 45, S1 (1973).

[2]Richard H. Capps, Phys. Rev. D 7, 3394 (1973).

[3]R. Odorico, Phys. Lett. 38B, 37 (1972).

[4]Some experimental evidence has been cited that favors $2^-$ over $0^-$ for the spin-parity of the $X(958)$. See V. I. Ogievetsky, W. Tybor, and A. N. Zaslavsky, Phys. Lett. 35B, 69 (1971); G. R. Kalbfleisch *et al.*, Phys. Rev. Lett. 31, 333 (1973). If this turns out to be the case, the $E$ may be the ninth member of the nonet, and the predictions of Ref. 2 do not apply.

PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

# Field-Theoretic Calculation of the Direct-Emission Amplitude in $K^\pm\to\pi^\pm\pi^0\gamma$

C. A. Dominguez and A. Zepeda

*Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politecnico Nacional, Apartado Postal 14-740, Mexico 14, D.F.*

(Received 21 May 1973)

We show that to order $eg^2G$ the direct-emission amplitude in the decay $K^\pm\to\pi^\pm\pi^0\gamma$ is logarithmically divergent.

The possible existence of a direct emission in the decay $K^\pm\to\pi^\pm\pi^0\gamma$ (Refs. 1–4) has recently attracted much attention after clear evidence for such a contribution was reported.[1,2] A short time ago Barshay and Hvegholm[5] computed the direct-emission amplitude to order $eg^2G$,[6] in perturbation theory, in a model in which the two pions rescatter through a $\rho$ meson in the direct channel. In their calculation the divergences of the pion loops cancel out and thus a finite result is obtained. However, the direct-channel contribution is not the only one arising at order $eg^2G$. In other words, given the interaction-Hamiltonian density considered in Ref. 5, the crossed-channel diagrams of that same order should in principle be considered also.

The purpose of the present paper is to point out that if all diagrams to order $eg^2G$ are included in the calculation the direct-emission amplitude turns out to be logarithmically divergent.

The considered interaction-Hamitonian density

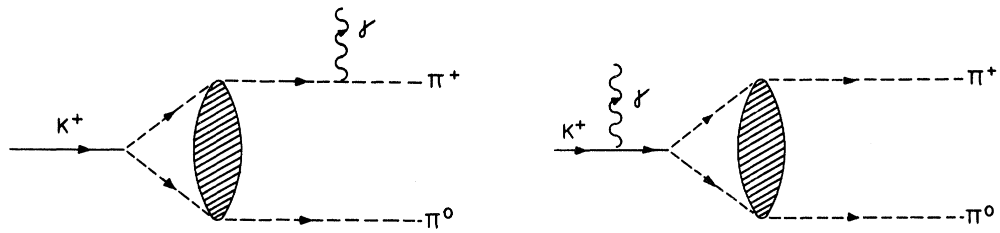


FIG. 1. Bremsstrahlung diagrams.

for $K^+ \to \pi^+\pi^0\gamma$ is

$$H = ie[\pi^-(\partial_\mu\pi^+) - (\partial_\mu\pi^-)\pi^+]A^\mu + ie[K^-(\partial_\mu K^+) - (\partial_\mu K^-)K^+]A^\mu + ie[\rho^{\nu-}(\partial_\mu\rho_\nu^+) - (\partial_\mu\rho_\nu^-)\rho^{\nu+}]A^\mu$$
$$+ ie[(\partial^\nu\rho_\mu^-)\rho_\nu^+ - \rho_\nu^-(\partial^\nu\rho_\mu^+)]A^\mu - ig[\pi^0(\partial_\mu\pi^+) - (\partial_\mu\pi^0)\pi^+]\rho^{\mu-} + ig[\pi^0(\partial_\mu\pi^-) - (\partial_\mu\pi^0)\pi^-]\rho^{\mu+}$$
$$+ ge\pi^0\pi^+\rho_\mu^- A^\mu + ge\pi^0\pi^-\rho_\mu^+ A^\mu + GK^+\pi^-\pi^0 + GK^-\pi^+\pi^0 \,, \qquad (1)$$

where the interactions between $\rho$ mesons, pions, and photons arise from a Yang-Mills-type coupling.

The matrix element for the decay can be written as

$$\langle \pi^0\pi^+\gamma | R | K^+\rangle = \frac{1}{2\pi^2}\,\frac{1}{(2E_\gamma 2E_k 2E_{\pi^+} 2E_{\pi^0})^{1/2}} \times (M_d + M_b),$$

where $M_d$ and $M_b$ are the direct-emission and the bremsstrahlung amplitudes, respectively. The interaction Hamiltonian Eq. (1) gives rise to the two general bremsstrahlung diagrams shown in Fig. 1,

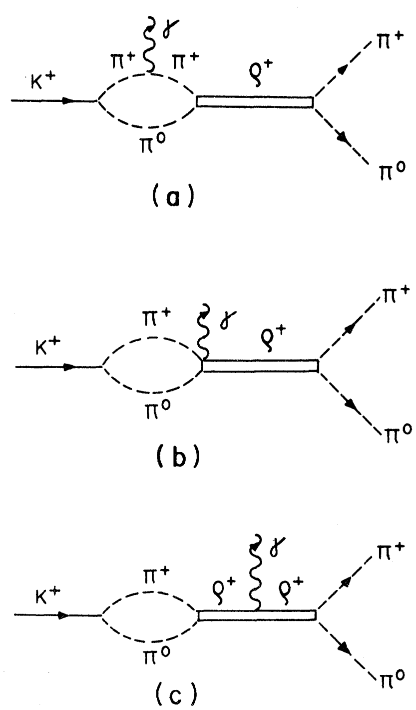


FIG. 2. Direct-emission diagrams.

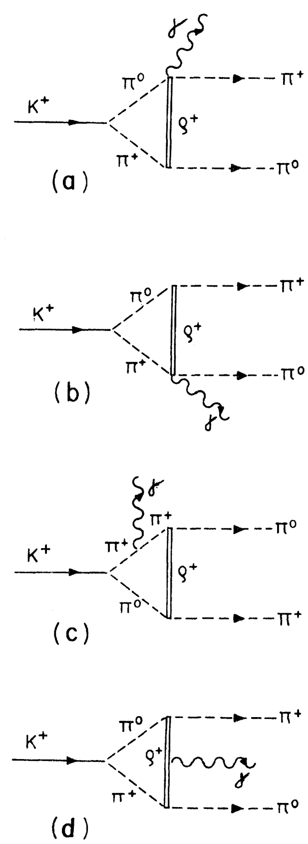


FIG. 3. Direct-emission diagrams.

and a straightforward calculation gives for $M_b$ the expression[7]

$$M_b = -ieG\left(\frac{k_+\cdot\epsilon}{k_+\cdot p} - \frac{k\cdot\epsilon}{k\cdot p}\right)e^{i\delta_0(M_K)},$$

where $k_+$, $k$, and $p$ are the $\pi^+$, $K$, and $\gamma$ four-momenta, $\epsilon$ the polarization vector of the $\gamma$, and $\delta_0(M_K)$ the isotopic-spin-2 $s$-wave pion-pion scattering phase shift.

On the other hand, the diagrams produced by Eq. (1) to order $eg^2G$, and relevant to $M_d$, are shown in Figs. 2 and 3. Those in Fig. 3 were not explicitly considered in Ref. 5. The divergent contribution to $M_d$ of each of the diagrams 2(a) and 2(b) is of the form

$$\pm ieg^2G\frac{1}{(4\pi)^2}\,\frac{(P\cdot\epsilon\, k\cdot p - k\cdot\epsilon\, P\cdot p)}{s - m_\rho^{\,2}}\,\frac{1}{2k\cdot p}\ln\frac{2\Lambda}{\mu},$$

where $P = k_+ - k_0$, $k_0$ being the $\pi^0$ four-momentum, $s = (k_+ + k_0)^2$, $\mu$ the pion mass, and $\Lambda$ the cutoff parameter. Since there is only a sign difference between the two divergent contributions, they cancel out. Figure 2(c) is finite and at most gives rise to a surface term.

We have calculated the divergent contributions to $M_d$ from the diagrams of Fig. 3, obtaining the total result

$$ieg^2G\frac{1}{4m_\rho^{\,2}}\left(\frac{\epsilon\cdot P}{p\cdot P} - \frac{\epsilon\cdot k}{p\cdot k}\right)P\cdot p\ln\frac{2\Lambda}{\mu}.$$

Thus, given the interaction-Hamiltonian density Eq. (1), the $s$ matrix to order $eg^2G$ is infinite.

We want to thank Dr. J. Hvegholm and Dr. M. Alexanian for very helpful discussions.

[1]R. J. Abrams *et al.*, Phys. Rev. Lett. 29, 1118 (1972).
[2]R. J. Abrams *et al.*, Phys. Rev. Lett. 30, 500 (1973).
[3]D. Ljung and D. Cline, Phys. Rev. D 8, 1307 (1973).
[4]R. R. Edwards *et al.*, Phys. Rev. D 5, 2720 (1972).
[5]S. Barshay and J. Hvegholm, Phys. Rev. Lett. 28, 1409 (1972).
[6]$g$ is the $\rho\pi\pi$ coupling constant and $G$ the weak-decay constant.
[7]H. Chew, Nuovo Cimento 26, 1109 (1962).