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Kaon Partial–Conservation–of–Axial-Vector–Current Anomaly and $K_L \rightarrow \gamma\gamma$ Decay

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The low-energy theorem is derived which relates the invariant vertex function of $K_L \rightarrow \gamma\gamma$ to the corresponding function of $\langle 2\gamma | [Q_5^6, L_w] | 0\rangle$, where it is assumed that Schwinger and seagull terms, if present, cancel against each other. A chiral Lagrangian model is studied which satisfies PCAC and which has an octet weak Lagrangian with $[Q_5^6, L_w]=0$. In this model, the effective $K_L \rightarrow \gamma\gamma$ coupling constant produced by certain baryon loop graphs is found to remain finite as the momentum of the kaon goes to zero. This contradicts the low-energy theorem and indicates the failure of seagull and Schwinger terms to cancel. The effective coupling produced by this anomaly is estimated to give a rate roughly comparable to the experimental one. Implications for mesonic effective-Lagrangian studies of K decays are discussed.

I. INTRODUCTION

Since its discovery by Bell and Jackiw[1] and by Adler,[2] the anomaly in partially conserved axial-vector current (PCAC) Ward identities has been the object of intensive study.[3] From investigation of Lagrangian field theories, some understanding has been gained of the source of the anomaly,[4] its renormalization (or lack of it),[5] and its generalizations to SU(3)×SU(3).[6] The problems that the axial-current anomaly create in proving renormalization of spontaneously-broken gauge theories have recently been discussed.[7] The existence of the anomaly does not depend upon existence of a perturbation expansion, as has been brought out in studies of short-distance behavior of the appropriate operator products.[8]

Several experimental tests for the existence of the anomaly have been proposed,[9] but the consequences are largely restricted to electromagnetic decays and electron-position annihilation processes. A possible test for the SU(3)×SU(3) version of the anomalies links the $\gamma \rightarrow 3\pi$ anomaly to the vector-current amplitude in K_{l4} decay,[10] but weak hadronic decays remain, for the most part, outside the problem of PCAC anomalies.

In this paper, we analyze the weak-electromagnetic process $K_L \rightarrow \gamma\gamma$ to determine the conditions under which an anomalous kaon PCAC effect could exist. In Sec. II, we examine the appropriate Ward identity and argue that if Schwinger and seagull terms cancel then the soft-kaon limit yields a low-energy theorem relating the invariant amplitude describing $K_L \to \gamma\gamma$ decay to the invariant amplitude for the matrix element

$$\langle 2 \text{ photons} | [Q_5^6, L_{\text{weak}}] | 0 \rangle$$

(Ref. 11). For the class of models where $[Q_5^6, L_w] = 0$, the $K_L \to \gamma\gamma$ amplitude is predicted to be zero in the soft-kaon limit. In Sec. III, a chiral Lagrangian model is studied which has PCAC for the pseudoscalars and which has an effective L_w which satisfies $[Q_5^6, L_w] = 0$. However, loop graphs[12] are found which violate the requirement that the $K_L \to \gamma\gamma$ invariant amplitude vanish as $P_K \to 0$, indicating a failure of the Schwinger and seagull terms to cancel.[13] The possible size of the effect is estimated in Sec. IV, and results are summarized and discussed in Sec. V. Details of the Ward identity analysis are presented in Appendix A, while in Appendix B a demonstration is given which shows that pseudoscalar (ps) and pseudovector (pv) coupling versions of the model give equivalent results for loop graphs.

II. WARD IDENTITY, KAON PCAC, AND $K_L \to \gamma\gamma$

In this section, we review the Ward identity appropriate to an off-shell pseudoscalar meson in the presence of a weak perturbation. For the case that $[Q_5^6, L_w^{\text{NL}}] = 0$, where Q_5^i are the axial charges and L_w^{NL} is the $\Delta S = 1$ nonleptonic weak Lagrangian, we present a formal argument that the invariant amplitude for the decay $K_L \to \gamma\gamma$ vanishes in the soft-K limit. This case, which includes the current-current octet Lagrangian $L_w \sim d_{6ab} j_\mu^a j^{\mu b}$,[14] is particularly transparent when looking for contradictions to the formal result in a calculation based on a chiral Lagrangian model.

We take PCAC in the operator form[15]

$$\partial_\mu A_i^\mu = F_\pi m_i^2 \phi_i, \quad i = 1, \ldots, 8 \tag{1}$$

and apply it to the matrix element

$$\langle \gamma(p), \gamma(q) | L_w(0) | \phi_6(k) \rangle,$$

which describes $K_L \to \gamma\gamma$ decay. A Lorentz tensor amplitude $G_{\alpha\beta}(k, p, q)$ is defined as follows:

$$\begin{aligned}
&(2\pi)^{9/2}(8k_0 p_0 q_0)^{1/2} \langle \gamma(p), \gamma(q) | L_w(0) | \phi_6(k) \rangle \\
&\quad \equiv e^\alpha(p) e^\beta(q) G_{\alpha\beta}(k, p, q) \\
&\quad = \frac{i(m_K^2 - k^2)}{m_K^2 F_\pi} \int d^4x\, e^{-ik\cdot x} (2\pi)^3 (4p_0 q_0)^{1/2} \\
&\qquad \times \langle \gamma(p), \gamma(q) | T(\partial A_6(x) L_w(0)) | 0 \rangle,
\end{aligned} \tag{2}$$

where the last step follows from the reduction formalism applied to K_L (Ref. 16) and the PCAC relation (1). Corresponding amplitudes $G_{\mu\alpha\beta}$ and $R_{\mu\alpha\beta}$ are defined as

$$\begin{aligned}
&\int d^4x\, e^{-ik\cdot x} \langle \gamma(p), \gamma(q) | T^*(A_\mu^6(x) L_w(0)) | 0 \rangle \\
&\quad \times (4p_0 q_0)^{1/2} (2\pi)^3 \\
&\qquad \equiv e^\alpha(p) e^\beta(q) G_{\mu\alpha\beta}(k, p, q) \\
&\qquad = \frac{F_\pi e^\alpha(p) e^\beta(q) k_\mu G_{\alpha\beta}}{m_K^2 - k^2} \\
&\qquad\quad + R_{\mu\alpha\beta}(k, p, q) e^\alpha(p) e^\beta(q).
\end{aligned} \tag{3}$$

The Ward identity then reads

$$\begin{aligned}
&k^\mu e^\alpha(p) e^\beta(q) [G_{\mu\alpha\beta}(k, p, q) + \text{possible seagulls}] \frac{(m_K^2 - k^2)}{F_\pi m_K^2} \\
&\quad - \frac{(m_K^2 - k^2)}{F_\pi m_K^2} (4p_0 q_0)^{1/2} (2\pi)^3 \left\langle \gamma(p), \gamma(q) \middle| \int d^3x\, e^{i\vec{k}\cdot\vec{x}} [A_0^6(\vec{x}, 0), L_w(0)] \middle| 0 \right\rangle = e^\alpha(p) e^\beta(q) G_{\alpha\beta}(k, p, q).
\end{aligned} \tag{4}$$

If it is assumed that seagull terms and Schwinger terms are either absent or cancel each other, then the covariant T product and the charge commutator can be used to write Eq. (4) as

$$\begin{aligned}
&e^\alpha(p) e^\beta(q) \left\{ G_{\alpha\beta}(k, p, q) + \frac{k^\mu}{F_\pi} R_{\mu\alpha\beta}(k, p, q) \right\} \\
&\quad = \frac{-i}{F_\pi} (4p_0 q_0)^{1/2} (2\pi)^3 \langle \gamma(p), \gamma(q) | [Q_5^6, L_w] | 0 \rangle.
\end{aligned} \tag{5}$$

The general covariant expansions for $G_{\alpha\beta}(p, q, k)$ and $R_{\mu\alpha\beta}(p, q, k)$ are very lengthy and are presented in Appendix A. We are only interested in the part of the braces in Eq. (5) which has the form

$$\mathfrak{M}(p\cdot q) p^\rho q^\sigma \epsilon_{\rho\sigma\alpha\beta},$$

when $k = p + q$. As shown in Appendix A, we can express Eq. (5) in terms of the invariant functions G_i and R_i which occur in the expansions of $G_{\alpha\beta}$ and $R_{\mu\alpha\beta}$, respectively. Displaying the important terms explicitly, we rewrite the Ward identity (5) as

$$e^{\alpha}(p)e^{\beta}(q)\Big[\Big(G_1+\frac{R_4 q\cdot k+\tilde{R}_4 p\cdot k+R_{12}k^2}{F_\pi}\Big)p^\rho q^\sigma\epsilon_{\rho\sigma\alpha\beta}+\Big(G_2+\frac{-\tilde{R}_1+\tilde{R}_{13}k^2+\tilde{R}_{14}p\cdot k+\tilde{R}_{15}q\cdot k}{F_\pi}\Big)k^\rho p^\sigma\epsilon_{\rho\sigma\alpha\beta}$$

$$+\Big(G_3+\frac{R_1-R_{13}k^2-R_{14}k\cdot q-R_{15}k\cdot p}{F_\pi}\Big)k^\rho q^\sigma\epsilon_{\rho\sigma\alpha\beta}+\cdots\Big]=\frac{f(p\cdot q)}{F_\pi}p^\rho q^\sigma\epsilon_{\rho\sigma\alpha\beta}e^{\alpha}(p)e^{\beta}(q), \qquad (6)$$

where the G_i and R_i depend on the kinematical invariants k^2, $s=(p+q)^2$, $t=(k-p)^2$, $u=(k-q)^2$. The functions $\tilde{G}_i$ are related to G_i by

$$\tilde{G}_i(k^2, s, t, u)=G_i(k^2, s, u, t) \qquad (7)$$

and similarly for R_i. The invariant function $f(p\cdot q)$ describes the matrix element of the commutator $[Q_5^6, L_w]$ and is defined as

$$-i(2\pi)^3(4p_0q_0)^{1/2}\langle\gamma(p),\gamma(q)|[Q_5^6, L_w]|0\rangle = f(p\cdot q)p^\rho q^\sigma\epsilon_{\rho\sigma\alpha\beta}e^{\alpha}(p)e^{\beta}(q). \qquad (8)$$

The content of the Ward identity is now contained in the set of relations

$$\begin{aligned} &F_\pi G_1+R_4 q\cdot k+\tilde{R}_4 p\cdot k+R_{12}k^2=f,\\ &F_\pi G_2-\tilde{R}_1+\tilde{R}_{13}k^2+\tilde{R}_{14}p\cdot k+\tilde{R}_{15}q\cdot k=0, \qquad (9)\\ &F_\pi G_3+R_1-R_{13}k^2-R_{14}k\cdot q-R_{15}k\cdot p=0\\ &\ldots .\end{aligned}$$

The invariant function appropriate to $K_L\to\gamma\gamma$ decay is regained by imposing energy-momentum conservation, $k=p+q$. We then have

$$G_{\alpha\beta}(k=p+q, p, q)=G(k^2=2p\cdot q)p^\rho q^\sigma\epsilon_{\rho\sigma\alpha\beta}, \qquad (10a)$$

where

$$G(k^2)\equiv G_1(k^2, k^2, 0, 0)+G_2(k^2, k^2, 0, 0)-G_3(k^2, k^2, 0, 0), \qquad (10b)$$

where the dependence on invariants was defined in Eq. (7). From Eq. (9) one now obtains

$$\begin{aligned} F_\pi G(k^2)=f(k^2)&+2R_1(k^2, k^2, 0, 0)-k^2R_4(k^2, k^2, 0, 0)\\ &-k^2R_{12}(k^2, k^2, 0, 0)-2k^2R_{13}(k^2, k^2, 0, 0)\\ &-k^2R_{14}(k^2, k^2, 0, 0)-k^2R_{15}(k^2, k^2, 0, 0). \end{aligned} \qquad (11)$$

Gauge invariance imposes the restriction

$$R_1+\tilde{R}_1=(R_9k\cdot q+\tilde{R}_9k\cdot p)+(R_2+\tilde{R}_2)p\cdot q, \qquad (12a)$$

which becomes

$$2R_1(k^2, k^2, 0, 0)=k^2R_9(k^2, k^2, 0, 0)+k^2R_{12}(k^2, k^2, 0, 0), \qquad (12b)$$

when $k=p+q$. Our final result is obtained by substituting (12b) into Eq. (11), and it reads

$$F_\pi G(k^2)=f(k^2)+k^2[R_2+R_9-(R_4+R_{12}+2R_{13}+R_{14}+R_{15})]. \qquad (13)$$

The R's are free of kinematical singularities as $k^2\to 0$ and we find

$$G(0)=f(0)/F_\pi. \qquad (14)$$

When $[Q_5^6, L_w]=0$ *the invariant function* f *is zero, and* $G(0)$ *vanishes for this case.*

As mentioned above, the octet weak Lagrangian $d_{6ab}j_\mu^a j^{\mu b}$ has the desired property since

$$[Q_5^i, L_w]=[Q^i, L_w]=if_{6ij}L_w^j. \qquad (15)$$

Lee[17] and Schechter[18] have studied a chiral dynamics model which includes baryons, pseudoscalar mesons, and a weak Lagrangian which satisfies Eq. (15). Since PCAC is built into this model and since $[Q_5^6, L_w]=0$, we expect the invariant amplitude for $K_L\to\gamma\gamma$ to vanish as $k^2\to 0$ if the assumptions of the argument presented in this section are satisfied. The baryon-loop contributions to $K_L\to\gamma\gamma$ are studied in Sec. III to see if this is the case.

III. EFFECTIVE LAGRANGIAN MODEL WITH $K_L\to\gamma\gamma$ ANOMALY

Following Lee,[17] we consider a chiral-dynamics Lagrangian which accommodates PCAC (though the details of the meson part of the Lagrangian are not considered here). A Lagrangian for a massive octet of baryons which is chiral SU(3)×SU(3) invariant[19] is

$$L_B=\mathrm{Tr}[(1-\alpha_1-\alpha_2)\bar{N}i\gamma\cdot\partial N+\alpha_1\bar{N}_1 i\gamma\cdot\partial N_1+\alpha_2\bar{N}_2 i\gamma\cdot\partial N_2-\tfrac{1}{2}m(\bar{N}_2N_1+\bar{N}_1N_2)], \qquad (16)$$

where

$$N_1=\frac{\sqrt{2}}{F_\pi}N(\sigma+i\phi\gamma_5)$$

and

$$N_2=\frac{\sqrt{2}}{F_\pi}(\sigma-i\phi\gamma_5)N. \qquad (17)$$

The fields ϕ and N are, respectively, pseudoscalar-meson and nucleon fields which transform as members of $(3^*, 3)+(3, 3^*)$ and $(8, 1)$. The axial transformations are

$$\delta M = \tfrac{1}{2} i \{\lambda_i, M\}\beta_i ,$$

$$M = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} (\lambda_i \sigma_i + i\lambda_i \phi_i) = \sigma + i\phi ,$$

$$\delta\psi = -f_{ijk}\beta_j \gamma_5 \psi_k , \quad N_i = (1+\gamma_5)\psi_i , \tag{18}$$

$$N = \frac{1}{\sqrt{2}} \sum \lambda_i N_i ,$$

so

$$\delta N = \tfrac{1}{2} i [\lambda_i, N]\beta_i , \quad \delta N_1 = \tfrac{1}{2} i \{\lambda_i, N_i\}\beta_i ,$$

$$\delta N_2 = -\tfrac{1}{2} i \{\lambda_i, N_2\}\beta_i .$$

Also $M^\dagger M = \frac{1}{2} F_\pi{}^2$ is imposed. The Lagrangian (16) contains both pseudovector (pv) and pseudoscalar (ps) meson-baryon couplings. The ps couplings can be eliminated by a redefinition of the baryon field. The unitary transformation

$$N = U(i\phi\gamma_5) B U^\dagger (i\phi\gamma_5) , \tag{19}$$

where

$$U(i\phi\gamma_5) = \left(\frac{F_\pi/\sqrt{2} + \sigma}{\sqrt{2}\, F_\pi} \right)^{1/2} \left(1 + i\phi\gamma_5 \frac{1}{F_\pi/\sqrt{2} + \sigma} \right) \tag{20}$$

assures that

$$\tfrac{1}{2} \mathrm{Tr}(\overline{N}_2 N_1 + \overline{N}_1 N_2) = \mathrm{Tr}\overline{B}B , \tag{21}$$

and produces the interaction Lagrangian:

$$L_I = \frac{(1-2\alpha_2)}{\sqrt{2}\, F_\pi} \mathrm{Tr}\overline{B}\gamma_5\gamma_\mu \partial^\mu \phi B - \frac{(1-2\alpha_1)}{\sqrt{2}\, F_\pi} \mathrm{Tr}\overline{B}\gamma_5\gamma_\mu B \partial^\mu \phi + \phi^4 \text{ and higher} . \tag{22}$$

The axial-vector current in this model is given by

$$A^i_\mu = (1-2\alpha_2)\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu \tfrac{1}{2}\lambda^i B - (1-2\alpha_1)\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu B \tfrac{1}{2}\lambda^i - F_\pi \partial_\mu \phi^i + \cdots , \tag{23}$$

where the dots indicate terms bilinear and higher in the meson fields and trilinear and higher in products of meson fields with baryon fields. The coupling coefficients are related to the standard ones by

$$\frac{g}{m} = \frac{(1-2\alpha_2)}{F_\pi} \tag{24a}$$

and

$$G_A/G_V = 1 - 2\alpha_2 \tag{24b}$$

or

$$F_\pi = \left(\frac{G_A}{G_V} \right) \frac{m}{g} , \tag{24c}$$

the Goldberger-Treiman relation, with g the pseudoscalar-meson to nucleon coupling constant, F_π the pion decay constant ≈ 94 MeV, and G_A/G_V the ratio of axial-vector weak current to vector weak current form factors. In this model the relation

$$\left(\frac{D}{F} \right)_{\text{axial}} = \left(\frac{D}{F} \right)_{\text{pv}} = \frac{\alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2} \tag{25}$$

holds.

The phenomenological nonleptonic weak Lagrangian for baryon decays which transforms as the $(\lambda_6)^\alpha_\beta$ member of an (8, 1) contains two independent nonderivative terms and twelve independent derivative terms. Those terms which have direct bearing on baryon-loop contributions to $K_L \to \gamma\gamma$ decay read, after the transformation (19) is made,

$$L_{\text{NL}} = \tfrac{1}{2} d\, \mathrm{Tr}\overline{B}\{\lambda_6, B\} + \tfrac{1}{2} f\, \mathrm{Tr}\overline{B}[\lambda_6, B] + b_1 \mathrm{Tr}\overline{B}\{\lambda_6 \partial_\mu \phi\}\gamma_5\gamma^\mu B + b_2 \mathrm{Tr}\overline{B}\gamma_5\gamma^\mu\{\lambda_6, \partial_\mu \phi\} , \tag{26}$$

where terms higher than first order in ϕ have been dropped. Charge couplings (and contact terms) involving the electromagnetic field $\mathcal{A}_\mu$ are generated by the minimal substitution $\partial_\mu \to \partial_\mu - ie\mathcal{A}_\mu[Q, \;]$, with $Q = \frac{1}{2}[\lambda_3 + (1/\sqrt{3})\lambda_8]$. Only the

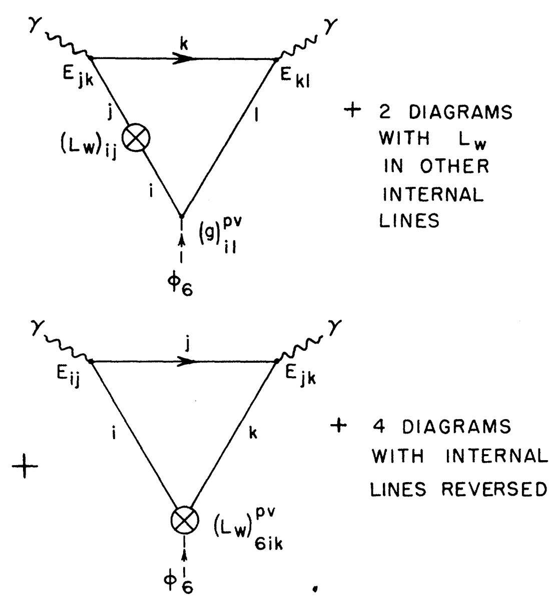

FIG. 1. Feynman diagrams for $K_L \to \gamma\gamma$ with the model described in the text. The pv label on the $\overline{B}B\Phi$ weak- and strong-coupling vertices refers to pseudovector couplings. The labels i, j, k, and l indicate the octet indices of the internal baryon lines while $(g)_{6il}$, $(L_w)_{ij}$, $(L_w)_{6ij}$, and E_{jk} are given by $(g/m)(Dd_{6il} + Ff_{6il})$, $(dd_{6ji} + iff_{6ij})$, $\sqrt{2}(b_1+b_2)d_{66k}d_{jik}$, and $e(f_{3jk} + 1/\sqrt{3} f_{8jk})$, respectively.

charge coupling of baryons to the photon,

$$L_{em} = -ie\,\mathrm{Tr}\bar{B}\not{a}[Q,B] = e\bar{B}_i\not{a}B_j\left(f_{ji3}+\frac{1}{\sqrt{3}}f_{ji8}\right), \tag{27}$$

enters in this calculation.

The $K_L \to \gamma\gamma$ loop graph contributions which are zeroth and first order in strong pv coupling are shown in Fig. 1. The amplitude $G_{\alpha\beta}(k,p,q)$, as defined in Sec. II, which is computed from these graphs, reads[20]

$$G^{\alpha\beta}(k,p,q) = -2\sqrt{2}\,e^2(b_1+b_2)[\Gamma^{\mu\alpha\beta}(k,p,q)+\Gamma^{\mu\beta\alpha}(k,q,p)]k_\mu + (g/m)e^2(fF+dD)[\tilde{\Gamma}^{\mu\alpha\beta}(k,p,q)+\tilde{\Gamma}^{\mu\beta\alpha}(k,q,p)]k_\mu \tag{28}$$

where $\Gamma^{\mu\alpha\beta}$ and $\tilde{\Gamma}^{\mu\alpha\beta}$ are given by

$$\Gamma^{\mu\alpha\beta}(k,p,q) = \mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5\gamma^\mu(\not{t}+\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-1}$$

$$\begin{aligned}\tilde{\Gamma}^{\mu\alpha\beta}(k,p,q) &= \mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5\gamma^\mu(\not{t}+\not{p}-m)^{-2}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-1}\\ &\quad+\mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5\gamma^\mu(\not{t}+\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-2}\gamma^\beta(\not{t}-\not{q}-m)^{-1}\\ &\quad+\mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5\gamma^\mu(\not{t}+\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-2}\\ &= \frac{d}{dm}\Gamma^{\mu\alpha\beta}.\end{aligned} \tag{29}$$

These amplitudes satisfy the relations

$$\begin{aligned}k_\mu\Gamma^{\mu\alpha\beta}(k,p,q) &= 2m\Gamma^{\alpha\beta}(k,p,q),\\ k_\mu\tilde{\Gamma}^{\mu\alpha\beta}(k,p,q) &= 2m\tilde{\Gamma}^{\alpha\beta}(k,p,q)+2\Gamma^{\alpha\beta}(k,p,q),\end{aligned} \tag{30}$$

with the definitions

$$\Gamma^{\alpha\beta}(k,p,q) = \mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5(\not{t}-\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-1},$$

$$\begin{aligned}\tilde{\Gamma}^{\alpha\beta}(k,p,q) &= \mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5(\not{t}-\not{p}-m)^{-2}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-1}\\ &\quad+\mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5(\not{t}-\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-2}\gamma^\beta(\not{t}-\not{q}-m)^{-1}\\ &\quad+\mathrm{Tr}\int\frac{d^4t}{(2\pi)^4}\gamma_5(\not{t}-\not{p}-m)^{-1}\gamma^\alpha(\not{t}-m)^{-1}\gamma^\beta(\not{t}-\not{q}-m)^{-2}\\ &= \frac{d}{dm}\Gamma^{\alpha\beta}.\end{aligned} \tag{31}$$

The symmetrized combinations $T^{\alpha\beta}$ are defined by

$$\begin{aligned}T^{\alpha\beta}(k,p,q) &\equiv \Gamma^{\alpha\beta}(k,p,q)+\Gamma^{\beta\alpha}(k,q,p)\\ &\equiv \epsilon^{\lambda\sigma\alpha\beta}p_\lambda q_\sigma T(k^2),\\ \tilde{T}^{\alpha\beta}(k,p,q) &\equiv \tilde{\Gamma}^{\alpha\beta}(k,p,q)+\tilde{\Gamma}^{\beta\alpha}(k,q,p)\\ &\equiv \epsilon^{\lambda\sigma\alpha\beta}p_\lambda q_\sigma\tilde{T}(k^2),\end{aligned} \tag{32}$$

and the invariant amplitudes have the values

$$T(0) = \frac{-i}{4\pi^2 m}$$

and

$$\tilde{T}(0) = \frac{i}{4\pi^2m^2} = \frac{d}{dm}T(0) \tag{33}$$

in the soft-kaon limit. Thus the more convergent amplitudes, which have the baryon "weak mass" transition in the internal loop, give contributions which are in agreement with the formal arguments of Sec. II, namely,

$$\begin{aligned}G(k^2)_{\text{"weak mass"}} &= 2\frac{g}{m}e^2(fF+dD)\left(m\frac{d}{dm}T+T\right)\\ &= -\frac{g}{m}e^2(fF+dD)\frac{k^2}{12m^2}\\ &\quad+k^4 \text{ and higher-order terms}.\end{aligned} \tag{34}$$

However, the *direct* parity-conserving phenomenological weak $\overline{B}B\phi$ coupling violates this formal result, and

$$G(k^2)_{\text{"direct"}} = -\frac{\sqrt{2}\,(b_1+b_2)}{\pi^2} + k^2 \text{ and higher-order terms}. \tag{35}$$

This is the anomaly-like behavior which we seek. Our assumption in Sec. II that seagull and Schwinger terms, if present, cancel against one another is not valid in this model. This is also a way to view the $\pi^0 \to \gamma\gamma$ anomaly.[13]

Though the effective Lagrangian which we are using is not renormalizable, the result (35) is of interest because it shows up in *finite* loops which are of the same order in strong and weak couplings as the usual pole approximations in chiral-dynamics calculations. Baryon nonleptonic decays can be used, for example, to estimate b_1 and b_2 in a calculation in which the couplings are present in the same orders as for $K_L \to \gamma\gamma$ loops. It is important to estimate the magnitude of this loop effect in order to assess the reliability of pole-model calculations of $K_L \to \gamma\gamma$.[21] If the anomaly can be sizable, perhaps phenomenological direct $K_L \to \gamma\gamma$ terms should be added to the weak Lagrangian, just as direct $\pi_0 \to \gamma\gamma$ terms are included which effectively represent the pion PCAC anomaly. Meson decay studies which attempt to determine the form of L_w^{NL} (Ref. 21) rely upon the absence of a direct $K_L \to \gamma\gamma$ coupling terms. Conclusions about the SU(3) content of L_w^{NL} would be weakened by the presence of such a term.

The result of Eq. (35) is superficially similar to the PCAC anomaly for $\pi^0 \to \gamma\gamma$ (which is present in this model also, and yields a π^0 width of 22 eV). However, though there is no explicit baryon mass dependence in Eq. (32), the parameters b_1 and b_2 are tied to a phenomenological evaluation of baryon nonleptonic processes. The coefficient of the π^0 anomaly, in contrast, can be expressed in terms of F_π and the fermion coupling to the axial-vector weak current. The possibility of interpreting this anomaly as a quark-loop effect does not seem likely in the present framework.

Because the Lagrangian is not renormalizable, there is no possibility of arguing that higher-order, multiloop graphs are free of anomalies and the argument for nonrenormalization[5] of the $K_L \to \gamma\gamma$ anomaly cannot be made. The contributions from phenomenological baryon magnetic form factors and direct strangeness-changing $\overline{B}B\gamma$ magnetic couplings[22] cannot be handled, since they lead to divergent graphs.

The result (35) does not lead to a complete calculation of $K_L \to \gamma\gamma$, but is an interesting effect which suggests a counterpart in renormalizable gauge-field models with hadrons[23] and which has implications for phenomenological Lagrangian studies of weak K decays.[21] Motivated by this latter consideration, we make a rough estimate of the triangle graph contributions to $K_L \to \gamma\gamma$ by determining the weak parameters d, f, b_1, and b_2 by a fit to hyperon s- and p-wave nonleptonic decay amplitudes.

IV. ESTIMATE OF BARYON-LOOP CONTRIBUTION

The effective weak Lagrangian of Lee[17] has fourteen independent terms but eight of the terms are SU(3)-breaking effects of order $\Delta m/m$, baryon mass splitting divided by baryon mass. We have ignored baryon mass differences up to this point and continue to do so in this section (except in denominators). The Lagrangian which describes weak baryon decay is then

$$\begin{aligned} L_w = {} & \tfrac{1}{2}(d-f)\left(\mathrm{Tr}\overline{B}B\lambda_6 + \frac{i}{\sqrt{2}\,F_\pi}\mathrm{Tr}\overline{B}B[\lambda_6,\phi]\right) \\ & + \tfrac{1}{2}(d+f)\left(\mathrm{Tr}\overline{B}\lambda_6 B + \frac{i}{\sqrt{2}\,F_\pi}\mathrm{Tr}\overline{B}[\lambda_6,\phi]B\right) \\ & + b_1\,\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu\{\lambda_6,\partial^\mu\phi\}B \\ & + b_2\,\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu B\{\lambda_6,\partial^\mu\phi\} \\ & + b_3\,\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu\partial^\mu\phi B\lambda_6 + b_4\,\mathrm{Tr}\overline{B}\gamma_5\gamma_\mu\lambda_6 B\partial^\mu\phi\,. \end{aligned} \tag{36}$$

As stressed by Lee, the last four terms do not vanish in the SU(3) limit nor do they obey the current-algebra relation between parity-violating s-wave amplitudes and parity-conserving "weak mass" transition amplitudes. In view of the well-known failure of "current algebra plus pole dominance" to correctly predict the ratios of parity-conserving to parity-violating amplitudes, the current algebra plus pole contributions must clearly be supplemented anyway. As was shown in Sec. III, these four derivative-coupling terms, which superficially vanish in the soft-meson limit, do *not* vanish as expected in the loop calculation.

The parameters d, f, and $b_1, \ldots, b_4$ can be fixed by comparison with the baryon nonleptonic decay amplitudes A and B defined by

$$\langle B'(p')\phi(k)\,|\,L_w(0)\,|B(p)\rangle \equiv \frac{i}{(2\pi)^{9/2}}\left(\frac{mm'}{8p_0k_0p_0'}\right)^{1/2}\overline{u}(p') \times (A + B\gamma_5)u(p)\,. \tag{37}$$

The results for the s-wave amplitudes are

$$A(\Sigma_-^-) = \frac{1}{2\sqrt{2}\,F_\pi}(f-d),$$

$$A(\Sigma_+^+) = 0,$$

$$A(\Sigma_0^+) = \frac{1}{4F_\pi}(d-f), \quad (38)$$

$$A(\Lambda_-^0) = \frac{1}{4F_\pi}\sqrt{\tfrac{1}{3}}\,(d+3f),$$

$$A(\Xi_-^-) = \frac{1}{4F_\pi}\sqrt{\tfrac{1}{3}}\,(d-3f),$$

and

$$B(\Sigma_-^-) = \frac{1}{\sqrt{2}}\left(\frac{G_A}{G_V}\right)\frac{m}{\Delta m}\left[\frac{F(f-d)}{F_\pi} - \frac{D}{3}\frac{(3f+d)}{F_\pi}\right] + 2mb_2,$$

$$B(\Sigma_+^+) = \frac{1}{\sqrt{2}}\left(\frac{G_A}{G_V}\right)\frac{m}{\Delta m}\left[D\frac{(f-d)}{F_\pi} - \frac{D}{3}\frac{(3f+d)}{F_\pi}\right] + 2mb_3,$$

$$B(\Sigma_0^+) = \tfrac{1}{2}\left(\frac{G_A}{G_V}\right)\frac{m}{\Delta m}\left[(f-d)\frac{(1-2F)}{F_\pi}\right] + \sqrt{2}\,m(b_3-b_2), \quad (39)$$

$$B(\Lambda_-^0) = \tfrac{1}{2}\left(\frac{G_A}{G_V}\right)\frac{m}{\Delta m}(\tfrac{1}{3})^{1/2}\left[-2D\frac{(f-d)}{F_\pi} + \frac{(3f+d)}{F_\pi}\right] - \frac{2m}{\sqrt{6}}(2b_1-b_2-b_3),$$

$$B(\Xi_-^-) = \tfrac{1}{2}\left(\frac{G_A}{G_V}\right)\frac{m}{\Delta m}(\tfrac{1}{3})^{1/2}\left[-2D\frac{(f+d)}{F_\pi} - (D-F)\frac{(3f-d)}{F_\pi}\right] + \frac{2m}{\sqrt{6}}(b_1+b_4-2b_2),$$

for p-wave amplitudes. A "mean mass" m and "mean mass splitting" Δm are taken for simplicity in expressing the B amplitudes. This is consistent with the disregard for mass splittings in the baryon-loop amplitudes. The values of the parameters and the assumed input values for G_A/G_V, m, Δm, and F_π are shown in Table I. A satisfactory description of the amplitudes is obtained as expected, since the s-wave amplitudes are the current-algebra results and are in satisfactory (10% or so) agreement with experiment. The addition of the b_i's preserves the $\Delta I = \frac{1}{2}$ rule among the $B(\Sigma)$ amplitudes, which are in 10–15% agreement with experiment, and at the same time allows one to lift each B magnitude to the experimental value.

To estimate the loop graph, it is sufficient to use the soft kaon approximation, $k \to 0$. The $O(m_K/m)^2$ correction from the graphs of Fig. 1 is only 4%. The result is

TABLE I. Predicted amplitudes, Lagrangian parameters, and $K_L \to \gamma\gamma$ branching ratio for the inputs $A(\Lambda_-^0)$, $A(\Sigma_-^-)$, $B(\Sigma_-^-)$, $B(\Sigma_+^+)$, $B(\Lambda_-^0)$, $B(\Xi_-^-)$,[a] $D/F = 1.5$,[b] $G_A/G_V = 1.23$, $\Delta m = 0.2$ GeV, and $m = 1.0$ GeV.

Hyperon decay amplitudes (dimensionless)	Predicted	Experimental[a]
$A(\Sigma_+^+)$	0	0.06 ± 0.02
$A(\Sigma_0^+)$	−1.36	-1.46 ± 0.06
$A(\Xi_-^-)$	−1.94	-2.02 ± 0.02
$B(\Sigma_0^+)$	13.9	12.2 ± 0.7

Lagrangian parameters $\times(\sqrt{2}F_\pi m_\pi^2/m_p^2 \times 10^{-5})^{-1}$ in GeV^{-1}	
$f = 3.96$	$f/d = -0.38$
$b_1 = 1.2$	$b_2 = -0.5$
$b_3 = 6.9$	$b_4 = 6.8$

$K_L \to \gamma\gamma$ branching ratio

$(K_L \to \gamma\gamma)/(K_L \to \text{all}) = 2 \times 10^{-4}$ (Expt. $= 5 \times 10^{-4}$), where $b_1 + b_2 = 1.5 \times 10^{-7}\ \text{MeV}^{-1}$

[a]Particle Data Group, Rev. Mod Phys. 45, S1 (1973).
[b]H. Filthuth, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969).

$$\Gamma_{K_L \to \gamma\gamma} \approx \alpha^2\left(\frac{m_K}{m}\right)^2[(b_1+b_2)m]^2\frac{1}{2\pi^3}m_K. \quad (40)$$

With the values listed in Table I, this yields

$$\frac{(\Gamma_{K_L \to \gamma\gamma})_{\text{theoretical}}}{\Gamma_{K_L \to \text{all}}} \approx 2 \times 10^{-4}$$

compared with the data compilation value 5×10^{-4}. We emphasize that no effort is being made to seriously calculate $K_L \to \gamma\gamma$, but only to get an idea of the order of magnitude of the theoretical effect discussed in Secs. II and III. Aside from unknown contributions such as magnetic couplings and higher-mass-resonance loops, a severe uncertainty

TABLE II. All input is the same as in Table I except that individual masses are put in everywhere in the baryon pole amplitudes (but not in the Goldberger-Treiman relation). Predicted amplitudes $A(\Sigma_+^+)$, $A(\Sigma_0^+)$, $A(\Xi_-^-)$, and $B(\Sigma_0^+)$ and f and D/F remain the same.

Lagrangian parameters $\times(\sqrt{2}F_\pi m_\pi^2/m_p^2 \times 10^{-5})^{-1}$ in GeV^{-1}	
$b_1 = 5.7$	$b_2 = 1.0$
$b_3 = 8.4$	$b_4 = 7.2$

$K_L \to \gamma\gamma$ branching ratio

$(K_L \to \gamma\gamma)/(K_L \to \text{all}) = 200 \times 10^{-4}$ (Expt. $= 5 \times 10^{-4}$), with $b_1 + b_2 = 15 \times 10^{-7}\ \text{MeV}^{-1}$

results from the sensitivity of pole-graph amplitudes in $B \to B'\phi$ to choice of mass inputs. For example, using the individual baryon mass values in Eq. (39) leads to the parameter values in Table II. The resulting $K_L \to \gamma\gamma$ branching ratio is an order of magnitude larger than the experimental one. We have established that, in this model, the rate calculated from the anomalous amplitude is comparable to the experimental $K_L \to \gamma\gamma$ rate.

V. SUMMARY AND CONCLUSIONS

We have taken a careful look at the Ward identity which, with kaon PCAC, provides a condition on the off-shell $K_L \to \gamma\gamma$ amplitude. We assumed that if seagull terms are present in the matrix element of the T produce $T(A_\mu^6(x)L_w(0))$ then they are canceled by corresponding matrix elements of Schwinger terms in the commutator $[A_6^0(\vec{x}, 0), L_w(0)]$. This assumption made it possible to state a low-energy theorem for the invariant amplitude $G(q\cdot p)$ appropriate to $K_L(k) \to \gamma(q)\gamma(p)$. For the case that $[Q_5^6, L_w]=0$, which is true, for example, in the model $L_w \sim d_{6ab} j_\mu^a j^{\mu b}$, the invariant amplitude $G(p\cdot q)$ should vanish in the soft-kaon limit, $k^2 = p\cdot q = 0$, according to the formal argument.

A chiral Lagrangian was then adopted which has baryon and pseudoscalar meson fields, operator PCAC for the eight axial-vector currents, and an effective current-current weak Lagrangian belonging to an (8, 1) representation of SU(3)×SU(3). The single-baryon loop amplitudes were examined and a contradiction to the formal arguments was found. That is, the parity-conserving $\bar{B}B\phi$ weak derivative coupling terms in L_w lead to baryon-loop graphs which do not vanish when the kaon momentum is set equal to zero. In an estimate of the size of this effect, we found that it can easily be comparable to the experimental $K_L \to \gamma\gamma$ rate.

We conclude that under certain conditions a soft-kaon theorem can be stated for the invariant amplitude for $K_L \to \gamma\gamma$, even though the Bose, Lorentz, gauge, and CP symmetry restrictions require that the matrix element be proportional to the momentum of the kaon. This is analogous to the PCAC analysis of $\pi^0 \to \gamma\gamma$. We find that a contradiction to the low-energy theorem is found in an effective Lagrangian which superficially satisfies the conditions of the formal argument. The source of the contradiction is the noncancellation of Schwinger and seagull terms in the manipulation of the T product. Because the estimate of the size of this anomalous kaon PCAC effect resulted in a $K_L \to \gamma\gamma$ rate comparable with the experimental one, we suggest that a *direct* $K_L \to \gamma\gamma$ (Ref. 24) coupling should be included in phenomenological chiral Lagrangian studies of weak and weak-electromagnetic K decays. For example, investigations which have argued that a $\underline{27}$ piece is needed in the current-current effective weak Lagrangian[21] have tacitly assumed that the direct coupling is small enough to ignore.

Because there is lack of solid evidence that kaon PCAC is reliable and because corrections to the basic effect which we find using kaon PCAC cannot be calculated in the model we use, this work does not provide a reliable calculation of baryon and kaon weak decays. Rather, we present it as demonstration of an effect which has implications for future calculations in a unified strong, electromagnetic, and weak renormalizable Lagrangian framework and which has direct bearing on parameterization of kaon decays in chiral-Lagrangian models.

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APPENDIX A

Recalling the definitions

$$\frac{i(m_K{}^2 - k^2)}{m_K{}^2 F_\pi}\int d^4x\, e^{-ik\cdot x}(2\pi)^3(4p_0q_0)^{1/2} \times \langle \gamma(p), \gamma(q)|T(\partial^\mu A_\mu^6(x)L_w(0))|0\rangle = e^\alpha(p)e^\beta(q)G_{\alpha\beta}(k,p,q)$$

and

$$(2\pi)^3(4p_0q_0)^{1/2}\int e^{-ik\cdot x}\langle \gamma(p),\gamma(q)|T^*(A_\mu^6(x)L_w(0))|0\rangle = e^\alpha(p)e^\beta(q)\left(F_\pi\frac{k_\mu G_{\alpha\beta}}{(m_K{}^2-k^2)}+R_{\mu\alpha\beta}\right),$$

we wish to write the covariant amplitudes $G_{\alpha\beta}(k,p,q)$ and $R_{\mu\alpha\beta}(k,p,q)$ in terms of explicit kinematical factors times scalar functions G which depend on the kinematical invariants

$$s=(p+q)^2,\quad t=(k-q)^2,\quad u=(k-p)^2,$$

and k^2. For $G_{\alpha\beta}(k,p,q)$ the desired expression is

$$\begin{aligned} G_{\alpha\beta}(k,p,q) = {} & G_1 q^\mu p^\nu \epsilon_{\mu\nu\alpha\beta} \\ & + G_2 k^\mu p^\nu \epsilon_{\mu\nu\alpha\beta} + G_3 k^\mu q^\nu \epsilon_{\mu\nu\alpha\beta} \\ & + (G_4 p_\beta + G_5 q_\beta + G_6 k_\beta)k^\mu q^\nu p^\sigma \epsilon_{\alpha\mu\nu\sigma} \\ & + (G_7 p_\alpha + G_8 q_\alpha + G_9 k_\alpha)k^\mu q^\nu p^\sigma \epsilon_{\beta\mu\nu\alpha}. \end{aligned} \tag{A1}$$

Bose symmetry requires the relations

$$G_1=\tilde{G}_1, \quad G_2=-\tilde{G}_2, \quad G_4=-\tilde{G}_8, \quad G_5=-\tilde{G}_7, \quad G_6=-\tilde{G}_9, \tag{A2}$$

where

$$\tilde{G}_i(s,t,u,k^2)=G_i(s,u,t,k^2).$$

Gauge invariance imposes two conditions:

$$G_2+p\cdot qG_7+q\cdot kG_9=0$$

and

$$G_3+p\cdot qG_5+p\cdot kG_6=0. \tag{A3}$$

Similar considerations for $R_{\mu\alpha\beta}(k,p,q)$ lead to the ponderous expression

$$\begin{aligned}
R_{\mu\alpha\beta}=\;&\epsilon_{\mu\alpha\beta\sigma}(R_1q^\sigma-\tilde{R}_1p^\sigma)+(R_2\epsilon_{\mu\alpha\sigma\rho}p_\beta-\tilde{R}_2\epsilon_{\mu\beta\sigma\rho}q_\alpha)q^\sigma p^\rho\\
&+(R_3\epsilon_{\mu\alpha\sigma\rho}q_\beta-\tilde{R}_3\epsilon_{\mu\beta\sigma\rho}p_\alpha)q^\sigma p^\rho+R_3\epsilon_{\mu\alpha\beta\sigma}k^\sigma+\epsilon_{\alpha\beta\rho\sigma}q^\rho p^\sigma(R_4q_\mu+\tilde{R}_4p_\mu)\\
&+(R_5\epsilon_{\mu\sigma\rho\beta}q^\sigma k^\rho q_\alpha+\tilde{R}_5\epsilon_{\mu\sigma\rho\alpha}p^\sigma k^\rho p_\beta)+(R_6\epsilon_{\mu\sigma\rho\alpha}q^\sigma k^\rho q_\beta+\tilde{R}_6\epsilon_{\mu\sigma\rho\beta}p^\sigma k^\rho p_\alpha)\\
&+(R_7\epsilon_{\mu\sigma\rho\beta}q^\sigma k^\rho p_\alpha+\tilde{R}_7\epsilon_{\mu\sigma\rho\alpha}p^\sigma k^\rho q_\beta)+(R_8\epsilon_{\mu\sigma\rho\alpha}q^\sigma k^\rho p_\beta+\tilde{R}_8\epsilon_{\mu\sigma\rho\beta}p^\sigma k^\rho q_\alpha)\\
&+(R_9\epsilon_{\mu\alpha\sigma\rho}q^\sigma p^\rho k_\beta-\tilde{R}_9\epsilon_{\mu\beta\sigma\rho}q^\sigma p^\rho k_\alpha)+(R_{10}\epsilon_{\mu\sigma\rho\alpha}p^\sigma k^\rho k_\beta+\tilde{R}_{10}\epsilon_{\mu\sigma\rho\beta}q^\sigma k^\rho k_\alpha)\\
&+(R_{11}\epsilon_{\mu\sigma\rho\beta}p^\sigma k^\rho k_\alpha+\tilde{R}_{11}\epsilon_{\mu\sigma\rho\alpha}q^\sigma k^\rho k_\beta)+R_{12}\epsilon_{\alpha\beta\rho\sigma}q^\rho p^\sigma k_\mu\\
&+(R_{13}\epsilon_{\alpha\beta\rho\sigma}q^\rho k^\sigma-\tilde{R}_{13}\epsilon_{\alpha\beta\rho\sigma}p^\rho k^\sigma)k_\mu+(R_{14}\epsilon_{\alpha\beta\rho\sigma}q^\rho k^\sigma q_\mu-\tilde{R}_{14}\epsilon_{\alpha\beta\rho\sigma}p^\rho k^\sigma p_\mu)\\
&+(R_{15}\epsilon_{\alpha\beta\rho\sigma}q^\rho k^\sigma p_\mu-\tilde{R}_{15}\epsilon_{\alpha\beta\rho\sigma}p^\rho k^\sigma q_\mu)+p^\rho k^\sigma q^\xi\\
&+(R_{16}\epsilon_{\alpha\rho\sigma\xi}p_\mu q_\beta-\tilde{R}_{16}\epsilon_{\beta\rho\sigma\xi}q_\mu p_\alpha+R_{17}\epsilon_{\alpha\rho\sigma\xi}p_\mu k_\beta-\tilde{R}_{18}\epsilon_{\beta\rho\sigma\xi}p_\mu k_\alpha\\
&\quad+R_{19}\epsilon_{\alpha\rho\sigma\xi}p_\beta k_\mu-\tilde{R}_{19}\epsilon_{\beta\rho\sigma\xi}q_\alpha k_\mu+R_{20}\epsilon_{\alpha\rho\sigma\xi}q_\beta k_\mu-\tilde{R}_{20}\epsilon_{\beta\rho\sigma\xi}p_\alpha k_\mu\\
&\quad+R_{21}\epsilon_{\alpha\rho\sigma\xi}p_\beta q_\mu-\tilde{R}_{21}\epsilon_{\beta\rho\sigma\xi}q_\alpha p_\mu+R_{22}\epsilon_{\mu\rho\sigma\xi}p_\alpha q_\beta-\tilde{R}_{22}p_\beta q_\alpha\\
&\quad+R_{23}\epsilon_{\mu\rho\sigma\xi}p_\alpha k_\beta-\tilde{R}_{23}\epsilon_{\mu\rho\sigma\xi}q_\beta k_\alpha+R_{24}\epsilon_{\mu\rho\sigma\xi}p_\beta k_\alpha-\tilde{R}_{24}\epsilon_{\mu\rho\sigma\xi}q_\alpha k_\beta).
\end{aligned} \tag{A4}$$

Now, eighteen conditions follow from imposing gauge invariance,

$$p^\alpha R_{\mu\alpha\beta}=0 \text{ and } q^\beta R_{\mu\alpha\beta}=0.$$

The details of most of these constraints, as well as the kinematical relations among the terms, are not of interest here. We quote only the gauge invariance conditions which are essential to the argument, namely,

$$R_1-\tilde{R}_2q\cdot p-\tilde{R}_9k\cdot p=0$$

and

$$\tilde{R}_1-R_2q\cdot p-R_9k\cdot q=0.$$

Forming $k^\mu R_{\mu\alpha\beta}$ from (A4) and using (A1), the low-energy theorem (5) takes the form shown in Eq. (6), where only the terms which have bearing on the $K_L\to\gamma\gamma$ amplitude are displayed. All others vanish identically when $k=p+q$ and are of no interest here.

Without the commutator amplitude $f(k^2)$, the form of Eq. (13) is identical to the corresponding result in $\pi^0\to\gamma\gamma$ analysis. When energy and momentum are conserved, the amplitude $R_{\mu\alpha\beta}$ has the form

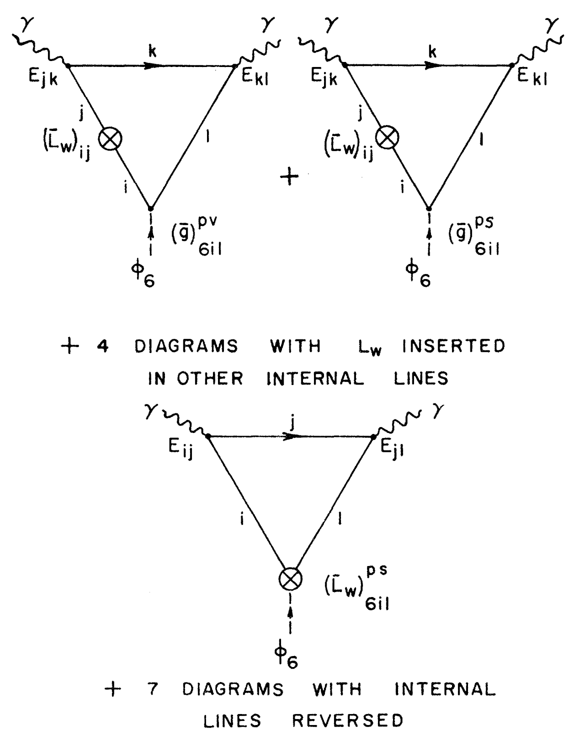

FIG. 2. Diagrams for $K_L\to\gamma\gamma$ in the mixed pv-ps form of the Lagrangian with nonderivative weak Lagrangian only. Couplings are defined in Appendix B.

$$R_{\mu\alpha\beta}(k,p,q)=r_1 k_\mu \epsilon_{\alpha\beta\sigma\rho} q^\sigma p^\rho + r_3(\epsilon_{\mu\alpha\sigma\rho} p_\beta - \epsilon_{\mu\beta\sigma\rho} q_\alpha) q^\sigma p^\rho + r_2(\epsilon_{\mu\alpha\sigma\rho} q_\beta - \epsilon_{\mu\beta\sigma\rho} p_\alpha) q^\sigma p^\rho + r_4 \epsilon_{\mu\alpha\beta\sigma}(q^\sigma - p^\sigma),$$

where

$$r_1(k^2)=R_4(k^2,k^2,0,0)+R_{12}(\cdots)+2R_{13}(\cdots)+R_{14}(\cdots)+R_{15}(\cdots)$$

and

$$r_3(k^2)=R_2(k^2,k^2,0,0)+R_9(\cdots).$$

The term in brackets in Eq. (13) is equal to (r_3-r_1), which is the same as the corresponding factor in the $\pi^0\to\gamma\gamma$ discussion of Bell and Jackiw,[1] for example.

APPENDIX B

In the chiral-invariant strong-interaction Lagrangian (16), there are both pv and ps couplings between the fields N and ϕ. The nonderivative weak Lagrangian, when expressed in terms of the N fields, is

$$L_w=\frac{(d-f)}{2}\mathrm{Tr}\left(\bar N N\lambda_6-\frac{i}{\sqrt2 F_\pi}\bar N N[\phi,\lambda_6]+\frac{i}{\sqrt2 F_\pi}[\phi,\bar N]\gamma_5 N\lambda_6-\frac{i}{\sqrt2 F_\pi}\bar N\gamma_5[\phi,N]\lambda_6\right)$$
$$+\frac{(d+f)}{2}\mathrm{Tr}\left(\bar N\lambda_6 N-\frac{i}{\sqrt2 F_\pi}[\phi,\lambda_6]N+\frac{i}{\sqrt2 F_\pi}[\phi,\bar N]\gamma_5\lambda_6 N-\frac{i}{\sqrt2 F_\pi}\bar N\gamma_5\lambda_6[\phi,N]\right)+\text{higher order in }\phi \qquad \text{(B1)}$$

and it also contains ps couplings. In this version the diagrams, shown in Fig. 2, which contain the ps couplings do not, by themselves, vanish as $k\to0$. However, the diagrams with *strong* ps coupling plus weak $N_i\to N_j$ internal line transition combine with the diagrams which have a direct $\Delta S=1$ *weak* ps coupling to produce the same structure as the graphs with strong pv coupling plus internal-line weak transitions. The sum of all graphs shown in Fig. 2 reproduces the result of the pv-coupling version of the same model. To see this, define the vertices

$$(\bar g)^{\rm pv}_{6ij}=-i\frac{(\alpha_1+\alpha_2)}{F_\pi}f_{6ij}-\frac{(\alpha_1-\alpha_2)}{F_\pi}d_{6ij},$$
$$(\bar L_w)_{ij}=(dd_{6ij}+if_{6ij}), \qquad \text{(B2)}$$
$$(\bar g)^{\rm ps}_{6ij}=2\frac{m}{F_\pi}f_{6ij},$$

and

$$(\bar L_w)^{\rm ps}_{6ij}=i\frac{2}{F_\pi}ff_{i6k}f_{k6j},$$

and fold in the propagator and momentum factors to obtain

$$i\frac{e^2}{F_\pi}[-f(\alpha_1+\alpha_2)+d(\alpha_1-\alpha_2)]\times[\tilde\Gamma^{\mu\alpha\beta}(p,q)+\tilde\Gamma^{\mu\beta\alpha}(q,p)]k_\mu \qquad \text{(B3a)}$$

from strong pv coupling,

$$2i\frac{me^2f}{F_\pi}[\tilde\Gamma^{\alpha\beta}(p,q)+\tilde\Gamma^{\beta\alpha}(q,p)] \qquad \text{(B3b)}$$

from strong ps coupling, and

$$2i\frac{e^2f}{F_\pi}[\Gamma^{\alpha\beta}(p,q)+\Gamma^{\beta\alpha}(q,p)] \qquad \text{(B3c)}$$

from direct weak ps coupling. The latter two terms combine to give

$$if\frac{e^2}{F_\pi}[2m\tilde\Gamma^{\alpha\beta}(p,q)+2\Gamma^{\alpha\beta}(p,q)+(\alpha\leftrightarrow\beta,p\leftrightarrow q)]=i\frac{e^2f}{F_\pi}k_\mu[\tilde\Gamma^{\mu\alpha\beta}(p,q)+\tilde\Gamma^{\mu\beta\alpha}(q,p)], \qquad \text{(B4)}$$

which has the same form as the pv-coupling term (B3a). The combination of (B3a) and (B4) yields the full amplitude

$$G^{\alpha\beta}(p,q)=\frac{e^2}{F_\pi}[f(1-\alpha_1-\alpha_2)+d(\alpha_1-\alpha_2)]\times[\tilde\Gamma^{\mu\alpha\beta}(p,q)+\tilde\Gamma^{\mu\beta\alpha}(q,p)]k_\mu$$

or

$$G^{\alpha\beta}(p,q)=e^2\frac{g}{m}[fF+dD]\times[\tilde\Gamma^{\mu\alpha\beta}+\tilde\Gamma^{\mu\beta\alpha}]k_\mu.$$

This is identical to the pv-coupling result obtained from the graphs in Fig. 1.

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