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Renormalizable SU(3) × SU(3) Model of the Weak Interaction*

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A new model of the weak interaction is presented. The renormalizable model, which is based on an SU(3) × SU(3) intermediate scalar boson coupling scheme, reproduces the successes of the standard $(V - A)$ theory for leptonic and semileptonic processes. Hadronic $\Delta Y = 1$ processes arise in the model exclusively from finite one-loop diagrams and obey an exact octet rule. The analogous $\Delta Y = 0$ graphs renormalize the quark masses to give finite mass differences within isotopic spin multiplets; when added to the electromagnetic mass differences within multiplets, this effect gives a good account of these mass splittings.

I. INTRODUCTION

Although the Fermi current-current weak interaction[1]

$$H_w(x) = \frac{G}{\sqrt{2}} J_\mu(x) J^{\mu\dagger}(x), \tag{1}$$

with

$$J^\mu = J_l^\mu + J_h^\mu, \tag{2}$$

$$J_l^\mu = \overline{\nu}_e \gamma^\mu (1-\gamma_5) e + \overline{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu, \tag{3}$$

$$J_h^\mu = J_{1+i2}^\mu \cos\theta + J_{4+i5}^\mu \sin\theta, \tag{4}$$

where the J_i^μ are an octet of currents,[2,3] has been very successful in its description of weak interactions, it cannot be considered a complete theory but must be viewed as a phenomenology of the lowest-order weak processes.

There have been many attempts to remedy this situation. In the intermediate-vector-boson theory the Fermi current-current interaction (1) arises in second order as an effective Hamiltonian in the local limit of a theory where the current $J^\mu(x)$ is coupled semiweakly to a vector boson field $W^\mu(x)$ according to

$$H_{sw}(x) = g J_\mu(x) W^\mu(x) + \text{H.c.} \tag{5}$$

In the limit $M_w \to \infty$ one recovers the Fermi theory in second order. Unfortunately, such a theory is still nonrenormalizable and so many quantities of interest must be expressed in terms of a cutoff mass.

Recently there has been renewed interest in finding a renormalizable theory of the weak interaction. One method that has been explored involves seeking a unified vector-boson theory of the weak and electromagnetic interactions, and such efforts have met with some success.[4,5] There have also been attempts to obtain renormalizability by introducing a scalar boson interaction.[6-8] Such considerations are based on the Fierz-Tanikawa reordering theorem[9,10]

$$\bar{\psi}_1\gamma^\mu(1-\gamma_5)\psi_2\bar{\psi}_3\gamma_\mu(1-\gamma_5)\psi_4 = -2\bar{\psi}_1(1+\gamma_5)\psi_3^c \times\bar{\psi}_2^c(1-\gamma_5)\psi_4, \quad (6)$$

which leads to the possibility that the Fermi current-current interaction arises in second order as the local limit of a semiweak interaction of a scalar boson with the densities $\bar{\psi}_i(1+\gamma_5)\psi_j^c$.

In what follows we shall present a simple SU(3) ×SU(3) scheme for a renormalizable intermediate scalar boson theory of the weak interaction. Among other things we will show that the theory (1) reduces to the usual $(V-A)$ results for nondiagonal leptonic and semileptonic processes in the local limit, (2) gives rise to hadronic weak interactions distinct in character from the leptonic and semileptonic interactions and obeying an exact octet rule (and therefore an exact $\Delta I=\frac{1}{2}$ rule), and (3) leads to mass splittings within isotopic multiplets which, when combined with calculations of electromagnetic mass differences, give a good account of these mass splittings.

II. THE $SU(3)_I \times SU(3)_{II}$ GROUP AND THE PROPOSED INTERACTION

It is clear that a weak-interaction theory based on the Fierz-Tanikawa identity (6) must be a theory of pointlike particles, otherwise the basic scalar densities $\bar{\psi}_1(1+\gamma_5)\psi_3^c$ would be so renormalized as to obscure the $(V-A)$ structure upon Fierz transformation. This is in accord with the mounting evidence that the hadrons are indeed composed of pointlike constituents. Of course, the exact nature of these constituents remains unclear, but in the absence of any other clear alternatives we shall take these constituents to correspond to the usual Gell-Mann quarks[11] $\mathcal{P}$, $\mathfrak{N}$, and λ. In the leptonic world the situation is clearer, as the known fundamental participants in the weak interactions are the pointlike particles ν_e, e^-, ν_μ, and μ^-.

The first requirement that we shall make in defining the properties of the weak interaction is to insist that all particles that participate in the interaction belong to an irreducible representation of a group which we denote $SU(3)_{II}$. On the basis of this requirement we assign $\mathcal{P}$, $\mathfrak{N}$, and λ to the triplet representation of $SU(3)_{II}$. In order to make an analogous assignment for the leptons, we are forced to postulate the existence of a heavy electron h_e^- and a heavy muon h_μ^- to complete the lepton doublets,[12] and assign ν_e, e^-, h_e^- and ν_μ^-, μ^-, h_μ^- to the $\underline{3}$ representation also. The Cabibbo angle is introduced by taking[13]

$$\xi = \mathfrak{N}\cos\theta + \lambda\sin\theta, \quad (7)$$

e^-, and μ^- to be the $(I_3)_{II}=-\frac{1}{2}$ partners of the $\mathcal{P}$, ν_e, and ν_μ, respectively. The three fundamental triplets are then given in Table I. We have denoted the three triplets by W_i $(i=1,2,3)$. A given particle within the ith triplet will be denoted by $W_{i\alpha}$ $(\alpha=1,2,3)$.

The next step which we take in defining the properties of the weak interaction is to require that each $SU(3)_{II}$ multiplet, of which W_1, W_2, and W_3 are examples, belong to an irreducible representation of the group $SU(3)_I$. Motivated by the observed universality of the weak interactions, we assign W_1, W_2, and W_3 to a fundamental $\underline{3}$ representation of $SU(3)_I$, which will contain all known weakly interacting particles, and completely define the weak interaction by seeking an interaction of this fundamental triplet

(1) which is invariant under $SU(3)_I\times SU(3)_{II}$,

(2) in which only the negative helicity parts of $W_{i\alpha}$ participate, and

(3) which is a Lorentz-invariant bilinear in the W's, linear in a scalar field, and without derivatives (for renormalizability).

The only such interaction is

$$H_w = \tfrac{1}{2}g\overline{W}_{i\alpha}(1+\gamma_5)W_{j\beta}^c f_{ijk}g_{\alpha\beta\gamma}S_{k\gamma} + \text{H.c.}, \quad (8)$$

where the $S_{k\gamma}$ are a set of scalar bosons and the f_{ijk} and $g_{\alpha\beta\gamma}$ are constants which make the interaction invariant under $SU(3)_I$ and $SU(3)_{II}$.

It follows from our assumptions that the scalar bosons $S_{k\gamma}$ belong to an irreducible representation of $SU(3)_I\times SU(3)_{II}$; since $\overline{W}_{i\alpha}W_{j\beta}^c$ contains the $\underline{3}^*$ and the $\underline{6}$ representations of both $SU(3)_I$ and $SU(3)_{II}$, to obtain an $[SU(3)_I\times SU(3)_{II}]$-invariant coupling the $S_{k\gamma}$ must belong to either the $(3^*, 3^*)$,

TABLE I. The $SU(3)_{II}$ quantum numbers of the triplets.

Triplet	Members	Y_{II}	I_{II}	I_{3II}
W_1 (electron)	ν_e	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
	e^-	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$
	h_e^-	$-\frac{2}{3}$	0	0
W_2 (muon)	ν_μ	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
	μ^-	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$
	h_μ^-	$-\frac{2}{3}$	0	0
W_3 (hadron)	p	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
	ξ	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$
	ζ	$-\frac{2}{3}$	0	0

$(3^*, 6)$, $(6, 3^*)$, or $(6, 6)$ representation of $SU(3)_I \times SU(3)_{II}$. The constants $f^{(3*)}_{ABC}$ and $f^{(6)}_{ABC}$, which couple the density $\overline{W}_A W^c_B$ to a triplet and a sextet of bosons [of either SU(3) group], may easily be computed, and if we define the boson matrix $\tilde{S}_{AB} = f_{ABC} S_C$ then the $\underline{3}^*$ and the $\underline{6}$ couplings are given by

$$\tilde{S}^{(3*)} = \begin{bmatrix} 0 & S_3^{(3*)} & S_2^{(3*)} \\ -S_3^{(3*)} & 0 & -S_1^{(3*)} \\ -S_2^{(3*)} & S_1^{(3*)} & 0 \end{bmatrix}, \tag{9}$$

$$\tilde{S}^{(6)} = \begin{bmatrix} S_1^{(6)} & (\frac{1}{2})^{1/2} S_2^{(6)} & (\frac{1}{2})^{1/2} S_4^{(6)} \\ (\frac{1}{2})^{1/2} S_2^{(6)} & S_3^{(6)} & (\frac{1}{2})^{1/2} S_5^{(6)} \\ (\frac{1}{2})^{1/2} S_4^{(6)} & (\frac{1}{2})^{1/2} S_5^{(6)} & S_6^{(6)} \end{bmatrix}, \tag{10}$$

where obviously the superscripts denote that the bosons belong to a k multiplet of SU(3).

Since

$$\overline{\psi}_1 (1+\gamma_5) \psi_2^c = \overline{\psi}_2 (1+\gamma_5) \psi_1^c , \tag{11}$$

the interaction must be totally symmetric. But the $\underline{3}^*$ couplings are antisymmetric and the $\underline{6}$ couplings are symmetric, so the over-all coupling must be of either the type $(3^*, 3^*)$ or $(6, 6)$. Simplicity would of course favor the $(3^*, 3^*)$ coupling, and since in fact the $(6, 6)$ coupling differs from it in relatively uninteresting ways we will assume that the economical $(3^*, 3^*)$ coupling is the correct one and write for the total weak Hamiltonian

$$H_w = \tfrac{1}{2} g \overline{W}_{i\alpha} (1+\gamma_5) W^c_{j\beta} f^{(3*)}_{ijk} f^{(3*)}_{\alpha\beta\gamma} S_{k\gamma} + \text{H.c.} \tag{12}$$

$$= \tfrac{1}{2} g \overline{W} (1+\gamma_5) \tilde{S}^{(3*,3*)} W^c + \text{H.c.} \tag{13}$$

Writing this out in full we obtain

$$\begin{aligned} H_w = g [& \overline{\nu}_e (1+\gamma_5) \mu^c S_{33} + \overline{\nu}_e (1+\gamma_5) h^c_\mu S_{32} - \overline{e} (1+\gamma_5) \nu^c_\mu S_{33} - \overline{e} (1+\gamma_5) h^c_\mu S_{31} - \overline{h}_e (1+\gamma_5) \nu^c_\mu S_{32} + \overline{h}_e (1+\gamma_5) \mu^c S_{31} \\ & + \overline{\nu}_e (1+\gamma_5) \xi^c S_{23} + \overline{\nu}_e (1+\gamma_5) \zeta^c S_{22} - \overline{e} (1+\gamma_5) \mathcal{P}^c S_{23} - \overline{e} (1+\gamma_5) \zeta^c S_{21} - \overline{h}_e (1+\gamma_5) \mathcal{P}^c S_{22} + \overline{h}_e (1+\gamma_5) \xi^c S_{21} \\ & - \overline{\nu}_\mu (1+\gamma_5) \xi^c S_{13} - \overline{\nu}_\mu (1+\gamma_5) \zeta^c S_{12} + \overline{\mu} (1+\gamma_5) \mathcal{P}^c S_{13} + \overline{\mu} (1+\gamma_5) \zeta^c S_{11} + \overline{h}_\mu (1+\gamma_5) \mathcal{P}^c S_{12} - \overline{h}_\mu (1+\gamma_5) \xi^c S_{11}] + \text{H.c.} \end{aligned} \tag{14}$$

The elementary vertices of H_w are given in Fig. 1. We shall take (13) as the fundamental Hamiltonian density for the weak interaction and explore its consequences in the succeeding development.

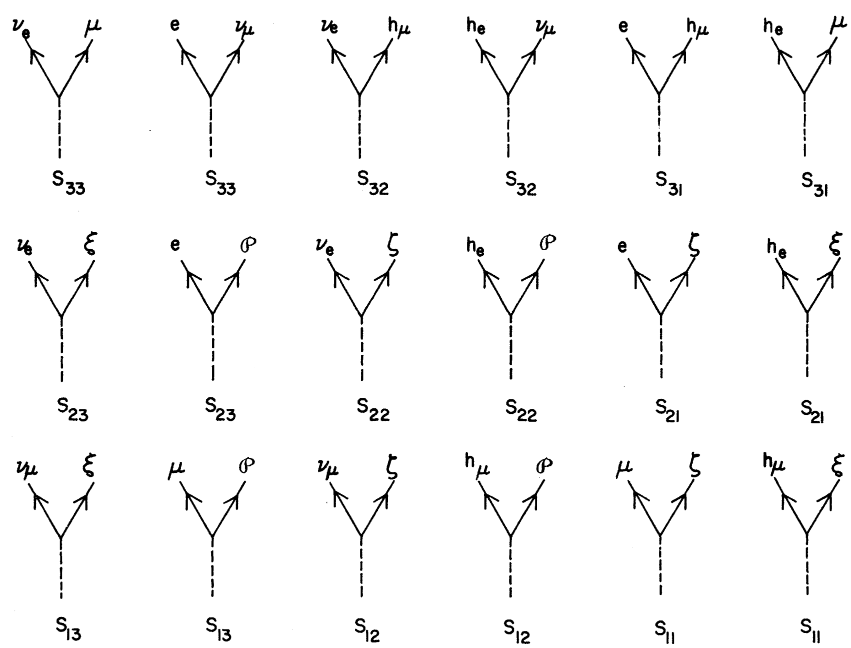

FIG. 1. The elementary vertices.

III. PURELY LEPTONIC PROCESSES

In second order in the local limit $M_s \to \infty$ the interaction (14) gives rise to the effective Hamiltonian for four-line leptonic processes involving known leptons:

$$H^{\text{eff}}_{w,l} = +\frac{g^2}{M_{33}{}^2}[\bar{\nu}_e(1+\gamma_5)\mu^c\bar{\nu}^c_\mu(1-\gamma_5)e + \bar{e}(1+\gamma_5)\nu^c_\mu\bar{\mu}^c(1-\gamma_5)\nu_e - \bar{\nu}_e(1+\gamma_5)\mu^c\bar{\mu}^c(1-\gamma_5)\nu_e - \bar{e}(1+\gamma_5)\nu^c_\mu\bar{\nu}^c_\mu(1-\gamma_5)e], \tag{15}$$

where $M_{k\gamma}$ denotes the mass of the boson $S_{k\gamma}$ and which upon using the Fierz rearrangement (6) and the identity (11), and setting

$$\frac{g^2}{2M_{33}{}^2} = \frac{G}{\sqrt{2}}, \tag{16}$$

becomes

$$H^{\text{eff}}_{w,l} = -(\tfrac{1}{2})^{1/2}G[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu + \bar{e}\gamma^\mu(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu - \bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\bar{\mu}\gamma_\mu(1-\gamma_5)\mu$$
$$- \bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\nu_\mu\bar{e}\gamma_\mu(1-\gamma_5)e]. \tag{17}$$

The first two terms are the usual off-diagonal leptonic terms of the current-current model leading to muon decay and associated processes. The second two terms are elastic terms corresponding to elastic $\nu_e\mu$ and $\nu_\mu e$ scattering and their associated processes. This is in contrast to the current-current model which predicts $\nu_e e$ and $\nu_\mu\mu$ elastic scattering but excludes these processes.[14] The experimental situation regarding any of these interactions is very uncertain at this time, although $\nu_\mu e$ scattering should be accessible to measurement in the next generation of neutrino experiments.[15]

Notice that even with $M_s \sim 100$ GeV, $g^2/4\pi$ is of the order of magnitude of α, so we expect a perturbation series to be viable. In addition, we expect that in some circumstances weak effects might be of the same order of magnitude as electromagnetic effects. This possibility will be discussed further in what follows.

IV. SEMILEPTONIC PROCESSES

In similar fashion the interaction (14) leads to the effective Hamiltonian for four-line semileptonic processes involving known leptons

$$H^{\text{eff}}_{w,\text{s.l.}} = -(\tfrac{1}{2})^{1/2}G[\bar{e}\gamma^\mu(1-\gamma_5)\nu_e\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\xi + \bar{\nu}_e\gamma^\mu(1-\gamma_5)e\bar{\xi}\gamma_\mu(1-\gamma_5)\mathcal{P} + (e\to\mu)]$$
$$+ (\tfrac{1}{2})^{1/2}G[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\{\bar{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathfrak{N} + \bar{\lambda}\gamma_\mu(1-\gamma_5)\lambda\} + \bar{e}\gamma^\mu(1-\gamma_5)e\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\mathcal{P} + (e\to\mu)]$$
$$+\frac{G}{\sqrt{2}}\left(\frac{M_s}{M_c}\right)^2[\bar{e}\gamma^\mu(1-\gamma_5)e\bar{\xi}\gamma_\mu(1-\gamma_5)\xi + (e\to\mu)], \tag{18}$$

where in anticipation of the ensuing discussion we have set

$$M_{12} = M_{13} = M_{22} = M_{23} = M_{32} = M_{33} \equiv M_s \tag{19}$$

and

$$M_{11} = M_{21} = M_{31} \equiv M_c\,. \tag{20}$$

The first term is the usual Cabibbo form of the semileptonic interactions which will reproduce all of the successes of the standard current-current semileptonic theory contained in (1).[16]

The second term is a $\Delta Y = 0$ neutral-current term consistent with the rather weak limits on such processes. For example, one has the limits[17]

$$Q_1 \equiv \frac{\sigma(\nu_\mu p \to \nu_\mu n\pi^+)}{\sigma(\nu_\mu p \to \mu^- p\pi^+)} < 0.16\,, \tag{21}$$

$$Q_2 \equiv \frac{\sigma(\nu_\mu n \to \nu_\mu n\pi^0) + \sigma(\nu_\mu p \to \nu_\mu p\pi^0)}{2\sigma(\nu_\mu n \to \mu^- p\pi^0)} < 0.14\,, \tag{22}$$

$$Q_3 = \frac{\sigma(\nu_\mu p \to \nu_\mu p)}{\sigma(\nu_\mu n \to \mu^- p)} < 0.24\,. \tag{23}$$

It is, however, difficult to draw any definite conclusions from these results since their interpretations are model-dependent. For example, if one assumes $I = \frac{3}{2}$ dominance of the processes in Q_1

and Q_2 and no reabsorptive effects, one can predict $Q_1 \simeq \frac{1}{9}$ and $Q_2 \simeq 1$; however, these assumptions are experimentally untenable, and if they are relaxed, these predictions are changed and the ratio Q_2 may be brought into accord with the experimental values.[17] Since the processes in Q_3 can also occur through either an $I=1$ or $I=0$ current, it is similarly model-dependent. However, within the context of the quark model one can predict $Q_3 \simeq \frac{1}{4}$, consistent with the experimental limit. Clearly, it is just a little too early to reach any definite conclusions regarding these terms.

On the other hand, the last term has been separated out because it would give rise to $\Delta Y=1$ neutral-current terms at a level ruled out experimentally if $M_c = M_s$.[18] We are led by this observation to conclude that the SU(3)$\times$SU(3) symmetry must be strongly broken by the masses of these bosons. Although there is no mechanism within the context of this theory to explain such mass breaking, there is similarly no mechanism to explain the broken masses of the weak spin-$\frac{1}{2}$ triplets in both $\mathrm{SU(3)_I}$ and $\mathrm{SU(3)_{II}}$ quantum numbers, and we ascribe these mass spectra to sources outside the scope of this theory. In accordance with this observation and the universality of the weak interactions, and in compliance with the experimental limits on such $\Delta Y=1$ neutral-current terms, we shall demand in what follows that all the bosons are (nearly) degenerate in mass with mass M_s except for the $U_{\mathrm{II}}=0$ members of the $\mathrm{SU(3)_{II}}$ triplets,[19] namely, S_{31}, S_{21}, and S_{11}, which we take to have masses $M_c \gtrsim 100 M_s$.[20] With this proviso we can neglect most processes mediated by these high-mass bosons to find that the effective semileptonic Hamiltonian is

$$H^{\mathrm{eff}\,\prime}_{w,\,\mathrm{s.l.}} = -(\tfrac{1}{2})^{1/2} G\{\bar{e}\gamma^\mu(1-\gamma_5)\nu_e \bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\xi + \bar{\nu}_e\gamma^\mu(1-\gamma_5)e\bar{\xi}\gamma_\mu(1-\gamma_5)\mathcal{P} - \bar{e}\gamma^\mu(1-\gamma_5)e\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\mathcal{P}$$
$$- \bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e[\bar{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathfrak{N} + \bar{\lambda}\gamma_\mu(1-\gamma_5)\lambda] + (e \to \mu)\} \tag{24}$$

in complete accord with experiment.[21]

In this regard, special note should be made of the absence of $\Delta Y=1$ neutral currents to neutrinos. Neutral currents to $e\bar{e}$ and $\mu\bar{\mu}$ have just been excluded by construction, but the absence of such currents to $\nu_e\bar{\nu}_e$ and $\nu_\mu\bar{\nu}_\mu$ is the result of a cancellation between graphs, as in Fig. 2. The cancellation is exact so long as $M_{22}=M_{23}$, but it is interesting to consider the possibility of a small breaking of the boson masses. The current experimental limit on such processes is[22]

$$\frac{\Gamma(K^+ \to \pi^+\nu\bar{\nu})}{\Gamma(K^+ \to \pi^0\bar{l}\nu)} < 2\times 10^{-3}, \tag{25}$$

which gives the result that the coupling for these processes is less than 4% of the coupling to charged currents, or, i.e., that

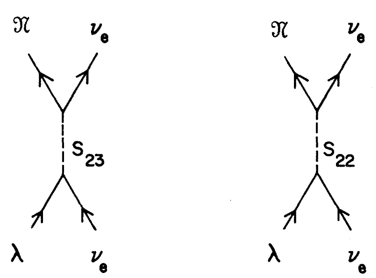

FIG. 2. The absence of $\Delta Y=1$ neutral currents to neutrinos.

$$\frac{\Delta M_s}{M_s} < 2\times 10^{-2}. \tag{26}$$

This is certainly a reasonable restriction which we will have occasion to recall later. In fact, if the mass splittings of the bosons were 10 MeV,[23] then for 30-GeV bosons we would only expect neutral $\Delta Y=1$ neutrino processes at the level

$$\left(\frac{2\Delta M_s}{M_s}\right)^2 \simeq 4\times 10^{-7}, \tag{27}$$

five orders of magnitude below the present experimental limit.

V. WEAK HADRONIC PROCESSES AND THE OCTET RULE

Up to now we have demonstrated how the model interaction (13) reproduces the successes of standard weak-interaction theory for leptonic and semileptonic processes and is at the same time a renormalizable theory that may be extended to all orders in $g^2/4\pi$. In the realm of the pure hadronic processes, however, the model gives results quite different from those of the standard theory. In particular we shall see that the model gives rise to an exact octet rule for $\Delta Y=1$ hadronic processes, in accord with observation.[24]

We begin by making the very simple observation that, because of the absence of a direct quark-quark-boson coupling in (14) there are no four-line quark processes in order g^2. In fact, to order g^2 the only hadronic processes allowed are those in

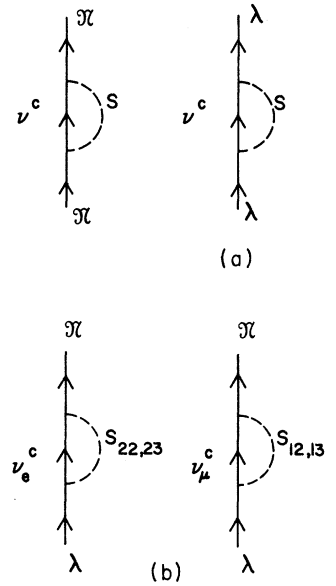

FIG. 3. (a) $\Delta Y = 0$ hadronic processes. (b) $\Delta Y = 1$ hadronic processes.

Fig. 3.[25] We immediately see that the graphs of Fig. 3(b) arise by joining the neutrino lines of the graphs of Fig. 2 and that in the limit of exact mass degeneracy of the bosons, there will be no $\Delta Y = 1$ hadronic processes. On the other hand, the symmetry is not exact and there will certainly be mass splittings.[23] In what follows we shall demonstrate that even though small mass splittings give rise to insignificant levels of $\Delta Y = 1$ neutral semileptonic processes, they can give rise to quite large levels of $\Delta Y = 1$ hadronic processes even though such amplitudes are proportional to ΔM; in fact, such a process can easily lead to

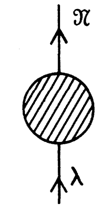

FIG. 4. The octet amplitude for $\Delta Y = 1$ hadronic processes.

$\Delta Y = 1$ rates larger than those predicted by the standard theory. This is advantageous since it has long been recognized that hadronic rates need some sort of order of magnitude enhancement over the predictions of the current-current model.[26]

With these preliminaries we now proceed to the following calculation: Any $\Delta Y = 1$ hadronic process can proceed via a one-line amplitude (which would have exact octet transformation properties) for changing a λ into an $\mathfrak{N}$ quark.[27] So we consider a line in a Feynman graph (not necessarily an external line, of course) as in Fig. 4. The amplitude A resulting from all the graphs of the type shown in Fig. 3(b) is

$$A = 2ig^2 \cos\theta \sin\theta \times \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - \mu_A{}^2 + i\epsilon} - \frac{1}{k^2 - \mu_B{}^2 + i\epsilon} \right) \times (1+\gamma_5) \frac{1}{\not p - \not k - m_l} (1-\gamma_5) , \tag{28}$$

where in the case at hand μ_A is the mass of S_{13} and S_{23}, μ_B is the mass of S_{12} and S_{22}, and m_l is the mass of the neutrino. Proceeding, one easily obtains

$$A = 4ig^2 \cos\theta \sin\theta (\mu_A{}^2 - \mu_B{}^2)(1+\gamma_5) \int \frac{d^4k}{(2\pi)^4} \frac{\not p - \not k}{(k^2 - \mu_A{}^2 + i\epsilon)(k^2 - \mu_B{}^2 + i\epsilon)[(p-k)^2 - m_l{}^2 + i\epsilon]} \tag{29}$$

$$= 4g^2 \sin\theta \cos\theta (\mu_A{}^2 - \mu_B{}^2)(1+\gamma_5) \not I , \tag{30}$$

where

$$I^\mu = i \int \frac{d^4k}{(2\pi)^4} \frac{p^\mu - k^\mu}{(k^2 - \mu_A{}^2 + i\epsilon)(k^2 - \mu_B{}^2 + i\epsilon)[(p-k)^2 - m_l{}^2 + i\epsilon]} . \tag{31}$$

It may easily be shown that in the case $\mu_A \simeq \mu_B \simeq M_s$, $m_l \simeq 0$, the integral is equal to

$$\frac{p^\mu}{32\pi^2 M_s{}^2} F(p/M_s) , \tag{32}$$

where for

$$\frac{p}{M_s} \leq 0.5 \tag{33}$$

$F(p/M_s)$ is practically independent of p/M_s and equal to unity. We therefore have, using $\mu_A{}^2 - \mu_B{}^2 \simeq 2M_s \Delta M_s$,

$$A = \frac{g^2}{4\pi^2 M_s{}^2} \cos\theta \sin\theta\, M_s \Delta M_s (1+\gamma_5) \not p , \tag{34}$$

or using $g^2/2M_s{}^2 = (\frac{1}{2})^{1/2} G$

$$A = \frac{G}{2\sqrt{2}\,\pi^2} \cos\theta \sin\theta\, M_s \Delta M_s (1+\gamma_5) \not p . \tag{35}$$

Of course this calculation does not in any way take into account the renormalization effects that the strong interactions will produce, and so this result may only be taken as indicative.

We can, however, immediately see the interesting result promised previously: Although the mass splitting by ΔM resulted in neutral $\Delta Y=1$ semileptonic processes of the order of $(G/\sqrt{2})\times(\Delta M_s/M_s)$, it results in $\Delta Y=1$ hadronic processes of the order of $(G/\sqrt{2})(M_s\Delta M_s)$ so that for reasonable values of ΔM_s and M_s, the hadronic processes occurring via symmetry breaking will be as large or larger than normal weak processes.

To make this observation quantitative, however, we must apply this calculation to a weak matrix element of observed particles. The simplest way to do this is to use soft-pion[28] techniques (which remain valid in this scheme, of course) to relate the amplitudes of the hadronic processes $H\to H'+\pi$ to the matrix elements of states of the same momentum and spin

$$\langle H'|H_w(0)|H\rangle \equiv (2\pi)^3 a_{H'H}\,. \tag{36}$$

In this way, one can extract from experiment that, for example,[29]

$$a_{p\Sigma^+}=0.48\times10^{-7}\ \mathrm{GeV}\,. \tag{37}$$

One can, on the other hand, easily calculate the amplitude $a_{p\Sigma^+}$ using the quark model. Assuming exact SU(3) and nonrelativistic quarks one easily obtains from (35)

$$a_{p\Sigma^+}=\frac{G\cos\theta\sin\theta\, M_s\Delta M_s M_\lambda}{2\sqrt{2}\,\pi^2}\,, \tag{38}$$

which with $M_s\Delta M_s M_\lambda \simeq 0.6\ \mathrm{GeV}^3$ (as would be the case if, for example, $M_s=30$ GeV, $\Delta M_s=10$ MeV, and $M_\lambda=2$ GeV) gives

$$a_{p\Sigma^+}\simeq 0.5\times10^{-7}\ \mathrm{GeV}\,, \tag{39}$$

in agreement with the result (37). In fact, of course, one may proceed in this way to calculate all of the amplitudes $a_{H'H}$ and thereby via the soft-pion results to calculate the parity-violating hyperon amplitudes. In this manner one obtains the results shown in the fourth column of Table II by making a best fit to the one parameter $M_s\Delta M_s M_\lambda \propto \alpha$. Considering that we have not included any renormalization effects, the results are in good agreement with the data.[24] This agreement is a reflection of the fact that these quark amplitudes are of the pure F type, offering an attractive explanation of the well-known F-type character of these decays.[30] Of course, the strong interactions will in general intercede to introduce D-type contributions as well. In the fifth column of Table II we have shown the result of allowing D couplings. The best fit in this case gives $D/F=-0.37$, bearing out the idea that the renormalization effects of the strong interactions are not overwhelming.[31] These results should be contrasted with those of the current-current theory, which do not follow the octet rule and are of the wrong magnitude.

Finally, it seems worth mentioning that because of the approximate $U_{\rm II}$-spin symmetry, fourth-order box diagrams contributing to the K_L^0-K_S^0 mass difference are very strongly suppressed. It follows that the K_L^0-K_S^0 mass difference occurs via the process

$$\bar{\mathfrak{N}}\lambda \to \{\bar{\mathfrak{N}}\mathfrak{N},\bar{\lambda}\lambda\}\to\bar{\lambda}\mathfrak{N} \tag{40}$$

by the mechanism of direct $\lambda\to\mathfrak{N}$ transition discussed here. One may therefore still use the octet rule for the effective weak Hamiltonian when dealing with $\Delta Y=2$ processes.[32]

VI. MASS DIFFERENCES WITHIN ISOMULTIPLETS

In Sec. V we have seen how the seemingly small graphs of Fig. 2 can become large when the neutrino lines are connected to make the self-energy-type graphs of Fig. 3(b). We will now discuss a similar effect that can arise from the graphs of Fig. 3(a). We will demonstrate that although the

TABLE II. Calculated and observed parity-violating hyperon decay amplitudes.

Amplitude	Formula I (bare quarks)	Formula II (renormalized quarks)	$10^7\times$ best fit I ($\alpha=3.6$)	$10^7\times$ best fit II ($D/F=-0.37$)	$10^7\times$ observed
$A(\Sigma_-^-)$	α	$(F-D)$	3.6	4.3	4.2 ± 0.1
$A(\Sigma_+^+)$	0	0	0.0	0.0	0.1 ± 0.1
$A(\Sigma_0^+)$	$-(\frac{1}{2})^{1/2}\alpha$	$-(\frac{1}{2})^{1/2}(F-D)$	−2.5	−3.1	-3.1 ± 0.2
$A(\Lambda_-^0)$	$(\frac{3}{2})^{1/2}\alpha$	$(\frac{3}{2})^{1/2}(F+\frac{1}{3}D)$	4.3	3.4	3.3 ± 0.1
$A(\Lambda_0^0)$	$-(\frac{3}{4})^{1/2}\alpha$	$-(\frac{3}{4})^{1/2}(F+\frac{1}{3}D)$	−3.1	−2.4	···
$A(\Xi_-^-)$	$-(\frac{3}{2})^{1/2}\alpha$	$-(\frac{3}{2})^{1/2}(F-\frac{1}{3}D)$	−4.3	−4.4	-4.5 ± 0.1
$A(\Xi_0^0)$	$(\frac{3}{4})^{1/2}\alpha$	$(\frac{3}{4})^{1/2}(F-\frac{1}{3}D)$	3.1	3.1	3.3 ± 0.1

graphs of Fig. 3(a) are infinite and must renormalize the $\mathcal{P}$, $\mathfrak{N}$, and λ masses, when combined with the analogous graphs of Fig. 5 involving the heavy $U_{\text{II}}=0$ bosons, they lead to *finite mass differences that make the $\mathcal{P}$ quark lighter than the $\mathfrak{N}$ or λ quarks*. The mass M_c of the heavy $U_{\text{II}}=0$ bosons enters the calculations as a natural "cutoff" on the mass differences.

We begin by noting that a graph of this type will indeed make the external particle lighter. It is easiest to see this in terms of the well-known result that the second-order mass renormalization of a particle $q_i(ps)$ with normalization

$$\langle q_j(p's')|q_i(ps)\rangle = \frac{E}{M}\delta_{ij}\delta_{s's}\delta^3(\vec{p}\,'-\vec{p})$$

is given by

$$\Delta M = \frac{(2\pi)^6}{E/M}\sum_{|n\rangle}\delta^3(\vec{p}_n-\vec{p})\,\mathrm{P}\frac{|\langle n|H_w(0)|q_i(ps)\rangle|^2}{E-E_n}\,. \tag{41}$$

In the rest frame of q_i it is obvious, since in lowest order the intermediate state must contain at least a boson of mass M_s, that $E-E_n$ will always be negative. It follows that each graph of Figs. 3(a) and 5 makes a negative contribution to ΔM; but the $\mathcal{P}$ quark is so renormalized by four graphs involving normal bosons, whereas the $\mathfrak{N}$ and λ quarks are each renormalized by two graphs involving normal bosons and two involving heavy $U_{\text{II}}=0$ bosons. It follows that the $\mathcal{P}$ and $\mathfrak{N}$ quarks will develop a mass difference

$$M(\mathcal{P}) - M(\mathfrak{N}) \equiv -\delta M\,, \tag{42}$$

where δM is a positive quantity. Such a lighter $\mathcal{P}$ quark would lead to the results for the mass differences within isomultiplets given in the second column of Table III.[33] Notice that $\Sigma^+ + \Sigma^- - 2\Sigma^0$ and $\pi^\pm - \pi^0$, the only cases in which calculations

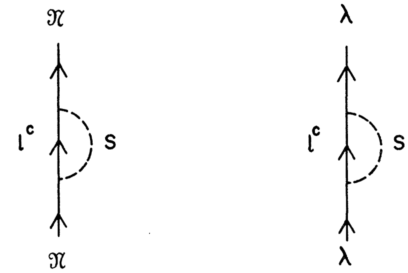

FIG. 5. $U_{\text{II}}=0$ boson mass-renormalization graphs.

of the electromagnetic mass differences ΔM_{em} agree with the observed mass differences, are unaffected by such a weak mass renormalization and that in all other cases the weak mass difference ΔM_w has the same sign as the observed ΔM. This indicates that perhaps the combination of ΔM_w and ΔM_{em} may account for these mass splittings, and we test this hypothesis in Table III.[34] As in the case of the hyperon amplitudes we have shown in a separate column of the table the result of including the D-type amplitudes introduced by strong renormalization effects. Considering the uncertainty in calculations of ΔM_{em}, the agreement that results is really quite startling.[35] Notice that once again the renormalization effects are small and that, in fact, we have $D/F=-0.31$ in this case, nearly the same value as in the hyperon amplitudes. This further substantiates the connection between the weak hyperon amplitudes and these mass splittings, since one would expect equal D/F ratios in the two cases.

But before putting too much credence into this hypothesis, we must first demonstrate that it is reasonable for a weak graph like those of Fig. 3(a) to give mass differences comparable to electromagnetic mass differences. We first remark that in principle this is not too surprising since, as

TABLE III. Calculated and observed values of isomultiplet mass differences. (Masses in MeV/c^2.)

Mass difference	ΔM_w I (bare quarks)	ΔM_w II (renormalized quarks)	ΔM_{em}	$\Delta M=\Delta M_w+\Delta M_{\text{em}}$ I ($\delta M=2.9$)	$\Delta M=\Delta M_w+\Delta M_{\text{em}}$ II ($D/F=-0.31$)	ΔM (observed)
$p-n$	$-\delta M$	$-(F+D)$	$+1.25$	-1.6	-1.3	-1.3
$\Sigma^+-\Sigma^0$	$-\delta M$	$-F$	0.4	-2.5	-3.2	-3.1 ± 0.2
$\Sigma^--\Sigma^0$	$+\delta M$	$+F$	$+1.65$	$+4.6$	$+5.2$	$+4.9\pm0.1$
$\Sigma^++\Sigma^--2\Sigma^0$	0	0	$+2.0$	$+2.0$	$+2.0$	$+1.8\pm0.2$
$\Xi^--\Xi^0$	$+\delta M$	$+(F-D)$	$+1.4$	$+4.3$	$+6.1$	$+6.6\pm0.7$
$\pi^\pm-\pi^0$	0	0	$+4.5$	$+4.5$	$+4.5$	$+4.5$
$K^\pm-K^0$ [a]	$-\delta M$	$-D'$	$+0.9$	-2.0	-3.9	-3.9 ± 0.1

[a] See Ref. 35.

pointed out in Sec. III, $g^2/4\pi$ may be a fairly large coupling constant. With this in mind, we proceed to a quantitative evaluation of the mass δM. Neglecting all the lepton masses δM will be given by the expression (recall the mass difference was defined to be $-\delta M$)

$$\delta M=-2ig^2\bar{u}(ps)\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2-M_s{}^2+i\epsilon}-\frac{1}{k^2-M_c{}^2+i\epsilon}\right)(1+\gamma_5)\frac{1}{\not p-\not k}(1-\gamma_5)u(ps) \tag{43}$$

$$=-4ig^2(M_s{}^2-M_c{}^2)\bar{u}(ps)(1+\gamma_5)\int\frac{d^4k}{(2\pi)^4}\frac{\not p-\not k}{(k^2-M_s{}^2+i\epsilon)(k^2-M_c{}^2+i\epsilon)[(p-k)^2+i\epsilon]}u(ps) \tag{44}$$

$$=4g^2(M_c{}^2-M_s{}^2)\bar{u}(ps)(1+\gamma_5)\not J u(ps)\,, \tag{45}$$

where

$$J^\mu=i\int\frac{d^4k}{(2\pi)^4}\frac{p^\mu-k^\mu}{(k^2-M_s{}^2+i\epsilon)(k^2-M_c{}^2+i\epsilon)[(p-k)^2+i\epsilon]}\,. \tag{46}$$

This is, of course, the same integral we encountered earlier in Eq. (31) with $\mu_A=M_s$, $\mu_B=M_c$, $m_l=0$, and $p=M_p$. It may be shown that for m_l and $M_p<<M_s$ as obtains here,

$$J^\mu=\frac{p^\mu}{32\pi^2M_s{}^2}\frac{\ln[(M_c/M_s)^2]}{(M_c/M_s)^2-1}\,, \tag{47}$$

so that

$$\delta M=\frac{g^2(M_c{}^2-M_s{}^2)}{8\pi^2M_s{}^2}\frac{\ln[(M_c/M_s)^2]}{(M_c/M_s)^2-1}M_{\mathcal{P}}\,, \tag{48}$$

where $M_{\mathcal{P}}$ is the mass of the $\mathcal{P}$ quark. Since $M_c>>M_s$ we may approximate to get the simple result that

$$\delta M=\frac{(GM_N{}^2)}{2\sqrt{2}\,\pi^2}\left(\frac{M_s}{M_N}\right)^2M_{\mathcal{P}}\ln\left(\frac{M_c}{M_s}\right), \tag{49}$$

which for $M_s{}^2M_{\mathcal{P}}\ln(M_cM_s{}^{-1})\simeq10^4$ GeV^3 (as would be the case if, for example, $M_s=30$ GeV, $M_{\mathcal{P}}=2$ GeV, and $M_c\simeq100M_s$) results in

$$\delta M\simeq3\text{ MeV}\,, \tag{50}$$

thereby substantiating the results of Table III and establishing a connection between isomultiplet mass splittings and the high mass of the $U_{\mathrm{II}}=0$ bosons. Quite similar arguments may be made regarding weak mass differences of the scalar bosons to justify the value of the mass splitting ΔM_s used in Secs. IV and V.

Finally, we wish to briefly discuss $\Delta Y=0$ parity-violating nuclear processes. As is the case with $\Delta Y=1$ hadronic processes, there seems to be considerable evidence for effects much larger than those predicted by the current-current model. Of course, one is again faced with the difficulty of drawing conclusions in the presence of strong interactions, but careful analyses seem to indicate discrepancies in *amplitudes* of about two orders of magnitude in the ^{181}Ta system[36] and of about three orders of magnitude in $np\to d\gamma$.[37] A detailed discussion of this situation is certainly beyond the scope of this article, but as is clear from inspection of Eqs. (45) to (49) one expects $\Delta Y=0$ parity-violating amplitudes in this model with a magnitude $GM_s{}^2$ as opposed to the current-current amplitudes of size $GM_N{}^2$, i.e., one expects these processes at a level roughly two to three orders of magnitude larger in amplitude than the predictions of the current-current model.

VII. CONCLUSION

The model of the weak interaction presented here has many desirable features. It reproduces the well-verified successes of the standard current-current theory for leptonic and semileptonic processes, and because the model is renormalizable the amplitude for these and all other weak processes may be calculated by perturbation theory to all orders. Weak hadronic processes in the theory have a quite different character from leptonic and semileptonic processes, in accord with experience, and not only obey an exact octet rule but can give amplitudes which are of the correct size, in contrast to the current-current model.[38] Finally, the model quite gratuitously gives mass splittings within isotopic-spin multiplets comparable to electromagnetic splittings which may help clear up the problem of "electromagnetic mass differences."

In closing, we feel obliged to mention once again the obvious fact that calculations of the sort presented in Secs. V and VI can only be indicative of the properties of this model; strong interactions and the unknown masses involved make it difficult to be more precise than this at this stage. In any event these calculations take the quark model rather seriously even though the relation of the quarks to possible hadron constituents remains obscure. On the positive side, the model does seem to incorporate in a simple renormalizable scheme the major features of the weak interaction, and as such may have value.

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[1]E. Fermi, Nuovo Cimento 11, 1 (1934); Z. Phys. 88, 161 (1934).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, *ibid.* 109, 1860 (1958).

[3]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[4]The idea behind these attempts is to introduce both photons and intermediate vector bosons as a multiplet of massless gauge fields and to then give the weak bosons a large mass via spontaneous symmetry breaking. The extension of these theories to hadrons is, however, unclear at this time.

[5]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971); G. 't Hooft, Nucl. Phys. B35, 167 (1971); B. W. Lee, Phys. Rev. D 5, 823 (1972); S. Y. Lee, Phys. Rev. D 6, 1701 (1972); 6, 1803 (1972).

[6]Y. Tanikawa, Phys. Rev. 108, 1615 (1957); Y. Tanikawa and S. Watanabe, *ibid.* 113, 1344 (1959).

[7]R. E. Pugh, Phys. Rev. D 5, 474 (1972), and references therein.

[8]R. L. Penner, Phys. Rev. D 6, 2059 (1972).

[9]M. Fierz, Z. Phys. 104, 553 (1937); see also Ref. 6.

[10]In Eq. (6), the ψ's are anticommuting quantum fields; ψ_a^c denotes the field of the particle a^c charge conjugate to a.

[11]M. Gell-Mann, Phys. Lett. 8, 214 (1964).

[12]Of course, we assume that the masses of the heavy leptons are sufficiently great so that they have not yet been detected. In fact this is a relatively weak constraint and we shall simply imagine $M_h > 1$ GeV.

[13]There is no reason why the same quark combination which is diagonal for strong SU(3) should be diagonal in weak SU(3).

[14]However, $\nu_e e$ and $\nu_\mu \mu$ elastic scattering are possible with the sextet coupling.

[15]Some months after this was written, the experimental situation is still uncertain, but there has been a striking development. A single neutral-current leptonic event, $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, has been seen [H. Faissner *et al.*, CERN-Gargamelle Collaboration, as reported by G. Myatt at the Bonn Conference, 1973 (unpublished)], corresponding to a cross section comparable to the one predicted here and in conflict with the predictions of the current-current model. The processes $\nu_\mu e \to \nu_\mu e$, $\bar{\nu}_e e \to \bar{\nu}_e e$, and $\nu_e e \to \nu_e e$ remain unseen, and although this indicates an upper limit for $\nu_\mu e \to \nu_\mu e$ somewhat below the prediction of this model, the results are not definitive. Should experiments continue to fail to see elastic $\nu_e e$ and $\bar{\nu}_e e$ scattering, one would have very strong evidence in favor of scalar boson theories.

[16]This remark requires qualification. In any weak interaction theory in which the currents are not fundamental, predictions using the conserved vector current can be obtained only with additional (albeit, reasonable) dynamical input.

[17]D. C. Cundy *et al.*, Phys. Lett. 31B, 478 (1970); W. Lee, Phys. Lett. 40B, 423 (1972); see also the report by Y. Cho [in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 195] and the comment by C. Baltay [*ibid.*, Vol. 2, p. 196]; for the reconcillation of theory with experiment for Q_2 see S. Meshkov and S. P. Rosen, Phys. Rev. Lett. 29, 1764 (1972).

[18]The recently reported value, for example, for $K_L^0 \to \mu^+\mu^-$ is $\Gamma(K_L^0 \to \mu^+\mu^-) = (1.2^{+0.8}_{-0.4}) \times 10^{-8}\, \Gamma(K_L^0 \to \text{all})$ [W. C. Carithers *et al.*, Phys. Rev. Lett. 31, 1025 (1973)].

[19]We define U_{II} spin for the weak $\text{SU}(3)_{\text{II}}$ symmetry in analogy with the strong SU(3) symmetry where U spin is the symmetry which assigns $\mathfrak{N}$ and λ to a U-spin doublet and $\mathcal{P}$ to a U-spin singlet.

[20]In fact, this mass splitting has measurable consequences which will be discussed in Sec. VI.

[21]The process $K_L^0 \to \mu^+\mu^-$ does occur in fourth order, but only at a level consistent with the experimental limit quoted in Ref. 18.

[22]See, for example, D. Haidt *et al.*, Phys. Rev. D 3, 10 (1971); J. H. Klems *et al.*, *ibid.* 4, 66. (1971).

[23]See Sec. VI for a possible mechanism for such a mass difference.

[24]The data in Table II are from an analysis by O. E. Overseth [Phys. Lett. 39B, 144 (1972)], in which he discusses the validity of the octet rule for hyperon decays and concludes that it is violated by less than 5%. This is consistent with an electromagnetic effect; see S. Okubo, R. E. Marshak, and V. S. Mathur [Phys. Rev. Lett. 19, 407 (1967)] in this regard.

[25]Although these are only one-line graphs, one should not suppose that they may merely be removed from the theory by a process of diagonalization and redefinition of the fields $\mathfrak{N}$ and λ since these fields are defined by the symmetry-breaking medium strong interaction in the absence of weak and electromagnetic interactions. This follows since here the $\Delta Y = 1$ weak hadronic interaction and the medium-strong interaction do not belong to the same octet (as they do, for example, in the tadpole model) even should the only source of chiral symmetry breaking be quark mass terms [see, e.g., the $\not{p}$ in Eq. (35)]. Thus in the case at hand one can show that even with an SU(3) × SU(3)-invariant strong interaction, the conjecture discussed by N. Cabibbo and L. Maiani [Phys. Rev. D 1, 707 (1970)] does not apply.

[26]See for examples: R. P. Feynman, in *Symmetries in Elementary Particle Physics,* edited by A. Zichichi (Academic, New York, 1965), p. 160; A. Salam, Phys. Lett. 8, 216 (1964); M. Breitenecher *et al.*, Z. Phys. 204, 345 (1967).

[27]Although couched in the language of divergent intermediate-vector-boson theory and current algebra, V. S. Mathur and P. Olesen [Phys. Rev. Lett. 20, 1527 (1968)] conjecture essentially this same mechanism for octet dominance. By placing this hypothesis in the framework of a renormalizable weak-interaction theory involving quarks, as we do here, their conjecture takes on substance and simplicity. It is also worth mentioning that, at least at the computational level, the resulting picture of octet dominance is very much like the tadpole model of S. Coleman and S. L. Glashow [Phys. Rev. 134, B671 (1964)].

[28]See, for example, C. G. Callan and S. B. Treiman, Phys. Rev. Lett. 16, 153 (1966). The soft-pion results depend on the relation $[F_i, H_w^{\text{p.c.}}] = [F_i^5, H_w^{\text{p.v.}}]$, where F_i and F_i^5 are the $\Delta Y = 0$ SU(3) charges and axial charges; this relation is easily proved by taking the $\Delta Y = 1$ hadronic amplitude to be represented by the low-energy phenomenological form $H_w = c\bar{\mathfrak{N}}(1+\gamma_5)i\gamma^\mu\partial_\mu\lambda + \text{H.c.}$ Notice that in the soft-pion limit for the process $H \to H' + \pi$ one is also taking $M_H = M_{H'}$. We shall comply

in what follows by doing all the calculations in the limit of SU(3) symmetry, but we keep in mind this difficulty in cases like $K \to \pi + \pi$.

[29]We normalize baryon states according to

$$\langle B_j(\vec{p}\,'s')|B_i(\vec{p}s)\rangle = \frac{E}{M}\delta_{ij}\delta_{s's}\delta^3(\vec{p}\,' - \vec{p}).$$

[30]See, for example, S. A. Bludman, in *Cargèse Lectures in Physics*, edited by M. Lévy (Gordon and Breach, New York, 1967), Vol. 1, p. 203.

[31]Similar methods may be applied to the decays $K \to 2\pi$. One finds, of course, that these amplitudes obey the octet rule; one also finds a reasonable fit to the amplitudes $a_{\pi K}$ in terms of the same parameter α used for the baryons.

[32]In the context of the nonrelativistic quark model this mechanism may, in fact, serve as a rationale for dynamical assumptions on the structure of the $\{\mathfrak{N}\bar{\mathfrak{N}}, \bar{\lambda}\lambda\}$ intermediate states that lead to a successful calculation of the $K_L^0 \to K_S^0$ mass difference. See N. Isgur, Phys. Rev. D 6, 393 (1972).

[33]The values quoted here come from constrained fits to all mass measurements; only in the case of the cascade splittings is this significantly different from the average value of measurements of the mass difference which yield the value 6.3 ± 0.7. See Particle Data Group, Phys. Lett. 39B, 1 (1972).

[34]The values for ΔM(electromagnetic) quoted in Table III for the baryons are taken from S. Coleman and H. J. Schnitzer, Phys. Rev. 136, B223 (1964). We have used the average of their two calculations. The values for ΔM(electromagnetic) for the mesons are from unpublished calculations by the author assuming SU(3) symmetry for the electromagnetic form factors and taking the pion form factor from recent analyses.

[35]The only case where ΔM(theoretical) in the bare quark case is more than 25% away from the observed value (see Ref. 33) is $M(K^\pm) - M(K^0)$ and we may rationalize this discrepancy as being due to different strong renormalizations of δM in the cases of mesons and baryons. In calculations of the amplitudes $a_{H'H}$ (which are exactly analogous) one finds that mesonic amplitudes are larger than their baryonic counterparts, as seems to be the case here also. Notice that in columns three and six of Table III, which are based on assuming only that ΔM_w has the transformation properties indicated by the quark model, this possible inequality of meson and baryon parameters is manifest.

[36]M. Gari *et al.*, Phys. Lett. 35B, 19 (1971).

[37]E. Hadjimichael and E. Fischbach, Phys. Rev. D 3, 755 (1971).

[38]Calculations by the author (to be published in Nuovo Cimento) indicate that the identification of one-loop diagrams as the mechanism responsible for the octet dominance of weak hadronic processes may be made in other renormalizable theories of the weak interaction.

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Why Does the Pomeron Look So Simple If It Is Really So Complicated?*

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The absorbed multiperipheral model is suggested as a vehicle for understanding the approximate constancy with energy of total cross sections that ultimately rise to saturate the Froissart bound. Two approximation schemes, which share as their lowest approximation a simple, factorizable Pomeron, are discussed. In a streamlined version of the model, an estimate of the squared energy s at which this simple picture breaks down is given by $\ln(s/s_0) = [\sigma_{tot}/(4\pi\alpha')]^{1/3} \cong 4.3$.

I. INTRODUCTION

We now know that the Pomeron is not a simple, isolated pole with an intercept of unity. Theoretically, the last link in the chain of argument showing that such a pole would be incompatible with unitarity has now been forged.[1] Experimentally, there are indications[2] that total cross sections may not have finite limits as the energy increases, or, if they do, that the values of those limits may be quite different from what had been previously expected. On the other hand, much of the phenomenology based on treating the Pomeron as a simple pole, for example the constancy of total cross sections and the factorization properties of inclusive distributions, seems to be at least approximately valid over a wide range of energy.[3] In this paper we present a discussion of the question of how the Pomeron can look like a simple pole and yet really be more complicated.

Several authors, for example those listed in Ref. 4, have suggested that it is useful to perform an expansion in powers of $\xi \ln s$, considered as a small parameter, where ξ is some measure of the Pomeron coupling to inelastic channels; to zeroth order in this expansion, the Pomeron would be a