PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

Tests of the Three-Triplet Models in the Deep-Inelastic Region

M. Koca*
International Centre for Theoretical Physics, Trieste, Italy

M. S. K. Razmi
Institute of Physics, University of Islamabad, Islamabad, Pakistan
(Received 29 November 1972)

Light-cone algebra is written for the currents transforming like (1,8), (8,1), and (8,8) representations of $SU(3)'' \otimes SU(3)'$, and is used to obtain the predictions of the Han-Nambu model and the Cabibbo-Maiani-Preparata model for the Bjorken form factors. Comparison is made with the usual light-cone current-algebra results.

I. INTRODUCTION

Following the advent of Bjorken's scaling hypothesis in deep-inelastic electron-nucleon scattering and its verification by the SLAC-MIT experiments, various theoretical models have been proposed to incorporate the ideas of scaling.[1] In particular, Fritzsch and Gell-Mann[2] have proposed a light-cone algebra abstracted from the quark model of Gell-Mann and Zweig. This algebra predicts a number of sum rules, linear relations, and bounds for the structure functions in the processes of interest. Although the experiments have not yet reached a stage where these results stand tested, some of these relations seem to be in agreement with the present data within experimental errors.

Now, it is well known that the Gell-Mann–Zweig (GMZ) quarks have some unpleasant features. If they obey the Fermi-Dirac statistics for spin-$\frac{1}{2}$ particles, one obtains a totally antisymmetric wave function in spin, isospin, and strangeness for the low-lying baryons. This contradicts the fact that the baryons lie in the symmetric representations $\underline{56}$ of SU(6). It has also been pointed out that with Gell-Mann–Zweig quarks, the $\pi^0 \to \gamma\gamma$ decay rate is about a factor of 9 too small.[3] Recently, though, Fritzsch and Gell-Mann (FGM)[4] have pointed out that it is possible to overcome these difficulties by postulating 9 fractionally charged quarks. They introduce three triplets of Gell-Mann–Zweig–type quarks, the three triplets having three different "colors." This new degree of freedom, "color," is such that all hadronic states are singlets with respect to color. This three-triplet model will henceforth be referred to as the FGM model.

In the meantime Han and Nambu[5] have proposed a three-triplet model (briefly called HN model) where the quarks are integrally charged and obey the Fermi-Dirac statistics. The HN model predicts the existence of particles with a new quantum number called "charm," which, however, have not yet been observed in any experiment. The possibility of the "charm" being a good quantum number has been investigated and the explanation given for the failure to observe such particles is that present experiments have not yet reached adequate energies.[6] At the same time, it has been shown that the HN model gives the correct $\pi^0 \to \gamma\gamma$ decay rate.[7] Further properties of this model have been investigated by Lipkin.[8]

A different version of the HN model has been proposed by Cabibbo, Maiani, and Preparata[9] (abbreviated to CMP). The CMP three-triplet model leads to finite radiative corrections to hadronic β decay.

Several tests have been proposed to see the differences between these models.[10] We use light-cone-algebra techniques in this paper to derive a number of sum rules, linear relations, and bounds for the structure functions in each of the three-triplet models mentioned earlier.

The plan of the paper is as follows: In Sec. II we review the three-triplet models and write down electromagnetic and weak currents for them. In Sec. III we introduce a generalization of the bilocal current algebra of FGM with the addition of the new $SU(3)''$ symmetry. Several relations valid in the three models are derived. In Sec. IV we discuss deep-inelastic muon pair production in electron-nucleon scattering, and concluding remarks are made in Sec. V. Some of the results (commutation relations of the electromagnetic and weak currents, positivity conditions, etc.) used in the text are derived in the Appendixes A and B.

II. THREE-TRIPLET MODELS

Here we give a brief resumé of the various three-triplet models discussed in the literature. We consider first the Han-Nambu model. Let $q_{\alpha 1}$,

$q_{\alpha 2}$, and $q_{\alpha 3}$ ($\alpha = \mathcal{P}, \mathfrak{N}, \lambda$) be the three fundamental triplets. We lump them together in a single multiplet field $q_{\alpha i}$, where both α and i run over 1, 2, 3. This enables us to introduce two distinct SU(3)'s, one acting on the index α and the other on i. Then $q_{\alpha i}$ form a $(3, 3^*)$ representation of the group $G = \mathrm{SU}(3)' \otimes \mathrm{SU}(3)''$. The familiar SU(3) group is the diagonal subgroup of G and its generators are given by $F_i = F_i' + F_i''$. Thus the operations of SU(3) are not independent of those of SU(3)′ and SU(3)″. The Gell-Mann–Nishijima formula

$$Q = (I_3' + \tfrac{1}{2}Y') + (I_3'' + \tfrac{1}{2}Y'') = Q' + Q'' \qquad (2.1)$$

leads to integral charges for the HN quarks which are $Q_{\alpha 1} = (1, 0, 0)$, $Q_{\alpha 2} = (0, -1, -1)$, $Q_{\alpha 3} = (1, 0, 0)$. A new quantum number $\mathcal{C}$ called "charm" is introduced by the formula $\frac{1}{3}\mathcal{C} = I_3'' + \frac{1}{2}Y''$, and $\mathcal{C}$ takes the values 1, −2, 1 for $q_{\alpha 1}$, $q_{\alpha 2}$, $q_{\alpha 3}$, respectively. $q_{\alpha 2}$ and $q_{\alpha 3}$ form an SU(2)″ doublet, while $q_{\alpha 1}$ is an SU(2)″ singlet. It is also assumed that the low-lying hadron states are SU(3)″ singlets so that SU(3)′ coincides with the usual SU(3).

Next, we have the model of Cabibbo, Maiani, and Preparata (CMP). The most important way in which it differs from the HN model is that the SU(3)′ coincides with the familiar SU(3) so that the operations of SU(3) do not depend on those of SU(3)″. The $q_{\alpha i}$ form a $(3, 3)$ representation of the group G. $q_{\alpha 1}$ and $q_{\alpha 2}$ form an SU(2)″ doublet and $q_{\alpha 3}$ is an SU(2)″ singlet. This gives $\mathcal{C} = 3Y''$. The Gell-Mann–Nishijima formula is modified to

$$Q = I_3 + \tfrac{1}{2}Y + \tfrac{1}{3}\mathcal{C} \qquad (2.2)$$

in order to give integral charges to the nine quarks, whose charges are now the same as in the HN model. $\mathcal{C}$ has the eigenvalues 1, 1, −2 for $q_{\alpha 1}$, $q_{\alpha 2}$, $q_{\alpha 3}$, respectively. All low-lying hadrons are assumed to have $\mathcal{C} = 0$ so that they satisfy the original Gell-Mann–Nishijima formula.

In the Fritzsch–Gell-Mann model the index i in $q_{\alpha i}$ refers to an SU(3) of color. I.e., we have $(\mathcal{P}, \mathfrak{N}, \lambda)$ type quarks, each in three different colors. The important assumption here is that all physical hadron states and hadronic currents are singlets under the SU(3) of "color." $q_{\alpha i}$ have fractional charges and $Q_{\alpha 1} = Q_{\alpha 2} = Q_{\alpha 3} = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$.

Let us now write down the electromagnetic and weak currents of the models. We write the field $q_{\alpha i}$ as a column vector

$$q = \begin{pmatrix} q_{\alpha 1} \\ q_{\alpha 2} \\ q_{\alpha 3} \end{pmatrix}, \qquad (2.3)$$

and the charge matrix can be written accordingly:

$$Q = \begin{pmatrix} Q_{\alpha 1} & 0 & 0 \\ 0 & Q_{\alpha 2} & 0 \\ 0 & 0 & Q_{\alpha 3} \end{pmatrix}. \qquad (2.4)$$

Furthermore, Q can be expressed in terms of two sets of λ and ρ matrices

$$Q = I \otimes [\tfrac{1}{2}\lambda^3 + (\tfrac{1}{3})^{1/2}(\tfrac{1}{2}\lambda^8)] - [\tfrac{1}{2}\rho^3 + (\tfrac{1}{3})^{1/2}(\tfrac{1}{2}\rho^8)] \otimes I, \qquad (2.5a)$$

$$Q = I \otimes [\tfrac{1}{2}\lambda^3 + (\tfrac{1}{3})^{1/2}(\tfrac{1}{2}\lambda^8)] + (\tfrac{1}{3})^{1/2}\rho^8 \otimes I, \qquad (2.5b)$$

for the HN and the CMP models, respectively. λ^i $(i = 1, \ldots, 8)$ are associated with the usual SU(3) group while ρ^i $(i = 1, \ldots, 8)$ are the generators of the so-called new SU(3)″.

Thus the respective electromagnetic currents will be of the form

$$J_\mu^{\mathrm{em}}(x) = \sqrt{6}J_\mu^{(0',3)}(x) + \sqrt{2}J_\mu^{(0',8)}(x) - \sqrt{6}J_\mu^{(3',0)}(x) - \sqrt{2}J_\mu^{(8',0)}(x), \qquad (2.6a)$$

$$J_\mu^{\mathrm{em}}(x) = \sqrt{6}J_\mu^{(0',3)}(x) + \sqrt{2}J_\mu^{(0',8)}(x) + 2\sqrt{2}J_\mu^{(8',0)}(x), \qquad (2.6b)$$

where we have introduced the generalized vector currents

$$J_\mu^{(i',i)}(x) = i\bar{q}(x)\gamma_\mu(\tfrac{1}{2}\rho^{i'} \otimes \tfrac{1}{2}\lambda^i)q(x). \qquad (2.7)$$

The currents $J_\mu^{(i',i)}(x)$ transform as (1, 8), (8, 1), and (8, 8) with respect to SU(3)″×SU(3)′. We notice that the electromagnetic currents in Eqs. (2.6a) and (2.6b) have two pieces transforming as a singlet and as an octet under the new SU(3)″ transformations. However, the singlet part, in SU(3)″ space, of the generalized currents are observables only, as far as hitherto observed hadrons are concerned. Since we are interested in the commutators of two electromagnetic currents, two octets will induce the singlet part. Therefore it is worthwhile to investigate the algebraic properties of the generalized currents. If the "charmed" states were detected, then the SU(3)″ octet part of the generalized currents would be observables for such states.

The electromagnetic current in the FGM model looks like

$$J_\mu^{\mathrm{em}}(x) = \sqrt{6}\,[J_\mu^{(0',3)} + (\tfrac{1}{3})^{1/2}J_\mu^{(0',8)}] \qquad (2.8)$$

in our notation. The main difference of Eq. (2.8) from Eqs. (2.6a) and (2.6b) is that the latter is an SU(3)″ singlet.

Following Budny *et al.*,[11] we assume that the weak current in the HN model is given by

$$J_\mu^+(x)=\sqrt{6}\{[J_\mu^{(0',1+i2)}(x)+J_\mu^{5(0',1+i2)}(x)]\cos\theta_C+[J_\mu^{(0',4+i5)}(x)+J_\mu^{5(0',4+i5)}(x)]\sin\theta_C$$
$$+[J_\mu^{(1'+i2',0)}(x)+J_\mu^{5(1'+i2',0)}(x)]\cos\theta_C+[J_\mu^{(4'+i5',0)}(x)+J_\mu^{5(4'+i5',0)}(x)]\sin\theta_C\}, \tag{2.9}$$

where

$$J_\mu^{5(i',i)}(x)=i\,\overline{q}(x)\gamma_\mu\gamma_5(\tfrac{1}{2}\rho^{i'}\otimes\tfrac{1}{2}\lambda^i)q(x) \tag{2.10}$$

is the generalized axial-vector current. It is clear that the weak current Eq. (2.9) transforms as (1, 8) + (8, 1) under SU(3)″ × SU(3)′.

The vector and the axial-vector currents in the CMP model are defined as follows[9]:

$$J_\mu^i=\tfrac{1}{2}i(\overline{q}_1\gamma_\mu\lambda^i q_1+\overline{q}_2\gamma_\mu\lambda^i q_2+\overline{q}_3\gamma_\mu\lambda^i q_3), \tag{2.11a}$$

$$J_\mu^{5i}=-\tfrac{1}{2}i(\overline{q}_1\gamma_\mu\gamma_5\lambda^i q_1+\overline{q}_2\gamma_\mu\gamma_5\lambda^i q_2-\overline{q}_3\gamma_\mu\gamma_5\lambda^i q_3). \tag{2.11b}$$

The hadron currents in weak interactions are built from the octet $J_\mu^i+J_\mu^{5i}$ according to Cabibbo's hypothesis. Then the weak current in our notation will be

$$J_\mu^+(x)=[\sqrt{6}J_\mu^{(0',1+i2)}(x)-(\tfrac{2}{3})^{1/2}J_\mu^{5(0',1+i2)}(x)-4(\tfrac{1}{3})^{1/2}J_\mu^{5(8',1+i2)}(x)]\cos\theta_C$$
$$+[\sqrt{6}J_\mu^{(0',4+i5)}(x)-(\tfrac{2}{3})^{1/2}J_\mu^{5(0',4+i5)}(x)-4(\tfrac{1}{3})^{1/2}J_\mu^{5(8',4+i5)}(x)]\sin\theta_C. \tag{2.12}$$

Here also, the weak current has an SU(3)″ octet part which manifests itself in the axial-vector form. In the FGM model the situation is simpler, the weak current is an SU(3)″ singlet only:

$$J_\mu^+(x)=\sqrt{6}\{[J_\mu^{(0',1+i2)}(x)+J_\mu^{5(0',1+i2)}(x)]\cos\theta_C+[J_\mu^{(0',4+i5)}(x)+J_\mu^{5(0',4+i5)}(x)]\sin\theta_C\}. \tag{2.13}$$

III. LIGHT-CONE APPROACH TO DEEP-INELASTIC STRUCTURE FUNCTIONS

It has been shown by Fritzsch and Gell-Mann[2] that the light-cone singularity of two local currents abstracted from free quark fields leads to Bjorken scaling. The commutator of two local currents on the light cone is no longer a local current but it is a bilocal current. However, it reduces to a local current when two space-time points coincide. The Fourier transform of its matrix element between nucleon states is the Bjorken form factor. Fritzsch and Gell-Mann have further shown that the bilocal currents form a closed algebra, provided all relative coordinates are lightlike. Here we write down the similar expressions for the generalized currents Eqs. (2.7) and (2.10). The light-cone commutators of two local currents are given by

$$[J_\mu^{(i',i)}(x),J_\nu^{(j',j)}(0)]\triangleq\tfrac{1}{4}\partial_\rho D(x)\{(d^{i'j'k'}d^{ijk}-f^{i'j'k'}f^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(A;x,0)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(S;x,0)]$$
$$+i(f^{i'j'k'}d^{ijk}+d^{i'j'k'}f^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(S;x,0)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(A;x,0)]\}, \tag{3.1a}$$

$$[J_\mu^{(i',i)}(x),J_\nu^{5(j',j)}(0)]\triangleq\tfrac{1}{4}\partial_\rho D(x)\{(d^{i'j'k'}d^{ijk}-f^{i'j'k'}f^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(A;x,0)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(S;x,0)]$$
$$+i(f^{i'j'k'}d^{ijk}+d^{i'j'k'}f^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(S;x,0)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(A;x,0)]\}, \tag{3.1b}$$

where $s_{\mu\nu\rho\sigma}=\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\rho}-\delta_{\mu\nu}\delta_{\rho\sigma}$, and $\triangleq$ means that equality must be understood when $x^2\simeq 0$. We have defined

$$J_\mu^{(k',k)}(S;x,0)\equiv J_\mu^{(k',k)}(x,0)+J_\mu^{(k',k)}(0,x), \tag{3.2a}$$

$$J_\mu^{(k',k)}(A;x,0)\equiv J_\mu^{(k',k)}(x,0)-J_\mu^{(k',k)}(0,x), \tag{3.2b}$$

and $D(x)=-(1/2\pi)\epsilon(x_0)\delta(x^2)$. The other commutators are related to Eqs. (3.1a) and (3.1b) by

$$[J_\mu^{5(i',i)}(x),J_\nu^{5(j',j)}(0)]=[J_\mu^{(i',i)}(x),J_\nu^{(j',j)}(0)], \tag{3.3a}$$

$$[J_\mu^{5(i',i)}(x),J_\nu^{(j',j)}(0)]=[J_\mu^{(i',i)}(x),J_\nu^{5(j',j)}(0)]. \tag{3.3b}$$

Furthermore, the commutators of bilocal currents can be calculated as

$$[J_\mu^{(i',i)}(u,v),J_\nu^{(j',j)}(x,y)]\triangleq\tfrac{1}{4}\partial_\rho D(v-x)(if^{i'j'k'}+d^{i'j'k'})(if^{ijk}+d^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(u,y)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(u,y)]$$
$$-\tfrac{1}{4}\partial_\rho D(u-y)(if^{i'j'k'}-d^{i'j'k'})(if^{ijk}-d^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(x,v)-\epsilon_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(x,v)], \tag{3.4a}$$

$$[J_\mu^{(i',i)}(u,v), J_\nu^{5(j',j)}(x,y)] \triangleq \tfrac{1}{4}\partial_\rho D(v-x)(if^{i'j'k'}+d^{i'j'k'})(if^{ijk}+d^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(u,y)+\epsilon_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(u,y)]$$
$$-\tfrac{1}{4}\partial_\rho D(u-y)(if^{i'j'k'}-d^{i'j'k'})(if^{ijk}-d^{ijk})[s_{\mu\nu\rho\sigma}J_\sigma^{5(k',k)}(x,v)-\epsilon_{\mu\nu\rho\sigma}J_\sigma^{(k',k)}(x,v)], \qquad (3.4b)$$

where $(u-v)^2\approx 0$, $(u-x)^2\approx 0$, $(u-y)^2\approx 0$, $(v-x)^2\approx 0$, $(v-y)^2\approx 0$, and $(x-y)^2\approx 0$ must be understood in (3.4a) and (3.4b). The other bilocal current commutators are given by the relations

$$[J_\mu^{5(i',i)}(u,v), J_\nu^{5(j',j)}(x,y)] = [J_\mu^{(i',i)}(u,v), J_\nu^{(j',j)}(x,y)], \qquad (3.5a)$$

$$[J_\mu^{5(i',i)}(u,v), J_\nu^{(j',j)}(x,y)] = [J_\mu^{(i',i)}(u,v), J_\nu^{5(j',j)}(x,y)]. \qquad (3.5b)$$

It is clear from Eqs. (3.4) and (3.5) that the bilocal currents form a closed algebra. If we restrict ourselves to the case of SU(3)″ singlet currents, then Eqs. (3.1)–(3.5) reproduce the FGM algebra.[2] By means of Eqs. (3.4a) and (3.4b) it is possible to express a commutator of two commutators of local currents in terms of bilocal currents. Such an algebra, for example, can be used to relate the matrix elements of the fourth power of the charge to the matrix elements of the square of the charge. We shall use Eqs. (3.4) and (3.5) in Sec. IV. In this section we are interested in the light-cone commutators of the electromagnetic current $[J_\mu^{\rm em}(x), J_\nu^{\rm em}(0)]$ and the weak current $[J_\mu^+(x), J_\nu^-(0)]$. The Fourier transforms of the spin-averaged nucleon matrix elements of these currents:

$$W_{\mu\nu}^{(\rm em)+-}(q^2, p\cdot q) = \frac{1}{4\pi}\int d^4x\, e^{-iq\cdot x}\langle p|[J_\mu^{(\rm em)+}(x), J_\nu^{(\rm em)-}(0)]|p\rangle, \qquad (3.6)$$

lead to the inelastic form factors F_2^{ep}, F_2^{en}, $F_2^{\nu p}$, $F_2^{\nu n}$, $F_3^{\nu p}$, and $F_3^{\nu n}$. Since the quarks are spin-$\frac{1}{2}$ fields, the Callan-Gross relation $F_2(\xi)=2\xi F_1(\xi)$ holds where $\xi=-(q^2/2p\cdot q)$.

The commutation relations of the electromagnetic and weak currents in the three models considered here are given in the Appendix A. We take their matrix elements between nucleon states of the same momentum and sum over spins. Following Fritzsch and Gell-Mann, we define the structure functions as follows:

$$\left\langle p\left|J_\sigma^{(i',i)}\binom{S}{A};x,0\right|p\right\rangle = p_\sigma\int d\xi\, e^{-i\xi p\cdot x}\binom{S^{i'i}(\xi)}{A^{i'i}(\xi)} + \text{trace terms}. \qquad (3.7)$$

From this point onward our discussion depends on the SU(3)″ transformation properties of the nucleons. We consider two cases. In the first case, nucleon is a pure SU(3)″ singlet, and in the second it has an SU(3)″ nonsinglet component also.

A. Case of a Pure SU(3)″ Singlet Nucleon

It is now clear that the structure functions $S^{i'i}(\xi)$ and $A^{i'i}(\xi)$ with $i'\neq 0$ vanish. Thus we get (on setting $\theta_C=0$):

(i) HN model.

$$F_2^{ep}(\xi)=\xi\left[\frac{4}{3}A^{00}(\xi)+\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.8a)$$

$$F_2^{en}(\xi)=\xi\left[\frac{4}{3}A^{00}(\xi)-\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.8b)$$

$$F_2^{\nu p}(\xi)=\xi[4A^{00}(\xi)-\sqrt6\,S^{03}(\xi)+\sqrt2\,A^{08}(\xi)], \qquad (3.8c)$$

$$F_2^{\nu n}(\xi)=\xi[4A^{00}(\xi)+\sqrt6\,S^{03}(\xi)+\sqrt2\,A^{08}(\xi)], \qquad (3.8d)$$

$$F_3^{\nu p}(\xi)=-4S^{00}(\xi)+\sqrt6\,A^{03}(\xi)-\sqrt2\,S^{08}(\xi), \qquad (3.8e)$$

$$F_3^{\nu n}(\xi)=-4S^{00}(\xi)-\sqrt6\,A^{03}(\xi)-\sqrt2\,S^{08}(\xi). \qquad (3.8f)$$

(ii) CMP model.

$$F_2^{ep}(\xi)=\xi\left[\frac{4}{3}A^{00}(\xi)+\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.9a)$$

$$F_2^{en}(\xi)=\xi\left[\frac{4}{3}A^{00}(\xi)-\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.9b)$$

$$F_2^{\nu p}(\xi)=\xi[2A^{00}(\xi)-\sqrt6\,S^{03}(\xi)+\sqrt2\,A^{08}(\xi)], \qquad (3.9c)$$

$$F_2^{\nu n}(\xi)=\xi[2A^{00}(\xi)+\sqrt6\,S^{03}(\xi)+\sqrt2\,A^{08}(\xi)], \qquad (3.9d)$$

$$F_3^{\nu p}(\xi)=\tfrac{2}{3}[S^{00}(\xi)-(\tfrac{3}{2})^{1/2}A^{03}(\xi)+(\tfrac{1}{2})^{1/2}S^{08}(\xi)], \qquad (3.9e)$$

$$F_3^{\nu n}(\xi)=\tfrac{2}{3}[S^{00}(\xi)+(\tfrac{3}{2})^{1/2}A^{03}(\xi)+(\tfrac{1}{2})^{1/2}S^{08}(\xi)]. \qquad (3.9f)$$

(iii) FGM model.

$$F_2^{ep}(\xi)=\xi\left[\frac{2}{3}A^{00}(\xi)+\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.10a)$$

$$F_2^{en}(\xi)=\xi\left[\frac{2}{3}A^{00}(\xi)-\frac{1}{\sqrt6}A^{03}(\xi)+\frac{1}{3\sqrt2}A^{08}(\xi)\right], \qquad (3.10b)$$

$$F_2^{\nu p}(\xi)=\xi[2A^{00}(\xi)-\sqrt{6}\,S^{03}(\xi)+\sqrt{2}A^{08}(\xi)], \tag{3.10c}$$

$$F_2^{\nu n}(\xi)=\xi[2A^{00}(\xi)+\sqrt{6}\,S^{03}(\xi)+\sqrt{2}A^{08}(\xi)], \tag{3.10d}$$

$$F_3^{\nu p}(\xi)=-2S^{00}(\xi)+\sqrt{6}A^{03}(\xi)-\sqrt{2}\,S^{08}(\xi), \tag{3.10e}$$

$$F_3^{\nu n}(\xi)=-2S^{00}(\xi)-\sqrt{6}A^{03}(\xi)-\sqrt{2}\,S^{08}(\xi). \tag{3.10f}$$

Using Eqs. (3.8) of the HN model, we get

$$6[F_2^{ep}(\xi)-F_2^{en}(\xi)]=\xi[F_3^{\nu p}(\xi)-F_3^{\nu n}(\xi)], \tag{3.11}$$

a relation which was first derived by Llewellyn Smith[12] within the GMZ quark-parton model. We also get

$$\int_0^1\frac{d\xi}{\xi}[F_2^{\nu n}(\xi)-F_2^{\nu p}(\xi)]=2 \quad \text{(Adler sum rule)}, \tag{3.12}$$

$$\int_0^1 d\xi[F_3^{\nu p}(\xi)+F_3^{\nu n}(\xi)]=-10. \tag{3.13}$$

In the CMP model, Eq. (3.11) is modified to

$$-2[F_2^{ep}(\xi)-F_2^{en}(\xi)]=\xi[F_3^{\nu p}(\xi)-F_3^{\nu n}(\xi)]. \tag{3.14}$$

The Adler sum rule is still valid, though the right-hand side of Eq. (3.13) changes to 2. Equations (3.11) and (3.12) are also true in the FGM model, whereas the right-hand side of (3.13) is now −6.

Using the positivity conditions derived in the Appendix B, we can obtain the relation[13]

$$\frac{1}{2}\leqslant\frac{F_2^{en}(\xi)}{F_2^{ep}(\xi)}\leqslant 2 \tag{3.15}$$

both in the HN and CMP models. On the other hand, the FGM model predicts the same ratio to lie between $\frac{1}{4}$ and 4.

Again, from the positivity conditions of Appendix B, the HN model gives

$$2\leqslant L(\xi)\equiv\frac{F_2^{\nu p}+F_2^{\nu n}}{F_2^{ep}+F_2^{en}}\leqslant\frac{10}{3}. \tag{3.16}$$

In the CMP model, we obtain an upper bound only on this quantity, namely

$$L(\xi)\leqslant 2. \tag{3.17}$$

In the FGM model also, one can write only an upper bound for such a ratio, viz.,

$$L(\xi)\leqslant\tfrac{18}{5}. \tag{3.18}$$

Finally, let us assume that nature follows the free-field theory of quarks. To be more specific, suppose that the stress-energy-momentum tensor $\theta_{\mu\nu}$ can be built from free quarks alone. Since the spin-averaged matrix element of $\theta_{\mu\nu}$ in any state of momentum p is given by $2p_\mu p_\nu$, we are able to write the following sum rule:

$$\int_0^1 d\xi\{6[F_2^{ep}(\xi)+F_2^{en}(\xi)]-[F_2^{\nu p}(\xi)+F_2^{\nu n}(\xi)]\}$$
$$=\tfrac{8}{3}\quad\text{for the HN model},$$
$$=4\quad\text{for the CMP model},$$
$$=\tfrac{4}{3}\quad\text{for the FGM model}. \tag{3.19}$$

B. Case of a Nucleon with an SU(3)″ Nonsinglet Component

In this case all the eighteen structure functions $S^{i'i}(\xi)$, $A^{i'i}(\xi)$ ($i', i=0, 3, 8$) are, in general, nonzero. The relations expressing the inelastic form factors $F_2(\xi)$ and $F_3(\xi)$ in terms of $S^{i'i}(\xi)$ and $A^{i'i}(\xi)$ can be immediately read off from Appendix A as before, though we do not stop to write them here. We note that none of the relations (3.11) to (3.13) can now be recovered in the HN model. Relation (3.11) modifies to a new one containing $(A^{38}+A^{83})$ also. The right-hand side of (3.12) is an integral over $(S^{03}+S^{30})$, whereas the isotopic-spin current in the HN model can be used to fix an integral over $(S^{03}-S^{30})$ only. Similarly, the right-hand side of (3.13) contains an integral over $(S^{08}+S^{80})$, whereas the hypercharge current in the HN model can be used to fix an integral over $(S^{08}-S^{80})$ only.

In the CMP model, on the other hand, the situation is relatively better in that both the relation (3.14) and the Adler sum rule are recovered. However, the analog of (3.13) in the CMP model (i.e., right-hand side equal to 2) cannot be recovered because the right-hand side contains integrals over S^{80} and S^{88} in addition to S^{00} and S^{08}, whereas the baryon number and hypercharge currents in the CMP model can fix only the latter two.

Perhaps, the most important difference of the general case from the special one considered earlier is that the inequality (3.15) for the ratio $F_2^{en}(\xi)/F_2^{ep}(\xi)$ now changes to

$$0\leqslant\frac{F_2^{en}(\xi)}{F_2^{ep}(\xi)}\leqslant\infty \tag{3.20}$$

in both the HN and CMP models, as can be explicitly checked from the positivity conditions of Appendix B.

For the ratio $L(\xi)$, we now get only a lower bound, namely

$$2\leqslant L(\xi) \tag{3.21}$$

in the HN model. In the CMP model, however, we reproduce the result (3.17).

Let us now turn our attention to a less general case in which we assume that the matrix elements $A^{i'i}$ ($i', i=3, 8$) are negligibly small so that we can

TABLE I. The sum rules in the Bjorken limit. Case A refers to a pure singlet nucleon and Case B_1 to a nucleon with a nonsinglet component also. Case B_2 is a special case of Case B_1, such that the matrix elements $A^{i'i}$ $(i', i = 3, 8)$ are all zero.

	FGM model	HN model Case A	Case B_1	Case B_2	CMP model Case A	Case B_1	Case B_2
$\frac{\xi(F_3^{\nu p} - F_3^{\nu n})}{(F_2^{ep} - F_2^{en})}$	6	6	...	6	−2	−2	−2
$\int_0^1 \frac{d\xi}{\xi}(F_2^{\nu n} - F_2^{\nu p})$	2	2	...	...	2	2	2
$\int_0^1 d\xi (F_3^{\nu p} + F_3^{\nu n})$	−6	−10	...	...	2	...	...
$K = \frac{F_2^{en}}{F_2^{ep}}$	$\frac{1}{4} \le K \le 4$	$\frac{1}{2} \le K \le 2$	$0 \le K \le \infty$	$\frac{1}{4} \le K \le 4$	$\frac{1}{2} \le K \le 2$	$0 \le K \le \infty$	$\frac{2}{5} \le K \le \frac{5}{2}$
$L = \frac{F_2^{\nu p} + F_2^{\nu n}}{F_2^{ep} + F_2^{en}}$	$L \le \frac{18}{5}$	$2 \le L \le \frac{10}{3}$	$2 \le L$	$L \le \frac{18}{5}$	$L \le 2$	$L \le 2$	$L \le \frac{18}{7}$
$M = \int_0^1 d\xi (F_2^{\nu p} + F_2^{\nu n})$	$M \le 2$	$\frac{4}{3} \le M \le \frac{10}{3}$	$M \le 4$	$M \le 4$	$M \le 2$	$M \le 2$	$M \le 2$

set them equal to zero. Consider first the HN model. We immediately recover the relation (3.11) which, of course, holds in the FGM model, also. We find, moreover, that instead of (3.15) or (3.20), we get the remarkable result that

$$\frac{1}{4} \le \frac{F_2^{en}(\xi)}{F_2^{ep}(\xi)} \le 4, \tag{3.22}$$

which again is a result of the FGM model. Further, the ratio $L(\xi)$ now satisfies the inequality

$$L(\xi) \le \tfrac{18}{5}, \tag{3.23}$$

a result which obtains in the FGM model, also. This similarity between the HN model (with $A^{38} = A^{83} = A^{33} = A^{88} = 0$) and the FGM model is noteworthy. The most important difference that still persists is that unlike the FGM model, we do not have the Adler sum rule in the HN model (except, of course, when the nucleon is a pure SU(3)″ singlet). In the CMP model, the corresponding inequalities are

$$\frac{2}{5} \le \frac{F_2^{en}(\xi)}{F_2^{ep}(\xi)} \le \frac{5}{2}, \tag{3.24}$$

$$L(\xi) \le \tfrac{18}{7}. \tag{3.25}$$

The various results obtaining in the special and general cases are collected in Table I.

IV. FURTHER TESTS OF THE MODELS

To test the FGM algebra of bilocal currents, several suggestions have been made. Various authors[14,15] have investigated the problem of the electroproduction of massive muon pairs in the context of FGM algebra. It is argued that the main contribution to the massive muon-pair production will be due to the diagram shown in Fig. 1.[14] In order to apply the algebra of bilocal currents to such processes, generalization of the Bjorken limit has been suggested.[2] In this section we want to compare the differential cross section for the scattering process in Fig. 1 with the electroproduction cross section. This amounts to comparing the matrix elements of the square of the charges with the matrix elements of the fourth power of the charges. More specifically, we compare the matrix elements, apart from the kinematical factors, of the following commutators on the light cone:

$$[J_\mu^{\text{cm}}(x), J_\nu^{\text{cm}}(0)] \tag{4.1}$$

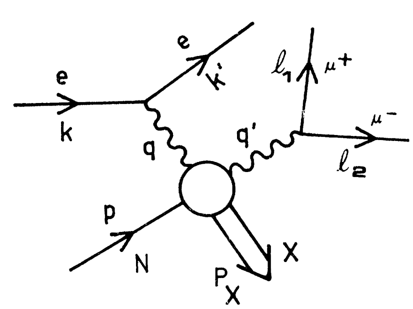

FIG. 1. Massive muon-pair production in electroproduction.

$$[[J_\alpha^{\rm cm}(z), J_\beta^{\rm cm}(y)], [J_\mu^{\rm cm}(x), J_\nu^{\rm cm}(0)]] \tag{4.2}$$

The commutator (4.1) is given by Eqs. (A1), (A3), and (A5) of the Appendix A for the three models.

The commutator of the commutators (4.2) can be evaluated using Eqs. (3.4a), (3.4b), (3.5a), and (3.5b). Apart from the tensor terms like sss, $s\epsilon\epsilon$, etc., and the singular δ functions, the remaining part of the expression (4.2) is the antisymmetrical bilocal vector current $\bar{J}_\sigma(A; x, 0)$ and the symmetrical axial-vector current $\bar{J}_\sigma(S; x, 0)$. The currents $\bar{J}_\sigma(A; x, 0)$ and $\bar{J}_\sigma(S; x, 0)$ are linear combinations of the currents $J_\sigma^{(i',i)}(A; x, 0)$ and $J_\sigma^{5(i',i)}(S; x, 0)$, respectively, with appropriate values of the indices (i', i). The HN and the CMP models give

$$\bar{J}_\sigma(A; x, 0) = \hat{J}_\sigma(A; x, 0). \tag{4.3}$$

This would be expected because in these models $Q^4 = Q^2$. The spin-averaged matrix elements of $\bar{J}_\sigma(A; x, 0)$ between two-nucleon states can be defined as

$$\langle p|\bar{J}_\sigma(A; x, 0)|p\rangle = p_\sigma \int d\xi\, e^{-i\xi p\cdot x} A_{Q^4}(\xi) + \text{trace terms}. \tag{4.4}$$

Then, as a consequence of Eq. (4.3), we find

$$A_{Q^4}^{p,n}(\xi) = A_{Q^2}^{p,n}(\xi) \tag{4.5}$$

or

$$\frac{A_{Q^4}^p - A_{Q^4}^n}{A_{Q^2}^p - A_{Q^2}^n} = 1. \tag{4.6}$$

On the other hand, this ratio in the FGM model is given by

$$\frac{A_{Q^4}^p - A_{Q^4}^n}{A_{Q^2}^p - A_{Q^2}^n} = \frac{5}{9}. \tag{4.7}$$

Now, $A_{Q^4}^p$ is directly proportional to the differential cross section

$$d^4\sigma/(dq^2 d\nu\, dE' d\Omega)$$

for

$$e + p \to e + (\mu^+ + \mu^-) + \text{anything},$$

and $A_{Q^2}^p$ is directly proportional to the differential cross section $d^2\sigma/(dE' d\Omega)$ for $e + p \to e + \text{anything}$. Thus Eqs. (4.6) and (4.7) are statements on the ratios of differential cross sections in the various models considered here.

V. DISCUSSION AND CONCLUSION

We have derived certain sum rules, various relations, and several bounds for the deep-inelastic structure functions in the context of light-cone algebra. Depending upon the assumed transformation properties of the nucleon with respect to SU(3)″, the results of the three models discussed here differ from one another. Let us first consider the assumption of a pure SU(3)″ singlet nucleon. Now, the nucleon has the charm quantum number equal to zero, so if we try to classify it in a representation of SU(3)″ other than the singlet representation, we will be faced with the problem of finding the charmed partners of the nucleon belonging to the same representation. It has been suggested[6] that the lowest charmed states may lie in the 2–3 BeV region. Thus if nucleon belongs to a nonsinglet representation of SU(3)″, we have the case of a very large mass splitting of the order of 1–2 BeV between the nucleon and its charmed partners. This is, of course, a very unsatisfactory situation since this would imply that SU(3)″ is badly broken. Thus we tend to believe that the nucleon is predominantly an SU(3)″ singlet plus, possibly, a small nonsinglet component. With a pure SU(3)″ singlet nucleon, we see that the Adler sum rule is common to all the three models. However, the Llewellyn Smith relation, Eq. (3.11), obtains in the HN and FGM models only. It is to be noted that the prediction of the HN model on the bounds of

$$M = \int_0^1 d\xi [F_2^{\nu p}(\xi) + F_2^{\nu n}(\xi)]$$

is in better agreement with the present experiment ($1.08 \pm 0.27 \leq M \leq 3.24 \pm 0.81$) than the others.[1] On the other hand, the present experimental value of the much celebrated ratio $F_2^{en}(\xi)/F_2^{ep}(\xi)$ is smaller than the lower bound of Eq. (3.15) so that the HN model may appear less favored in comparison with the FGM model so long as the nucleon is treated as a pure SU(3)″ singlet. One severe test will be the sum rule for the quantity:

$$\int_0^1 d\xi [F_3^{\nu p}(\xi) + F_3^{\nu n}(\xi)],$$

which is equal to −10, −6, and 2 in the HN, FGM, and CMP models, respectively.

When the assumption of a pure singlet nucleon is relaxed to admit that of a nonsinglet component also, there is a substantial change in the results of the HN and CMP models, not the least important of them being the ratio $F_2^{en}(\xi)/F_2^{ep}(\xi)$. The new bounds, inequalities (3.20), are not in contradiction with the experimental results. In fact, when we consider the less general case of $A^{38} = A^{83} = A^{33} = A^{88} = 0$, a number of important results (including the bounds on the ratio F_2^{en}/F_2^{ep}) are common to the HN and FGM models. However, the two models still differ in terms of integral sum rules of the type (3.12), (3.13), and (3.19).

A further useful test will be comparison of electroproduction experiments with the electroproduction of muon pairs. In the HN and CMP models, the fourth power of the charge is equal to its square. The FGM model does not give such a simple relation.

One should expect the three-triplet models of HN and CMP to give different results. This is because the electromagnetic currents, Eqs. (2.5a), (2.5b), and the weak currents (2.9) and (2.12) contain additional terms which are not singlets with respect to the new SU(3). These SU(3)″ octet terms excite the charmed states. Therefore, our results are based on the assumption that charmed particles are produced in deep-inelastic nucleon scattering. If no charmed particles are produced in processes of interest, the predictions of the HN and CMP models coincide with those of the colored triplets of FGM. It must also be noted that the predictions of the HN and CMP models are expected to hold when experimental energies appreciably exceed the threshold for excitations of charmed states. The lower bound of (3.15) is violated in the present scaling region. Since the ratio F_2^{en}/F_2^{ep} should be independent of the energy $-p\cdot q/M$ it is possible to conclude that, in the framework of the HN and CMP models, scaling must be broken in the transition from presently available energies to extremely high energies. However, if the nucleon wave function has a nonsinglet SU(3)″ component also, no such breakdown need take place. The NAL experiments on deep-inelastic lepton-nucleon scattering will shed light on the problems which we have discussed here.

ACKNOWLEDGMENTS

The authors would like to thank Professor G. Furlan for helpful comments and for reading the manuscript, and Professor Riazuddin for useful comments. One of us (M.K.) is grateful to Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

APPENDIX A: COMMUTATION RELATIONS OF THE ELECTROMAGNETIC AND WEAK CURRENTS

Using Eqs. (3.1) to (3.3) and the expressions for the electromagnetic and weak currents in the three models, we obtain the following commutation relations near the light cone:

(i) HN model.

$$[J_\mu^{\rm em}(x), J_\nu^{\rm em}(0)] \triangleq \partial_\rho D(x)[s_{\mu\nu\rho\sigma}\hat{J}_\sigma(A; x, 0) + \epsilon_{\mu\nu\rho\sigma}\hat{J}_\sigma^5(S; x, 0)], \qquad \text{(A1a)}$$

where

$$\hat{J}_\sigma = \tfrac{8}{3}J_\sigma^{(0',0)} + (\tfrac{2}{3})^{1/2}(J_\sigma^{(0',3)} + J_\sigma^{(3',0)}) + \tfrac{1}{3}\sqrt{2}\,(J_\sigma^{(0',8)} + J_\sigma^{(8',0)}) - 2(\tfrac{1}{3})^{1/2}(J_\sigma^{(3',8)} + J_\sigma^{(8',3)}) - 2J_\sigma^{(3',3)} - \tfrac{2}{3}J_\sigma^{(8',8)}. \qquad \text{(A1b)}$$

$\hat{J}_\sigma^5(S; x, 0)$ has the same form as $\hat{J}_\sigma(A; x, 0)$ except that we are now dealing with axial-vector currents instead of the vector ones, and that we must now make the replacement $A \to S$. Note that when we sandwich the commutator between nucleon states of the same momentum and sum over spins, $\hat{J}_\sigma^5$ gives no contribution:

$$[J_\mu^+(x), J_\nu^-(0)] \triangleq \partial_\rho D(x)[s_{\mu\nu\rho\sigma}J_\sigma(x, 0) + \epsilon_{\mu\nu\rho\sigma}\tilde{J}_\sigma(x, 0)] + \cdots . \qquad \text{(A2a)}$$

The omitted terms contain the axial-vector currents, and

$$J_\sigma(x, 0) = 8J_\sigma^{(0',0)}(A; x, 0) + 2\sqrt{6}\,[J_\sigma^{(0',3)}(S; x, 0) + J_\sigma^{(3',0)}(S; x, 0)] + 2\sqrt{2}\,[J_\sigma^{(0',8)}(A; x, 0) + J_\sigma^{(8',0)}(A; x, 0)]$$
$$+ 4[J_\sigma^{(1',1)}(A; x, 0) + J_\sigma^{(2',2)}(A; x, 0)]. \qquad \text{(A2b)}$$

$\tilde{J}_\sigma(x, 0)$ can be obtained from $J_\sigma(x, 0)$ by making the substitution $A \to S$ on the right-hand side of (A.2b). Note that the matrix elements of $J^{(1',1)}$ and $J^{(2',2)}$ between states carrying the same charge are zero.

(ii) CMP model.

$$[J_\mu^{\rm em}(x), J_\nu^{\rm em}(0)] \triangleq \partial_\rho D(x)[s_{\mu\nu\rho\sigma}\hat{J}_\sigma(A; x, 0) + \epsilon_{\mu\nu\rho\sigma}\hat{J}_\sigma^5(S; x, 0)], \qquad \text{(A3a)}$$

where

$$\hat{J}_\sigma = \tfrac{8}{3}J_\sigma^{(0',0)} + (\tfrac{2}{3})^{1/2}J_\sigma^{(0',3)} + \tfrac{1}{3}\sqrt{2}\,(J_\sigma^{(0',8)} - 2J_\sigma^{(8',0)}) + 4(\tfrac{1}{3})^{1/2}J_\sigma^{(8',3)} + \tfrac{4}{3}J_\sigma^{(8',8)}. \qquad \text{(A3b)}$$

$\hat{J}_\sigma^5$ can be obtained from $\hat{J}_\sigma$ as explained earlier.

$$[J_\mu^+(x), J_\nu^-(0)] \triangleq \partial_\rho D(x)[s_{\mu\nu\rho\sigma}J_\sigma(x, 0) + \epsilon_{\mu\nu\rho\sigma}\tilde{J}_\sigma(x, 0)] + \cdots , \qquad \text{(A4a)}$$

where

$$J_\sigma(x,0)=4J_\sigma^{(0',0)}(A;x,0)+2\sqrt{6}J_\sigma^{(0',3)}(S;x,0)+2\sqrt{2}J_\sigma^{(0',8)}(A;x,0) \quad \text{(A4b)}$$

and

$$\tilde{J}_\sigma(x,0)=-\tfrac{2}{3}[2J_\sigma^{(0',0)}(S;x,0)+\sqrt{6}J_\sigma^{(0',3)}(A;x,0)+\sqrt{2}J_\sigma^{(0',8)}(S;x,0)+4\sqrt{2}J_\sigma^{(8',0)}(S;x,0)+4\sqrt{3}J_\sigma^{(8',3)}(A;x,0)+4J_\sigma^{(8',8)}(S;x,0)]. \quad \text{(A4c)}$$

(iii) FGM model.

$$[J_\mu^{\text{em}}(x),J_\nu^{\text{em}}(0)]\doteq\partial_\rho D(x)[s_{\mu\nu\rho\sigma}\hat{J}_\sigma(A;x,0)+\epsilon_{\mu\nu\rho\sigma}\hat{J}_\sigma^5(S;x,0)], \quad \text{(A5a)}$$

where

$$\hat{J}_\sigma=\tfrac{4}{3}J_\sigma^{(0',0)}+(\tfrac{2}{3})^{1/2}J_\sigma^{(0',3)}+\tfrac{1}{3}\sqrt{2}J_\sigma^{(0',8)} \quad \text{(A5b)}$$

and $\hat{J}_\sigma^5$ can be obtained from $\hat{J}_\sigma$ by the usual replacement.

$$[J_\mu^+(x),J_\nu^-(0)]\doteq\partial_\rho D(x)[s_{\mu\nu\rho\sigma}J_\sigma(x,0)+\epsilon_{\mu\nu\rho\sigma}\tilde{J}_\sigma(x,0)]+\cdots, \quad \text{(A6a)}$$

where

$$J_\sigma(x,0)=4J_\sigma^{(0',0)}(A;x,0)+2\sqrt{6}J_\sigma^{(0',3)}(S;x,0)+2\sqrt{2}J_\sigma^{(0',8)}(A;x,0). \quad \text{(A6b)}$$

$\tilde{J}_\sigma(x,0)$ can be obtained from $J_\sigma(x,0)$ by the substitution $A \leftrightarrow S$ on the right-hand side of (A6b).

APPENDIX B: POSITIVITY CONDITIONS

To fix ideas, let us consider the relation (3.10a) obtaining in the FGM model. We write this symbolically as

$$F_2^{ep}(\xi)=\tfrac{1}{2}\xi A_{Q^2}(\xi), \quad \text{(B1a)}$$

where

$$Q^2=\tfrac{4}{3}M^{00}+(\tfrac{2}{3})^{1/2}M^{03}+\tfrac{1}{3}\sqrt{2}M^{08},$$

with

$$M^{i'i}=\tfrac{1}{2}\rho_{i'}\otimes\tfrac{1}{2}\lambda_i. \quad \text{(B1b)}$$

Since the scale function $F_2^{ep}(\xi)$ is positive, we conclude that $A_{Q^2}\geq 0$.

Let us now introduce nine matrices $N_1,\ldots,N_9$ with matrix elements given by

$$(N_i)_{jk}=\delta_{ij}\delta_{ik} \quad (i,j,k=1,\ldots,9). \quad \text{(B2)}$$

These nine matrices have non-negative eigenvalues and satisfy $N_i^2=N_i$ $(i=1,\ldots,9)$. We can express the matrices N_i as linear combinations of the nine matrices $M^{i'i}$ $(i',i=0,3,8)$. We have

$$\begin{aligned}
N_1&=\tfrac{2}{3}M^{00}+(\tfrac{2}{3})^{1/2}(M^{03}+M^{30})+\tfrac{1}{3}\sqrt{2}(M^{08}+M^{80})+(\tfrac{1}{3})^{1/2}(M^{38}+M^{83})+M^{33}+\tfrac{1}{3}M^{88},\\
N_2&=\tfrac{2}{3}M^{00}-(\tfrac{2}{3})^{1/2}M^{03}+(\tfrac{2}{3})^{1/2}M^{30}+\tfrac{1}{3}\sqrt{2}(M^{08}+M^{80})+(\tfrac{1}{3})^{1/2}M^{38}-(\tfrac{1}{3})^{1/2}M^{83}-M^{33}+\tfrac{1}{3}M^{88},\\
N_3&=\tfrac{2}{3}M^{00}+(\tfrac{2}{3})^{1/2}M^{30}-\tfrac{2}{3}\sqrt{2}M^{08}+\tfrac{1}{3}\sqrt{2}M^{80}-2(\tfrac{1}{3})^{1/2}M^{38}-\tfrac{2}{3}M^{88},\\
N_4&=\tfrac{2}{3}M^{00}+(\tfrac{2}{3})^{1/2}M^{03}-(\tfrac{2}{3})^{1/2}M^{30}+\tfrac{1}{3}\sqrt{2}(M^{08}+M^{80})-(\tfrac{1}{3})^{1/2}M^{38}+(\tfrac{1}{3})^{1/2}M^{83}-M^{33}+\tfrac{1}{3}M^{88}, \quad \text{(B3)}\\
N_5&=\tfrac{2}{3}M^{00}-(\tfrac{2}{3})^{1/2}(M^{03}+M^{30})+\tfrac{1}{3}\sqrt{2}(M^{08}+M^{80})-(\tfrac{1}{3})^{1/2}(M^{38}+M^{83})+M^{33}+\tfrac{1}{3}M^{88},\\
N_6&=\tfrac{2}{3}M^{00}-(\tfrac{2}{3})^{1/2}M^{30}-\tfrac{2}{3}\sqrt{2}M^{08}+\tfrac{1}{3}\sqrt{2}M^{80}+2(\tfrac{1}{3})^{1/2}M^{38}-\tfrac{2}{3}M^{88},\\
N_7&=\tfrac{2}{3}M^{00}+(\tfrac{2}{3})^{1/2}M^{03}+\tfrac{1}{3}\sqrt{2}M^{08}-\tfrac{2}{3}\sqrt{2}M^{80}-2(\tfrac{1}{3})^{1/2}M^{83}-\tfrac{2}{3}M^{88},\\
N_8&=\tfrac{2}{3}M^{00}-(\tfrac{2}{3})^{1/2}M^{03}+\tfrac{1}{3}\sqrt{2}M^{08}-\tfrac{2}{3}\sqrt{2}M^{80}+2(\tfrac{1}{3})^{1/2}M^{83}-\tfrac{2}{3}M^{88},\\
N_9&=\tfrac{2}{3}M^{00}-\tfrac{2}{3}\sqrt{2}(M^{08}+M^{80})+\tfrac{4}{3}M^{88}.
\end{aligned}$$

If we now define currents $J_\sigma^{N_i}(x)\sim i\bar{q}(x)N_i\gamma_\sigma q(x)$ and the corresponding bilocal operators, it follows in analogy with (B1a) that $F_2^{N_i}(\xi)=\frac{1}{2}\xi A_{N_i}(\xi)$. Since $F_2^{N_i}(\xi)$ is positive, we have $a_i\equiv A_{N_i}\geq 0$ for the proton as well as the neutron matrix elements. In other words, we have nine positivity conditions, which can be immediately written down from (B3). For example, the first relation of (B3) gives

$$a_1=[\tfrac{2}{3}A^{00}+(\tfrac{2}{3})^{1/2}(A^{03}+A^{30})+\tfrac{1}{3}\sqrt{2}(A^{08}+A^{80})+(\tfrac{1}{3})^{1/2}(A^{38}+A^{83})+A^{33}+\tfrac{1}{3}A^{88}]\geq 0,$$

and similarly for the others.

It is useful to express $A^{i'i}$ $(i',i=0,3,8)$ in terms of a_k $(k=1,\ldots,9)$. Thus we get

$$\begin{aligned}
A^{00}&=\tfrac{1}{6}(a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9),\\
(\tfrac{2}{3})^{1/2}A^{03}&=\tfrac{1}{6}(a_1-a_2+a_4-a_5+a_7-a_8),\\
(\tfrac{2}{3})^{1/2}A^{30}&=\tfrac{1}{6}(a_1+a_2+a_3-a_4-a_5-a_6),\\
\sqrt{2}A^{08}&=\tfrac{1}{6}(a_1+a_2-2a_3+a_4+a_5-2a_6+a_7+a_8-2a_9),\\
\sqrt{2}A^{80}&=\tfrac{1}{6}(a_1+a_2+a_3+a_4+a_5+a_6-2a_7-2a_8-2a_9), \quad \text{(B4)}\\
(\tfrac{1}{3})^{1/2}A^{38}&=\tfrac{1}{12}(a_1+a_2-2a_3-a_4-a_5+2a_6),\\
(\tfrac{1}{3})^{1/2}A^{83}&=\tfrac{1}{12}(a_1-a_2+a_4-a_5-2a_7+2a_8),\\
A^{33}&=\tfrac{1}{4}(a_1-a_2-a_4+a_5),\\
A^{88}&=\tfrac{1}{12}(a_1+a_2-2a_3+a_4+a_5-2a_6-2a_7-2a_8+4a_9).
\end{aligned}$$

*On leave of absence from the Middle East Technical University, Ankara, Turkey.

[1]For a review, see C. H. Llewellyn Smith, Phys. Rep. **3C**, 263 (1972).
[2]H. Fritzsch and M. Gell-Mann, California Inst. of Technology Report No. CALT-68-297, and in *Proceedings of International Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann, Jerusalem, 1971).
[3]S. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser *et al.* (MIT Press, Cambridge, Mass. 1970).
[4]See Ref. 2 and M. Gell-Mann, CERN Report No. CERN-TH-1543, CERN, 1972 (unpublished).
[5]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).
[6]J. C. Pati and C. H. Woo, Phys. Rev. D **3**, 1173 (1971).
[7]S. Okubo, Phys. Rev. **179**, 1629 (1969).
[8]H. J. Lipkin, Phys. Rev. Lett. **28**, 63 (1972); Phys. Rev. D **7**, 1850 (1973).
[9]N. Cabibbo, L. Maiani, and G. Preparata, Phys. Lett. **25B**, 132 (1967).
[10]H. Suura, T. F. Walsh, and Bing-Lin Young, Nuovo Cimento Lett. **4**, 505 (1972).
[11]R. Budny, T. Chang, and D. Choudhury, Nucl. Phys. **B44**, 618 (1972).
[12]C. H. Llewellyn Smith, Nucl. Phys. **B17**, 277 (1970).
[13]M. Gronau and Y. Zarmi, Phys. Rev. D **6**, 396 (1972).
[14]J. Iliopoulos and E. A. Paschos, Phys. Rev. D **6**, 1340 (1972).
[15]H. Fritzsch, Nuovo Cimento **12A**, 1016 (1972).

PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

Decay Spectra of Particles and Resonances Produced in a Central Plateau

R. Michael Barnett and D. Silverman*
University of California, Irvine, Irvine, California 92664
(Received 6 April 1973)

We consider the two-body decay of the spinless resonance or particle produced in a central plateau, with an arbitrary transverse-momentum spectrum. The spectrum of the decay products is calculated exactly as an integral over the spectrum of the centrally produced resonance or particle. Special forms applicable to large and small momentum transfer are presented along with an accurate inversion formula. We show how the large-transverse-momentum behavior of the resonance production is replicated in the decay products. The decay $\pi^0 \to \gamma + \gamma$ is considered in detail.

I. INTRODUCTION

The recent verification of the existence of a central plateau in the CERN Intersecting Storage Rings (ISR) experiments[1] allows us to probe deeper into the detailed mechanism of pionization. In previous papers[2,3] we have investigated the properties of the pionization spectrum in $q_\perp{}^2$ resulting from the internal-damping structure. As originally discussed by Amati, Stanghellini, and Fubini[4] (ASF), the pions in the central plateau arise from fireballs or resonances produced in a chain of peripheral pion exchanges. In this paper we calculate the inclusive spectrum of a particle resulting from decay of a spinless two-particle resonance which is peripherally produced in a central-plateau region. The generality of the calculation allows it to be applied also to the case of a π^0 produced in the central plateau, which then decays into two photons. It can then be used to infer the π^0 spectrum from the γ spectrum.

Our work is an extension of the treatment of these problems as recently considered by others[5-10] Our formulation (1) includes an exact treatment of the kinematics and integrations; (2) is applicable to any $q_\perp{}^2$ spectrum of produced resonances or π^0's; (3) applies to both large and small $q_\perp$; (4) has the integrations performed analytically, not numerically; (5) gives a unified treatment of massive and massless final particles. The formulation includes many of the earlier results as limiting or special cases.

The calculation proceeds by considering a resonance of momentum q and mass $q^2 \equiv m^2$ being produced in a central plateau with a spectrum $\rho(q^2, q_\perp{}^2)$ independent of longitudinal momentum. This then decays into two particles of masses μ_1 and μ_2 so that $q = q_1 + q_2$. Since only one particle q_1 is observed in the single-particle spectrum, we must integrate over the momentum of q_2. It is convenient to work with

$$\begin{aligned} \eta &\equiv (q_1^\perp + q_2^\perp)^2 + m^2 , \\ \eta_1 &= (q_1^\perp)^2 + \mu_1{}^2 , \\ \eta_2 &= (q_2^\perp)^2 + \mu_2{}^2 , \\ m_0{}^2 &\equiv m^2 - \mu_2{}^2 + \mu_1{}^2 , \end{aligned} \tag{1.1}$$

where the $\perp$ denotes two-dimensional transverse vectors. The integration over q_2 is performed by converting to integrals over η, η_2, and the rapidity $y_2 = \sinh^{-1}(q_2^\parallel/\eta_2{}^{1/2})$. The η_2 and y_2 integrations are performed exactly for infinite energy, and the integral over η, with the general function $\rho(\eta) \equiv \rho(m^2, q_\perp{}^2)$ remains.