PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

Weak-Interaction Effects in $\eta \to \pi^+\pi^-\gamma$*

Peter Herczeg† and Paul Singer‡
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 18 June 1973)

Parity-violating effects due to the strangeness-conserving nonleptonic weak interaction are studied in the process $\eta(X^0) \to \pi^+\pi^-\gamma$. The circular polarization and the parity-violating linear polarization are calculated in the framework of the Cabibbo theory by using a pole model for the strong and electromagnetic interactions. The isoscalar ($\sim \cos^2\theta$) and isovector ($\sim \sin^2\theta$) components of the weak Hamiltonian are found to give comparable contributions. The magnitude of the circular polarization is obtained to be $(1-5)\times 10^{-9}$ and the linear polarization, being dependent on final-state interactions, is smaller by nearly two orders of magnitude. In theories with a large isovector component in the weak nonleptonic Hamiltonian (theories involving neutral currents or second-class currents) the circular polarization can be larger, an estimate giving $(1-3)\times 10^{-8}$.

I. INTRODUCTION

During the past several years, parity-nonconserving effects were observed in numerous nonleptonic nuclear transitions. Although the agreement between theory and experiment is not at all satisfactory at the present time, the identification of the source of parity violation with a first-order strangeness-conserving nonleptonic weak interaction appears now to be well established.[1]

The study of the $\Delta S=0$ nonleptonic weak interactions can provide important information on the structure of weak interactions. In particular, it enables one, in principle, to discriminate among various models of weak interactions which give identical testable predictions for the strangeness-changing nonleptonic weak processes.[2] To date, the only experimental data concerning the nonleptonic strangeness-conserving weak Hamiltonian $H_{\rm NL}^{\Delta S=0}$ come from observations of parity mixing in nuclei. Analysis of these effects can supply knowledge on the matrix elements $\langle NX|H_{\rm NL}^{\Delta S=0}|N\rangle$, where $|N\rangle$ is a single nucleon state and $X=\pi$, 2π, ρ, ω, and ϕ.[2] With the exception of the $n+p \to D+\gamma$ reaction,[3] the experiments performed up to now have been, however, in complex nuclei, where the interpretation of the results is clouded by complications arising from nuclear structure. Theoretical estimates have been carried out also for several simpler systems, like πN scattering,[4] photopion reactions,[5] electron-proton,[6] and proton-proton scattering.[7] In these processes one also probes weak nucleonic matrix elements.

Recently we have considered the possibility of studying the effects of the strangeness-conserving nonleptonic weak interactions in strangeness-conserving nonleptonic decays of mesons.[8] From the great variety of possible decays only a few, such as for example $\eta \to \pi^+\pi^-\gamma$, $\rho^+ \to \pi^+\gamma$, and $\omega, \rho \to 3\pi$ turn out to be sensitive to the parity-violating part of $H_{\rm NL}^{\Delta S=0}$, because of constraints imposed by angular momentum conservation and CP invariance.[8,9]

In the present paper we consider in detail the implications of $H_{\rm NL}^{\Delta S=0}$, for the decay $\eta(X^0) \to \pi^+\pi^-\gamma$.[9] From dimensional considerations alone, the size of the parity-violating effects is expected to be of the order of Gm^2, where G is the Fermi constant and m is an effective mass relevant to the process, possibly of the order of a vector or axial-vector meson mass, so that $Gm^2 \simeq 10^{-5}-10^{-6}$. Actually, as will be described in Sec. II, the weak amplitude in this process is reduced by centrifugal-barrier effects. This disadvantage is to be weighted against the relative ease of selecting a narrow-width particle, in comparison with other decays, like $\rho^+ \to \pi^+\gamma$ and $\omega, \rho \to 3\pi$, where on the other hand the parity-violating effects are enhanced due to the suppression of the parity-conserving transition.[9]

In Sec. II we present a general discussion of the $\eta(X^0) \to \pi^+\pi^-\gamma$ transition without assuming C, P, or CP invariance, with particular attention to parity-violating effects.

Parity violation is found to manifest itself in a nonvanishing photon circular polarization and a nonvanishing photon linear polarization at 45° with respect to the decay plane. The latter effect is possible only on the account of final-state interactions.

In Sec. III we describe a model which enables us to express the parity-violating $\eta \to \pi^+\pi^-\gamma$ amplitude in terms of a few matrix elements of $H_{\rm NL}^{\Delta S=0}$ between pseudoscalar-meson and vector-meson single-particle states. The parity-violating $N \to NV$ (V =vector meson) amplitudes can be expressed in terms of the same matrix elements, provided that pseudoscalar-pole dominance holds for the weak NNV vertices.[10] In this framework, parity-violating effects in $\eta \to \pi^+\pi^-\gamma$ and in nuclear transitions will thus be related. In the last section we estimate the magnitude of the parity-violating polarization effects in the Cabibbo model and also comment on their possible size in other theories of weak interactions. The estimated circular polarization is of the order of 10^{-9} in the Cabibbo theory and can be one order of magnitude larger in theories with an enhanced isovector weak coupling. The linear polarization being proportional to the π-π phase shifts, is smaller by one to two orders of magnitude. The experimental study of such small effects appears to be remote at the present time. However, in view of the large (~ factor of 20) discrepancies between theory and experiment noted in nuclear physics,[1] and the uncertainties inherent to the present calculation, even an experimental upper limit would be of value in limiting the magnitude of the weak matrix elements involved.

II. SYMMETRY-VIOLATING EFFECTS

Let us consider the decay $\eta \to \pi^+\pi^-\gamma$ without assuming C, P, or CP invariance. As the $\pi^+\pi^-\gamma$ state has charge conjugation $(-1)^{l+1}$ ($l \equiv$ angular momentum of the dipion) and $C(\eta) = +1$, the C-conserving transitions correspond to photons of odd multipolarity with the pions being in an odd angular momentum state, while transitions in which the photon and the dipion are in even angular momentum states violate charge-conjugation invariance. The parity of the final state is $(-1)^{l+\lambda}$ $[(-1)^{l+\lambda+1}]$ for $E\lambda$ $(M\lambda)$ transitions, so that magnetic and electric transitions are parity-conserving and parity-violating, respectively.

We shall write the decay amplitude as a sum of amplitudes $M^{\Delta P, \Delta C}$ describing transitions characterized by definite change in charge conjugation and parity:

$$M = M^{++} + M^{+-} + M^{-+} + M^{--}, \tag{2.1}$$

where M^{+-}, for example, is P-conserving and C-violating, etc. The most general Lorentz- and gauge-invariant amplitudes M^{ij} $(i, j = +, -)$ have the following form[11]:

$$M^{++} = \frac{1}{m_{++}{}^3} iF^{++}(s, (k\cdot q)^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma, \tag{2.2}$$

$$M^{+-} = \frac{1}{m_{+-}{}^5} F^{+-}(s, (k\cdot q)^2)(k\cdot q)\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma, \tag{2.3}$$

$$M^{-+} = \frac{1}{m_{-+}{}^3} iF^{-+}(s, (k\cdot q)^2) \times [(\epsilon\cdot p_+)(k\cdot p_-) - (\epsilon\cdot p_-)(k\cdot p_+)], \tag{2.4}$$

$$M^{--} = \frac{1}{m_{--}{}^5} F^{--}(s, (k\cdot q)^2)(k\cdot q) \times [(\epsilon\cdot p_+)(k\cdot p_-) - (\epsilon\cdot p_-)(k\cdot p_+)]. \tag{2.5}$$

In Eqs. (2.2)–(2.5) ϵ, k, p_+, and p_- are the photon polarization vector, the γ, π^+, and π^- four-momenta, respectively, and $\epsilon_{\mu\nu\rho\sigma}$ is the completely antisymmetric unit tensor of fourth rank. F^{++}, F^{+-}, F^{-+}, and F^{--} are, in general, complex functions of

$$s \equiv (p_+ + p_-)^2$$

and

$$(k\cdot q)^2 \equiv [k\cdot(p_+ - p_-)]^2 .$$

M_{ij} are effective masses used to make F^{ij} dimensionless. In the absence of final-state interactions, CPT invariance requires all the functions F^{ij} to be real.

The amplitude M^{++} describes the main decay mode and is responsible for the observed decay rate. M^{+-} has been considered[12] in connection with the conjecture that charge-conjugation invariance may be violated in the electromagnetic interactions.[13] The CP-violating effect is an asymmetry in the momentum spectra of the π^+ and π^- due to interference between M^{++} and M^{+-}. The experimental results obtained so far are consistent with zero asymmetry.[14] However, the size of the asymmetry depends not only on the magnitude of F^{+-} but also on the π-π phase shifts and is moreover further suppressed by centrifugal-barrier effects, the lowest contributing angular momentum state of the dipion being a D wave. As a consequence, the existing experimental limits on the asymmetry[14] still allow the order of magnitude of F^{+-} to be as large as that of F^{++}.

The amplitudes M^{-+} and M^{--} are parity-nonconserving. As mentioned in the introduction, experiments studying parity violation in nuclear interactions give good evidence for parity conservation in the strong and in the electromagnetic interactions and indicate parity violation at the level of first-order weak interaction. Accordingly, the magnitude of F^{--} is expected to be of the order of $\sqrt{\alpha}GM_{--}{}^2$, where M_{--} is the characteristic mass for this decay. If CP is violated in the electromagnetic or in the $\Delta S = 0$ nonleptonic weak interactions, F^{-+} could be of the same order of magnitude. In both cases, study of parity-violating effects would provide information on the $\Delta S = 0$ nonleptonic weak interaction.

The observables in the decay $\eta \to \pi^+\pi^-\gamma$ are the photon and the pion momenta and the polarization of the photon. Parity-violating effects arise from interference between $M^+ = M^{++} + M^{+-}$ and $M^- = M^{-+} + M^{--}$. Summing in $|M^+ + M^-|^2$ over photon polarization one finds that the interference term vanishes, so that there are no parity-violating effects in $\eta \to \pi^+\pi^-\gamma$ decay if the photon polarization is unobserved.

Next we consider the photon polarization density matrix ρ_{ij} $(ij = 1, 2)$. We choose the two independent polarization vectors $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ to coincide with the vectors

$$\frac{\vec{p}_+ \times \vec{k}}{|\vec{p}_+ \times \vec{k}|}$$

and

$$\frac{(\vec{p}_+ \times \vec{k}) \times \vec{k}}{|(\vec{p}_+ \times \vec{k}) \times \vec{k}|},$$

respectively, in the dipion center-of-mass system. The contribution of the various terms in $|M|^2$ to the Stokes parameters $P_1 = \rho_{11} - \rho_{22}$, $P_2 = \rho_{12} + \rho_{21}$, and $P_3 = i(\rho_{12} - \rho_{21})$ are summarized in Table I. One sees that the parity-conserving amplitudes alone give no contribution to P_2 or P_3. This statement is true irrespective of final-state interactions and remains valid also in the presence of parity-conserving CP-violating interactions. In particular, interference between M^{++} and M^{+-} has no effect on P_2 and P_3. Observation of circular polarization, or a nonvanishing photon linear polarization at 45° with respect to the decay plane in $\eta \to \pi^+\pi^-\gamma$ is therefore evidence for parity violation. We note that if CP is conserved, the latter effect appears only in the presence of final-state interactions.

In the rest of this paper we shall disregard the effects due to possible CP violation.[15] Then, the expressions for P_3 and P_2 read as follows:

$$P_3 \equiv \frac{W_L(\theta, s) - W_R(\theta, s)}{W_L(\theta, s) + W_R(\theta, s)} = -\frac{m_{++}{}^3}{m_{--}{}^5} \frac{\mathrm{Re}(F^{++}F^{--*})}{|F^{++}|^2} (m_\eta^2 - s) \times \left(\frac{s - 4m_\pi^2}{s}\right)^{1/2} \cos\theta, \quad (2.6)$$

$$P_2 \equiv \frac{W_1'(\theta, s) - W_2'(\theta, s)}{W_1'(\theta, s) + W_2'(\theta, s)} = \frac{m_{++}{}^3}{m_{--}{}^5} \frac{\mathrm{Im}(F^{++}F^{--*})}{|F^{++}|^2} (m_\eta^2 - s) \times \left(\frac{s - 4m_\pi^2}{s}\right)^{1/2} \cos\theta, \quad (2.7)$$

where $W_L(\theta, s)$ and $W_R(\theta, s)$ are the distributions of left and right circularly polarized photons, while $W_1'(\theta, s)$ and $W_2'(\theta, s)$ refer to photons polarized in the directions

$$\vec{\epsilon}_1{}' = \frac{1}{\sqrt{2}} (\vec{\epsilon}_1 + \vec{\epsilon}_2)$$

TABLE I. Contribution of various terms in $|M|^2$ to the Stokes parameters. Terms of order G^2 are not included. θ is the angle between the momenta of the positive pion and of the photon in the dipion center-of-mass system, $s = (p_+ + p_-)^2$.

	Violates	P_1	P_2	P_3
$\|M^{++}\|^2$	...	≈ 1	...	...
$\|M^{+-}\|^2$	...	$\frac{m_{++}{}^6}{m_{+-}{}^{10}} \frac{1}{4} \left\|\frac{F^{+-}}{F^{++}}\right\|^2 \left(\frac{s-4m_\pi^2}{s}\right)(m_\eta^2 - s)^2 \cos^2\theta$	...	...
$M^{++}M^{-+*} + M^{++*}M^{-+}$	P, CP	...	$-2\frac{m_{++}{}^3}{m_{-+}{}^3} \frac{\mathrm{Re}(F^{++}F^{-+*})}{\|F^{++}\|^2}$	$2\frac{m_{++}{}^3}{m_{-+}{}^3} \frac{\mathrm{Im}(F^{++}F^{-+*})}{\|F^{++}\|^2}$
$M^{+-}M^{-+*} + M^{+-*}M^{-+}$	C, P	...	$\frac{m_{++}{}^6}{m_{-+}{}^3 m_{+-}{}^5} \frac{\mathrm{Im}(F^{-+}F^{+-*})}{\|F^{++}\|^2}(m_\eta^2 - s)\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\cos\theta$	$\frac{m_{++}{}^6}{m_{-+}{}^3 m_{+-}{}^5} \frac{\mathrm{Re}(F^{-+}F^{+-*})}{\|F^{++}\|^2}(m_\eta^2 - s)\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\cos\theta$
$M^{++}M^{+-*} + M^{++*}M^{+-}$	C, CP	$\frac{m_{++}{}^3}{m_{+-}{}^5} \frac{1}{2} \frac{\mathrm{Im}(F^{++*}F^{+-})}{\|F^{++}\|^2} \frac{(m_\eta^2 - s)^2}{s}(s - 4m_\pi^2)^{1/2}\cos\theta$	...	...
$M^{++}M^{--*} + M^{++*}M^{--}$	C, P	...	$\frac{m_{++}{}^3}{m_{--}{}^5} \frac{\mathrm{Im}(F^{++}F^{--*})}{\|F^{++}\|^2}(m_\eta^2 - s)\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\cos\theta$	$\frac{m_{++}{}^3}{m_{--}{}^5} \frac{\mathrm{Re}(F^{++}F^{--*})}{\|F^{++}\|^2}(m_\eta^2 - s)\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\cos\theta$
$M^{+-}M^{--*} + M^{+-*}M^{--}$	P, CP	...	$-\frac{1}{2}\frac{m_{++}{}^6}{m_{--}{}^5 m_{+-}{}^5} \frac{\mathrm{Re}(F^{--*}F^{+-})}{\|F^{++}\|^2}(m_\eta^2 - s)^2\left(\frac{s-4m_\pi^2}{s}\right)\cos^2\theta$	$\frac{1}{2}\frac{m_{++}{}^6}{m_{--}{}^5 m_{+-}{}^5} \frac{\mathrm{Im}(F^{--*}F^{+-})}{\|F^{++}\|^2}(m_\eta^2 - s)^2\left(\frac{s-4m_\pi^2}{s}\right)\cos^2\theta$

and

$$\vec{\epsilon}_2' = \frac{1}{\sqrt{2}}(-\vec{\epsilon}_1 + \vec{\epsilon}_2),$$

respectively. We note that there are no net polarizations; upon integrating over θ both P_2 and P_3 vanish.

The size of the polarizations depends on the functions F^{++} and F^{--}. Information on the former is obtainable from the $\eta \to \pi^+\pi^-\gamma$ decay spectrum and rate. We shall estimate the latter in a model calculation.

III. MODEL FOR THE PARITY-VIOLATING AMPLITUDE

The parity-violating part of the matrix element under consideration is given by

$$M^{--} = \langle \pi^+(p_+)\pi^-(p_-)\gamma(k)|H^{\Delta S=0}_{\text{NL,p.v.}}|\eta(l)\rangle, \qquad (3.1)$$

where $H^{\Delta S=0}_{\text{NL,p.v.}}$ is the parity-violating component of the strangeness-conserving nonleptonic weak Hamiltonian. We shall assume $H^{\Delta S=0}_{\text{NL,p.v.}}$ to be CP-invariant.

As the $\eta \to \pi^+\pi^-\gamma$ is a direct transition,[16] a detailed treatment of M^{--} requires a model for the structure-dependent vertex $F^{--}(s, (q\cdot k)^2)$, which would take into account the effects of the strong interactions. To this end, we shall employ an effective Lagrangian containing vector (V), axial vector with normal and abnormal charge conjugation (A and B),[17] and pseudoscalar (P) particles, with all the allowed trilinear couplings among them[18] and treat it in the tree approximation.[19] The electromagnetic interactions are included using the vector-meson-dominance formalism of Kroll, Lee, and Zumino.[20] The weak vertices are approximated by pseudoscalar, vector, and axial-vector pole diagrams, so that finally the weak interaction appears in each of the diagrams as a transition operator between single-particle states.[21] states.[21] We shall take zero widths for all particles involved obtaining thus only Re F^{--} with our model.

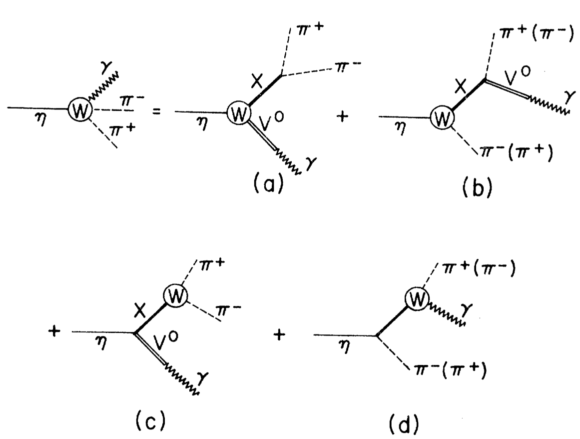

FIG. 1. The four classes of diagrams considered for the weak $\eta \to \pi^+\pi^-\gamma$ amplitude. X stands for any of the allowed single-particle contributions from the P, V, A, and B multiplets. The blobs represent the weak transition.

The diagrams contributing to the weak $\eta \to \pi^+\pi^-\gamma$ amplitude can be classified into four general groups shown in Fig. 1. The blobs W represent the weak transition. Independently of the detailed structure of W, one can easily see that the groups of diagrams (a), (c), and (d) do not contribute to M^{--}. In the case of group (a) and (c) this is due to the fact that they cannot give D-wave pions in the final state, while for group (d) the requirement of C, P, and isospin invariance make the strong coupling vanish for X being any of P, V, A, or B.

Proceeding now to consider the Feynman diagrams belonging to group (b), we find that the only ones that survive are those depicted in Fig. 2. It is interesting to observe that the PVV and PPV couplings which are the main underlying interactions in parity-conserving radiative meson decays,[22,21] do not contribute at all to the parity-violating $\eta \to \pi^+\pi^-\gamma$ transition. This is due to the fact that in the weak P-V transitions only the fourth component of the vector field contributes.

The weak vertices entering into our calculation are defined as

$$\langle P(k)|H^{\Delta S=0}_{\text{NL,p.v.}}(0)|V(k)\rangle = \frac{1}{(2\pi)^3}\frac{1}{(4E_P E_V)^{1/2}} G_{PV}\epsilon^{(V)}_\mu k_\mu, \qquad (3.2)$$

where G_{PV}, the effective coupling for the corresponding pseudoscalar (P) to vector (V) transition, is generally a function of k^2.

For the strong couplings BVP we use the symmetry structure of the effective Lagrangian of Gatto and Maiani[23] (built from nonets of A, B,

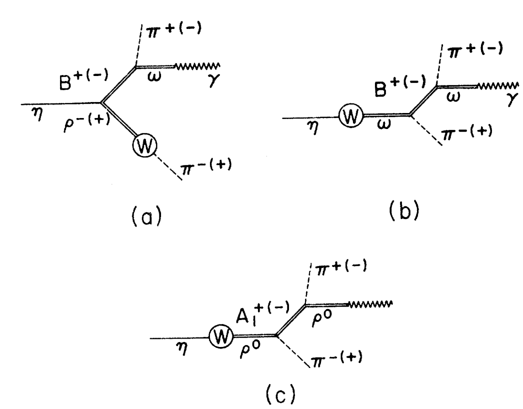

FIG. 2. Diagrams contributing to the weak $\eta \to \pi^+\pi^-\gamma$ amplitude.

and V, and the pseudoscalar octet P), which leads to a vanishing $B\to\phi\pi$ transition.[24,25] Then we are left only with the contribution of ω in diagram (b) of Fig. 2, as indicated on the graph. For the BVP couplings we use the following expression:

$$\langle B(p)P(p-q)|V(q)\rangle = g\epsilon^{(B)}\cdot\epsilon^{(V)} - h(\epsilon^{(B)}\cdot q)(\epsilon^{(V)}\cdot p)\,, \tag{3.3}$$

where $\epsilon^{(B)}(p)$ and $\epsilon^{(V)}(q)$ are the polarization vectors of the B and V mesons. The BVP coupling can also be written in a form more suitable for use at the electromagnetic vertex as follows:

$$\begin{aligned}\langle B(p)P(p-q)|V(q)\rangle &= g_T[(\epsilon^{(B)}\cdot\epsilon^{(V)})(p\cdot q) - (\epsilon^{(B)}\cdot q)(\epsilon^{(V)}\cdot p)]\\ &\quad + g_L\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(q_\lambda - \frac{p\cdot q}{p^2}p_\lambda\right)\epsilon_\lambda^{(B)}\epsilon_\mu^{(V)}\,.\end{aligned} \tag{3.4}$$

On the mass shell for all three particles, the relation between the various coupling constants is given by

$$g_T = \frac{2g}{m_B^2 + m_V^2 - m_\pi^2}\,, \tag{3.5}$$

$$g_L = -\frac{2g}{m_B^2 + m_V^2 - m_\pi^2} - h\,. \tag{3.6}$$

Identical expressions are used for the AVP couplings, with an A meson replacing the B meson. In view of the lack of knowledge of the actual behavior of the vertices involved, we have used expressions (3.3) and (3.4) assuming constant couplings, thus neglecting any possible off-mass-shell effects. These, however, are not likely to alter significantly our results.[26,27]

Using for the vector-meson–photon vertices an effective Lagrangian[20]

$$\mathcal{L}_{\text{eff}}^{\text{e.m.}} = e\left[\frac{m_\rho^2}{f_\rho}\epsilon_\mu^{(\rho)} + \frac{m_\omega^2}{f_\omega}\epsilon_\mu^{(\omega)} + \frac{m_\varphi^2}{f_\varphi}\epsilon_\mu^{(\varphi)}\right]A_\mu \tag{3.7}$$

and with the aid of (3.3) and (3.4), we find that the parity-violating $\eta\to\pi^+\pi^-\gamma$ amplitude is given by

$$\begin{aligned}M^{--} &= \frac{eg_T^{(B\omega\pi)}G_{\rho\pi}}{m_\rho^2 f_\omega}[(p_-\cdot k)(p_+\cdot\epsilon) - (p_+\cdot k)(p_-\cdot\epsilon)]\\
&\quad\times\left\{\frac{g^{(B\rho\eta)}}{m_\pi^2 - m_B^2 + 2p_+\cdot k} - \frac{g^{(B\rho\eta)}}{m_\pi^2 - m_B^2 + 2p_-\cdot k} - \frac{h^{(B\rho\eta)}(p_-\cdot p_+ + p_-\cdot k)}{m_\pi^2 - m_B^2 + 2p_+\cdot k} + \frac{h^{(B\rho\eta)}(p_-\cdot p_+ + p_-\cdot k)}{m_\pi^2 - m_B^2 + 2p_-\cdot k}\right\}\\
&\quad + \frac{eg_T^{(B\omega\pi)}G_{\omega\eta}}{m_\omega^2 f_\omega}[(p_-\cdot k)(p_+\cdot\epsilon) - (p_+\cdot k)(p_-\cdot\epsilon)]\\
&\quad\times\left\{\frac{g^{(B\omega\pi)}}{m_\pi^2 - m_B^2 + 2p_+\cdot k} - \frac{g^{(B\omega\pi)}}{m_\pi^2 - m_B^2 + 2p_-\cdot k} - \frac{h^{(B\omega\pi)}p_\eta\cdot(p_+ + k)}{m_\pi^2 - m_B^2 - 2p_+\cdot k} + \frac{h^{(B\omega\pi)}p_\eta\cdot(p_- + k)}{m_\pi^2 - m_B^2 + 2p_-\cdot k}\right\}\\
&\quad + \frac{eg_T^{(A_1\rho\pi)}G_{\eta\rho}}{m_\rho^2 f_\rho}[(p_-\cdot k)(p_+\cdot\epsilon) - (p_+\cdot k)(p_-\cdot\epsilon)]\\
&\quad\times\left\{\frac{g^{(A_1\rho\pi)}}{m_\pi^2 - m_{A_1}^2 + 2p_+\cdot k} - \frac{g^{(A_1\rho\pi)}}{m_\pi^2 - m_{A_1}^2 + 2p_-\cdot k} - \frac{h^{(A_1\rho\pi)}p_\eta\cdot(p_+ + k)}{m_\pi^2 - m_{A_1}^2 + 2p_+\cdot k} + \frac{h^{(A_1\rho\pi)}p_\eta\cdot(p_- + k)}{m_\pi^2 - m_{A_1}^2 + 2p_-\cdot k}\right\}.\end{aligned} \tag{3.8}$$

In view of the fact that $2p_+\cdot k/m_B^2 - m_\pi^2$, $2p_-\cdot k/m_B^2 - m_\pi^2$, $2p_+\cdot k/m_{A_1}^2 - m_\pi^2$, and $2p_-\cdot k/m_{A_1}^2 - m_\pi^2$ are of the order of 10^{-1} or less in this process, we can expand to first order in these variables to obtain the simplified expression

$$\begin{aligned}M^{--} &= \frac{2eg_T^{(B\omega\pi)}}{f_\omega(m_B^2 - m_\pi^2)^2}[(p_-\cdot k)(p_+\cdot\epsilon) - (p_+\cdot k)(p_-\cdot\epsilon)][k\cdot(p_- - p_+)]\\
&\quad\times\left\{\frac{g^{(B\rho\eta)}G_{\rho\pi}}{m_\rho^2} + \frac{g^{(B\omega\pi)}G_{\omega\eta}}{m_\omega^2} - \frac{h^{(B\rho\eta)}G_{\rho\pi}}{2m_\rho^2}(s - m_B^2 - m_\pi^2) - \frac{h^{(B\omega\pi)}G_{\omega\eta}}{2m_\omega^2}(m_B^2 + 2m_\eta^2 - m_\pi^2 - s)\right\}\\
&\quad + \frac{2eg_T^{(A_1\rho\pi)}G_{\eta\rho}}{f_\rho m_\rho^2(m_{A_1}^2 - m_\pi^2)^2}[(p_-\cdot k)(p_+\cdot\epsilon) - (p_+\cdot k)(p_-\cdot\epsilon)][k\cdot(p_- - p_+)]\\
&\quad\times\{g^{(A_1\rho\pi)} - \tfrac{1}{2}h^{(A_1\rho\pi)}(m_{A_1}^2 + 2m_\eta^2 - m_\pi^2 - s)\}\,.\end{aligned} \tag{3.9}$$

We can now identify $F^{--}(s, (q\cdot k)^2)$ of Eq. (2.5) to be given in our model by

$$F^{--}(s,\,(k\cdot q)^2)=\frac{2eg_T^{(B\,\omega\pi)}}{f_\omega(m_B{}^2-m_\pi{}^2)^2}\bigg[\frac{g^{(B\eta\rho)}G_{\rho\pi}}{m_\rho{}^2}+\frac{g^{(B\,\omega\pi)}G_{\omega\eta}}{m_\omega{}^2} -\frac{h^{(B\rho\eta)}G_{\rho\pi}}{2m_\rho{}^2}(s-m_B{}^2-m_\pi{}^2)-\frac{h^{(B\,\omega\pi)}G_{\omega\eta}}{2m_\omega{}^2}(m_B{}^2+2m_\eta{}^2-m_\pi{}^2-s)\bigg]$$
$$+\frac{2eg_T^{(A_1\rho\pi)}G_{\eta\rho}}{f_\rho m_\rho{}^2(m_{A_1}{}^2-m_\pi{}^2)^2}\big[g^{(A_1\rho\pi)}-\tfrac{1}{2}h^{(A_1\rho\pi)}(m_{A_1}{}^2+2m_\eta{}^2-m_\pi{}^2-s)\big]\,. \qquad (3.10)$$

As is evident from (3.10), F^{--} depends in this approximation only on the s variable. Thus only the lowest allowed multipole transition ($E2$) contributes to the parity-violating amplitude.

IV. ESTIMATE OF THE PARITY-VIOLATING POLARIZATIONS

To obtain an estimate for P_2 and P_3 we need the values of the various strong couplings appearing in F^{--} as well as in the parity-conserving form factor F^{++}.

For the parity-conserving amplitude M^{++} we use the ρ-dominance model[28] for the π-π $l=1$ channel, which has been found experimentally[29] to give an accurate description of this decay. Thus, we can write

$$F^{++}(s,\,(k\cdot q)^2)=\frac{f_{\eta\pi\pi\gamma}}{s-m_\rho{}^2}\ , \qquad (4.1)$$

where $f_{\eta\pi\pi\gamma}$ is an effective $\langle\pi\pi\gamma|\eta\rangle$ coupling which is determined to be $f_{\eta\pi\pi\gamma}=2.8e/m_\pi$. In obtaining this value, we use the decay width calculated from (2.2) and (4.1) as well as the experimental values[25] for the absolute rate of $\eta\to\gamma\gamma$ and the relative ratio $\Gamma(\eta\to\pi\pi\gamma)/\Gamma(\eta\to\gamma\gamma)$. We note, as seen from Eq. (4.1) that only the lowest allowed multipole transition, i.e., ($M1$), contributes to the decay.

The magnitude of F^{--} depends, aside from the weak matrix elements to be discussed later, on the strong couplings $g^{(B\eta\rho)}$, $h^{(B\eta\rho)}$, $g^{(B\omega\pi)}$, $h^{(B\omega\pi)}$, $g^{(A_1\rho\pi)}$, and $h^{(A_1\rho\pi)}$ as well as on f_ω and f_ρ. The latter are determined up to a relative sign, from the electromagnetic leptonic decays of ρ^0 and ω^0 to be[30] $f_\rho{}^2/4\pi=2.26\pm0.25$ and $f_\omega{}^2/4\pi=18.4\pm1.8$. For the coupling constants $g^{(B\eta\rho)}$ and $h^{(B\eta\rho)}$ there is no direct experimental information. We shall rely here on the effective Lagrangian of Ref. 23, from which we obtain

$$g^{(B\eta\rho)}=\frac{1}{\sqrt{3}}\,g^{(B\pi\,\omega)}=\frac{1}{\sqrt{3}}g_B$$

and

$$h^{(B\eta\rho)}=\frac{1}{\sqrt{3}}h^{(B\,\omega\pi)}\equiv\frac{1}{\sqrt{3}}\,h_B\ .$$

The couplings g_B, h_B, $g_A(\equiv g^{(A_1\rho\pi)})$, and $h_A(\equiv h^{(A_1\rho\pi)})$ can be determined in principle from the measurement of the decay width and the D/S ratio in the appropriate $B\to\omega\pi$ and $A\to\rho\pi$ decays. Unfortunately, the experimental situation regarding these quantities is unsettled at the present time.

Concerning the B meson, experiments[25,31] seem to confirm it as a bona fide resonance, with a width of about 130 MeV. On the other hand, the magnitude of the D/S ratio is as yet uncertain. Henceforth, we shall use for the ratio h_B/g_B a set of values

$$r_B\equiv m_B{}^2h_B/g_B=0,\,1,\,2.5,\,6$$

lying within the range of the available experimental results,[31] as well as of theoretical predictions.[32] For g_B we shall take the value corresponding to $\Gamma_B=130$ MeV and to a given h_B/g_B ratio (for $h_B/g_B=0$ one has $g_B{}^2/4\pi=1.06$ GeV^2).

The present status of the A_1 is rather obscure, even its resonance character being questioned. Although a large group of experiments have indicated a width of around 130 MeV, there are also recent results[33] pointing to a larger width of 200–300 MeV. Concerning the D/S ratio, while an early experiment[34] indicated an appreciable value for it, the more recent experiments[34] are consistent with a fraction of the D-wave amplitude not exceeding a few percent. This would be consistent with the current algebra predictions,[27] provided the width turns out to be in the 100-MeV range. For our purpose, we shall assume the existence of an A_1 axial-vector meson with a width of 130 MeV and take the range of values

$$\Gamma_A=m_{A_1}{}^2h_A/g_A=0,\ 2.3,\ 2.5,\ 5$$

($m_{A_1}{}^2h_A/g_A=1.3$ is the current algebra value[27] for $\delta=-\frac{1}{2}$). With $h_A/g_A=0$, for example, one has $g_A{}^2/4\pi=0.61$ GeV^2.

Inserting numerical values into expressions (4.1), (3.10), (2.6), and (2.7), one has

$$P_3=-\frac{(m_\eta^2-s)(s-4m_\pi^2)^{1/2}(s-m_\rho^2)}{\sqrt{s}}$$
$$\times\cos\theta\left[0.035\frac{g_B^2}{4\pi}G_{\rho\pi}\left(1-\frac{s-1.5}{2}\frac{h_B}{g_B}\right)+0.058\frac{g_B^2}{4\pi}G_{\omega\eta}\left(1+\frac{s-2.1}{2}\frac{h_B}{g_B}\right)\right.$$
$$\left.\pm0.38\frac{g_A^2}{4\pi}G_{\rho\eta}\left(1+\frac{s-1.7}{2}\frac{h_A}{g_A}\right)\right]\cos(\delta_1-\delta_2)\,, \tag{4.2}$$

$$P_2=-P_3\tan(\delta_1-\delta_2)\,. \tag{4.3}$$

In the above expressions, the sign of f_ω was chosen arbitrarily to be positive and the ± sign in the last expression refers to the unknown sign of f_ρ/f_ω. For photon energies of interest here (i.e., $E_\gamma\gtrsim50$ MeV), the D-wave pion phase shift δ_2 is practically zero and the P-wave phase shift δ_1 is less than a few degrees. Thus $\cos(\delta_1-\delta_2)\simeq1$ and P_2 is smaller than P_3 by nearly two orders of magnitude.

When expressed in the η rest system in terms of the commonly-used variables E_γ and

$$x=\frac{1}{\sqrt{3}\,Q}(E_{\pi^+}-E_{\pi^-}),$$

with

$$Q=M_\eta-M_{\pi^+}-M_{\pi^-},$$

E_i being the particle energies,

$$P_3=-x(0.15+0.56E_\gamma)\left[0.035\frac{g_B^2}{4\pi}G_{\rho\pi}\left(1+\frac{1.2+1.1E_\gamma}{2}\frac{h_B}{g_B}\right)+0.058\frac{g_B^2}{4\pi}G_{\omega\eta}\left(1-\frac{1.8+1.1E_\gamma}{2}\frac{h_B}{g_B}\right)\right.$$
$$\left.\pm0.38\frac{g_A^2}{4\pi}G_{\rho\eta}\left(1-\frac{1.4+1.1E_\gamma}{2}\frac{h_A}{g_A}\right)\right]. \tag{4.4}$$

In Eqs. (4.2) and (4.4), all dimensional quantities are expressed in powers of GeV. We note that $G_{(VP)}$ has dimension of mass. In the $\eta\to\pi^+\pi^-\gamma$ decay $E_\gamma^{\max}=0.20$ GeV and $x_{\max}=0.2$. As one sees in (4.4), the coefficient of the A_1 contribution is larger by one order of magnitude than the contribution from the B meson. This is due to the factors $f_\rho/f_\omega\simeq\frac{1}{3}$, $m_B^{-4}/m_A^{-4}\simeq\frac{1}{2}$, $g_A^2/g_B^2\simeq\frac{1}{2}$. As a result, the isovector term gives a contribution comparable to the contribution of the isoscalar component, even though the strength of the weak transition is smaller by an order of magnitude because of the $\sin^2\theta$ factor [cf. Eq. (4.7c)].

We turn now to consider the weak matrix elements G_{VP}. The form of the nonleptonic weak Hamiltonian is not as well established as that responsible for semileptonic processes. In general, the Hamiltonian will contain both isotopic-spin (I) even and odd terms. The process $\eta\to\pi^+\pi^-\gamma$ is sensitive to $I\leq3$ terms only. Furthermore, $G_{\omega\eta}$ receives contributions only from the isoscalar part of $H_{\rm NL,p.v.}^{\Delta S=0}$ and $G_{\rho\eta}$ depends only on its isovector part. As a consequence of the assumption of CP invariance, the I-even and I-odd components of $H_{\rm NL,p.v.}^{\Delta S=0}$ are respectively of odd and even G parity. Thus, only the I-even component of $H_{\rm NL,p.v.}^{\Delta S=0}$ contributes to $G_{\rho\pi}$ and the photons involved in the transitions induced by the I-even (odd) components of $H_{\rm NL,p.v.}^{\Delta S=0}$ are of isoscalar (isovector) nature.

To be more specific, we shall assume the Cabibbo form for $H_{\rm NL}$:

$$H_{\rm NL}=-\frac{G}{\sqrt{2}}\frac{1}{2}(J_\mu J_\mu^\dagger+J_\mu^\dagger J_\mu)\,, \tag{4.5}$$

where

$$J_\mu=\cos\theta_C\,J_{(110)\mu}+\sin\theta_C\,J_{(\frac{1}{2}\frac{1}{2}1)\mu}\,,$$
$$J_{(abc)\mu}=J_{(I\,I_3\,Y)\mu}\,.$$

The strangeness-conserving part of (4.5) is

$$H_{\rm NL}^{\Delta S=0}=-\frac{G}{\sqrt{2}}\frac{1}{2}(\cos^2\theta_C\{J_{(110)\mu},J^\dagger_{(110)\mu}\}_+ +\sin^2\theta_C\{J_{(\frac{1}{2}\frac{1}{2}1)\mu},J^\dagger_{(\frac{1}{2}\frac{1}{2}1)\mu}\}_+)\,. \tag{4.6}$$

The $\cos^2\theta$ part is a mixture of $T=0$ and 2 components, while the $\sin^2\theta$ part involves $T=0$ and $T=1$. Consequently, the isovector component of $H_{\rm NL}^{\Delta S=0}$ is suppressed by the factor $\cot^2\theta\simeq20$.

The determination of $G_{\rho\pi}$, $G_{\omega\eta}$, and $G_{\rho\eta}$ involves calculating matrix elements of bilinear products of currents between the corresponding single-meson states. There is no method at present which would allow a reliable calculation of this sort. In order to get some feeling for the magnitude of these matrix elements, we shall consider estimates presented in the literature.

In a recent paper,[10] in connection with the calculation of the parity-violating NNV vertex, McKellar and Pick estimated the magnitude of $G_{\rho\pi}$ using the Hamiltonian (4.5). Retaining only the vacuum as the intermediate state between the weak currents (the so-called factorization approximation), the value obtained is

$$G_{\rho\pi} = (f_\pi m_\rho^2/f_\rho)G\cos^2\theta \simeq 1.56\times 10^{-7}\ \text{GeV}.$$

Inclusion of the next intermediate state (the one-pion contribution) increases this value by nearly two orders of magnitude. Estimates restricting the number of contributing intermediate states are therefore, as emphasized by the authors, very volatile. Another estimate for $G_{\rho\pi}$ can be obtained[10] by combining the pseudoscalar-meson-dominance model for the NNV vertex with an $SU(6)_W$ model[35] for the NNP and NNV vertices, and assuming that the $\{27\}$ component of the H_{NL} has zero-matrix elements between single-baryon states. This approach leads to a value of

$$G_{\rho\pi} = 8.9\times 10^{-7}\ \text{GeV} \qquad (4.7a)$$

which is about six times larger than that of the factorization approximation. It also relates $G_{\omega\eta}$ to $G_{\rho\pi}$, giving[10]

$$G_{\omega\eta} = \frac{7\sqrt{3}}{45} G_{\rho\pi} = 2.4\times 10^{-7}\ \text{GeV}. \qquad (4.7b)$$

Applying the same framework to calculate $G_{\rho\eta}$, we obtain

$$G_{\rho\eta} = \frac{11\sqrt{3}}{20}\tan^2\theta_C\, G_{\rho\pi} = 0.42\times 10^{-7}\ \text{GeV}. \qquad (4.7c)$$

We note that the factorization approximation [with Eq. (4.5)] gives

$$G_{\omega\eta} = G_{\rho\eta} = 0\,.$$

We have evaluated P_3 [Eq. (4.4)] with the values (4.7a)–(4.7c) for the weak matrix elements, for various E_γ and for all combinations of the selected values of r_A and r_B. We find that the absolute value of the circular polarization is increasing slowly with energy for all pairs of r_A, r_B, the variation in the energy interval $50 < E_\gamma < 170$ MeV not exceeding a factor of about 2. On the other hand, the polarization is more sensitive to the magnitude of the strong couplings and also to the sign of the ratio (f_ρ/f_ω). The results for $(1/x)P_3$ at $E_\gamma = 150$ MeV are presented in Table II, where P_3^+ (P_3^-) correspond to f_ρ/f_ω being positive (negative). (We remind that the allowed range for x is limited by the boundary of the Dalitz plot in the E_γ, x variables. For example, for convenient γ energies of 100 and 150 MeV, the x variable cannot exceed ~0.15 and ~0.20, respectively).

TABLE II. Circular polarization at $E_\gamma = 150$ MeV for various values of the strong coupling parameters. $P_3^{+(-)}$ are the circular polarizations in the Cabibbo theory, corresponding to positive (negative) sign of f_ρ/f_ω (f_ω chosen positive). $P_3'^{+(-)}$ are the circular polarizations for the case when the isovector contribution is enhanced by a factor $\cot^2\theta = 20$. x is the Dalitz variable $x = (E_{\pi^+} - E_{\pi^-})/\sqrt{3}\,Q$.

	$r_A=0$ $r_B=0$	$r_A=0$ $r_B=1$	$r_A=0$ $r_B=2.5$	$r_A=0$ $r_B=6$	$r_A=1.3$ $r_B=0$	$r_A=1.3$ $r_B=1$	$r_A=1.3$ $r_B=2.5$	$r_A=1.3$ $r_B=6$	$r_A=2.5$ $r_B=0$	$r_A=2.5$ $r_B=1$	$r_A=2.5$ $r_B=2.5$	$r_A=2.5$ $r_B=6$	$r_A=5$ $r_B=0$	$r_A=5$ $r_B=1$	$r_A=5$ $r_B=2.5$	$r_A=5$ $r_B=6$
$10^8 P_3^+/x$	−1.3	−1.6	−2.1	−3.4	−1.1	−1.4	−1.9	−3.3	−0.93	−1.2	−1.7	−3.0	−0.43	−0.69	−1.1	−2.5
$10^8 P_3^-/x$	−0.9	−1.2	−1.6	−3.0	−1.1	−1.3	−1.8	−3.2	−1.3	−1.5	−2.0	−3.4	−1.8	−2.1	−2.5	−3.9
$10^8 P_3'^+/x$	−5.6	−5.9	−6.4	−7.8	−1.6	−1.9	−2.3	−3.7	2.6	2.3	1.9	0.50	12.8	12.5	12.0	10.6
$10^8 P_3'^-/x$	3.4	3.1	2.7	1.3	−0.65	−0.91	−1.4	−2.7	−4.9	−5.1	−5.6	−7.0	−15.0	−15.3	−15.7	−17.1

As seen from Table II, except for those cases in which the particular combination of r_A and r_B values leads to a near cancellation of the isovector and isoscalar contributions, the circular polarization turns out to be generally negative and of the order of $(1-5)\times10^{-9}$ (with f_ω chosen arbitrarily positive).

It is of interest to consider the magnitude of the circular polarization assuming that the effective strength of the $T=1$ transition is increased by a factor $\cot^2\theta\simeq 20$, which could be the case in theories with neutral currents [36] or with second-class currents.[37] Provided the same values are used for $\cos^{-2}\theta\, G_{\rho\pi}$, $\cos^{-2}\theta\, G_{\omega\eta}$, and $\sin^{-2}\theta\, G_{\rho\eta}$ as before, we find now that the circular polarization (given as P'_+, P'_- in Table II) is in some cases larger by about one order of magnitude than in the Cabibbo theory. Thus if $r_A\gtrsim 5$, there appears to be an interesting possibility of discriminating between the Cabibbo theory and theories with a large $\Delta T=1$, $\Delta S=0$ component in the Hamiltonian.

The values we have obtained for the polarizations are apparently beyond the present experimental feasibility. We would like to note, however, that the discrepancy between theory and experiments on parity-violating transitions in nuclear physics may hint to larger values for the G_{VP} transitions than used here (the G_{VP} matrix elements are related in the pole model[10] to the weak NNV vertices). Thus, even an upper limit on the parity-violating effects discussed here would be valuable.

In concluding, we remark that our formulas can be used to calculate the parity-violating effects in the $X^0\to\pi^+\pi^-\gamma$ decay, when information is available on the various couplings involved. Due to the larger phase space available, the X^0 decay is more favorable for the parity-violating transition and hence the effects could be larger.

ACKNOWLEDGMENTS

We are grateful to Professor Lincoln Wolfenstein for many helpful discussions. One of us (P.S.) would like to thank the members of the Physics Department at Carnegie-Mellon University for their kind hospitality during the academic year 1972–1973.

---

*Research supported in part by the U. S. Atomic Energy Commission.

†Address from September 1, 1973: Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544.

‡Permanent address: Technion-Israel Institute of Technology, Haifa, Israel.

[1]E. M. Henley, in *Proceedings of the International Conference on Few-Particle Problems in the Nuclear Interaction, Los Angeles, 1972,* edited by I. Šlaus *et al*. (North-Holland, Amsterdam, 1972); D. H. Wilkinson, *ibid.* and references quoted there.

[2]See, e.g., E. Fischbach and D. Tadić, Phys. Rep. 6C, 123 (1973); M. Gari, *ibid.* 6C, 317 (1973); R. J. Blin-Stoyle, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969); E. M. Henley, Ann. Rev. Nucl. Sci. 19, 367 (1969), and references quoted there.

[3]V. M. Lobashov *et al*., Zh. Eksp. Teor. Fiz. Pis'ma Red. 11, 124 (1970) [JETP Lett. 11, 76 (1970)]; Nucl. Phys. A197, 241 (1972).

[4]B. H. J. McKellar and P. Pick, Nucl. Phys. B22, 625 (1970).

[5]D. U. L. Yu, E. M. Henley, T. Keliher, and W. J. Pardee, Bull. Am. Phys. Soc. 17, 493 (1972).

[6]E. M. Henley, A. H. Huffman, and D. U. L. Yu, Phys. Rev. D 7, 943 (1973).

[7]M. Simonius, Phys. Lett. 41B, 415 (1972); V. R. Brown, E. M. Henley, and F. R. Krejs, Phys. Rev. Lett. 30, 770 (1973).

[8]P. Herczeg and P. Singer, Bull. Am. Phys. Soc. 18, 25 (1973).

[9]The parity-violating effects in radiative and nonradiative vector-meson decays will be discussed in a separate publication.

[10]B. H. J. McKellar and P. Pick, Phys. Rev. D 6, 2184 (1972).

[11]The metric used is $g_{00}=-g_{11}=-g_{22}=-g_{33}=1$. In the system of units we use $\hbar=c=1$; $(e^2/4\pi)=\frac{1}{137}$.

[12]B. Barrett and T. Truong, Phys. Rev. 147, 1161 (1966).

[13]J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965); S. Barshay, Phys. Lett. 17, 78 (1965).

[14]J. J. Thaler *et al*., Phys. Rev. Lett. 29, 313 (1972). The asymmetry parameter obtained in this experiment is $A=(0.44\pm0.60)$.

[15]The decay $\eta\to\pi^0\pi^0\gamma$ has then no parity-violating effects to order G and its rate is proportional to G^2.

[16]Inner brehmsstrahlung radiation can occur only if there is a CP-violating transition $\eta\to\pi^+\pi^-$.

[17]By A-type axial-vector meson we denote the octet or nonet whose $T=1$ member is the A_1 meson with $I^G(J^P)C_n=1^-(1^+)(+)$, and by B-type axial-vector meson we denote the octet (or nonet) whose $T=1$ member is the B meson with $I^G(J^P)C_n=1^+(1^+)(-)$.

[18]The inclusion of only pseudoscalar, vector, and axial-vector (of both kinds) mesons in the Lagrangian is an approximation in itself. These particles are the minimum set necessary to allow for the possibility of both even- and odd-isospin parts of the weak Hamiltonian to contribute. In a more elaborate treatment one could also include the effects of tensor mesons ($J^P=2^+$), which seem to be natural candidates in view of the final D-wave state of the pions. Hopefully, their neglect does not alter appreciably our numerical estimates. In any case, an attempt to include their contribution is not feasible at present, in view of lack of knowledge concerning their couplings to vector and

axial-vector mesons.

[19]See, e.g., B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1968).

[20]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[21]A similar approach has been used recently to describe a class of charged and neutral K-meson weak radiative decays by M. Moshe and P. Singer, Phys. Rev. Lett. 27, 1685 (1971); Phys. Rev. D 6, 1379 (1972); and R. F. Sarraga and H. J. Munczek, *ibid.* 4, 2884 (1971).

[22]L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Lett. 21, 707 (1968); P. Singer, Phys. Rev. D 1, 86 (1970).

[23]R. Gatto and L. Maiani, Phys. Lett. 26B, 95 (1967).

[24]Experimentally, $B\to\phi\pi/B\to$ all is indeed less than 1.5%. (See Ref. 25.)

[25]Particle Data Group, Rev. Mod. Phys. 45, 51 (1973).

[26]In general, a vector-current (V_ν)–axial-vector-current (A_μ)–pseudoscalar-particle vertex can be written as $T_{\mu\nu}=i\int d^4x\, e^{-ipx}\theta(x_0)\langle P(l)|[A_\mu(x),V_\mu(0)]|0\rangle \equiv B_1 g_{\mu\nu}+B_2 q_\mu q_\nu+B_3 p_\mu p_\nu+B_4 q_\mu p_\nu+B_5 p_\mu q_\nu$, with $l=p-q$. When all particles are on the mass shell, only B_1 and B_4 contribute and are determined from the rate and the S/D ratio in the decay $B(A)\to VP$. It would appear therefore that in our calculation we neglect several unknown form factors. At a closer look one finds, however, that the contribution of B_3 and B_5 vanishes in the radiative process calculated here (due to gauge invariance). Moreover, if B_2 is not anomalously large and has no strong off-mass-shell dependence, the contribution of the second term is of order $m_P{}^2/m_B{}^2$ compared to that of the other terms. Thus, in using the $B(A)VP$ vertices of Eqs. (3.3) and (3.4), our approximation consists mainly only in neglecting the (hopefully mild) variation of B_1 and B_4 off the mass shell. In addition, for $A_{1\rho\pi}$ these assertions can be confirmed by the explicit off-mass-shell dependence of the form factors in existing models (Ref. 27).

[27]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); S. G. Brown and G. B. West, *ibid.* 180, 1613 (1969).

[28]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. 8, 261 (1962); L. M. Brown and P. Singer, *ibid.* 8, 460 (1962); 16, 424E (1966).

[29]F. Crawford and L. Price, Phys. Rev. Lett. 16, 333 (1966); P. J. Litchfield *et al.*, Phys. Lett. 24B, 486 (1967); A. M. Cnops *et al.*, *ibid.* 26B, 398 (1968); J. J. Thaler, Columbia University Thesis, Nevis-194, 1972 (unpublished); J. G. Layter *et al.*, Phys. Rev. D 7, 2565 (1973); 7, 2569 (1973).

[30]D. Benaksas *et al.*, Phys. Lett. 39B, 289 (1972); 42B, 507 (1972).

[31]G. Ascoli *et al.*, Phys. Rev. Lett. 20, 1411 (1968); A. Werbrouck *et al.*, Nuovo Cimento Lett. 6, 1267 (1970); M. Afzal *et al.*, DESY Report No. 72/63 (unpublished); R. L. Ott, Berkeley thesis, LBL-1547 (unpublished).

[32]P. Shen and K. Kang, Phys. Rev. D 7, 269 (1973); F. J. Gilman and M. Kugler, Phys. Rev. Lett. 30, 518 (1973); G. Fourez, Phys. Rev. 178, 2454 (1969). In the first of these papers a value of $m_B{}^2h_B/g_B\simeq 2.5$ is obtained, while in the second paper the value of 1.4, corresponding to $g_L^B\simeq 0$, is suggested.

[33]J. Crennell *et al.*, Phys. Rev. Lett. 24, 781 (1970); G. Rinauds *et al.*, Nuovo Cimento 5A, 239 (1971); J. W. Lamsa *et al.*, Nucl. Phys. B41, 388 (1972).

[34]J. Ballam *et al.*, Phys. Rev. D 1, 94 (1970).

[35]B. H. J. McKellar and P. Pick, Phys. Rev. D 7, 260 (1973).

[36]For example, B. d'Espagnat, Phys. Lett. 1, 209 (1963) [see also R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, in *The Eightfold Way,* edited by M. Gell-Mann and Y. Ne'eman (Benjamin, New York, 1964), p. 254]; G. Segrè, Phys. Rev. 173, 1730 (1968); T. D. Lee, *ibid.* 171, 1731 (1968); R. J. Oakes, Phys. Rev. Lett. 20, 1539 (1968). The suppression of the isovector term by the factor $\sin^2\theta$ is absent also in several unified gauge theories of the weak and electromagnetic interactions. [cf. D. Bailin, A. Love, and D. V. Nanopoulos, Rutherford Laboratory report, 1973 (unpublished).

[37]R. J. Blin-Stoyle and P. Herczeg, Phys. Lett. 23, 376 (1966).