Can We Measure Parton-Parton Cross Sections?*

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Multiple-core production in hadron-hadron collisions directly measures (in the context of the parton model) differential cross sections for parton-parton collisions. To obtain ~20% accuracy in reconstruction of the parton-parton kinematics requires transverse momentum of the cores $\gtrsim$ 5 GeV/c. Experiments at NAL as well as the CERN ISR appear feasible. What might be learned from such studies is discussed.

I. INTRODUCTION

Observation at the CERN Intersecting Storage Rings (ISR) of a large yield of high-transverse-momentum hadrons[1,2,3] invites many speculations regarding the production mechanism. Among the many mechanisms suggested, the parton model[4] provides a simple intuitive way of thinking about the process and, most importantly, suggests other experimental ways of investigating this kind of dynamics. It is not at all clear that even the present data, especially those[3] on the increasing K/π and p/π ratio with increasing $p_\perp$, can be accommodated by the simple parton picture (Appendix C contains the basis for these doubts). Nevertheless, we shall in this paper uncritically accept the simple parton model. Our apology is simply that the main purpose of this paper, to motivate the usefulness of multiple-core measurements, is to a large extent independent of model, but very simply expressed in parton terms.

Thus we shall suppose that an event containing high-transverse-momentum secondary hadrons is the consequence of a large-angle two-body[5] collision of two partons present in the incident hadrons; these partons then "fragment" into high-transverse-momentum secondary hadrons. This general picture has been described in a previous paper,[6] hereafter called BBK. There the emphasis rested on the electromagnetic interactions of the partons. However, it appears from the data that for the foreseeable future the electromagnetic contribution is overwhelmed by purely strong-interaction processes. This possibility was already entertained in BBK, and crude estimates of single-particle inclusive distributions were made, assuming that partons exchange $J=1$ neutral gluons ($g^2/4\pi \lesssim 1$) as well as photons ($e^2/4\pi \simeq \frac{1}{137}$). Fits to the data using such a model have even been tried[7] and are not too bad, but scarcely convincing. A testable prediction is a scaling behavior given by dimensional analysis,

$$E\frac{d\sigma}{d^3p} = \frac{\text{constant}}{p_\perp{}^4} F\left(\frac{2p}{\sqrt{s}}, \theta_{\text{c.m.}}\right)$$

$$= \frac{\text{constant}}{s^2} f\left(\frac{2p}{\sqrt{s}}, \theta_{\text{c.m.}}\right). \tag{1.1}$$

Experimentally the exponent of s appears to be closer to 3 than 2,[8] but it may be premature to draw definite conclusions. Another possible problem for parton models may lie in the aforementioned particle ratios, as discussed in Appendix C. However, parton-model explanations of these inclusive spectra will be capable of great resiliency under experimental stress. There are three major uncertainties, associated with each major step in the simple parton-model calculation. We recall the three elements of the recipe here, along with the problems they present:

I. Choose a collinear frame of reference in which the initial projectiles move relativistically and in opposite directions. Replace each projectile A, B by a beam of massless, noninteracting partons $\{i, j\}$ whose momentum distributions scale:

$$\frac{dN_i^A}{dp} = \frac{1}{p}f_{iA}\left(\frac{p}{P}\right), \tag{1.2}$$

where P is the projectile momentum and p the parton longitudinal momentum.

The uncertainty here is that we do not know even the composition of the parton beams. The charged-parton composition and momentum distributions are in principle determined from electroproduction and neutrino-production experiments. However, about 50% of the proton momentum is not carried by charged partons.[9] The remaining momentum is often speculated to be carried by neutral isoscalar $J=0$ or $J=1$ gluons, with unknown momentum distribution functions.

II. Regard the collision as a 2-body collision of a parton from each beam, the cross section depending only on s' and t' of the interacting parton pair and independent of the rest of the environment

of "spectator" partons.

This uncertainty is the worst: We do not know the strong parton-parton interactions. If the interactions involve no dimensional coupling constants or large masses, then the scaling behavior of Eq. (1.1) may be expected.[6] But even in the absence of a large mass parameter, trilinear couplings of $J = 0$ gluons do introduce a dimension. Violation of dimensional scaling could be accounted for in that way.[10] Also, the parton interchange model of Blankenbecler, Brodsky, and Gunion[11] reminds one that mechanisms other than direct parton-parton collisions should also contribute; their model violates strongly the dimensional scaling in Eq. (1).

III. The momentum of each parton emerging after the high-transverse-momentum collision is approximately equal to the sum of the momenta of the hadrons emerging in the direction of the struck parton. While the transverse momentum of these hadrons relative to the initial beams is large, relative to the direction of the parent parton, their transverse momentum is small. The inclusive distribution of these hadrons scales in the same way as Eq. (1.1):

$$\frac{dN_{Ai}}{dP'} = \frac{1}{P'} g_{Ai}\left(\frac{P'}{p'}\right) , \qquad (1.3)$$

where P' is hadron momentum and p' is parton momentum.

The uncertainty associated with this final step is that we do not know the nature of this "parton fragmentation" into hadrons. The above picture, as described in detail in BBK and elsewhere,[12,13] is at present not established. It is true that data from e^+-e^- colliding beams, electroproduction, and neutrino-induced production can tell us about "fragmentation" of charged partons. However, those data yield no information about what the neutral partons do.

How can we extricate ourselves from such a mess? One way is to abandon the parton-model description for these collisions, for which there is (apart from lepton-induced phenomena) little real support. But it is probable that almost any model will be faced with similar uncertainties. Hence it may be that we shall have to explore much more than single-particle or two-particle inclusive distribution functions to make real headway. In particular, experiments which study multiple-core structures appear to be an especially attractive way to circumvent the uncertainties in Step III above, and to attack Step II at the experimental level by directly *measuring* the strong parton-parton interactions. It is the main purpose of this paper to examine this possibility and outline what might be learned from such studies. While this is done here in the context of the parton model, it should become clear that such measurements are likely to yield fundamental information for a wide variety of dynamical models.

II. A MULTIPLE-CORE EXPERIMENT

An idealized multiple-core experiment consists of a bank of hadron calorimeters or other devices surrounding the interaction region, capable of measuring in each event the total amount of energy emerging into small elements of solid angle.[14] The model we have described predicts that the energy will be localized into four cores for ISR center-of-mass kinematics, or three cores for stationary target kinematics (Fig. 1). The axes of the cores point along the directions of the struck partons, and the angular size of a core of total energy E should be of order $\Delta\theta \sim 0.5$ GeV/E. Evidently it is of central importance to verify the existence, coplanarity, etc, of these cores. But here we shall anticipate that this will turn out to be correct. For even setting parton interpretations aside, any other choice of distribution for the outgoing energy (fan-shaped, nonplanar, isotropic, etc.) seems even more unprecedented than one composed of cores.

The purpose of studying the production of these cores is that the four-momentum of a core is approximately the same as the four-momentum of the parent parton. (The uncertainty is of order 0.5 GeV/$p_\perp$ for a core with transverse momentum $p_\perp$, as we shall discuss later.) Thus a measurement of the angle and energy distribution of the cores is essentially a measurement of $d\sigma/dt$ for parton-parton scattering. A simple calculation
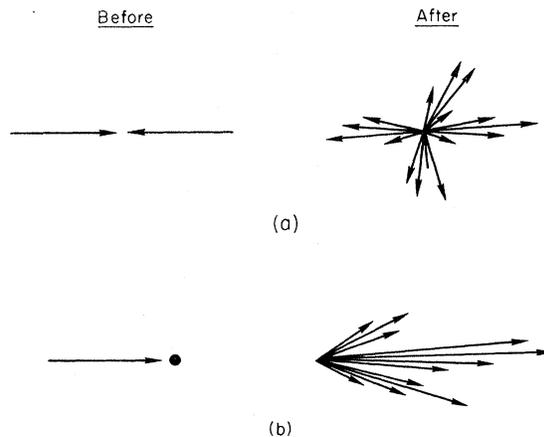

FIG. 1. (a) Schematic structure of a multiple-core event, as seen in the center-of-mass frame. (b) Schematic structure of a multiple-core event, as seen for stationary-target kinematics (conventional laboratory frame).

using the naive parton model yields the formula

$$E_1 E_2 \frac{d\sigma}{d^3 p_1 d^3 p_2} \approx \frac{1}{\pi} \sum_{i,j} f_{iA}(x_1) f_{jB}(x_2) \frac{d\sigma_{ij}(s', t')}{dt'}$$
$$\times \delta^2(\vec{p}_{1\perp} + \vec{p}_{2\perp}). \qquad (2.1)$$

In this formula $(E_1, \vec{p}_1)$ and $(E_2, \vec{p}_2)$ are the energy and momenta of the cores.[15] The functions $f_{iA}(x_1)$ and $f_{jB}(x_2)$ are defined in Eq. (1.2). The kinematical variables s' and t' describing the parton-parton collision are defined by the formula

$$s' \cong p_\perp^2 (1 + \tan\tfrac{1}{2}\theta_1 \cot\tfrac{1}{2}\theta_2)(1 + \tan\tfrac{1}{2}\theta_2 \cot\tfrac{1}{2}\theta_1),$$
$$(2.2)$$

$$t' \cong -p_\perp^2 (1 + \tan\tfrac{1}{2}\theta_1 \cot\tfrac{1}{2}\theta_2),$$

and the longitudinal fractions x_1 and x_2 of the incident partons are given by

$$x_1 \cong \frac{p_\perp}{\sqrt{s}} (\cot\tfrac{1}{2}\theta_1^* + \cot\tfrac{1}{2}\theta_2^*),$$

$$x_2 = \frac{p_\perp}{\sqrt{s}} (\tan\tfrac{1}{2}\theta_1^* + \tan\tfrac{1}{2}\theta_2^*)$$

(center-of-mass kinematics),

$$(2.3)$$

$$x_1 \approx \frac{M p_\perp}{s} (\cot\tfrac{1}{2}\theta_1 + \cot\tfrac{1}{2}\theta_2),$$

$$x_2 \approx \frac{p_\perp}{M} (\tan\tfrac{1}{2}\theta_1 + \tan\tfrac{1}{2}\theta_2)$$

(stationary-target kinematics).

Some care must be taken in doing the kinematics for this process, and the above equations are expecially convenient in that regard. In particular, Eq. (2.2) is invariant under longitudinal Lorentz transformation and is equally valid in the center-of-mass frame or in the frame in which one of the initial hadrons is at rest.

The reason for such care in calculating the kinematics is that while the parton four-momentum is assumed to be null, i.e., $p^2 \approx 0$, and while the momentum P^μ of the jet of hadrons is taken to be approximately that of the parton, it does not follow that $P^2 \cong 0$. Indeed $P^2 \gtrsim O(P_\perp)$ because of the small, but unavoidable, uncertainty in determining P^μ. The optimum theoretical accuracy in measuring P^μ is found by going to the frame in which the jet emerges at right angles to the incident beams. There we have

$$P^\mu = p^\mu + [O(0.5 \text{ GeV})]^\mu. \qquad (2.4)$$

This follows because only hadrons in the jet which emerge with $|\vec{p}| = |\vec{p}_\perp| \gg 350$ MeV may be safely identified as a "parton fragment," and hadrons

with $|\vec{p}| \lesssim 350$ MeV may, with high probability, belong to the jets oriented along the incident beams.[16] These wee hadrons cannot, even in principle, be identified as a constituent of any one jet. Taking the vector $[O(0.5 \text{ GeV})]^\mu$ to be timelike and ≈ 0.5 GeV, we estimate

$$P^2 \approx (1 \text{ GeV}) \times |\vec{P}_\perp|. \qquad (2.5)$$

Thus as $P_\perp \to \infty$, also $P^2 \to \infty$. It definitely does not vanish. The moral is that from the measurement of the jets one must first reconstruct the parton null four-vector p_μ (within the uncertainties inherent in the reconstruction, of order 0.5 GeV/$p_\perp$) and only then do the kinematics. There may be another more fundamental lesson to be learned as well; this is discussed in Appendix A.

III. EXPERIMENTAL UNCERTAINTIES

At this point it is evident that there are *in principle* difficulties and uncertainties in determining in a given event the important quantities x_1, x_2, s', t' for the parton-parton collision. In practice, in a given event one observes a localization of energy within an angle $\Delta\theta$ of the parton direction and (after perhaps some small background subtraction) one estimates the parton energy in terms of the hadron energy observed within $\Delta\theta$. It is clearly of importance to ascertain what fraction of the parton momentum is found, on the average, in those hadrons. We may estimate it as follows: If the hadron fragments are distributed longitudinally according to Eq. (1.3), and if their transverse-momentum distribution is of typical exponential form, then

$$\sum_A E' \frac{dN_{iA}}{dP' dP_\perp^2} = \tfrac{1}{2} a^2 e^{-aP_\perp} g_i\left(\frac{P'}{p}\right). \qquad (3.1)$$

It follows that the mean fraction $\epsilon_i(E, \theta)$ of parton energy E found in the hadrons emerging within an angle θ of the parton direction is an integral over Eq. (3.1):

$$\epsilon_i(E, \theta) = \int_0^1 dx \, g_i(x)[1 - (1 + aEx\theta)e^{-aEx\theta}]. \quad (3.2)$$

For the "reasonable"[6],[17] choices $g_i(x) = 2(1 - x)$ and $a = 6$ GeV^{-1}, Eq. (3.2) is plotted in Fig. 2. In general, for large θ ($aE\theta \gg 1$)

$$\epsilon(\theta) \approx 1 - \frac{2g(0)}{aE\theta} \approx 1 - \frac{0.7 \text{ GeV}}{E\theta}. \qquad (3.3)$$

The approach to unity is quite slow. In particular, the "missing energy" $E[1 - \epsilon(E, \theta)]$ emerging in the large angle ($>\theta$) hadrons is independent of parton energy, and for $\theta \sim 30°$ (~ 1 sr of solid angle) is of order 1 GeV. This result is consistent with and supportive of the comments made earlier. In order to obtain a determination of parton momen-

tum good to ~20%, it therefore appears that a par-
ton transverse momentum $\gtrsim 5$ GeV is required. A
20% accuracy is perhaps a good match to the ac-
curacy of energy determination obtainable in cal-
orimeters.

However, this discussion of accuracy and back-
grounds, etc., is full of a theorist's naiveté. Fluc-
tuations of individual events away from the mean
behavior, combined with the steep fall in event
rate with increasing $p_\perp$, may create a serious
problem.[18] An investigation of this may well re-
quire Monte Carlo simulations of individual mul-
tiple-core events. Toward this end, a sensible
way to proceed might be to suppose that the hadron
fragments associated with a parton of momentum
p are identical to the products of a γ-nucleon or
π-nucleon inelastic collision at laboratory momen-
tum p. This way of proceeding has some support
from the deep-inelastic electroproduction experi-
ments with hadron final states observed.[19] The
energetic electroproduced hadrons (at small ω)
may be regarded as "parton fragments." But the
behavior of these hadron final states appears to be
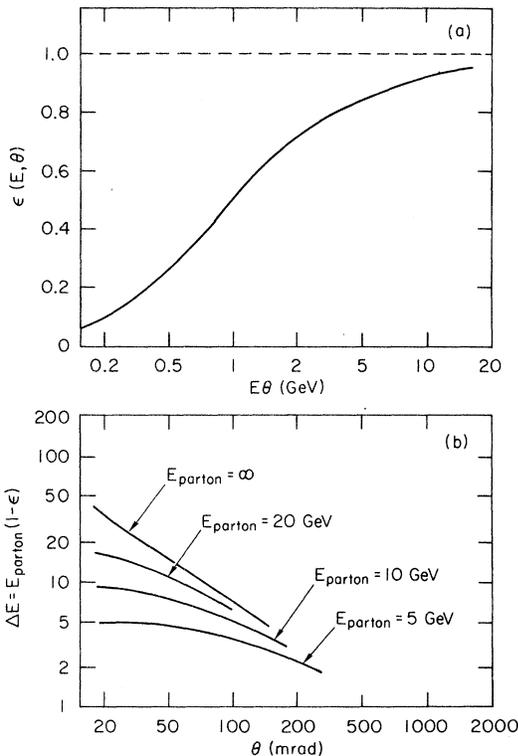very similar to those in photoproduction (or π-N)

FIG. 2. (a) Fraction of parton energy E emerging in
hadrons at angle $\leq \theta$ with respect to the parton direction.
(b) Missing energy ΔE associated with hadrons emitted
at angle greater than θ with respect to the direction of
the parent parton.

interactions at the same hadron center-of-mass
energy.

IV. EXTRACTION OF PHYSICS FROM MULTIPLE-CORE MEASUREMENTS

What can we hope to learn from multiple-core
measurements? Clearly the advantage over inclu-
sive studies lies in the elimination of uncertainty
regarding Step III and reduction of the problems in
Step II to the direct experimental study of the par-
ton-parton cross section $d\sigma/dt'$ as a function of
s' and t'. The uncertainty of Step I regarding the
structure of the incident parton beams is the main
obstacle. However, with some luck some of this
can be overcome. From Eq. (2.1) we see that the
quantity

$$\sum_{i,j=1}^{N} f_{iA}(x_1) f_{jB}(x_2) \frac{d\sigma_{ij}(s',t')}{dt'} \equiv \vec{f}_B^T(x_2) F(s',t') \vec{f}_A(x_1)$$

(4.1)

is what can be observed. If x_1 and x_2 are not too
small (>0.3?), a simple and reasonable hypothesis
is that the important partons consist of u and d
quarks and a single gluon g. Furthermore, if we
neglect the (probably small) difference between
uu and ud cross sections, we may average over
u and d distribution functions and obtain an effective
dimensionality N of 2 for the vector space spanned
by the vectors $\vec{f}_{A,B}(x)$. It then seems not impossible
to extract from data at fixed s' and t', but variable
x_1 and x_2, the 2-vectors $\vec{f}_A(x_1)$ and $\vec{f}_B(x_2)$, where

$$\vec{f}_A(x) = \begin{pmatrix} f_{qA}(x) \\ f_{gA}(x) \end{pmatrix}$$

$$= \begin{pmatrix} f_{uA}(x) + f_{dA}(x) \\ f_{gA}(x) \end{pmatrix}.$$

(4.2)

Of considerable help is the fact that $f_q(x)$ is already
reasonably well determined at large x from electro-
production data,[20] and the energy-conservation con-
straint

$$\sum_{i=1}^{n} f_{iA}(x) dx = 1$$

(4.3)

serves to normalize the distribution function $f_{gA}(x)$.
Furthermore the hypothesis of equality of uu and
ud interactions may be checked (at NAL) by com-
paring pp and pn interactions.

Such a simple working hypothesis as the above
model might well not survive an onslaught of data.
However, as new partons are added to the descrip-
tion their effect can be checked (again at NAL) by
changing projectiles. For example, if $\bar{u}$ and $\bar{d}$ are
needed to interpret data on pp collisions, then they
should be of much greater importance in $\bar{p}p$ col-
lisions or πp collisions. Likewise, strange-quark
interactions may be isolated by studying $K^\pm p$ inter-

actions and comparing them with $\pi^\pm p$. Evidently an empirical approach is indicated here, with the data guiding the theoretical considerations along.

The expected rate for multiple-core events is evidently very good at the CERN ISR. It is still quite acceptable for high-energy secondary beams at NAL. To make this estimate, we first suppose that the inclusive distribution of partons (or cores) at ISR energies falls roughly as a power:

$$E \frac{dN_{\text{parton}}}{d^3P} \bigg|_{\theta_{\text{c.m.}}=90°} \approx \frac{C}{P^n} , \tag{4.4}$$

with $n \sim 8$ for the momentum range (3–9 GeV) of interest. We take the inclusive distribution function for finding a π^0 of momentum p emerging from this parton to be

$$p \frac{dN_{\pi^0}}{dp} \approx \frac{2}{3}\left(1 - \frac{p}{P}\right) \equiv g_{\pi^0}\left(\frac{p}{P}\right) . \tag{4.5}$$

Folding this over Eq. (4.4) gives

$$E \frac{dN_{\pi^0}}{d^3p} \bigg|_{\theta_{\text{c.m.}}=90°} \cong \frac{C}{p^8} \int_0^1 dx\, x^{n-3} g_{\pi^0}(x) \tag{4.6}$$

or

$$E \frac{dN_{\pi^0}}{d^3p} \cong \frac{2}{3(n-1)(n-2)}\left(E \frac{dN_{\text{parton}}}{d^3p}\right) . \tag{4.7}$$

That is, for $n \approx 8$, the ratio of π^0 daughters to parent partons of the same momentum is $\sim \frac{1}{63}$. From the observed π^0 spectrum,[1] and integrating over all $p_\perp > 5$ GeV, we get

$$\frac{d\sigma_{\text{parton}}}{d\Omega}(p_\perp > 5 \text{ GeV};\ \theta_{\text{c.m.}} = 90°) \approx 6 \times 10^{-32} \text{ cm}^2/\text{sr}. \tag{4.8}$$

Finally, taking half the full solid angle ($60° < \theta_{\text{c.m.}} < 120°$) gives a total

$$\sigma^{\text{effective}}(p_\perp > 5 \text{ GeV};\ |\cos\theta^*| \leq \tfrac{1}{2}) \approx 3 \times 10^{-31} \text{ cm}^2 \tag{4.9}$$

at ISR energies. At NAL, one needs $E_{\text{incident}} \gtrsim 200$ GeV to comfortably attain $p_\perp > 5$ GeV. Under these circumstances we have to require $x_1 \gtrsim 0.5$ and $x_2 \gtrsim 0.5$, and the factors $f_A(x_1)f_B(x_2)$ could suppress the above estimate by a factor 10–100. However, the distribution function $f_{qM}(x)$ for a meson projectile M is anticipated[6,17,21] to behave as $(1 - x)$, not $(1 - x)^3$, for large x, giving perhaps considerably less suppression than for a baryon projectile. But even the conservative estimate $\sigma \gtrsim 3 \times 10^{-32}$ cm^2 is still sufficiently large to make an experiment very feasible.

V. CONCLUSIONS

Multiple-core experiments appear to be a feasible and attractive way to bypass some of the dif- ficulties of parton-model interpretations of inclusive hadron spectra at high transverse momentum. We believe that these experiments have an intrinsically fundamental flavor, and will likewise be useful if some other model of high-$p_\perp$ phenomena turns out to better describe the data. For example, in the Blankenbecler, Brodsky, Gunion parton-interchange model,[11] the multiple-core experiment appears to measure directly the square of a single two-body parton-parton wave function at large relative $p_\perp$. (However, we have not succeeded in determining the precise quantity measured, but hope to return to this question at some later time.)

Finally, one must not forget that these measurements are an excellent way to search for any heavy objects ($m \gtrsim 10$ GeV) which are produced (not necessarily through strong interactions) in parton-parton collisions. The prototype is the $W^\pm$, decaying into only hadrons via an intermediary parton-antiparton pair. This would show up as a narrow s-channel resonance in $d\sigma/dt'(s', t')$. Similarly associated production of two massive objects each decaying into hadrons via parton-antiparton pairs would be seen as production of 4 cores of high $p_\perp$ and high relative $p_\perp$. And, of course, leptons (including neutrinos) are special kinds of partons and can be included in the game.[22]

Indeed, in looking toward that future day when center-of-mass energy is measured in many hundreds of GeV, the separation of individual channels or even individual hadrons in pp collisions will diminish in importance, both because of difficulties in experimental resolution and of decrease in intrinsic interest. Conversely the study of the production of groups of hadrons representing individual partons greatly increases in importance, certainly for weak and electromagnetic interactions and probably for strong interactions as well. For $p_\perp \gtrsim 50$ GeV, the 1% accuracy of parton kinematics compares well with the accuracy of present day hadron-hadron two-body kinematics. Thus the physics as $s \sim 1-10$ TeV^2 at the parton level might well be approached in a way much like the physics of hadrons at $s \sim 1-10$ GeV^2. Were such dreams to come true, the parton, while impossible to observe in isolation, would in an operational sense attain a reality comparable to what is ascribed to hadrons. Whether that would make them more real in a fundamental sense is a question best left to philosophers.

specifically, the NAL proposal of the Harvard-Penn-Wisconsin collaboration[23] and of Frisch et al.,[24] as well as the ideas of W. Willis,[15] cover much of the same ground as this paper. I have benefited very much from discussions with D. Ritson, D. Cline, R. Cool, H. Frisch, B. Winstein, W. Willis, W. Selove, A. Erwin, and F. Turkot, as well as my theoretical colleagues at SLAC and NAL. In particular I thank S. Ellis for interesting discussions; his paper[7] with M. Kisslinger also explores many of the issues discussed here in considerable detail.

APPENDIX A: IS THE PARTON MASS ZERO OR INFINITE?

As we discussed in Sec. II, the identification of the momentum $\sum_i p_i^\mu$ of a group of high-$p_\perp$ hadrons with the parton momentum raises the point that, while the parton four-momentum is generally considered (and in this paper specifically assumed) to be null, $(\sum p_i^\mu)^2$ is certainly not zero, and in fact is $\gtrsim |\vec{p}_\perp|$. We demonstrated that this feature does not stop one from identifying the parton four-momentum up to a fractional uncertainty of order $(1\ \mathrm{GeV})/p_\perp$.

This situation invites consideration of the same phenomenon at the parton level itself. In other words we may ask, in what sense is the parton four-momentum really null? The answer may well be similar: *No matter how large the momentum p that a parton carries, it emits and reabsorbs wee partons at a rate independent of its momentum. Therefore it is typically off energy shell by an a amount $O(1)$ and off mass shell by an amount $O(p)$, even as $p \to \infty$.*

The presence of these interactions involving wee partons appears desirable in order to cope with the problems raised by Kogut, Sinclair, and Susskind[25] in connection with the process $e^+e^- \to$ hadrons. In that process one expects first that the massive virtual photon materializes into an energetic parton-antiparton pair which subsequently evolves into hadrons. However, if the partons possess fractional charge (as suggested by electroproduction and neutrino data), the energetic partons must do something in a time scale $O(1)$ or else find themselves isolated at distances large compared with 10^{-13} cm. The emission of wee partons can create enough spoor of parton-antigluons pairs along their outgoing trajectories to allow a polarization current to flow, neutralizing the fractional charge.

On the other hand, for electroproduction (viewed in the laboratory frame) the distances for the free parton propagation can be large ($\approx 2\omega \times 10^{-14}$ cm). However, the most favored assumption (epitomized in Fig. 3) is that of free parton propagation over such distances, which in the light of the above
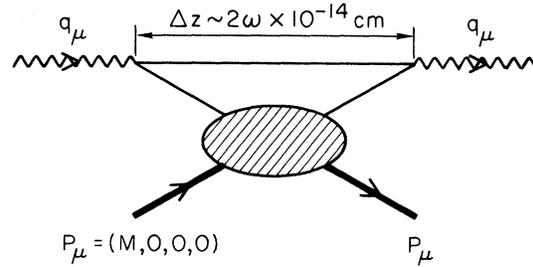

FIG. 3. "Handbag" diagram often used to interpret scaling behavior in deep-inelastic electroproduction.

comments becomes suspect. The problem is how to allow emission of wee partons without destroying the impulse approximation and scaling behavior at large ω. Perhaps $I = 0$ wee vector-gluon emission, which only puts an eikonal phase on the wave function of the energetic parton, is in the right direction. However, I certainly do not claim a clear understanding of these problems, and raise this issue here mainly to underline, first, that parton propagation may well be more subtle than what is illustrated in Fig. 3, and second, that the more mundane problem of the empirical identification of the null four-momentum of a parton involves a closely analogous situation.[26]

APPENDIX B: ANGULAR CORRELATIONS

Of current experimental interest at the CERN ISR is the determination of the angular correlation
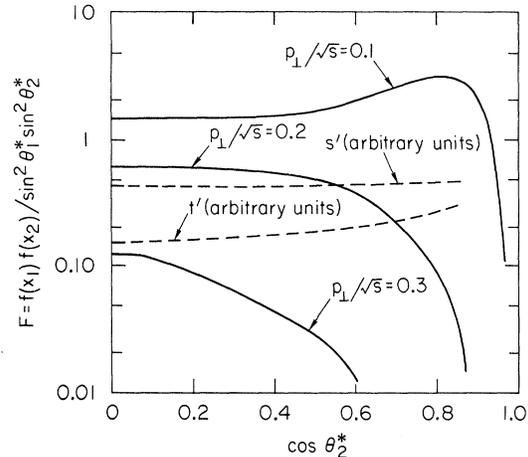

FIG. 4. The factor

$$F(\theta_1^*, \theta_2^*, p_\perp/\sqrt{s}) = f(x_1)f(x_2)/\sin^2\theta_1^*\sin^2\theta_2^*$$

which controls the angular distribution and correlation of high-$p_\perp$ cores here plotted for $\theta_1^* = 90°$ and $p_\perp/\sqrt{s}$ =0.1, 0.2, and 0.3 as a function of $\cos\theta_2^*$. Also shown are s' and t' (arbitrary units) as a function of $\cos\theta_2^*$ for $\theta_1^* = 90°$. Notice the insensitivity of s' and t' with angle. We have chosen $f(x) = 2\nu W_2^{ed}(x)$; hence gluon contributions are neglected here.
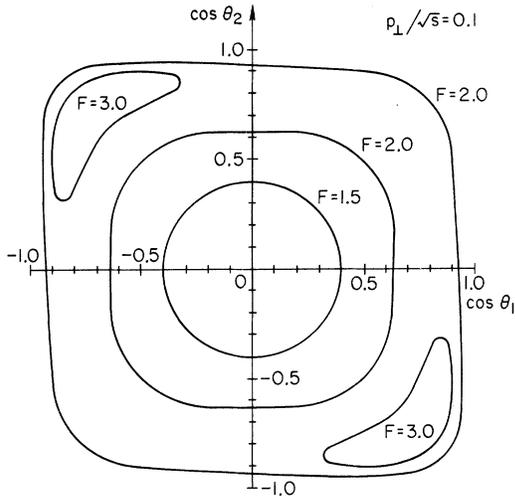
FIG. 5. Contour plot of the function

$$F = f(x_1) f(x_2)/\sin^2\theta_1 \sin^2\theta_2$$

as a function of $\cos\theta_1$ and $\cos\theta_2$ for $p_\perp/\sqrt{s} = 0.1$.

FIG. 7. Contour plot of F, as in Fig. 5, for $p_\perp/\sqrt{s} = 0.3$.

of two high-$p_\perp$ cores. Equation (2.1) may be applied to this question. We rewrite it as follows:

$$\frac{d\sigma_{AB}}{dp_\perp^2(\cos\theta_1)d(\cos\theta_2)} = \sum_{ij} \frac{f_{iA}(x_1)f_{jB}(x_2)}{\sin^2\theta_1 \sin^2\theta_2} \frac{d\sigma_{ij}}{dt'}(s', t').$$

$$(B1)$$

For purposes of estimation, we may take $f_{iA}(x)$ and $f_{jB}(x)$ to be equal to the structure function νW_2 as measured in deep-inelastic electroproduction. The main variation in the yield comes from the
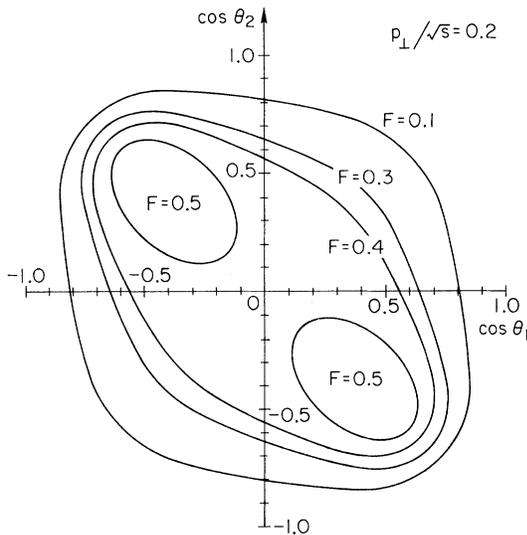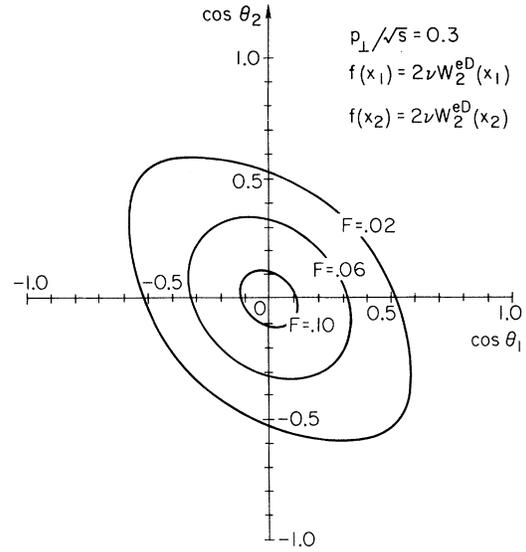
factor $f(x_1)f(x_2)/\sin^2\theta_2$, which is plotted in Figs. 4–8 for typical choices of $p_\perp/\sqrt{s}$ and θ_1. We have also plotted s' and t', which are not strong functions of $\cos\theta_2$. Thus the angular dependence is not critically dependent on the s' and t' dependence of the parton-parton cross section.

In any ISR experiment which triggers on a single high-$p_\perp$ secondary (which we here consider a constituent of produced parton 1), that particle will on the average carry a major fraction (~70–80%?)

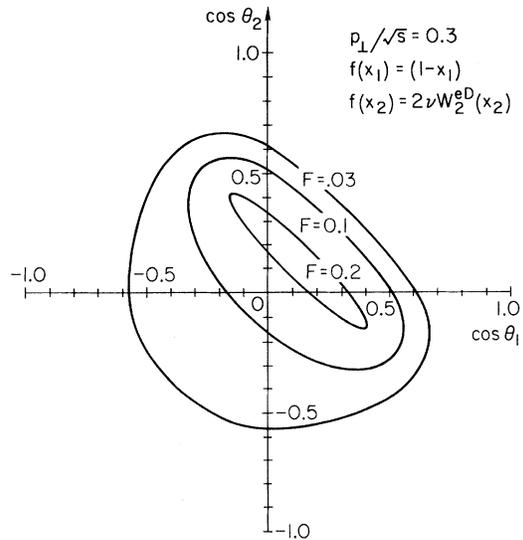FIG. 6. Contour plot of F, as in Fig. 5, for $p_\perp/\sqrt{s} = 0.2$.

FIG. 8. Contour plot of F, as in Fig. 5, for $p_\perp/\sqrt{s} = 0.3$ and $f(x_1)$ replaced by a conjectured "mesonic" structure function: $f(x_1) = (1 - x_1)$.

of the total parton, or jet, momentum [this follows from Eq. (4.6)]. Thus the quantities $p_{\perp}/\sqrt{s}$ and θ_1 for parton 1 are reasonably well determined. *Any energetic particle of high $p_{\perp}$ detected in the opposite hemisphere should point approximately along the direction θ_2 (and of course approximately coplanar with particle 1). High-$p_{\perp}$ particles produced on the same side of the intersecting beams should have low ($\lesssim 0.5$ GeV) relative transverse momentum, inasmuch as they belong in the same parton jet.

APPENDIX C: PARTICLE RATIOS

In the text we intimated that measurements of the inclusive hadron distributions alone can neither destroy nor establish the parton picture because of the large freedom in choice of partons, how they interact, and how they fragment. This view is too pessimistic. There is already possible trouble in sight. The ratios K/π, p/π^+, and $\bar{p}/\pi^-$ have been observed[3] to increase with increasing $p_{\perp}$, up to $p_{\perp} \sim 3$ GeV. If we regard hadrons with $p_{\perp} \sim 3$ GeV as *bona fide* parton fragments, this indicates that parton-fragmentation products in $e^+e^- \rightarrow$ hadrons or electroproduction should likewise be relatively rich in heavy particles. Were the ISR particle ratios to become extremely large, we would find such an embarrassingly large p/π^+ ratio in e^+e^- annihilation to make it unlikely that the model is correct.

To crudely estimate the status of this situation, let us again suppose that the inclusive distribution function of high-$p_{\perp}$, 90° partons, or cores, produced in hadron-hadron collisions at ISR energies ($\sqrt{s} \sim 50$ GeV) falls as a power of $p_{\perp}$, as given by Eq. (4.4). Following the arguments leading to Eq. (4.6), we obtain for a rough estimate

$$\frac{p}{\pi^+} = \frac{\int_0^1 dx\, x^{n-3}\bar{g}_p(x)}{\int_0^1 dx\, x^{n-3}\bar{g}_{\pi^+}(x)}, \tag{C1}$$

where the barred average is over the types of partons i produced in the p-p collision. It follows

that for some values of x (and most likely large values of x) the $\bar{p}/\pi^- = p/\pi^+$ ratio in e^+e^- annihilation must be as large as that observed at the ISR (roughly 50% at $p_{\perp} \sim 3 \pm 0.5$ GeV).

Kogut and I estimated[17] the $\bar{p}/\pi^-$ ratio for e^+e^- annihilation using "correspondence" arguments, and found a ratio smaller by over an order of magnitude. Would a much larger ratio be "unreasonable?" Guided mainly by the survival instinct we may try to revise upward the normalization of the proton inclusive distribution, given in Figure 16 of that paper. If we multiply it by a factor 20, and estimate the integrals from Eq. (C1) using $n = 8$, we obtain $p/\pi^+ \sim 20\%$. Considering the large amount of guesswork, this is perhaps satisfactory. But the p/π^+ ratio at small x in the colliding-beam process becomes as big as 50%. To rationalize this, one can appeal to the Feynman conjecture that the mean baryon number found in the parton-fragmentation region is of order $\frac{1}{3}$. Then such a large p/π ratio can be approached.

The whole situation looks on the one hand uncomfortable and on the other hand quite spectacular in its implications for e^+e^- annihilation. However, it would be premature to bury the parton-fragmentation hypothesis for hadron-hadron collisions at the present time. The easiest excuse is that, as in the BBK picture,[6,7] the inclusive hadron distributions at the ISR contain two components, the tail of the low-$p_{\perp}$ distribution and the high-$p_{\perp}$ parton fragments, and that $p_{\perp} \sim 3$ GeV is still controlled by the low-$p_{\perp}$ tail. Yet two points appear quite clear: First, the problem of p/π ratios and K/π ratios deserves most careful attention, both theoretically and experimentally. Second, if the p/π ratio continues to rise with increasing $p_{\perp}$, the picture put forward in this paper will become increasingly difficult to support.

Even were this to turn out to be the case, we still hold to the view that the multiple-core experiments will be of great value in yielding fundamental information on what is responsible for the high-$p_{\perp}$ events.

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