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SU(3) for Constituent and Current Quarks: How Different Are They?*

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Algebraic properties of the transformation between constituent- and current-quark bases are discussed in the case when SU(3) is broken for the current quarks. In a simple model, it is shown that $SU(4)_W$ may still be an exact symmetry in the constituent-quark basis. Still within the context of the model, the question of whether the transformation actually does distinguish two SU(3) algebras in nature, one for current and one for constituent quarks, is investigated. It is shown that the so-called σ terms of meson-baryon scattering provide a means of determining if there is such a distinction.

I. INTRODUCTION

The existence of two SU(6) algebras generated by "good charges" (those whose matrix elements at infinite momentum do not vanish) has been recognized for some time, although the distinction between them has not always been clearly drawn. The generators of $SU(6)_{W,\text{current}}$ are essentially the integrated weak and electromagnetic current densities and related operators.[1] The other algebra, $SU(6)_{W,\text{strong}}$, is an approximate symmetry of the strong-interaction Hamiltonian[2] and it is not known if its generators can be written as integrals of local operators. Although the two SU(6) algebras are isomorphic, they are not the same; however, they are closely related by the conserved-vector-current (CVC) hypothesis which identifies an SU(3) subalgebra in $SU(6)_{W,\text{current}}$ to one in $SU(6)_{W,\text{strong}}$. In general, hadron states at infinite momentum do not transform irreducibly under the group $SU(6)_{W,\text{current}}$ and the problem of discovering the ensuing representation mixing has been attacked[3] in the past with some success, although not in a systematic fashion. Some time ago Gell-Mann suggested[4] that the two algebras may be related by a unitary transformation, V. If so, then by finding such a V, one would solve the mixing problem. Less ambitiously, by determining some of the general structure of V, one would obtain some properties of the mixing that might be useful.

Following this suggestion, Melosh[5] has shown that in the free-quark model the two $SU(6)_W$ algebras may be related by a unitary transformation V_{free}. Assuming that the algebraic structure of the correct unitary transformation V is similar to that of V_{free}, one may make predictions for pionic decays of meson and baryon resonances,[6] recover many of the good results of the old $SU(6)_W$ scheme for matrix elements of weak charges, and correct some of the poor results (such as the prediction of $G_A/G_V=-\frac{5}{3}$).

A fundamental question in this context is how different can one expect the correct transformation V to be from the explicitly constructed model transformation V_{free}? For example, $V_{\text{free}}=\exp(iY_{\text{free}})$, where Y_{free} is bilinear in quark fields. In general, writing $V=\exp(iY)$, we might expect to find Y possessing higher-order terms in quark fields. Moreover, the extremely simple property of V_{free} that the transformed axial charge $V_{\text{free}}^{-1}Q_i^5V_{\text{free}}$ transforms as a sum of $(8,1)-(1,8)$ and $(3,\bar{3})-(\bar{3},3)$ representations of $SU(3)\times SU(3)_{\text{strong}}$ might not be generally true. It is surely astonishing that the series should terminate in but two terms.

We would like to point out here that (at least in a simple model) even if the generators of $SU(6)_{W,\,\mathrm{current}}$ are not conserved at the $SU(3)_{\mathrm{current}}$ level, one may still have an exactly conserved $SU(4)_{W,\,\mathrm{strong}}$ and a unitary transformation V which relates the algebras and which may be explicitly constructed so that its algebraic properties are easily read off. In other words, the SU(3) subalgebras of the two $SU(6)_W$ algebras which are identified by Melosh [since he assumes exact CVC for all eight SU(3) generators] are now split into two separate SU(3) algebras. The SU(2) generators commute with the Hamiltonian; the two SU(3) algebras now coincide only in their respective isotopic-spin SU(2) subalgebras. This discussion is presented in Sec. II; in Sec. III we examine the question of whether the transformation V actually does distinguish two SU(3) algebras in nature. This is done by studying the transformation properties of the scalar densities which occur in the context of the model used in Sec. II. We find that the so-called σ terms of meson-baryon scattering provide a means of determining if V does make the distinction mentioned above and also that they indicate the magnitude of this difference between the two SU(3) algebras.

II. BREAKING THE SYMMETRY

Consider the Hamiltonian

$$H=\int d^3x\, q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\partial}+\beta M)q(x)\,, \tag{1}$$

where $M=m(\lambda_0+c\lambda_8)$. This Hamiltonian has been studied by Gell-Mann, Oakes, and Renner[7] in the context of chiral SU(3)×SU(3) symmetry breaking, although the distinction between the two SU(3) ×SU(3) algebras ("currents" and "strong") was not made by them. Clearly H does not commute with $SU(3)_{\mathrm{current}}$ since there is a term proportional to c which transforms like an octet component of $(3,\bar{3})+(3,\bar{3})$. However, H is invariant under isotopic-spin rotation. The question now arises, how does H transform under $SU(3)_{\mathrm{strong}}$? The answer is that it depends on what the operators of $SU(3)_{\mathrm{strong}}$ are. The $SU(6)_{W,\,\mathrm{current}}$ generators are defined and satisfy the commutation relations of their algebra independent of the existence of any symmetry of the hadrons. What is the transformation V which gives the $SU(6)_{W,\,\mathrm{strong}}$ generators in terms of the others? As a guide we will assume that V should go over smoothly into the transformation V_{free} of Melosh in the limit $c\to 0$. This does not determine V uniquely. Nevertheless, we believe it interesting that there exists such a V which can be constructed so as to give a set of operators W^i_α which commute with H for $i=1,2,3$.

The construction proceeds as follows. First one defines the generators F^i_α of the $SU(6)_{W,\,\mathrm{current}}$ algebra:

$$\begin{aligned}F^i&=\int d^3x\, q^\dagger(x)\tfrac{1}{2}\lambda_i q(x)\,,\\ F^i_{x,y}&=\int d^3x\, q^\dagger(x)\beta\sigma_{x,y}\tfrac{1}{2}\lambda_i q(x)\,,\\ F^i_z&=\int d^3x\, q^\dagger(x)\sigma_z\tfrac{1}{2}\lambda_i q(x)\,.\end{aligned} \tag{2}$$

The $SU(6)_{W,\,\mathrm{strong}}$ generators W^i_α are then given by

$$W^i_\alpha=VF^i_\alpha V^{-1} \tag{3}$$

and the Hamiltonian is given above in Eq. (1). Write $V=\exp(iY)$. We assert that the operator

$$Y=\tfrac{1}{2}\int d^3x\, q^\dagger(x)\arctan\left(\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{M}\right)q(x) \tag{4}$$

"rotates away" the $\vec{\alpha}_\perp\cdot\vec{\partial}_\perp$ term in H; that is,

$$V^{-1}HV=\int d^3x\, q^\dagger(x)[-i\alpha_z\partial_z+\beta(M^2+\partial_\perp{}^2)^{1/2}]q(x)\,. \tag{5}$$

The transformed Hamiltonian $V^{-1}HV$ manifestly commutes with all F^i_α for $i=1,2,3$. It is then trivial to show that H commutes with all W^i_α for $i=1,2,3$. Furthermore, it is clear that in the limit $c\to 0$ this Hamiltonian reduces to that of Melosh, as does the transformation V.

Now let us analyze the structure of V in some detail. Under the group $SU(6)_{W,\,\mathrm{current}}\times O(3)$ we see that Y is a sum of the uncharged $\Delta J_z=0$, $\Delta L_z=\pm1$ members of a 35 (transforming like a combination of ω and φ with helicities ±1). Although it only changes L_z by ±1, the power series generated by an expansion in $(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)$ shows that Y contains operators with $\Delta J=0,1,2,\ldots,$ and so can mix essentially all angular momenta. Furthermore, the presence in Y of a λ_8 term leads to a mixing of SU(3) multiplets other than the usual 1, 8, and 10 representations. For example, whereas the Melosh transformation connects a state of 56, $L_z(W)=0$ to representations containing $\bar{q}q$ pairs in SU(3) singlets with "exotic" quantum numbers as $J^{PC}=0^{--}$, our transformation additionally introduces $\bar{q}q$ pairs transforming as $I=Y=0$ octet members under $SU(3)_{\mathrm{current}}$, thus leading to "exotic" SU(3) representations such as 27, 10, etc. In the picturesque language of the quark model, this means that the nucleon, for example, is composed of only three nonstrange "constituent" quarks, but a complicated mixture of an infinite number of strange and nonstrange "current" quarks. (A phenomenological mixing operator with the general properties of our V has recently been proposed by Buccella *et al.*,[8] although from a very different viewpoint.)

Vertex functions are analyzed as usual in the following way. Matrix elements are of the form

$$\langle A,\text{strong}|O|B,\text{strong}\rangle = \langle A,\text{current}|V^{-1}OV|B,\text{current}\rangle \tag{6}$$

where

$$|B,\text{current}\rangle \equiv V^{-1}|B,\text{strong}\rangle, \tag{7}$$

so that the state $|B,\text{current}\rangle$ transforms the same way under the F^i_α as $|B,\text{strong}\rangle$ does under the W^i_α. Therefore, the matrix element is easily analyzed if one simply computes $V^{-1}OV$. We have done this for several cases and find (i) the transformed axial charges $F^{i,5}$ transform as the sum of $(8,1)-(1,8)$, $L_z=0$ and $(3,\overline{3})-(\overline{3},3)$, $L_z=\pm1$ so long as $i\neq 8$, while $F^{8,5}$ transforms partly as an SU(3) singlet as well; (ii) the transformed polar charges F^i transform, to order c, as $(8,1)+(1,8)$, $L_z=0$ except that for $i=4,5,6,7$, there is some additional $(3,\overline{3})+(\overline{3},3)$, $L_z=\pm1$ as well (of order c); (iii) the matrix elements of axial and polar charges taken between states of an $\mathrm{SU}(6)_{W,\,\text{strong}}$ multiplet are reduced from their old $\mathrm{SU}(6)_W$ values in the way found by Buccella *et al.*; in particular, the strangeness-changing polar charge is renormalized only to order c^2 as the Ademollo-Gatto theorem[9] requires. The magnitude of the renormalizations is not simply related to the magnitude of the symmetry-breaking parameter c, but also depends on the size of $\langle\partial_\perp{}^2\rangle$, for example. Corrections to SU(3) may therefore be qualitatively different from SU(6) multiplet to SU(6) multiplet without contradicting the universality of the constant c; (iv) the integrated divergence of the axial current, $\int d^3x\,\partial^\mu\mathfrak{F}^{i,5}_\mu$ commutes with V for $i=1,2,3,8$; for $i=4,5,6,7$, however, it transforms partly as $\int d^3x\,\partial^\mu\mathfrak{F}^{i,5}_\mu$ and partly as $L_z=\pm1$, much as in the case of the polar charge. Note that $\partial^\mu\mathfrak{F}^{i,5}_\mu$ does not transform as an octet, in spite of appearances. Write $\partial^\mu\mathfrak{F}^{i,5}_\mu=a^iv^i$ (no sum on i), where the a^i are constants and the v^i are pseudoscalar densities; then what is true is that $\int d^3x v^i$ transform as $\underline{35}$, $L_z=0$ under $\mathrm{SU}(6)_{W,\,\text{current}}$ and approximately so under $\mathrm{SU}(6)_{W,\,\text{strong}}$. Since the v^i are "bad" operators these properties are more likely to be model-dependent and have less of a chance to be validly abstracted from the model.

Some comments are in order here.

(a) It is clear that results previously[5,6] derived using only the general property (i) above (which, of course, holds in the $c=0$ limit) will also obtain in the present context. This means, for example, that axial-charge renormalization, the D/F ratio for the nucleon coupling to the axial current, and pionic decay amplitudes are all unaffected by the SU(3)-symmetry breaking considered here. The general form of

$$Y=\int d^3x\, q^\dagger(x) y(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)q(x) \tag{8}$$

is sufficient for these purposes.

(b) Recall our earlier statement that the transformation V is not uniquely determined even if we require a smooth transition as $c\to0$ to the transformation V_{free} of Melosh. That is because we need not rotate away any of the λ_8 terms in H with the operator V; for example, an operator can be found which rotates away the $\lambda_0\vec{\alpha}_\perp\cdot\vec{\partial}_\perp$ term leaving some $\lambda_8\vec{\alpha}_\perp\cdot\vec{\partial}_\perp$ in the transformed Hamiltonian. Any such transformation reduces to V_{free} for $c\to0$; however, the resulting symmetry of the Hamiltonian will be only SU(2).

III. THE SCALAR DENSITIES

Now we would like to treat the problem of determining if the transformation V, relating the constituent- and current-quark bases, actually does allow one to distinguish two SU(3) algebras. That is, we pose the question: Is SU(3) split as well as broken? We will continue in the context of the $(3,\overline{3})$ model described above, since it has shown itself to be a useful phenomenological model in the past. The basic objects to be studied are the scalar operators u_i, which are defined as usual by

$$u_i=\int d^3x\, q^\dagger(x)\beta\frac{\lambda_i}{2}q(x), \tag{9}$$

where $i=0,1,\ldots,8$; u_0 and u_8 appear in the Hamiltonian, Eq. (1), and break the chiral SU(3)×SU(3) invariance. Although the transformation properties of the u_i under $\mathrm{SU}(3)_{\text{currents}}$ may be read off directly from the above expression, it is not so obvious how they transform under the algebra of $\mathrm{SU}(3)_{\text{strong}}$. Of course, we may compute $V^{-1}u_iV$ explicitly using the transformation defined by Eq. (4), and we find

$$V^{-1}u_iV=\int d^3x\, q^\dagger(x)\beta\left[\frac{\kappa+1+i\vec{\gamma}_\perp\cdot\vec{\partial}_\perp/M}{[2\kappa(\kappa+1)]^{1/2}}\,\frac{\lambda_i}{2}\times\frac{\kappa+1-i\vec{\gamma}_\perp\cdot\vec{\partial}_\perp/M}{[2\kappa(\kappa+1)]^{1/2}}\right]q(x), \tag{10}$$

where

$$\kappa=[1+(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)^2/M^2]^{1/2} \tag{11}$$

and M is as in Eq. (1). More generally, however, we may write

$$V^{-1}u_i V = \int d^3x\, q^\dagger(x)\beta(A + iB\vec{\gamma}_\perp \cdot \vec{\partial}_\perp) \times \frac{\lambda_i}{2}(A - iB\vec{\gamma}_\perp \cdot \vec{\partial}_\perp)q(x), \quad (12)$$

where A and B are linear combinations of λ_0 and λ_8, and

$$A^2 + B^2\partial_\perp{}^2 = 1 \quad (13)$$

to ensure that V is unitary. Proceeding in the above manner, we could examine the vector and pseudoscalar operators (which will have similar forms) and the axial-vector operators (which will have a form similar to that of the scalar operators). Except for the scalar operators, we have discussed these above; in the following, we will concentrate our attention upon the scalars u_i.

Note first of all that $u_1, \ldots, u_8$ do not form an octet under $SU(3)_{\text{strong}}$ in general. In particular, u_1 and u_4 are not in the same octet. Each of the u_i for $i = 1, \ldots, 7$ transforms like a member of an octet under $SU(3)_{\text{strong}}$, but not the same octet. Moreover, u_8 transforms like a linear combination of singlet and octet, as does u_0. Explicitly, one finds

$$V^{-1}u_i V = \int d^3x\, \overline{q}(x)[A^2 - B^2\partial_\perp{}^2 + 2iAB\vec{\gamma}_\perp \cdot \vec{\partial}_\perp]\frac{\lambda_i}{2}q(x) \quad (14)$$

for $i = 1, 2, 3, 8, 0$, while for the other values of i the expression is more complicated, owing to the noncommutativity of A and B with λ_i. From this we see that the u_i are complicated objects, transforming under $SU(3)\times SU(3)_{\text{strong}}$ as part $(3, \overline{3}) + (\overline{3}, 3)$ and part $(1, 8) + (8, 1)$ in spite of the fact that they are simple objects when acted upon by the generators of $SU(3)\times SU(3)_{\text{currents}}$, transforming as $(3, \overline{3}) + (\overline{3}, 3)$ as usual.

In view of these rather complex transformation properties of the scalars u_i, it is worthwhile investigating how much of the original results of Gell-Mann *et al.* still obtains. Chiral $SU(3)\times SU(3)$ symmetry is broken by two physical effects: mass splitting within SU(3) multiplets and finite masses of the pseudoscalar mesons. In the model of Gell-Mann *et al.* these two effects are related in a simple and definite manner, and the parameter c appearing in Eq. (1) was estimated from the experimental values of the pseudoscalar meson masses, together with smoothness assumptions on the matrix elements of u_i between pseudoscalar mesons off their mass shells. In their calculation, Gell-Mann *et al.* did not distinguish between the two possible SU(3) algebras, and so the u_i were assumed to transform as the index i would indicate. We will proceed in an almost identical manner, except that we will carefully avoid identifying the SU(3) algebras for constituent and current quarks.

First consider the matrix elements of the u_i between members of the pseudoscalar meson octet P_j in the low-energy limit, neglecting the dependence on $t = (p' - p)^2$:

$$\begin{aligned} F_{ijk}(m_k{}^2) &= -\lim_{p\to 0}\langle P_i(p)|u_j|P_k(p')\rangle \\ &\quad -2if_i\langle 0|[F_i^5, u_j]|P_k(p')\rangle \\ &\quad -2f_i d_{ijl}\langle 0|v_l|P_k(p')\rangle \\ &\cong F_{ijk}(m_i{}^2) = -\lim_{p'\to 0}\langle P_i(p)|u_j|P_k(p')\rangle \\ &= -2f_k d_{ljk}\langle P_i(p)|v_l|0\rangle\,. \end{aligned} \quad (15)$$

Now, since we are assuming that the pseudoscalar mesons form an octet under $SU(3)_{\text{strong}}$, but the u_i do not as we discussed above, we are not able to write

$$F_{ijk}(t) = \alpha(t)\delta_{j0}\delta_{ik} + \beta(t)d_{ijk} \quad (16)$$

as Gell-Mann *et al.* do. Nevertheless, using different values for i, j, k, we still find

$$\begin{aligned} &-2f_\pi\langle 0|v_K|K\rangle = -2f_K\langle \pi|v_\pi|0\rangle\,, \\ &-2f_\pi\langle 0|v_8|\eta\rangle(\tfrac{1}{3})^{1/2} - 2f_\pi\langle 0|v_0|\eta\rangle(\tfrac{2}{3})^{1/2} = -2f_\eta\langle \pi|v_\pi|0\rangle(\tfrac{1}{3})^{1/2}\,, \end{aligned} \quad (17)$$

$$2f_K\langle 0|v_8|\eta\rangle\frac{1}{2\sqrt{3}} - 2f_K\langle 0|v_0|\eta\rangle(\tfrac{2}{3})^{1/2} = 2f_\eta\langle K|v_K|0\rangle\frac{1}{2\sqrt{3}}\,,$$

which then implies

$$\begin{aligned} &\langle 0|v_0|\eta\rangle = 0\,, \\ &\frac{\langle 0|v_K|K\rangle}{f_K} = \frac{\langle 0|v_\pi|\pi\rangle}{f_\pi} = \frac{\langle 0|v_8|\eta\rangle}{f_\eta}\,. \end{aligned} \quad (18)$$

Now consider the matrix elements of the axial-vector divergences:

$$\begin{aligned} 2f_i\langle 0|\partial_\mu \mathfrak{F}_{i\mu}^5|P^i\rangle &= m_i{}^2 \\ &= -2f_i\langle 0|[F_i^5, u_0 + cu_8]|P^i\rangle\,, \end{aligned} \quad (19)$$

from which it follows directly that

$$\begin{aligned} m_\pi{}^2 &= 2f_\pi\frac{\sqrt{2} + c}{\sqrt{3}}\langle 0|v_\pi|\pi\rangle\,, \\ m_K{}^2 &= 2f_K\frac{\sqrt{2} - \frac{1}{2}c}{\sqrt{3}}\langle 0|v_K|K\rangle\,, \\ m_\eta{}^2 &= 2f\,\frac{\sqrt{2} - c}{\sqrt{3}}\langle 0|v_8|\eta\rangle\,. \end{aligned} \quad (20)$$

Putting the results of Eqs. (17)–(20) together, we find

$$c=-2\,\frac{f_\pi^{\,2}m_K^{\,2}-f_K^{\,2}m_\pi^{\,2}}{f_\pi^{\,2}m_K^{\,2}+\frac{1}{2}f_K^{\,2}m_\pi^{\,2}}\approx-1.31\,, \tag{21}$$

$$3(m_\eta^{\,2}/f_\eta^{\,2})+(m_\pi^{\,2}/f_\pi^{\,2})=4m_K^{\,2}/f_K^{\,2}\,. \tag{22}$$

Using the experimental values for pseudoscalar-meson masses, this last result, Eq. (22), implies that $f_\eta\approx f_K$. Note that we get all the results of Gell-Mann *et al.* except for the equality of f_π and f_K, which in fact is violated experimentally by about 25%.

As an aside, we notice that if we make the more questionable approximation of taking the limit $p\to 0$ in Eq. (17) we then find

$$\langle 0|u_8|0\rangle=0\,, \tag{23}$$

$$(2f_K^{\,2})\langle 0|u_0|0\rangle=m_K^{\,2}\,, \tag{24}$$

corresponding to a vacuum that is $SU(3)_{\text{current}}$-invariant, rather than $SU(3)_{\text{strong}}$, since u_8 is a mixture of $SU(3)_{\text{strong}}$ singlet and octet. From a slightly different viewpoint, an argument has been given by Renner[10] which results in

$$\frac{\langle 0|u_8|0\rangle}{\langle 0|u_0|0\rangle}=\sqrt{2}\,\frac{f_K^{\,2}m_\pi^{\,2}(\sqrt{2}-\frac{1}{2}c)-f_\pi^{\,2}m_K^{\,2}(\sqrt{2}+c)}{\frac{1}{2}f_K^{\,2}m_\pi^{\,2}(\sqrt{2}-\frac{1}{2}c)+f_\pi^{\,2}m_K^{\,2}(\sqrt{2}+c)}, \tag{25}$$

and if we use our result Eq. (21) for the constant c, then we again are led to Eq. (23). Both Renner's argument and the one we used to obtain Eq. (23) rest on smoothness assumptions that may well be invalid. We thus reject these results, Eqs. (23)–(25), while maintaining the previous results of this work.

Further tests of this model for chiral-symmetry breaking may be derived from low-energy theorems of meson-baryon scattering. Whereas the low-energy values of the crossing-odd amplitudes are determined by the model-independent current-algebra commutators, the crossing-even amplitudes are dependent upon the so-called σ terms which are highly model-dependent. In particular, for πN scattering

$$\begin{aligned}\sigma_{\pi N}&=\langle \pi N\,|\,T\,|\,\pi N\rangle|_{q_\pi=0}\\&=-\tfrac{1}{3}(\sqrt{2}+c)(4f_\pi^{\,2})\langle N|\sqrt{2}\,u_0+u_8|N\rangle\,,\end{aligned} \tag{26}$$

and for K^+N scattering

$$\begin{aligned}\sigma_{K^+N}&=\langle K^+N|\,T\,|K^+N\rangle|_{q_K=0}\\&=-\tfrac{1}{3}(\sqrt{2}-\tfrac{1}{2}c)(4f_K^{\,2})\\&\quad\times\langle N|\sqrt{2}\,u_0+\tfrac{1}{2}\sqrt{3}\,u_3-\tfrac{1}{2}u_8|N\rangle\,.\end{aligned} \tag{27}$$

These results are valid at an unphysical point of zero energy for zero-mass mesons, and there is thus an important question of how to extrapolate from this point to a physical point where connection may be made with experimental quantities. Various techniques have been used, and the results fall into two major categories: those approximately in agreement with the analysis of von Hippel and Kim[11] and those approximately in agreement with that of Cheng and Dashen.[12]

Taking the value of c to be known, and using the experimental values for the baryon mass differences to compute the matrix elements of cu_i for $i=1,\ldots,8$ [assuming the u_i transform as an octet under $SU(3)_{\text{strong}}$], there remains but one parameter free: the matrix element of u_0 between baryons. Von Hippel and Kim[11] based their estimate of

$$\langle N|u_0|N\rangle\approx 215\text{ MeV} \tag{28}$$

mainly on KN and $\bar{K}N$ data, since with $c\approx-\sqrt{2}$ the πN scattering lengths are extremely small and make the determination of $\sigma_{\pi N}$ quite difficult. Using this result for $\langle N|u_0|N\rangle$ and a value for c of -1.31, for example, one would predict

$$\sigma_{\pi N}=15\text{ MeV}\,. \tag{29}$$

On the other hand, Cheng and Dashen[12] have used πN scattering data to estimate

$$\sigma_{\pi N}\approx 110\text{ MeV}\,, \tag{30}$$

which is an order of magnitude in disagreement with the previous result.

Various approaches to resolve this discrepancy have been proposed. First, the Cheng-Dashen result has been questioned and independent estimates of $\sigma_{\pi N}$ have been made using πN scattering data as well as using KN scattering data together with the assumed value for c and SU(3) applied to matrix elements of the u_i. All but one of these estimates[13] give

$$\sigma_{\pi N}\approx 40\text{ MeV}\,. \tag{31}$$

A recent paper of Carter *et al.*[14] in which some new data on πN scattering at low energies are analyzed quotes a value

$$\sigma_{\pi N}=86\pm 12\text{ MeV}\,, \tag{32}$$

which is essentially in agreement with the Cheng-Dashen value. In a second direction of approach to the problem, the result of Cheng and Dashen has been adopted and the other estimates have been rejected on the basis of an expected rapid variation of off-mass-shell quantities which do not occur in the Cheng-Dashen method but which, for example, may occur naturally if a scale- and chiral-invariant model approach is used.[15] Unfortunately, in this approach, one requires the existence of particles (dilatons) which have little or no experimental support; dilatation invariance is probably an extremely badly broken symmetry.

There is a third way to resolve the problem.

We would like to point out here that the discrepancy between a large value of $\sigma_{\pi N}$ as obtained from πN scattering data and a small value of $\sigma_{\pi N}$ as obtained from KN scattering data may be understood if there is a large splitting of SU(3), i.e., if $\mathrm{SU}(3)_{\mathrm{strong}}$ and $\mathrm{SU}(3)_{\mathrm{currents}}$ are significantly different. We also find that a value of $\sigma_{\pi N}$ of 40 MeV or so, which one might characterize as intermediate in size, indicates a non-negligible splitting of SU(3).

Our starting point is the earlier discussion of the transformation properties of the scalar operators u_i, which we alluded to above. In particular, we are interested in the operators u_0, u_3, and u_8 since it is linear combinations of these quantities which appear in $\sigma_{\pi N}$ and σ_{KN}. As we can see from Eq. (14), we must consider the transformed scalars $V^{-1}u_i V$, which have the general form (for $i=0, 3, 8$)

$$V^{-1}u_i V = m \int d^3x\, \overline{q}(x) U(\tfrac{1}{2}\lambda_i) q(x) \tag{33}$$

and we will choose to parametrize U as follows:

$$U=(a\lambda_0+b\lambda_8/\sqrt{2})(\tfrac{3}{2})^{1/2}\,. \tag{34}$$

Of course, a and b are effectively just numbers if we restrict our attention to matrix elements of the u_i between baryon octet states only. The mass parameter m serves to determine the scale of the matrix element. It is then easy to show that

$$\begin{aligned} V^{-1}u_0V &= m\int d^3x\,\overline{q}(x)\tfrac{1}{2}(a\lambda_0+b\lambda_8/\sqrt{2})q(x)\,,\\ V^{-1}u_3V &= m\int d^3x\,\overline{q}(x)\tfrac{1}{2}(a+b)\lambda_3 q(x)\,,\\ V^{-1}u_8V &= m\int d^3x\,\overline{q}(x)\tfrac{1}{2}[\sqrt{2}\,b\lambda_0+(a-b)\lambda_8]q(x)\,, \end{aligned} \tag{35}$$

from which the $\mathrm{SU}(3)_{\mathrm{strong}}$ transformation properties of the scalars $u_{0,3,8}$ may be read off directly. The question now concerns the magnitude of b/a, which determines how much the two SU(3) algebras differ. As we mentioned above, the σ terms for πN and KN scattering are determined by different linear combinations of u_0, u_3, and u_8; furthermore, the linear combination u_0+cu_8 is determined by the baryon mass differences. We therefore must study matrix elements of three quantities:

$$\begin{aligned} V^{-1}(\sqrt{2}\,u_0+u_8)V &= \tfrac{1}{2}m\int d^3x\,\overline{q}(x)[\sqrt{2}(a+b)\lambda_0+a\lambda_8]q(x)\,,\\ V^{-1}(\sqrt{2}\,u_0+\tfrac{1}{2}\sqrt{3}\,u_3-\tfrac{1}{2}u_8)V &= \tfrac{1}{2}m\int d^3x\,\overline{q}(x)\,[\sqrt{2}(a-\tfrac{1}{2}b)\lambda_0+\tfrac{1}{2}(a+b)\sqrt{3}\,\lambda_3-\tfrac{1}{2}(a-3b)\lambda_8]q(x)\,,\\ V^{-1}(u_0+cu_8)V &= \tfrac{1}{2}m\int d^3x\,\overline{q}(x)[(a+\sqrt{2}\,cb)\lambda_0+(ca-cb+b/\sqrt{2})\lambda_8]q(x)\,. \end{aligned} \tag{36}$$

Assuming that c is known, we may determine these three matrix elements from $\sigma_{\pi N}$, σ_{KN}, and the baryon mass differences. This in turn will determine the ratio b/a. If we fix σ_{KN} from von Hippel and Kim (this is not a controversial number),[16] and fix the matrix elements $\langle N|u_8|N\rangle$ and $\langle N|u_3|N\rangle$ from the baryon mass differences, then b/a is determined by $\sigma_{\pi N}$ (see Table I).

It is clear from Table I that a value for $\sigma_{\pi N}$ of 40 MeV or so can be easily accommodated by a small but nonzero splitting of the SU(3) algebras corresponding to b/a of about 0.2; moreover, if one were to accept the large value for $\sigma_{\pi N}$ as given by Cheng and Dashen or by Carter *et al.*,[12,14] then one could accommodate this with a rather large splitting corresponding to b/a of 0.5.

In Table I we have given the value of $\langle N|u_0+cu_8|N\rangle$ for each value of b/a. Note that we have chosen our sign convention such that the contribution of this term to the nucleon mass is the negative of the table entry. This means that if we accept this mechanism for explaining the discrepancy between the small σ term expected from analysis of KN scattering with the rather large one obtained from analysis of πN scattering, then the matrix element of u_0 between nucleons is small compared with the nucleon matrix element of the $\mathrm{SU}(3)\times\mathrm{SU}(3)$-invariant piece of the Hamiltonian, which of course must make up the rest of the nucleon mass. This would be very unwelcome in the theory of broken scale invariance,[15] while it would be quite acceptable in other schemes, for example, the tadpole model of scalar mesons dominating matrix elements of the u_i as discussed by Renner.[10]

TABLE I. The ratio b/a for different values of $\sigma_{\pi N}$.

$\sigma_{\pi N}$ ($c=-1.25$) in MeV	b/a	$\langle N\vert u_0 - cu_8\vert N\rangle$ in MeV
18	−0.2	−47
25	0	0
37	0.2	78
55	0.4	148
96	0.5	215
180	0.6	495

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Extrapolation of the π-N Amplitude to the π-π Cut

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The problem of extrapolating, at a fixed energy, a scattering amplitude determined from phase-shift analysis onto the t- or u-channel cuts is studied in detail by following Cutkosky's work on reproducing kernels. The Riesz representation theorem is used to estimate and minimize the bounds on extrapolation errors. Percentage errors of extrapolating the present six-partial-wave π-N amplitude to various points beyond the two-pion cut are calculated explicitly. These errors arise *from extrapolation only* and are independent of the empirical values of the phase shifts.

I. INTRODUCTION

It has long been an appealing idea to extract dynamical information by directly extrapolating the empirically determined π-N amplitude from the physical region onto the two-pion cut. Such information can be used, for instance, to calculate the two-pion contribution to N-N scattering and the π-π scattering amplitude. In view of the increasingly accurate results on π-N phase shifts available, it is of practical interest to ask whether such a program can already be carried out reliably. The difficulty is of course that different kinematical regions for various channels are involved and several extrapolations are necessary to perform such theoretical calculations. The most urgent question is whether the first step of the extrapolations from the πN physical region ($s>0, t<0$) to its forward unphysical region ($s>0, t>0$) can be safely carried out. The empirically determined quantities are the first few terms of the Legendre-polynomial expansion over the physical region of the