# General SU(3) Particle Mixing Among Mesons in SU(3) Sum Rules*

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(Received 2 July 1973)

The features of general SU(3) particle mixing among mesons are studied in the framework of asymptotic SU(3), chiral SU(3)⊗SU(3) charge algebra, and the hypothesis of asymptotic level realization of SU(3) in the algebra $[A_i, A_j]=if_{ijk}V_k$. In the level scheme of a simple quark model, our framework requires that the mixings between the corresponding members of two nonets with the same $J^{PC}$ but different $L$ excitations should take place in such a way that their net effect disappears from the SU(3) sum rules, leaving the previously obtained nonet structure of bosons intact. We thus need to consider, in effect, only the usual mixings between the $I=Y=0$ members of each nonet (with the same $L$) and also, less importantly, the mixings between the $I=\frac{1}{2}$ members of two nonets with the same $J^P$ and $L$ but opposite $C$. These mixing parameters are fixed in our theoretical framework.

In broken SU(3) symmetry, SU(3) particle mixing must always be considered in order to have a correct description of physical ("in" or "out") particles. We summarize in this note the result of our investigation as to the question: Is there any simple regularity among the various possible types of SU(3) particle mixing?

Consider, as a guide, the $\bar{q}q$ quark model of bosons with $L$ excitation, i.e., $0^{-+}$, $1^{--}$ ($L=0$); $1^{+-}$, $0^{++}$, $1^{++}$, $2^{++}$ ($L=1$); $2^{-+}$, $1^{--}$, $2^{--}$, $3^{--}$ ($L=2$), .... For each $J^{PC}$ and $L$ we will have a nonet [SU(3) singlet and octet]. Since SU(3) is broken, SU(3) particle mixing can take place between various levels: (i) mixing between the $I=Y=0$ members of a nonet, such as the $\omega$-$\phi$ and $f$-$f'$ mixing; (ii) mixing between two nonets with the same $J^{PC}$ but different $L$ (this is no longer academic because of the recent discovery of $\rho'$ which may belong to the $1^{--}$ with $L=2$); (iii) mixing between the $I=\frac{1}{2}$ members of two nonets with the same $J^P$ and $L$ but opposite $C$ (for example, $K_A$ and $K_B$ of the $L=1$, $1^{++}$, $1^{+-}$ nonets).

We study general SU(3) mixing in the framework of asymptotic SU(3) and the chiral SU(3)⊗ SU(3) charge algebra[1] supplemented by the presence of the exotic commutation relations[2,3] (CR) such as

$$[\dot{V}_{K^0}, V_{K^0}]=[\dot{V}_{K^0}, A_{\pi^-}]=[\dot{V}_{K^0}, A_{K^0}]=0, \text{ etc.}$$

($\dot{V}_{K^0}=dV_{K^0}/dt$, $V_{K^0}=V_6+iV_7$, etc.).

Nonet bosons are denoted by $B_{\alpha,s}$ ($\pi_s, K_s, \eta_s, \eta_s'$), where $s$ stands for $J^{PC}, L$, etc. Asymptotic SU(3) assumes that the annihilation operator $a_{\alpha,s}(\vec{k},\lambda)$ of $B_{\alpha,s}$ ($\lambda$ denotes helicity) transforms linearly under SU(3) in the limit $\vec{k}\to\infty$.[3,4] Namely, we write

$$[V_i, a_{\alpha,s}(\vec{k},\lambda)]=i\sum_{\beta,t} u_{i\alpha\beta,st}a_{\beta,t}(\vec{k},\lambda)+\delta u_{i\alpha}, \quad (1)$$

picking up *all* the possible terms linear in $a_{\beta,t}(k)$, where $t$ has the same $J^{PC}$ or $J^P$ as $s$. We assume that as $\vec{k}\to\infty$, $\delta u_{i\alpha}\to 0$ sufficiently fast.[3] Therefore, at $\vec{k}\to\infty$, $a_{\alpha,s}(\vec{k},\lambda)$ can be linearly related to the (hypothetical) SU(3) representation operator $a_{j,s}(\vec{k},\lambda)$, satisfying $[V_i,a_{j,s}]=if_{ijl}a_{l,s}$ for $j=1,2,\ldots,$ 8 and equal to zero for $j=0$. We thus write

$$a_{\alpha,s}(\vec{k},\lambda)=\sum_{j,t}C_{\alpha j,st}a_{j,t}(\vec{k},\lambda), \quad \vec{k}\to\infty.$$

$C_{\alpha j,st}$ involves the SU(3) mixing parameters. With the exotic CR, $[\dot{V}_{K^0}, V_{K^0}]=0$, one can show[3,4] that the above-defined mixing plays exactly the same role as the usual one in the SU(3) mass formulas. With the CR, $[V_i,A_j]=if_{ijk}A_k$, our asymptotic SU(3) also permits our mixing parameters to play a role similar to the usual ones in the coupling constants[3]:

(A): "the matrix elements of $A_i$,

$$\langle B_{\alpha,s}(\vec{k},\lambda)|A_i|B'_{\beta,t}(\vec{k},\lambda')\rangle$$

with $\vec{k}\to\infty$, can be parametrized by the usual prescription of exact SU(3) plus mixing."

We now look at the mixings of the type (ii) between the $I=1$ members and $I=\frac{1}{2}$ members of $L=0$ $1^{--}(\rho, K^*, \phi, \omega)$ and $L=2$ $1^{--}(\rho', K^{*\prime}, \phi', \omega')$. The $\rho$-$\rho'$ mixing angle $\theta_\rho$ is given by (suppressing $\lambda$)

$$a_\rho(\vec{k})=\cos\theta_\rho a_\rho(\vec{k}, L=0)+\sin\theta_\rho a_\rho(\vec{k}, L=2),$$

$$a_{\rho'}(\vec{k})=-\sin\theta_\rho a_\rho(\vec{k}, L=0)+\cos\theta_\rho a_\rho(\vec{k}, L=2), \quad \vec{k}\to\infty.$$

In the SU(3) limit,

$$a_\rho\to a_\rho\,(L=0), \quad a_{\rho'}\to a_\rho\,(L=2).$$

Similarly we define the $K^*$-$K^{*\prime}$ mixing angle $\theta_{K^*}$.

We now define

$\alpha \equiv \cos(\theta_\rho - \theta_{K^*})$,

$\beta \equiv \sin(\theta_\rho - \theta_{K^*})$,

$m_{K^*}{}^2 \equiv K^{*2}$,

$m_\rho{}^2 \equiv \rho^2, \ldots$.

We also define in the limit $\vec{k} \to \infty$

$\langle \pi^0 | A_{\pi^-} | \rho^+(\vec{k})\rangle \equiv f$,

$\langle K^0 | A_{\pi^-} | K^{*+}(\vec{k})\rangle \equiv i$,

$\langle \pi^0 | A_{\pi^-} | \rho'^+(\vec{k})\rangle \equiv g$,

and

$\langle K^0 | A_{\pi^-} | K^{*\prime +}(\vec{k})\rangle \equiv j$.

(In the following we exclude the trivial case $g = j = 0$.)

Sandwich, for example, the CR's

$[V_{K^0}, A_{\pi^-}] = 0$

and

$[\dot{V}_{K^0}, A_{\pi^-}] = 0$

between the states (with $\vec{k} \to \infty$) (a) $\langle K^{*0}|$ and $|\rho^+\rangle$; (b) $\langle K^{*0}|$ and $|\rho^{+\prime}\rangle$; (c) $\langle K^{*0}|$ and $|\pi^+\rangle$; (d) $\langle K^{*\prime 0}|$ and $|\pi^+\rangle$. With asymptotic SU(3) we obtain from (A)

(a) $(\frac{1}{2})^{1/2} f = -i\alpha - j\beta$,

$(K^2 - \pi^2)(\frac{1}{2})^{1/2} f = (K^{*2} - \rho^2)(-i\alpha) + (K^{*\prime 2} - \rho^2)(-j\beta)$,

(b) $(\frac{1}{2})^{1/2} g = i\beta - j\alpha$,

$(K^2 - \pi^2)(\frac{1}{2})^{1/2} g = (K^{*2} - \rho'^2)(i\beta) + (K^{*\prime 2} - \rho'^2)(-j\alpha)$,

(c) $(\frac{1}{2})^{1/2}(f\alpha) + (\frac{1}{2})^{1/2}(-g\beta) = -i$,

$$(K^{*2} - \rho^2)(\tfrac{1}{2})^{1/2}(f\alpha) + (K^{*2} - \rho'^2)(\tfrac{1}{2})^{1/2}(-g\beta) = (K^2 - \pi^2)(-i),$$

(d) $(\frac{1}{2})^{1/2}(f\beta) + (\frac{1}{2})^{1/2}(g\alpha) = -j$,

$$(K^{*\prime 2} - \rho^2)(\tfrac{1}{2})^{1/2}(f\beta) + (K^{*\prime 2} - \rho'^2)(\tfrac{1}{2})^{1/2}(g\alpha) = (K^2 - \pi^2)(-j).$$

We now add the recently proposed hypothesis of asymptotic level realization[3,5] of SU(3) in the hitherto unutilized CR,

$[A_i, A_j] = i f_{ijk} V_k$.

Sandwich it between $\langle B_{\alpha,s}(\vec{k}, \lambda)|$ and $|B_{\beta,s}(k, \lambda)\rangle$ with $\vec{k} \to \infty$. Then the right-hand side will produce pure numbers $g_{\alpha\beta}$. We seek a simple pattern by which the sets of single-particle intermediate states (inserted between the $A_i$ and $A_j$) realize the ratios of $g_{\alpha\beta}$ produced by varying the SU(3) indices $\alpha$ and $\beta$. We assume that, among the whole set of intermediate states, the set of states belonging to each level $L$, $L = 0, 1, 2, \ldots$, *separately* realize the ratios in question.

Three points may be worth mentioning: (i) We do not seek the saturation of the CR. Instead, we are interested in realizing the *asymptotic* SU(3) contents of the CR (represented by the ratios of $g_{\alpha\beta}$) by levels. (ii) We prefer to invoke only the concept of levels and the nonet structure of the quark model. We expect that some regularities come out without imposing further assumptions [$SU(6)_W$, etc.]. (iii) We cope with broken SU(3) by using asymptotic SU(3).

Consider for illustration the CR $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$ and choose

$\langle B_{\alpha,s}(\lambda)| = \langle \rho^+(\lambda)|$,

$|B_{\beta,s}(\lambda)\rangle = |\rho^+(\lambda)\rangle$

and also

$\langle B_{\alpha,s}(\lambda)| = \langle K^{*+}(\lambda)|$,

$|B_{\beta,s}(\lambda)\rangle = |K^{*+}(\lambda)\rangle$.

We consider the level realization of SU(3) of this CR at the ground-state level, $L = 0$. If we choose $\lambda = 0$, only the $0^{-+}$ state of the ground states needs to be considered and we obtain a constraint,

(e) $j^2 = 2i^2$.

Repeat the argument for the $L = 2$ $1^{--}$ (i.e., replace $\rho$ and $K^*$ by $\rho'$ and $K^{*\prime}$); we obtain

(f) $g^2 = 2j^2$.

Successive elimination from Eqs. (a)–(f) yields

$$K^2 - \pi^2 = K^{*2} - \rho^2 = K^{*\prime 2} - \rho'^2, \tag{2}$$

$$\alpha = 1, \quad \beta = 0, \quad \text{or} \quad \theta_{K^*} = \theta_\rho \equiv \theta. \tag{3}$$

Therefore, the SU(6) result[6] $K^2 - \pi^2 = K^{*2} - \rho^2$ emerges even in the presence of general SU(3) mixing. The confirmation of the mass of $K^{*\prime}$ is awaited. The constraint obtained, $\theta_{K^*} = \theta_\rho$, looks reasonable, especially if we note the fact that $\rho = K^*$ and $\rho' = K^{*\prime}$ before SU(3) breaking is introduced. It implies that, although $\theta$ itself *should not be zero*, the *net* effect of type-(ii) mixing nevertheless disappears from our sum rules, since $\theta_{K^*}$ and $\theta_\rho$ always appear in the combination $\theta_{K^*} - \theta_\rho$. For example, with partial conservation of axial-vector current (PCAC) for $A_\pi$, Eqs. (a)–(d) reproduce the old broken-SU(3) result,[3]

$$(g_{K^*K\pi}/g_{\rho\pi\pi}) = (K^*/\rho),\ (g_{K^{*\prime}K\pi}/g_{\rho'\pi\pi}) = (K^{*\prime}/\rho'). \tag{4}$$

The mixing between the $I = Y = 0$ members of these nonets is much more complicated. It has the form

$$a_{\beta,s}(\vec{k},\lambda)=\sum C_{\beta j,st}a_{j,t}(\vec{k},\lambda)\,,\quad \vec{k}\to\infty$$

where $(\beta, s)=\phi, \omega, \phi', \omega'$. $C$ is a member of SO(4). We parametrize[7] $C$ by

$$C=e^{i\chi J_3}e^{i\chi' K_3}e^{i\delta J_1}e^{i\gamma K_1}e^{i\mu K_1}e^{i\nu K_2}\,,$$

where $J_s'$ and $K_s'$ are the generators of angular momentum and of "boosts," respectively. This general parametrization has the advantage that it is simpler to handle and that each angle permits a simple physical interpretation. Namely, $\chi$ and $\chi'$ will be the $\omega$-$\phi$ and $\omega'$-$\phi'$ mixing angles, respectively, when all other angles vanish. $\mu$, $\alpha$, $\delta$, and $\nu$ will also reduce to the $\phi$-$\phi'$, $\phi$-$\omega'$, $\omega$-$\phi'$, $\omega$-$\omega'$ mixing angles respectively in a similar way. Again we make use of the result (A) and the exotic CR's, deriving sum rules corresponding to Eqs. (a)-(d). For the level realization of SU(3), we make use of the *full* algebra,

$$[A_i, A_j]=if_{ijk}V_k.$$

We now choose $\lambda=\pm1$ for the external $1^{--}$ states ($L=0$ and $L=2$). Then only the $1^{--}$ states appear in $L=0$ intermediate states and we obtain constraints similar to Eqs. (e) and (f). Corresponding to Eq. (3), we obtain

$$\theta+\mu=0\,,\quad \gamma=\delta=0\,. \tag{5}$$

$\theta+\mu=0$ implies that the magnitudes of octet ($L=0$)-octet ($L=2$) mixing angles are all the same and $\gamma=\delta=0$ means that singlet ($L=0$) and octet ($L=2$) and also octet ($L=0$) and singlet ($L=2$) do not, *in effect*, mix. Therefore, the *net* result is again that the angles $\theta$, $\mu$, $\gamma$, and $\delta$ do not appear in the sum rules. $\nu$ never enters from the outset. The only mixing angles appearing are the $\chi$ and $\chi'$ which represent the $\omega$-$\phi$ and $\omega'$-$\phi'$ mixing angles, respectively. They appear as follows. From $[\dot{V}_{K^0}, V_{K^0}]=0$ we obtain the familiar mass formulas for $1^{--}$,

$$\begin{aligned}&3\phi^2-4K^{*2}+\rho^2=3\sin^2\chi\,(\phi^2-\omega^2)\,,\\&3\phi'^2-4K^{*\prime 2}+\rho'^2=3\sin^2\chi'\,(\phi'^2-\omega'^2)\,.\end{aligned} \tag{6}$$

Corresponding to Eq. (4), we obtain with PCAC for $A_\pi$

$$\begin{aligned}\frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}}&=\frac{g_{\phi\rho'\pi}}{g_{\omega\rho'\pi}}\\&=\tan\chi\,(\rho^2-\omega^2)(\rho^2-\phi^2)^{-1}\\&\simeq 0.053\,,\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{g_{\phi'\rho'\pi}}{g_{\omega'\rho'\pi}}&=\frac{g_{\phi'\rho\pi}}{g_{\omega'\rho\pi}}\\&=\tan\chi'\,(\rho'-\omega'^2)(\rho'^2-\phi'^2)^{-1}\,.\end{aligned} \tag{8}$$

For $1^{--}$ ($L=0$) we obtain, after eliminating $\chi$ and coupling constants,[8] Schwinger's nonet mass relation[9]

$$(\omega^2-\rho^2)=-4(K^{*2}-\rho^2)(2K^{*2}-\phi^2-\rho^2)\times(\rho^2+3\phi^2-4K^{*2})^{-1}\,. \tag{9}$$

Note that these sum rules are obtained in the *presence* of general SU(3) mixing. Equations (7) and (9) which exhibit the almost ideal nonet structure of $1^{--}$ ($L=0$) nonet are well satisfied experimentally.

So far the type-(iii) mixing did not enter into our discussion. Unlike the type-(ii) mixing discussed, it appears *explicitly* in the sum rules. The $K_A-K_B$ mixing ($L=1, 1^{++}, 1^{+-}$) has been treated before.[10] The result is unchanged in the presence of type-(ii) mixing because of the argument presented here. It was shown that the $K_A$-$K_B$ mixing can take place *only* to the extent that the ideal nonet structure for the $1^{--}$ ($L=0$) is violated. Therefore, $K_A$-$K_B$ mixing angle $\theta$ cannot be very large ($\lesssim 10°$). In fact we obtain[10]

$$-(K_A{}^2-K_B{}^2)\cos\theta\doteq(K_B{}^2-A_1{}^2)-(K^{*2}-\rho^2)\,, \tag{10}$$

which reduces, when $\theta=0$, to $K_A{}^2-A_1{}^2=K^{*2}-\rho^2$ [compare with Eq. (2)].

By repeating our argument for higher-lying mesons [again using the asymptotic level realization of SU(3) at the ground-state level], we find that the same pattern of general SU(3) mixing persists[7] also among higher-lying mesons.

In summary, we find that due to our result [Eqs. (3) and (5)] the type-(ii) mixings, though present, will not appear in the SU(3) sum rules, and we need to consider, *in effect*, only the usually considered type-(i) mixings. As shown, for example, by Eq. (10), type-(iii) mixings appear explicitly but the effect will not be so important, since they can take place only to the extent that the ideal nonet structure of the $1^{--}$ ($L=0$) mesons (i.e., $\rho=\omega$, $K^{*2}-\phi^2=\rho^2-K^{*2}$, $g_{\phi\rho\pi}=0$, etc.) is actually violated.

Modification of our result by, for example, the presence of radially excited states is possible but is expected to be less important in boson cases.

For baryons the situation looks more complicated and we expect that the type-(ii) mixing may play a more important role.

We thank our colleagues at the University of Maryland for useful discussions. Two of us (S. O. and S. M.) thank Aspen Center for Physics which provided the opportunity of collaboration for the early phase of this work. One of us (T. L.) would like to thank Professor L. Greenberg for discussions on SO(4).

*Work supported in part by the National Science Foundation under Grant No. NSF GP 8748.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

# SU(3) for Constituent and Current Quarks: How Different Are They?*

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(Received 16 July 1973)

Algebraic properties of the transformation between constituent- and current-quark bases are discussed in the case when SU(3) is broken for the current quarks. In a simple model, it is shown that $SU(4)_W$ may still be an exact symmetry in the constituent-quark basis. Still within the context of the model, the question of whether the transformation actually does distinguish two SU(3) algebras in nature, one for current and one for constituent quarks, is investigated. It is shown that the so-called $\sigma$ terms of meson-baryon scattering provide a means of determining if there is such a distinction.

## I. INTRODUCTION

The existence of two SU(6) algebras generated by "good charges" (those whose matrix elements at infinite momentum do not vanish) has been recognized for some time, although the distinction between them has not always been clearly drawn. The generators of $SU(6)_{W,\text{current}}$ are essentially the integrated weak and electromagnetic current densities and related operators.[1] The other algebra, $SU(6)_{W,\text{strong}}$, is an approximate symmetry of the strong-interaction Hamiltonian[2] and it is not known if its generators can be written as integrals of local operators. Although the two SU(6) algebras are isomorphic, they are not the same; however, they are closely related by the conserved-vector-current (CVC) hypothesis which identifies an SU(3) subalgebra in $SU(6)_{W,\text{current}}$ to one in $SU(6)_{W,\text{strong}}$. In general, hadron states at infinite momentum do not transform irreducibly under the group $SU(6)_{W,\text{current}}$ and the problem of discovering the ensuing representation mixing has been attacked[3] in the past with some success, although not in a systematic fashion. Some time ago Gell-Mann suggested[4] that the two algebras may be related by a unitary transformation, $V$. If so, then by finding such a $V$, one would solve the mixing problem. Less ambitiously, by determining some of the general structure of $V$, one would obtain some properties of the mixing that might be useful.

Following this suggestion, Melosh[5] has shown that in the free-quark model the two $SU(6)_W$ algebras may be related by a unitary transformation $V_{\text{free}}$. Assuming that the algebraic structure of the correct unitary transformation $V$ is similar to that of $V_{\text{free}}$, one may make predictions for pionic decays of meson and baryon resonances,[6] recover many of the good results of the old $SU(6)_W$ scheme for matrix elements of weak charges, and correct some of the poor results (such as the prediction of $G_A/G_V=-\frac{5}{3}$).

A fundamental question in this context is how different can one expect the correct transformation $V$ to be from the explicitly constructed model transformation $V_{\text{free}}$? For example, $V_{\text{free}}=\exp(iY_{\text{free}})$, where $Y_{\text{free}}$ is bilinear in quark fields. In general, writing $V=\exp(iY)$, we might expect to find $Y$ possessing higher-order terms in quark fields. Moreover, the extremely simple property of $V_{\text{free}}$ that the transformed axial charge $V_{\text{free}}^{-1}Q_i^5V_{\text{free}}$ transforms as a sum of $(8,1)-(1,8)$ and $(3,\bar{3})-(\bar{3},3)$ representations of $SU(3)\times SU(3)_{\text{strong}}$ might not be