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Electron-Positron Annihilation in Stagnant Field Theories*

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We discuss the ratio $R(s) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in stagnant (asymptotically free) field theories. The leading correction to the scaling limit is determined.

There has been a great deal of interest recently in stagnant (asymptotically free) field theories.[1-5] These are field theories in which the origin in the coupling-constant plane is ultraviolet-stable.[6,7] Indeed, there are indications[8,9] that if Bjorken's scaling phenomenon[10] is to be understood in the framework of field theory at all the field theory would have to be stagnant. It appears now that the only[11] stagnant theory is the non-Abelian gauge theory of Yang and Mills.[12]

In this paper we would like to discuss the annihilation process $e^+e^- \to$ "γ" $\to$ any hadrons within[13] the framework of local field theories. The appropriate object to study is the vacuum polarization tensor

$$\pi_{\mu\nu}(q) \equiv -i\int d^4x\, e^{iqx}\langle 0|\, T^*J_\mu(x)J_\nu(0)\,|0\rangle$$
$$\equiv (q_\mu q_\nu - g_{\mu\nu}q^2)\pi(q^2),$$

where J_μ denotes the electromagnetic current. The total annihilation cross section σ_T is given by the absorptive part of π:

$$\sigma_T(s) \equiv (32\pi^2\alpha^2/s)\,\text{Abs}\,\pi(s).$$

It is customary to consider the ratio

$$R(s) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-).$$

We will discuss this process in the language of the Callan-Symanzik equation.[14] The discussion differs from the usual analysis of the asymptotic behavior of Green's functions in two important respects.

(A) The vacuum polarization function is subtractively renormalizable[15] while Green's functions are multiplicatively renormalizable. This leads to an inhomogeneous Callan-Symanzik equation in the asymptotic region rather than a homogeneous one as is appropriate for Green's functions.

(B) The experimentally measured total annihilation cross section is the absorptive part of π.

We will be concerned with the consequences of these two facts.

For definiteness let us consider a Yang-Mills model[16] of three quark triplets with a global symmetry SU(3)×SU(3)′ and a gauge symmetry SU(3)′. The eight gauge bosons are massless at the Lagrangian level but are presumed to become massive due to some miraculous nonperturbative[17] mechanisms (which are not understood at present). The electromagnetic current is an SU(3)′ singlet:

$$J_\mu = \sum_{a=1}^{3} \overline{q}_a \gamma_\mu Q q_a .$$

Q is some charge matrix. We treat electromagnetism only to lowest order but strong interaction to all orders.[17] The unrenormalized vacuum polarization function is a function of four variables: $\pi_0(q/m, \mu/m, \Lambda/m, g)$. Here m is the quark mass, Λ is the cutoff, and g is the Yang-Mills coupling constant. The massless gauge boson propagator has to be renormalized at some arbitrary momentum q_0 such that $q_0^2=\mu^2$. The renormalized vacuum polarization function is given by

$$\pi_R(q^2/m^2,\mu/m,q)\equiv\lim_{\Lambda\to\infty}\pi_0(q^2/m^2,\mu/m,\Lambda/m,g) - \pi_0(0,\mu/m,\Lambda/m,g).$$

When we make the usual mass variation[14] on π_R to obtain the Callan-Symanzik equation the mass variation on the subtraction $\pi_0(0,\mu/m,\Lambda/m,g)$ contributes an inhomogeneous term $C(\mu/m,g)$, quite apart from the usual scalar insertion term which vanishes in the asymptotic region according to Weinberg's theorem.[18] In this way we obtain

$$\left[m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\pi_R^{\rm as}(q^2/m^2,\mu/m,g) = C(\mu/m,g), \quad (1)$$

where $\pi_R^{\rm as}$ denotes the asymptotic form of π_R.

$$C(\mu/m,g)\equiv\lim_{\Lambda\to\infty}-\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}\right)\pi_0(0,\mu/m,\Lambda/m,g) \quad (2)$$

is the cutoff-independent inhomogeneous term alluded to above. Equation (1) is solved by a slight variation of standard techniques. The mass ratio μ/m will always be kept fixed. Equation (1) may be rewritten as

$$\left[-z\frac{\partial}{\partial z}+\beta(g)\frac{\partial}{\partial g}\right]\pi_R^{\rm as}(z^2,\mu/m,g)=C(\mu/m,g)$$

or

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial\rho}\right]\pi_R^{\rm as}(z^2,\mu/m,g)=C(\mu/m,g)$$

with $t=-\ln z$.

$$\rho(g)=\int_{g_A}^{g}\frac{dg'}{\beta(g')},$$

with g_A a completely arbitrary constant. The solution is

$$\pi_R^{\rm as}(z^2,\mu/m,g)=\Phi(t-\rho(g))+\int_{g_B}^{g}dg'\,\frac{C(\mu/m,g')}{\beta(g')},$$

where $\Phi(x)$ is an arbitrary function and g_B is an arbitrary constant. We eliminate Φ, g_A, and g_B to obtain

$$\pi_R^{\rm as}(\xi^2z^2,\mu/m,g)=\pi_R^{\rm as}(z^2,\mu/m,g(\xi)) - \int_g^{g(\xi)}dg'\,\frac{C(\mu/m,g')}{\beta(g')}, \quad (3)$$

where $g(\xi)$ is defined by

$$\int_g^{g(\xi)}\frac{dg'}{\beta(g')}=\ln\xi. \quad (4)$$

With z fixed to be, say, =1 the asymptotic behavior of $\pi_R^{\rm as}$ is described by Eq. (3).

In Yang-Mills theory β has the perturbative expansion

$$\beta(g)=-(b_0g^3+\cdots),$$

where b_0 is a positive,[3,4] model-dependent but definitely calculable number. Equation (4) determines

$$g(\xi)^2\to\frac{1}{2b_0\ln\xi}\ \text{as}\ \xi\to\infty.$$

Evaluating Eq. (3) we find

$$\pi_R^{\rm as}(\xi^2z^2,\mu/m,g)\to\pi_R^{\rm as}(z^2,\mu/m,0)-C(\mu/m,0)\ln\xi + \text{corrections to be discussed}. \quad (5)$$

The logarithmic term follows from the presence of an inhomogeneous term $C(\mu/m,g)$ in the Callan-Symanzik equation and is present in Eq. (5) precisely in order that the annihilation total cross section scale, since the absorptive part of $\pi(s)$ is from Eq. (5)

$$\text{Abs}\,\pi_R^{\rm as}(s)=-\tfrac{1}{2}C(\mu/m,0)+\text{correction terms}. \quad (6)$$

It is important to realize that $C(\mu/m,0)$ is evaluated with the strong interaction turned off. We may now simply consult a textbook to discover that

$$\pi_0(0,\mu/m,\Lambda/m,0)=-\frac{1}{12\pi}\Sigma Q_i^2\ln(\Lambda/m),$$

where Q_i is the charge of the ith fundamental fermion. Hence

$$-12\pi C(\mu/m,0)=\Sigma Q_i^2. \quad (7)$$

Let us turn our attention to the correction terms indicated in Eqs. (5) and (6). These corrections stem from two sources. Firstly, in Eq. (3) we may expand the first term for small $g(\xi)^2\sim(2b_0\ln\xi)^{-1}$ thus:

$$\pi_R^{\rm as}(z^2,\mu/m,g(\xi))=\pi_R^{\rm as}(z^2,\mu/m,0) + f(z^2,\mu/m)g^2(\xi)+\cdots = \text{const.}+\text{const.}/\ln\xi+\cdots.$$

This generates a correction term to $R(s)$ which vanishes like $(\ln s)^{-2}$. The more interesting correction to Eq. (5) comes from a more accurate determination of the second term in Eq. (3). For instance, to the next order we have

$$\beta(g)=-(b_0g^3+b_1g^5+\cdots)$$

and

$$C(\mu/m,g)=(C_0+C_1g^2+\cdots).$$

Proceeding thus, we find

$$\pi_R^{as}(s, \mu/m, g) \to f_1 \ln(-s/s_0) + f_2 \ln\ln(-s/s_0) + \cdots + \text{constant} + \text{terms which vanish}. \tag{8}$$

s_0 is some scale not determined by the Callan-Symanzik equation. It is to be emphasized that the quantities f_i are in principle completely determined from a perturbative computation of β and C up to some appropriate order in g. This is of course the reason why it is so attractive to consider stagnant field theories.

Let us evaluate the first correction coefficient f_2. A straightforward calculation up to terms of order $\ln\ln\xi$ gives

$$\int_g^{g(\xi)} dg' \frac{C(\mu/m, g')}{\beta(g')} \to C_0 \ln\xi + \frac{C_1}{2b_0} \ln\ln\xi + \cdots . \tag{9}$$

Hence the ratio $(f_2/f_1) = (C_1/C_0 b_0)$. An evaluation of this ratio without undue labor is made possible by two marvelous sets of circumstances: firstly, that b_1 does not enter to this order, and secondly, that the nonlinearity of the Yang-Mills interaction is not yet manifest in π_R to order g^2. This happy observation allows us to simply extract the coefficient C_1/C_0 from the calculation of Jost and Luttinger[19] provided that the group-theoretic factor (s_3 in the notation of Ref. 2) appropriate to the theory is taken into account. We can already make a statement independent of the specific group-theoretic choice in the Yang-Mills theory: namely that the discovery of Jost and Luttinger that $(c_1/c_0) > 0$ implies that $(f_2/f_1) > 0$. For illustration let us consider the SU(3)×SU(3)′ model described previously. In this model

$$(C_1/C_0) = \frac{1}{3}\sum_{i=1}^{8} (\operatorname{tr} \tfrac{1}{2}\lambda^i \tfrac{1}{2}\lambda^i)(3g^2/16\pi^2) = g^2/4\pi^2 . \tag{10}$$

Referring to the result and the notation of Ref. 3 we find

$$\beta(g) = \frac{g^3}{16\pi^2}\left[-\tfrac{11}{3}C_2(G) + \tfrac{4}{3}T(R)\times 3\right]. \tag{11}$$

Here $C_2(\text{SU}(3)) = 3$ and $T(\underline{3}) = \frac{1}{2}$. Hence $(f_2/f_1) = \frac{4}{9}$. This leads to[20]

$$R(s) \to (\Sigma Q_i^2)\left(1 + \frac{4}{9}\frac{1}{\ln(s/s_0)} + \cdots\right) \tag{12}$$

as $s \to +\infty$. We may emphasize that the number $\frac{4}{9}$ is a number calculated with strong interaction taken fully into account. The present experimental data points have larger error bars, but there is a definite trend for $R(s)$ to rise slowly with s over the range ~1 GeV^2 to ~16 GeV^2 for which data have been published.[21] If one believes in Eq. (12) one would have to conclude that the asymptotic region has not yet been reached.

The scaling behavior of the annihilation total cross section should be compared to the asymptotic behavior of Green's functions [recall fact (A)]. The Green's functions Γ satisfy a homogeneous equation of the form

$$\left(\xi\frac{\partial}{\partial\xi} + \beta(g)\frac{\partial}{\partial g} - \gamma(g)\right)\Gamma = 0 . \tag{13}$$

In stagnant theories the solution is

$$\Gamma \to (\ln\xi)^{-c/2b_0}, \tag{14}$$

where $\gamma(g) = cg^2 + \cdots$.

The possibility that the world is describable by stagnant field theories is a very appealing one. It is of obvious importance to see if the experimental information on e^+e^- annihilation and deep-inelastic scattering is indeed consistent with the behavior outlined in Eqs. (12) and (14), respectively.

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Corrections to Equivalent-Photon Approximation for Two-Photon Processes in Colliding Beams*

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The cross section for the two-photon process $ee \to ee\,\Gamma$ in colliding electron beams, in the limit $E \to \infty$ with E^2/s fixed (E being the energy per beam of the colliding electrons of mass m, and $\sqrt{s}$ the invariant mass of the state Γ), is well known to have a leading term of order $[\ln(4E^2/m^2)]^2$. This leading term is given correctly by the well-known Weizsäcker-Williams approximation or the equivalent-photon approximation. There are sizable corrections coming from nonleading terms, of which the first, namely the term of order $\ln(4E^2/m^2)$, has been obtained *for an arbitrary hadronic final state* Γ. The approximations made in getting the Weizsäcker-Williams result and the correction terms due to each of them have been exhibited.

I. INTRODUCTION

A considerable amount of attention has been given[1,2] in the past few years to the so-called two-photon processes in colliding electron beams, i.e., reactions of the form $e+e \to e+e+\Gamma$ as shown in Fig. 1. The two-photon processes are expected to be comparable in rate to the one-photon or annihilation processes shown in Fig. 2 even at present machine energies, and will eventually dominate. Since the final electrons in the two-photon process tend to be nearly in the forward directions, it is often useful to integrate over them. There are well-known approximations for studying this integrated cross section which, in the limit $E \to \infty$ with E^2/s fixed (E being the energy per beam of the colliding electrons of mass m, $\sqrt{s}$ the invariant mass of the produced state Γ), has a leading term of order L^2, where $L = \ln(4E^2/m^2)$. Unfortunately, there does not appear to be a uniform nomenclature for these approximations in the literature; we shall adopt the following: The equivalent-photon approximation involving the complete Dalitz-Yennie function [Eq. (3.3) in Ref. 1] we shall call the double equivalent-photon approximation (DEPA). By keeping only the leading term, namely, the term of order L^2 (we are treating $L_s \equiv \ln(4E^2/s)$ to be of order L^0), we get an approximation which we will call the Weizsäcker-Williams approximation (WWA) [Eq. (3.3) in Ref. 1 with g and h terms