Spin Structure of Exchange Processes*

Kui-Chuen Lam and Michael J. Moravcsik
Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403
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It is shown that the spin structure of a given reaction with a dynamics of one-particle or one-Reggeon exchanges can be reduced to a few simple cases, and hence one can easily devise experimental tests for the validity of all such dynamics in terms of specific and simple polarization experiments.

Various versions of one-particle or one-Reggeon exchange models form the most prevalent simple dynamical schemes in present-day particle physics. The purpose of this note is to provide a method for experimental tests of such models. It pertains to polarization experiments in four-particle reactions, which have recently received increasing attention as it becomes more and more evident that differential cross sections alone provide only very weak constraints on various models.

We are going to prove the following very powerful theorem: In the scattering of four external particles of arbitrary spins, with the exchange of a particle of arbitrary spin, the amplitude structure of spin-j exchange is identical to that of spin-$(j+1)$ or spin-$(j+2)$ exchange having the same normality at the respective vertex.

By "amplitude structure" (or "spin structure") we mean which of the spin amplitudes in the reaction matrix are zero and which are not.

The wide-ranging implications of the theorem are obvious: For a given reaction one can devise tests of all the infinite number of one-particle exchanges (no matter what the spin of the exchanged particle is) by simply looking at the spin structure of spin-0 exchange and spin-$\frac{1}{2}$ exchange (or in some cases spin-1 exchange or spin-$\frac{3}{2}$ exchange). Furthermore, since Reggeons can be pictured as representing a series of particles with spins two units apart, the same tests will also apply to them. Their polarization structure will be the same as the structure of the exchanges of any of the particles lying on that Regge trajectory. The tests themselves can be easily constructed once it is known which of the spin amplitudes vanish. Examples for this will be given at the end of this note.

Furthermore, the deviations from these predictions of the one-particle or one-Reggeon exchange also give us a handle on the additional terms coming from the cuts in the complex plane. The large class of predictions offered here thus gives us a large variety of ways to study the nature of these cut contributions.

Theorems similar in spirit to the present one have been formulated before in connection with specific processes.[1] The present theorem is much more general in scope.

The proof of the theorem will use a notation similar to that of Scadron.[2] The type of couplings used will be similar to the ones used in the literature,[3] and we will use the same coupling at the vertices for all exchanged particles except for the obvious modifications due to the different values of the spin. This assures us that we can consider a Regge trajectory as a sequence of exchanges of these particles.

The proof then rests on the following reduction procedure: Let us write, using the Rarita-Schwinger formalism,

$$T_{\mu_1\mu_2\cdots;\nu_1\nu_2\cdots}=\sum_{\Lambda}\epsilon^{\Lambda}_{\mu_1\mu_2\cdots}(k)\bar{\epsilon}^{\Lambda}_{\nu_1\nu_2\cdots}(k),$$

where T is the numerator of the propagator of a massive particle, and we have

$$\epsilon^{\Lambda}_{\mu_1\mu_2\cdots\mu_J}(k)=\epsilon^{\Lambda}_{\mu_2\mu_1\cdots\mu_J}(k),$$

where $J=[j]$ (integer part of the spin) and Λ is the polarization along some axis.

Then by complete symmetry of the μ's (or ν's) we have

$$2\gamma^{\mu_1}\gamma^{\mu_2}\cdots T_{\mu_1\mu_2\cdots;\nu_1\nu_2\cdots}=\gamma^{\mu_1}\gamma^{\mu_2}\cdots T_{\mu_1\mu_2\cdots;\nu_1\nu_2\cdots}+\gamma^{\mu_1}\gamma^{\mu_2}\cdots T_{\mu_2\mu_1\cdots;\nu_1\nu_2\cdots},$$

and so, by the anticommutation relations for the γ matrices,

$$\gamma^{\mu_1}\gamma^{\mu_2}\cdots T_{\mu_1\mu_2\cdots;\nu_1\nu_2\cdots}=g^{\mu_1\mu_2}\cdots T_{\mu_1\mu_2\cdots;\nu_1\nu_2\cdots}.$$

The degraded expression has two fewer γ matrices. This can be repeated until there is no more γ matrix, or only one (excluding γ^5). Thus contraction with γ matrices reduces the rank by two. We also note that contraction with momentum reduces the rank by one. Let us denote by

N_1 the number of indices coupled to the external particles at vertex 1, and to the γ matrix at that vertex (if there remains one after reduction). We can then contract $J-N_1$ indices with momenta and by the $g^{\mu\nu}$'s generated by the reduction and group into classes the various terms in the Λ sum according to the resulting spin-function polarization λ of the following contraction:

$$p^{\mu_1}\cdots p^{\mu_{J-N(1)}}\epsilon^{\Lambda}_{\mu_1\cdots\mu_J}(k)=\sum_{\lambda}C^{\lambda}\epsilon^{\lambda}_{\mu_{J-N(1)+1}\cdots\mu_J}(k)\,.$$

Doing similarly on the other vertex, we get new classes from the present ones with an extra label $\bar{\lambda}$. For spin-$(j+n)$ exchange we repeat the procedure.

Let us denote the classes thus obtained by $\mathcal{C}^j(\lambda,\bar{\lambda})$ and $\mathcal{C}^{j+n}(\lambda,\bar{\lambda})$. We see immediately that there is a one-to-one correspondence between these classes:

$$\mathcal{C}^j(\lambda,\bar{\lambda})\leftrightarrow\mathcal{C}^{j+n}(\lambda,\bar{\lambda})\,.$$

Therefore the total amplitudes A^j are "identical" in the sense used in the statement of the theorem [this identity being denoted by (=)]

$$A^j\equiv\sum_{\lambda,\bar{\lambda}}a^j(\lambda,\bar{\lambda})\mathcal{C}^j(\lambda,\bar{\lambda})\,(=)\,\sum_{\lambda,\bar{\lambda}}a^{j+n}(\lambda,\bar{\lambda})\mathcal{C}^{j+n}(\lambda,\bar{\lambda})\equiv A^{j+n}\,.$$

This completes the proof. The presence of charge-conjugation matrices does not alter the proof. The above proof is given in terms of the Rarita-Schwinger formalism. The result is, however, intuitively obvious in other languages also. For example, the result is really the basis of the assumption that Regge trajectories exist, and without it the idea of grouping particles into trajectories would make little sense. The theorem, however, does not depend on the Regge model being right. The theorem is implicit in work done previously,[4] and so the contribution of the present paper is mainly to make the result explicit and to relate it to the problem of finding experimental tests for dynamical models.

We will now illustrate the usefulness of the theorem by considering first the reaction $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}$, where the $\frac{1}{2}$'s denote spins. The spin structure of this reaction has been discussed[5] in the context of the study of the general polarization structure of particle reactions with arbitrary spins.[6] We will use the terminology of those studies, which is also summarized in the Appendix for quick reference.

We will write the parity-conserving M matrix as

$$\begin{aligned}M=&A_{00}+A_{11}\vec{\sigma}^{(1)}\cdot\hat{l}\,\vec{\sigma}^{(2)}\cdot\hat{l}+A_{22}\vec{\sigma}^{(1)}\cdot\hat{m}\vec{\sigma}^{(2)}\cdot\hat{m}\\&+A_{33}\vec{\sigma}^{(1)}\cdot\hat{n}\vec{\sigma}^{(2)}\cdot\hat{n}\\&+B^+(\vec{\sigma}^{(1)}\cdot\hat{m}+\vec{\sigma}^{(2)}\cdot\hat{m})+B^-(\vec{\sigma}^{(1)}\cdot\hat{m}-\vec{\sigma}^{(2)}\cdot\hat{m})\\&+C^+(\vec{\sigma}^{(1)}\cdot\hat{l}\,\vec{\sigma}^{(2)}\cdot\hat{n}+\vec{\sigma}^{(1)}\cdot\hat{n}\vec{\sigma}^{(2)}\cdot\hat{l})\\&+C^-(\vec{\sigma}^{(1)}\cdot\hat{l}\,\vec{\sigma}^{(2)}\cdot\hat{n}-\vec{\sigma}^{(1)}\cdot\hat{n}\vec{\sigma}^{(2)}\cdot\hat{l})\,.\end{aligned}$$

In the case where all four particles are identical,[7] for 0^- exchange $B^+=0$, for both the forward and the backward pole. Thus only the product sets[6] with signature[6] *eee* remain, and hence all observables other than those in subclasses[6] *eee* and *ooo* vanish identically.

According to our theorem, this result also holds for the exchange of 2^-, 4^-, etc., and hence also for the Regge exchange containing these quantum numbers. This immediately opens up a huge class of experiments that can test the presence, absence, or dominance of such exchanges. For example (for notation see Ref. 6),

$$\begin{aligned}L(0,0;0,m)&=L(0,m;0,0)=L(m,0;0,0)\\&=L(0,0;m,0)=0\,,\\L(n,l;0,0)&=L(n,0;l,0)=L(l,n;0,0)\\&=L(l,0;n,0)=L(l,0;0,n)=\text{etc.}=0\end{aligned}$$

(that is, all observables with some combination of one l and one n vanish) if the reaction proceeds only through such exchanges. The first of these, namely that the polarization vanishes, is already well known.

Now we turn to a different example: $0+\frac{1}{2}\to 1+\frac{1}{2}$, which is, for example, vector-boson production in pion-nucleon collisions. With parity conservation this reaction has six amplitudes:

$$\begin{aligned}M=&A_1\vec{S}\cdot\hat{l}\vec{\sigma}\cdot\hat{l}+A_2\vec{S}\cdot\hat{l}\vec{\sigma}\cdot\hat{n}+A_3\vec{S}\cdot\hat{n}\vec{\sigma}\cdot\hat{l}\\&+A_4\vec{S}\cdot\hat{n}\vec{\sigma}\cdot\hat{n}+A_5\vec{S}\cdot\hat{m}+A_6\vec{S}\cdot\hat{m}\vec{\sigma}\cdot\hat{m}\,.\end{aligned}$$

A direct calculation of the one-pion-exchange contribution to this process shows that for that dynamics $A_5=A_6=0$. To explore the consequence of this on the 144 experimental observables appears a formidable task, but it is in fact easy in terms of the once-factorizable subclass–product-set structure[8] of this reaction. Writing out the component–term-set combinations, and imposing on them $A_5=A_6=0$, one finds that the subclasses indicated in Table I vanish. The simplest observables which vanish are therefore

$$\begin{aligned}L(0,n;n,0)&=L(0,0;n,n)=L(0,l;n,0)\\&=L(0,0;n,l)=L(0,l;l,0)\\&=L(0,0;l,l)=L(0,n;l,0)\\&=L(0,0;l,n)=0\,.\end{aligned}$$

TABLE I. Observable subclasses vanishing in one-pion exchange for the reaction $0+\frac{1}{2}\to 1+\frac{1}{2}$.

Over-all subclass	Component subclasses			
eee	*eoo*	*eoo*	*oee*	*oee*
eee	*ooe*	*ooe*	*eeo*	*eeo*
eoe	*eoo*	*eeo*	*oee*	*ooe*
eoe	*ooe*	*oee*	*eeo*	*eoo*
oeo	*eoo*	*ooe*	*oee*	*eeo*
oeo	*ooe*	*eoo*	*eeo*	*oee*
ooo	*eoo*	*oee*	*oee*	*eoo*
ooo	*ooe*	*eeo*	*eeo*	*ooe*

According to our theorem these are then also tests for higher-spin boson exchanges and corresponding Reggeon exchanges.

These examples, however specific, demonstrate that the theorem in this note can serve as a basis for a far-reaching development of complete experimental prescriptions for the testing of various exchange processes. Further work along this line is in progress and will be reported elsewhere.

APPENDIX

Though the formalism used in this paper has been used extensively in previous papers, we summarize it here for convenience.

The M matrix M_0 of a reaction

$$A+B\to C+D \tag{A1}$$

can be factorized in terms of M matrices M_1 and M_2 of the reactions

$$A+0\to C+0 \tag{A2}$$

and

$$0+B\to 0+D \tag{A3}$$

as

$$M_0\,(=)\,M_1\otimes M_2\,, \tag{A4}$$

where (=) means equality for all purposes which are independent of the dynamics of the reactions, and $\otimes$ denotes outer product in spin space.

The M matrix of (A2) can then be written as

$$M=\sum_{J,r} a_J^r\, S_{[J]}(s,s'):T_{[J]}^r\,, \tag{A5}$$

where $S_{[J]}$ is an irreducible spin tensor of rank J in the space spanned by particle A with spin s and particle C with spin s'; the $T_{[J]}^r$ is a momentum tensor of rank J (and within a given rank indexed by r), depending on the momenta which describe the kinematics of the reaction; the colon means complete contraction over all tensorial indices; a_J^r is an amplitude, a rank-zero tensor (indexed by J and r), depending on the rank-zero tensors one can form out of the momenta that describe the kinematics of the reaction, and containing all of the dynamical information. The values of J run from $|s'-s|$ to $s'+s$. The form of (A5) also holds relativistically, though not manifestly so.

The description of the kinematics of the reaction can be done in terms of various different sets of momentum vectors. The set used here is

$$\hat{l}\equiv\frac{\vec{q}\,'-\vec{q}}{|\vec{q}\,'-\vec{q}|}\,,\qquad \hat{m}\equiv\frac{\vec{q}\,'\times\vec{q}}{|\vec{q}\,'\times\vec{q}|}\,,\qquad \hat{n}\equiv\hat{l}\times\hat{m}\,, \tag{A6}$$

where $\vec{q}$ and $\vec{q}\,'$ are the center-of-mass momenta of A and C, respectively.

The experimental observables for the reaction (A2) are defined as

$$L(S_{[J_I]}:T_{[J_I]}^{r_I};S_{[J_F]}:T_{[J_F]}^{r_F})\equiv \mathrm{Tr}\{MS_{[J_I]}:T_{[J_I]}^{r_I}M^\dagger S_{[J_F]}:T_{[J_F]}^{r_F}\}\,, \tag{A7}$$

where $S_{[J_I]}:T_{[J_I]}^{r_I}$ is the spin-momentum tensor contraction specifying the preparation of the initial beam of A particles, and $S_{[J_F]}:T_{[J_F]}^{r_F}$ describes the polarization to be measured for the final-state particle C. Observables for the composite reaction (A1) can be synthesized by a simple product of observables for the constituent reactions

$$L\{S_{[J_A]}:T_{[J_A]}^{r_A},\,S_{[J_B]}:T_{[J_B]}^{r_B};\,S_{[J_C]}:T_{[J_C]}^{r_C},\,S_{[J_D]}:T_{[J_D]}^{r_D}\}$$
$$(=)\,L\{S_{[J_A]}:T_{[J_A]}^{r_A};S_{[J_C]}:T_{[J_C]}^{r_C}\}\,L\{S_{[J_B]}:T_{[J_B]}^{r_B};S_{[J_D]}:T_{[J_D]}^{r_D}\}\,. \tag{A8}$$

For simplifying the notation, the arguments of the observables will be given only in terms of the momenta in the T's. Thus, for example, we will write

$$L\{S_{[1]}:T_{[1]}(m);S_{[2]}:T_{[2]}(ln)\}\equiv L(m;ln)\,. \tag{A9}$$

The 0 argument will represent an unpolarized state.

These experimental observables are identical with, or simple linear combinations of, the actual quantities measured in real experiments, for example,

$$\frac{d\sigma}{d\Omega} = L(0,0;0,0)\,, \tag{A10}$$

the differential cross section;

$$P = L(0,0;m,0)\,, \tag{A11}$$

the (vector) polarization of particle C perpendicular to the reaction plane;

$$c_{NN} = L(0,0;m,m)\,, \tag{A12}$$

the spin correlation perpendicular to the reaction plane; etc.

In terms of (A5), the observables (A7) can be written as

$$L\{S_{[J_I]}:T^{r_I}_{[J_I]};S_{[J_F]}:T^{r_F}_{[J_F]}\} = \sum_{J_1 J_2 r_1 r_2} a^{r_1}_{J_1} a^{r_2}_{J_2} X^{r_1 r_I r_2 r_F}_{J_1 J_I J_2 J_F}\,, \tag{A13}$$

where the X's are the four-traces

$$X^{r_1 r_I r_2 r_F}_{J_1 J_I J_2 J_F} \equiv \mathrm{Tr}\{S_{[J_1]}:T^{r_1}_{[J_1]}\,S_{[J_I]}:T^{r_I}_{[J_I]}(S_{[J_2]}:T^{r_2}_{[J_2]})^{\dagger}S_{[J_F]}:T^{r_F}_{[J_F]}\}\,, \tag{A14}$$

which are dynamics-independent numbers, tabulated for a variety of cases in Ref. 9.

If we denote $x \equiv \Pi^4_{i=1}(2s_i+1)$, where s_i is the spin of particle i in the four-particle reaction (A1), the number of a^r_J's in (A5) (assuming for the moment only rotation invariance and no other symmetry) is x. Thus (A13) can be viewed as an x^2-by-x^2 matrix, in which x^2 experimental observables are related to the x^2 bilinear combinations of amplitudes by the x^4 coefficients $X^{r_1 r_I r_2 r_F}_{J_1 J_I J_2 J_F}$. Due to the judicious choice of the spin tensors and of the momentum tensors characterizing the kinematics, the matrix of X's contains mostly zeros. In fact, this large matrix is thus decomposed into small matrices along its main diagonal, thus breaking up the problem into many independent smaller problems. The observables within one of these small submatrices are said to form a subclass, and the bilinear products of amplitudes within one of these small submatrices are said to form a product set. For a reaction that cannot be further decomposed into smaller constituents along (A4) (i.e., for irreducible reactions) there are eight subclass–product-set pairs. For a reaction decomposable once into two irreducible constituents, there are 32 subclass–product-set pairs. In this latter case about 97% of the X matrix is thus zero. There are easy rules to determine which of the observables belong together in a subclass and which amplitude products belong together in a product set. In particular, what matters is the signature of the subclass or product set, which tells whether the number of l's, m's, or n's (respectively) contained in the arguments of the observables, or in the terms of the M matrix belonging to the two amplitudes in the bilinear products, is even (e) or odd (o). Thus there are eight signatures: *eee*, *eeo*, *eoe*, *oee*, *eoo*, *oeo*, *ooe*, and *ooo*. For once-decomposable reactions the signatures of both constituents matter, but the 64 different pairs of signatures are grouped pairwise into 32 different sets.

The recognition of the subclass–product-set structure greatly simplifies the relationship between the observables and the amplitudes, and hence facilitates the study of tests of conservation laws and of specific dynamical models, as well as of the value of a new proposed experiment. More details on these topics can be found in the already published papers.[10]

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Electron-Positron Annihilation in Stagnant Field Theories*

A. Zee†

The Rockefeller University, New York, New York 10021

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We discuss the ratio $R(s) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in stagnant (asymptotically free) field theories. The leading correction to the scaling limit is determined.

There has been a great deal of interest recently in stagnant (asymptotically free) field theories.[1-5] These are field theories in which the origin in the coupling-constant plane is ultraviolet-stable.[6,7] Indeed, there are indications[8,9] that if Bjorken's scaling phenomenon[10] is to be understood in the framework of field theory at all the field theory would have to be stagnant. It appears now that the only[11] stagnant theory is the non-Abelian gauge theory of Yang and Mills.[12]

In this paper we would like to discuss the annihilation process $e^+e^- \to$ "γ" $\to$ any hadrons within[13] the framework of local field theories. The appropriate object to study is the vacuum polarization tensor

$$\pi_{\mu\nu}(q) \equiv -i \int d^4x\, e^{iqx}\langle 0|\, T^*J_\mu(x)J_\nu(0)\,|0\rangle$$
$$\equiv (q_\mu q_\nu - g_{\mu\nu}q^2)\pi(q^2)\,,$$

where J_μ denotes the electromagnetic current. The total annihilation cross section σ_T is given by the absorptive part of π:

$$\sigma_T(s) \equiv (32\pi^2\alpha^2/s)\,\text{Abs}\pi(s)\,.$$

It is customary to consider the ratio

$$R(s) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)\,.$$

We will discuss this process in the language of the Callan-Symanzik equation.[14] The discussion differs from the usual analysis of the asymptotic behavior of Green's functions in two important respects.

(A) The vacuum polarization function is subtractively renormalizable[15] while Green's functions are multiplicatively renormalizable. This leads to an inhomogeneous Callan-Symanzik equation in the asymptotic region rather than a homogeneous one as is appropriate for Green's functions.

(B) The experimentally measured total annihilation cross section is the absorptive part of π.

We will be concerned with the consequences of these two facts.

For definiteness let us consider a Yang-Mills model[16] of three quark triplets with a global symmetry $SU(3)\times SU(3)'$ and a gauge symmetry $SU(3)'$. The eight gauge bosons are massless at the Lagrangian level but are presumed to become massive due to some miraculous nonperturbative[17] mechanisms (which are not understood at present). The electromagnetic current is an $SU(3)'$ singlet:

$$J_\mu = \sum_{a=1}^{3} \overline{q}_a \gamma_\mu Q q_a\,.$$

Q is some charge matrix. We treat electromagnetism only to lowest order but strong interaction to all orders.[17] The unrenormalized vacuum polarization function is a function of four variables: $\pi_0(q/m, \mu/m, \Lambda/m, g)$. Here m is the quark mass, Λ is the cutoff, and g is the Yang-Mills coupling constant. The massless gauge boson propagator has to be renormalized at some arbitrary mo-