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Experimental Suppressions of Resonance Couplings as Gaugelike Conditions from Dual Resonance Models*

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The paper deals with the observed phenomenon of dynamical suppression of a number of resonance couplings [$f'(1520)\pi\pi$, $K^{**}(1420)K\eta$, etc.], new instances of which keep appearing as data accumulate. A few of such effects have been conventionally understood in terms of the quark model, which is however inadequate to account for all of them. We consider a specific dual-resonance model, which is suggested by the approximate linearity in the (s,t,u) plane observed for the zero trajectories of certain two-body amplitudes. Relevant experimental evidence concerning zero behavior has been already discussed elsewhere; we present here a short and updated review of it. We show that such a dual model predicts in a natural way many of the observed coupling suppressions, and possibly—once generalized—all of them. It also accounts partially, in an analogous manner, for the decoupling of exotic resonances. The way in which the dual constraints of the model and the resonance decouplings are interrelated exhibits a structural similarity with the well-known interconnection between charge conservation and decoupling of longitudinal photons in electromagnetic interactions. The model is supposed to apply in its present form only to amplitudes in which linearity of zeros is observed, but there are elements to argue that dual constraints of the same nature are active in any hadronic amplitude. Central in obtaining the results is the interplay of the dual model and unitary symmetries, which is much stronger than in conventional dual models, where such symmetries intervene only externally through Chan-Paton factors.

I. INTRODUCTION

Since the original proposal of duality[1] based on features exhibited by the early Saclay πN phase shifts of 1965[2] the experimental state of two-body processes at low and intermediate energies has witnessed enormous improvements, both in accuracy and range covered by data. In spite of that, the evolution of dual models in the meantime has been guided very little by new fresh experimental input, following an independent and increasingly more sophisticated course.[3,4] The many difficulties faced by these models, though, have not made further inspiration from data unnecessary.

It is the purpose of this paper to point out that certain effects observed experimentally can be associated with characteristic features of dual-resonance models, and provide concrete indications about how to interpret and develop such models. We have already called attention in previous publications (see especially Ref. 5) to the approximate linearity of dynamical zeros in the plane of s, t, and u variables observable in some hadronic amplitudes. New data and analyses appearing in the meantime have strengthened further the evidence presented there. The focus of this paper is on a specific dual resonance model dictated by this property, which is supposed to apply to amplitudes (with simple spin configurations) in which the property is observed to hold—scattering of two pseudoscalar mesons, and invariant amplitude $A(s, t)$ in meson-baryon two-body processes. The model is expected to provide a sensible description of the couplings of leading resonances appearing in these amplitudes. The dual constraints of the model are particularly tight and this fact is responsible for a certain quality of nonlinearity of it (Sec. III). Because of such a nonlinearity the interplay of the model and of SU(2) [or SU(3)] symmetry leads in a natural way to the vanishing of some resonance couplings, which in all the cases met so far are observed to be actually suppressed experimentally. Even decouplings of exotic resonances are partially accounted for in this manner. The way in which the dual constraints of the model and the decouplings are interrelated happens to have a structural similarity with the interconnection existing in electromagnetic interactions between the coupling-constant relations implied by the conservation of the electric charge and the decoupling of longitudinal photons associated with gauge invariance. Some of the mentioned effects of coupling suppressions have been conventionally understood in the past by means of the quark model. New effects of this type keep emerging as data accumulate (see Table I), and while for several of them there is no foreseeable interpretation in terms of the quark model others appear to be directly in conflict with it. The dual resonance model that we consider has, as a matter of fact, a number of points of contact with the quark model—as well as points of difference. The model appears to have interconnections with unitary symmetries too. This is not altogether surprising, recalling previous theoretical work concerning duality bootstraps. Such interconnections, though, are considerably more impressive in the specific dual model under study. A general point that we believe emerges from the paper is that the interplay of dual-resonance constraints and unitary symmetries is considerably more articulated than what appears in its conventional description involving Chan-Paton factors.[6] Although the model is supposed to apply in its present form only to a few amplitudes with simple spin configurations, there are elements which encourage us to believe that more general constraints of the same nature operate in any hadronic amplitude.

Section II contains an updated review of the experimental evidence concerning linear behavior of amplitude zeros. In Sec. III the dual resonance model ensuing from the requirement of linearity of zeros is presented and discussed. Section IV deals with a number of applications of the model. In Sec. V the interconnections of the model with unitary symmetries and the quark model are examined. Section VI deals with the similarity between the phenomenon of suppression of resonance couplings and the decoupling of longitudinal photons in electromagnetic interactions. Section VII contains a summary and the conclusions. A first quick perusal can be limited to Secs. III, IV, VI and VII.

II. LINEARITY OF AMPLITUDE ZEROS

We have extensively discussed in previous publications (see especially Ref. 5) experimental evidence concerning regularities in the behavior of amplitude zeros in two-body processes. This section serves the twofold purpose of presenting new evidence which has appeared in the meantime, especially in connection with $\pi\pi$ and $K\pi$ scatterings, and of providing a short and self-consistent exposition of some of the relevant experimental facts, in order to clarify the motivation of the dual model for resonance couplings presented in Sec. III.

A. Meson-Baryon Two-Body Processes

$K^- p \to \bar{K}^0 n$ is a process particularly suited to illustrate the existence and the relevance of regularities in the behavior of amplitude zeros at low energy. At all energies its angular distribution (Fig. 9) exhibits dips at $u \simeq 0, -0.8, -1.7$ $(\text{GeV}/c)^2$, when these values are physically accessible. When the same values of u correspond to the forward direction the forward peak is observed to leave the place to a turnover. Conventional Breit-Wigner plus background fits to these data are of course possible. But this approach would not explain the constancy in u of the dips, and their apparent insensitivity to the boundary of the physical region. The dips cannot even be considered as low-energy propagations of high-energy dips related to u-channel Regge exchanges, since the u channel is exotic (Z^*) in this process.

Lacking conventional explanations for what appears to be a remarkable and simple fact, one is prompted to look for new types of understanding. One is offered by the zero trajectories generated in the unphysical region, at the intersections of resonances contributing to the s and t channels (Fig. 2). Near an intersection like that the amplitude $F(s, t)$ can be expanded

$$(s-M_s^2)(t-M_t^2)F(s,t) = C + r_s(t-M_t^2) + r_t(s-M_s^2) + O(\cdots), \tag{1}$$

where M_s and M_t are the masses of the two poles. The constant term C originates unwanted singularities in the residues of the s- and t-channel poles, which must be regular polynomials in the remaining Mandelstam variables. Therefore it must vanish. This fact implies that $F(s, t)$, locally, is zero along a line whose orientation is determined by the ratio of r_s and r_t, which are the values of the s- and t-channel resonance residues, respectively, at the intersection (e.g., if $r_s = r_t$, the local orientation of the zero trajectory is at constant u). Let us now make the assumption that zero trajectories generated in this way move approximately as straight lines in the real (s,t,u) plane. In order to have proper locations for the zeros of resonance residues, and since slopes of Regge trajectories are equal to $\simeq 1$ GeV^{-2} in both s and t channels, the linear trajectories of zeros must then move at constant u and with a spacing among them of $\Delta u \simeq 1$ GeV^2 (Fig. 2). This corresponds exactly to the observed behavior of dips in $K^- p \to \bar{K}^0 n$ (Fig. 1).[7,8]

The schematic pattern of resonances and zeros of Fig. 2 coincides with that resulting from the Veneziano formula[9] $V(s,t) = \Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))/\Gamma(1-\alpha(s)-\alpha(t))$. From this is clear that the idea of linear propagation of amplitude zeros is naturally associated with dual resonance models. Such models however, as they are commonly

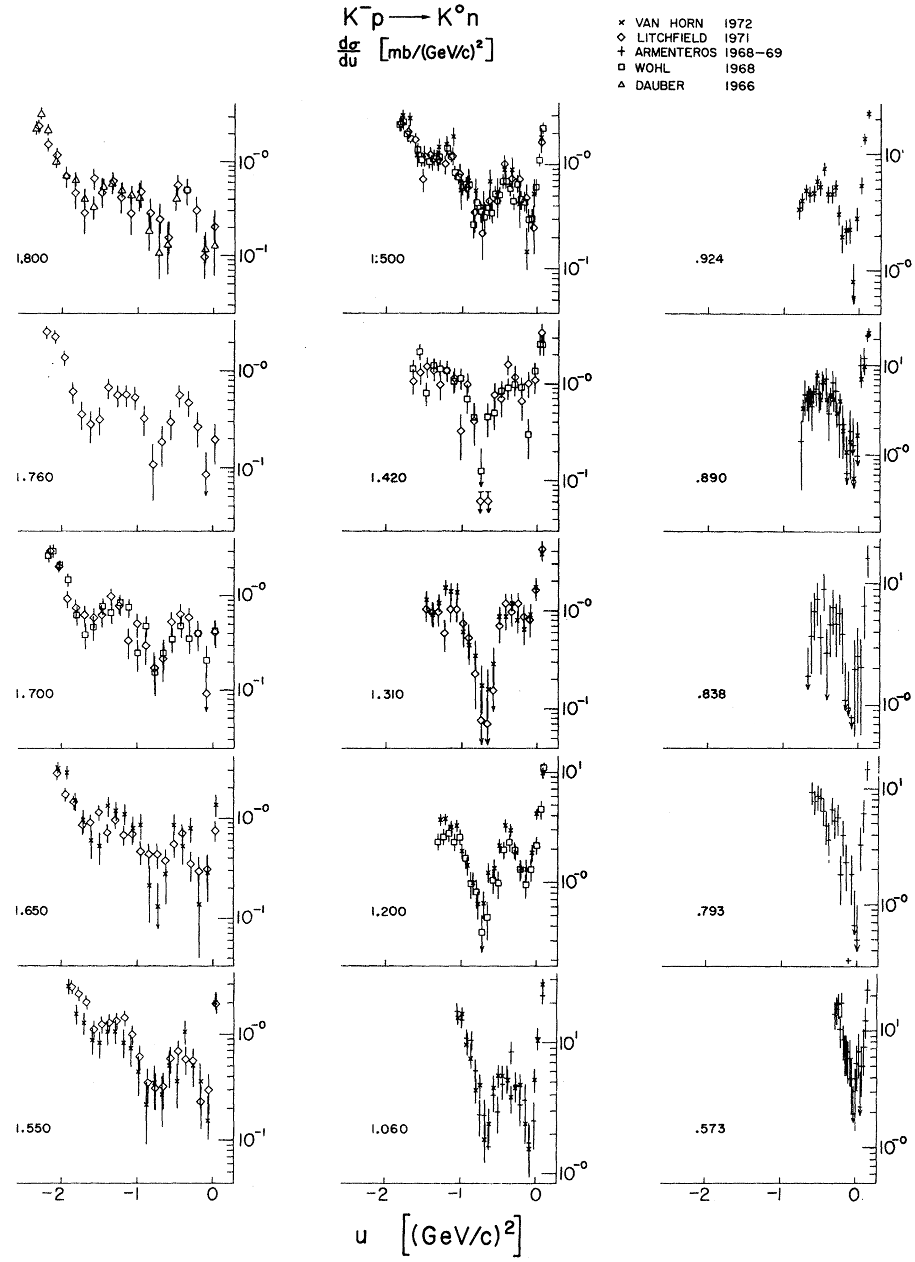

FIG. 1. Angular distributions of $K^-p \to \bar{K}^0 n$ at low laboratory momenta (indicated in GeV/c in each frame) plotted versus u. The figure shows that such distributions exhibit dips at $u \simeq 0$, -0.8, and -1.7 (GeV/c)^2. See Ref. 82 for data.

conceived at present, do not necessarily have this property. This is especially true when dealing with processes having resonances in all the three channels s, t, and u. Such processes must be described by superpositions of Veneziano terms and, by summing, the property of having linear zeros, originally shared by the single Veneziano terms, is generally lost. We shall discuss this point at length in Sec. III. What should be clear now is that, as far as $K^-p \to \bar{K}^0 n$ is concerned, the observed regularity of its dips might be accounted for by a rather generic dual model. In order to find evidence calling for linearity of zeros as an independent requirement one has to consider processes with no exotic channels. The pattern of zeros which should be observed in these processes are those indicated by $\text{II}^\pm$ and $\text{III}^\pm$ in Fig. 3. These patterns together with pattern I, which corresponds to the simple Veneziano formula, constitute all the theoretically possible patterns of

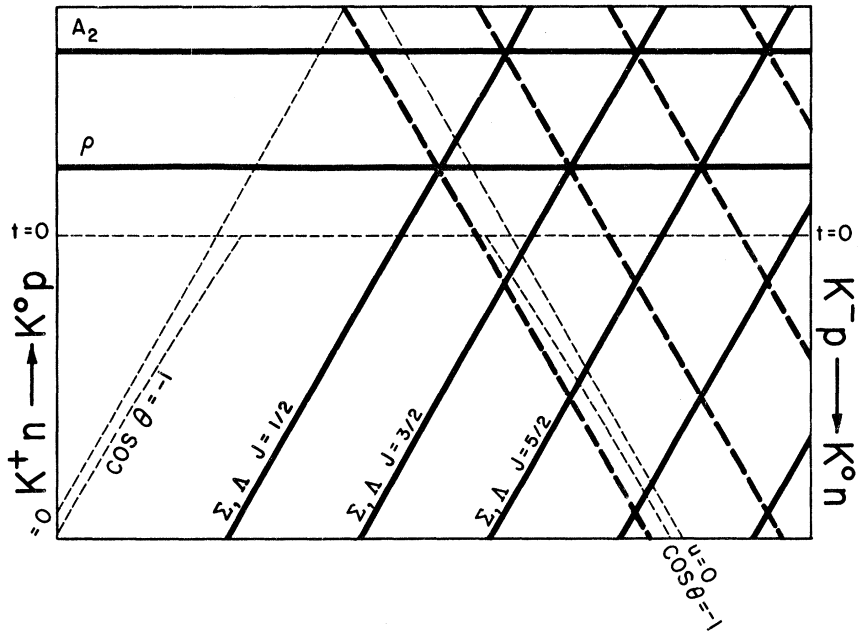

FIG. 2. Schematic plot in the (s,t,u) plane of the resonance and zero structure of $K^-p \to \bar{K}^0 n$. Solid lines represent resonances, heavy dashed lines zeros.

linear zeros, once resonances are required to be ordered in Regge families. Each pattern of zeros is associated with a different pattern of resonances, and therefore a rough knowledge of the type of resonance spectrum showing up in a process determines uniquely the pattern of zeros expected for it.

Pattern II^+ should apply to $K^-p \to \pi^0\Lambda$ (Fig. 4). A unique and interesting feature of this pattern is the presence of zeros at constant $s-t$, with a spacing of ~2 GeV^2 in this variable. Corresponding effects are actually visible in the differential cross section of $K^-p\to\pi^0\Lambda$ at $s-t\simeq 5$ and 3 GeV^2, and in the differential cross section of $\pi^-p\to K^0\Lambda$ at $\simeq 1$ GeV^2, in the same variable (see Ref. 5). The effects at $s-t\simeq 5$ and 3 GeV^2 appear more neatly in the invariant amplitude $A(s,t)$ [with the usual decomposition $T=i\bar{u}(-A(s,t)+i\gamma\cdot QB(s,t))u$] reconstructed from phase shifts, as shown in Fig. 5. In this figure, since also dips at constant u are expected, we plot distributions at constant u and $s-t$ in order to single out structure at fixed $s-t$ and u, respectively. $N(s)$, in the figure, is the integral over $-1\leq\cos\theta\leq+1$ of $|A(s,t)|^2$. The removal of this factor from $|A(s,t)|^2$ is aimed at eliminating modulation caused by resonances, since the distributions are not at constant momentum. Essentially the same structure is observable however also without removing $N(s)$. Besides the effects in the $s-t$ distributions, in which we are especially interested, also a dip at $u\simeq -0.2$ $(\mathrm{GeV}/c)^2$ is visible—low-energy continuation of the similar structure observed at high energy.

Pattern III^- is expected to apply to $\pi^\pm p$ elastic scatterings. Zeros at constant t and constant u with ~2 $(\mathrm{GeV}/c)^2$ spacing are consequently expected to appear in these two processes. Figure 6 shows distributions versus u of the real and imaginary parts of the amplitude $A(s,t)$ of $\pi^+p\to\pi^+p$. In all these distributions a zero at constant $u\simeq -0.4$ $(\mathrm{GeV}/c)^2$ can be neatly observed. The distributions are at fixed t in order to remove the effects of existing zeros, which stay constant in this variable. The zero, which at higher energies is related to the well-known dip at $u\simeq -0.2$ $(\mathrm{GeV}/c)^2$, is clearly visible also at $t=0$, when it leaves the physical region. If linearly extrapolated into the unphysical region, it crosses the intersection of $\rho(765)$ and $\Delta(1235)$, which occurs at $u\simeq -0.3$ $(\mathrm{GeV}/c)^2$ (Fig. 7). One can follow in a similar way (see Ref. 5) zeros at constant t [$t\simeq -0.6$ and -2.8 $(\mathrm{GeV}/c)^2$], whose extrapolations into the unphysical region do also cross intersections of important poles, as shown in Fig. 7.

The wealth of data and the good quality of the phase shifts available for πN scattering make it possible to clarify some points concerning zero regularities. From the comparison of $\pi^+p\to\pi^+p$, $\pi^-p\to\pi^-p$, and $\pi^-p\to\pi^0n$ it appears clear that linearity of zeros holds to the extent to which the spectrum of prominent resonances approaches the

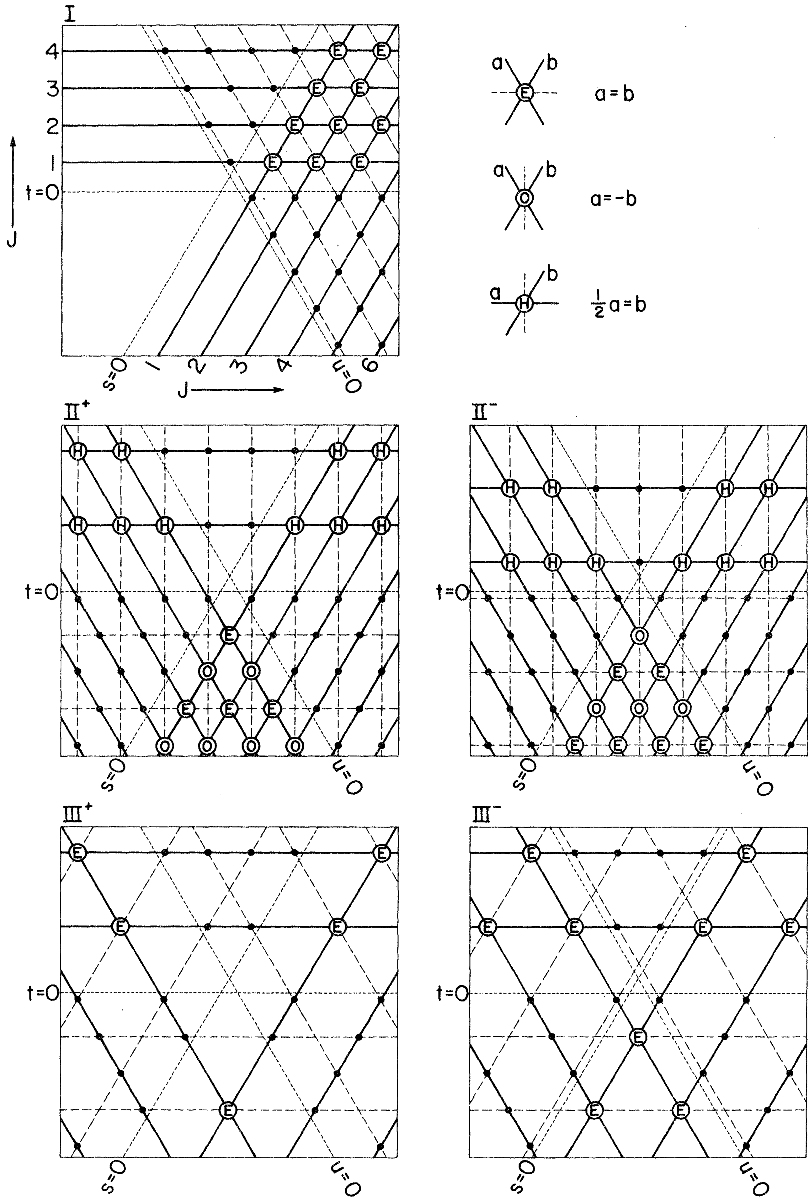

FIG. 3. Patterns of Regge families of poles (solid lines) and linear zeros (heavy dashed lines) in the (s,t,u) plane. Black points represent zeros of resonance residues; their number on a resonance line is equal to the maximum angular momentum J of the corresponding resonance tower. In the top-right corner relations holding between resonance residues at resonance intersections—labelled by E, O, and H according to the orientation of the local zero—are reported.

stylized resonance patterns of Fig. 3. In $\pi^+p\to\pi^+p$, where the Regge family of Δ_δ resonances dominates [$\Delta_{3/2^+}(1235)$, $\Delta_{7/2^+}(1950)$, $\Delta_{11/2^+}(2420)$], the behavior of zeros is very regular and close to what expected. In $\pi^-p\to\pi^-p$ and $\pi^-p\to\pi^0n$, where leading resonances having equal angular momenta but substantially shifted in mass are present [e.g., $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$], in disagreement with the mass degeneracy conditions of the resonance patterns of Fig. 3, strong perturbations in the behavior of zeros are observed. This interconnection is not difficult to understand after recalling that zero trajectories are also responsible for the Legendre zeros of resonance residues (Fig. 3). Even in the best cases, local deviations of zero positions within ~0.2–0.3 GeV^2 from those expected assuming exact linearity are relatively common. They are comparable in magnitude to the observed deviations of mass squares of leading resonances from the values expected

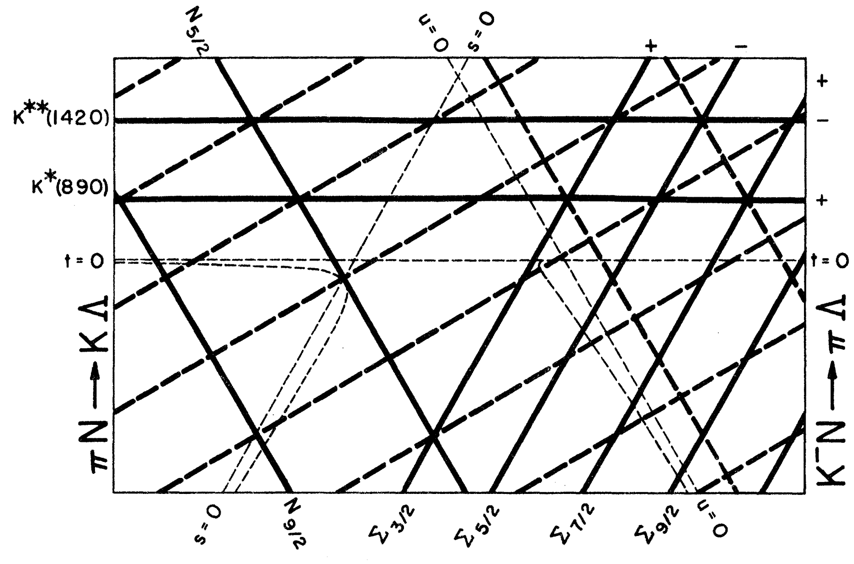

FIG. 4. Pattern of resonances and zeros in the (s, t, u) plane applying to $K^-p \to \pi^0\Lambda$.

from exact linearity and universal slope for Regge trajectories.

A close study of the resonance and zero structure of $\pi^{\pm}p$ elastic scattering reveals that the interlocking of zeros and intersections of leading resonances, according to pattern III$^-$ of Fig. 3, can occur systematically only in the invariant amplitude $A(s,t)$ and not in $B(s,t)$. For $A(s,t)$, besides other reasons, this systematic interlocking is possible because the f^0 resonance, according to experimental indications, does not contribute to this amplitude. $A(s,t)$, indeed, coincides at high energy and fixed t with the s-channel helicity-flip amplitude $f(\Delta\lambda_s = 1)$, and data for $\pi^{\pm}p$ and pp elastic polarizations and spin-rotation parameters indicate that the coupling of f^0 to $\bar{N}N$ with s-channel helicity flip is compatible with zero.[10] As one can realize from Fig. 7, in order to make the interlocking possible, also $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$ in $\pi^-p \to \pi^-p$ (together with their Regge recurrences) must disappear. These two resonances give contributions of opposite signs to $A(s,t)$, and their magnitudes turn out to be approximately equal experimentally. If the two resonances had the same mass, therefore, they would cancel each other identically in $A(s,t)$ since their residues in this amplitude have the same linear shape. These experimental facts, which at the moment may look as curious accidents, will be discussed at length in Sec. IV. For the moment we remark that, as a matter of fact, the resonance and zero structure of $A(s,t)$ in $\pi^{\pm}p$ elastic scattering is well described schematically by one of the patterns (III$^-$) of Fig. 3, whereas this is not true for $B(s,t)$. The idea that the pat-

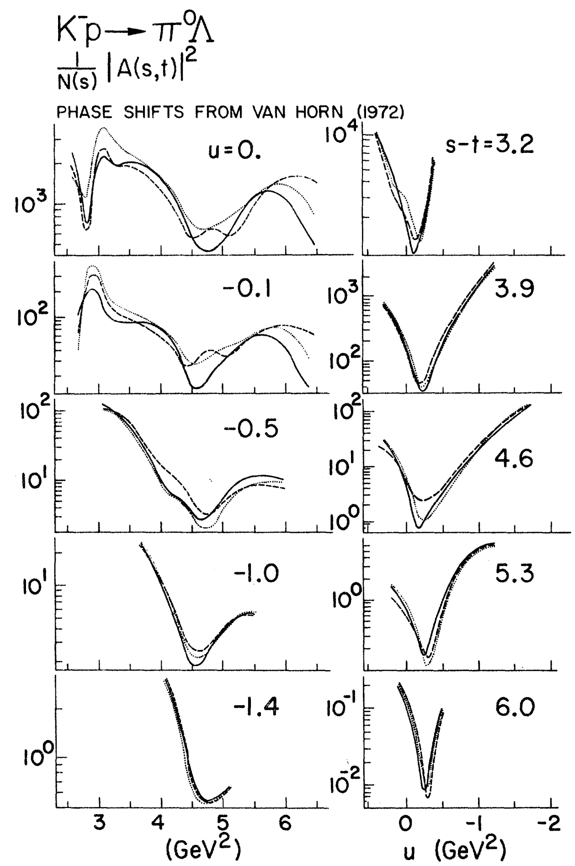

FIG. 5. Distributions of the quantity $|A(s,t)|^2/N(s)$ for $K^-p \to \pi^0\Lambda$, where $N(s)$ is defined in the text, versus the variables $s-t$ and u, at constant values of u and $s-t$, respectively. The three curves correspond to different phase-shift solutions of Ref. 73. The figure shows the presence of dips at $s-t \simeq 2.7$, 4.7, and $u \simeq -0.2$ GeV^2.

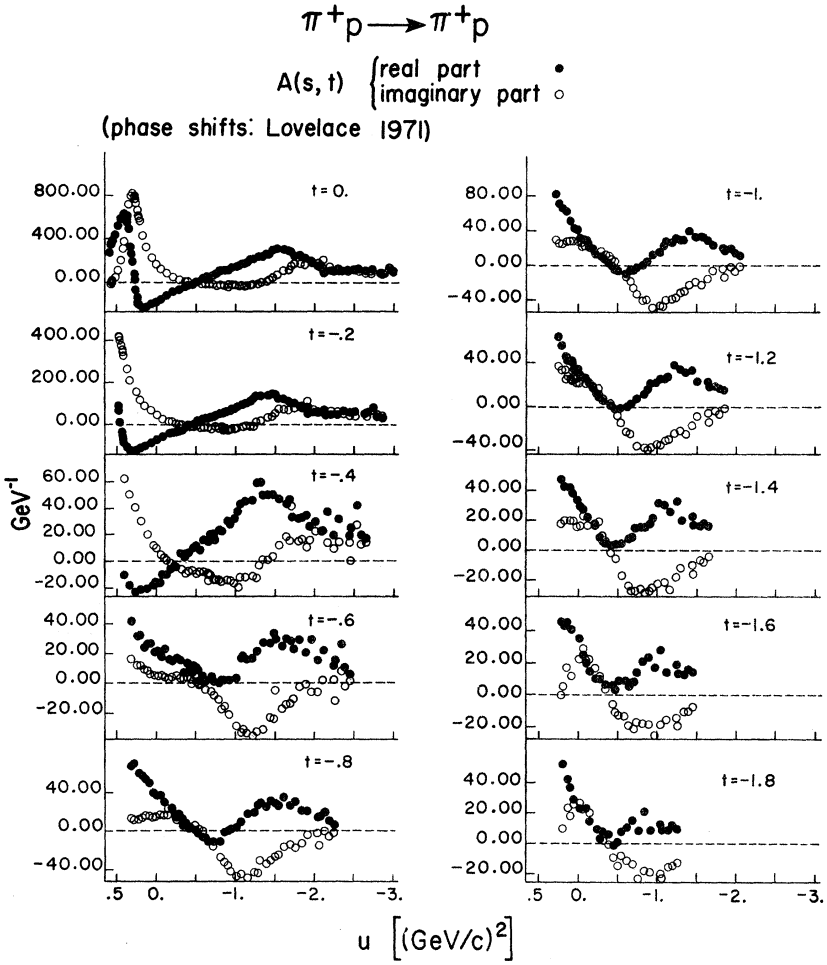

FIG. 6. Real and imaginary parts of the invariant amplitude $A(s,t)$ of $\pi^+p \rightarrow \pi^+p$ plotted versus u at constant values of t (phase shifts from Ref. 32). The figure shows the existence of a zero at $u \simeq -0.4$ (GeV/c)^2. At $t = -1.8$ (GeV/c)^2 the laboratory momentum at which the zero is observed is approximately 1.7 GeV/c.

terns of Fig. 3 are physically significant only for $A(s,t)$ and not necessarily for $B(s,t)$ is supported also by $K^-p \rightarrow K^-p$ phase shifts. These indicate the existence of a family of zeros at constant u in $A(s,t)$, as in $K^-p \rightarrow \bar{K}^0 n$, but a more confused situation for $B(s,t)$ (see Ref. 5). One can see, more in general, that with any reasonable resonance spectrum the positions of the zeros of the resonance residues in $B(s,t)$ (uniquely determined by the mass, spin, and parity of the resonances) would not fit into the diagrams of Fig. 3. Another phenomenological fact relevant in this context is that at high energy $f(\Delta\lambda_s = 1)$ is observed to have zeros corresponding to wrong-signature nonsense (WSNS) values of $\alpha(t)$, whereas this is not true for $f(\Delta\lambda_s = 0)$.[11,12] In the diagrams of Fig. 3 the zeros at high energy have WSNS positions. Therefore even from this point of view such diagrams appear appropriate for a description of $A(s,t)$, which coincides with $f(\Delta\lambda_s = 1)$ at high energy and fixed t, but are inadequate for $B(s,t)$, which corresponds to a combination [namely, the difference of $f(\Delta\lambda_s = 1)$ and $f(\Delta\lambda_s = 0)$].

B. $\pi\pi$ And $K\pi$ Scatterings

The data appearing in the last one or two years have qualitatively improved the experimental state of these processes both in accuracy and range of mass covered. The spherical harmonic moments $\langle Y_l^m \rangle$, which are commonly used for the description of these data, exhibit considerable structure, which is particularly marked in $\langle Y_1^m \rangle$. Some of the available data for this quantity in a number of $\pi\pi$ and $K\pi$ processes are plotted in Fig. 8. For each process the expected pattern of zeros is drawn on the left.[13] Effects from zeros are expected to show up very visibly in $\langle Y_1^0 \rangle$. In the mass range of interest this quantity is an approx-

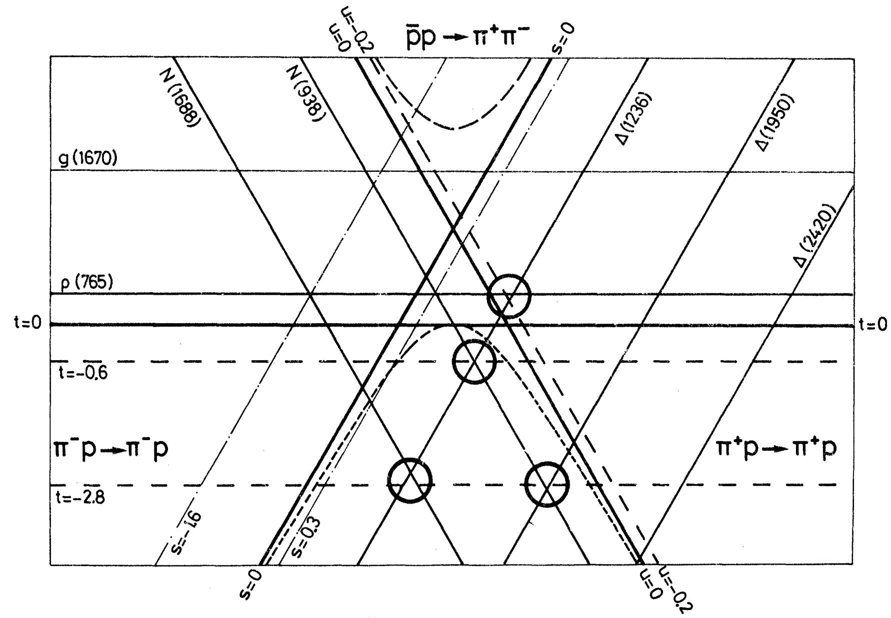

FIG. 7. Plot in the (s, t, u) plane of prominent zeros and resonances in $\pi^{\pm}p$ elastic scattering, showing the coincidence of zero positions with resonance intersections.

imate measure of the difference between the population of events in the forward and the backward peaks. When zeros propagate from the unphysical region into the physical region, they have necessarily to cross forward and backward peaks and then to suppress them. The entry of a zero into the physical region through the forward direction is then expected to produce a steep negative variation in $\langle Y_1^0 \rangle$, as mass increases, while the entry of a zero through the backward direction should produce a corresponding positive variation. The patterns of Fig. 3 give definite predictions for the approximate locations of the effects and for their signs. The entries of zeros into the physical region in the mass range of interest are denoted by heavy dots in the diagrams of Fig. 8. According to them, near ~ 1 GeV a zero should enter through the forward direction in $\pi^+\pi^- \to \pi^+\pi^-$, $K^+\pi^- \to K^+\pi^-$, $K^+\pi^- \to K^0\pi^0$, whereas a zero should enter through the backward direction in $\pi^-\pi^0 \to \pi^-\pi^0$. One then expects to observe near ~1 GeV negative jumps in $\langle Y_1^0 \rangle$ in the first three processes and a positive variation in $\langle Y_1^0 \rangle$ in the fourth. This is exactly what is observed in the data plotted in the right-hand frames.

In $K^+\pi^- \to K^0\pi^0$, $\langle Y_1^0 \rangle$ becomes considerably negative after the effect, whereas in $\pi^+\pi^- \to \pi^+\pi^-$ and $K^+\pi^- \to K^+\pi^-$, which are elastic processes, $\langle Y_1^0 \rangle$ does not reach very negative values. This is understandable, since unitarity does always require the existence of a finite forward peak in these processes. When a zero crosses the forward direction of an elastic process, it cannot remain exactly real but it must acquire a finite imaginary part. This does not prevent the zero from remaining near the forward peak and depressing it. Phase-shift analyses of $\pi^+\pi^- \to \pi^+\pi^-$ explicitly confirm [14,15] the existence of such a zero entering the physical region at ~1 GeV, as shown in Fig. 9.[16]

According to the diagrams of Fig. 8 at higher masses other zeros should enter the physical region. These zeros originate from the intersections of wider resonances, and therefore their effects are expected to be weaker, though still of the same type of those discussed before. In $\pi^+\pi^- \to \pi^+\pi^-$ a zero is expected to enter ~1 GeV^2 above the entry of the previous one, i.e., at about ~1.5 GeV. High-statistics data for $\pi^+\pi^- \to \pi^+\pi^-$ at 17.2 GeV/c[17] clearly exhibit the effect[18]—a flattening of the forward peak at ~1.5 GeV similar to that observed at ~1 GeV (Fig. 10). Also for $K^+\pi \to K^+\pi^-$, which has a zero pattern similar to that of $\pi^+\pi^- \to \pi^+\pi^-$, available data, taken at 12 GeV/c,[19] show at ~1.5 GeV the presence of an effect weaker but similar to that observable at ~1 GeV (Fig. 11).

The interpretation of the effect in $\pi^+\pi^- \to \pi^+\pi^-$ at ~1 GeV deserves a few more comments. This effect occurs at a value of the mass coinciding with that of the $\bar{K}K$ threshold (980 MeV). The cross section for $\bar{K}K$ production rises very sharply at threshold, and it is natural to think of a connection through unitarity of this effect with the sharp structure observed in $\langle Y_1^0 \rangle$. Some authors

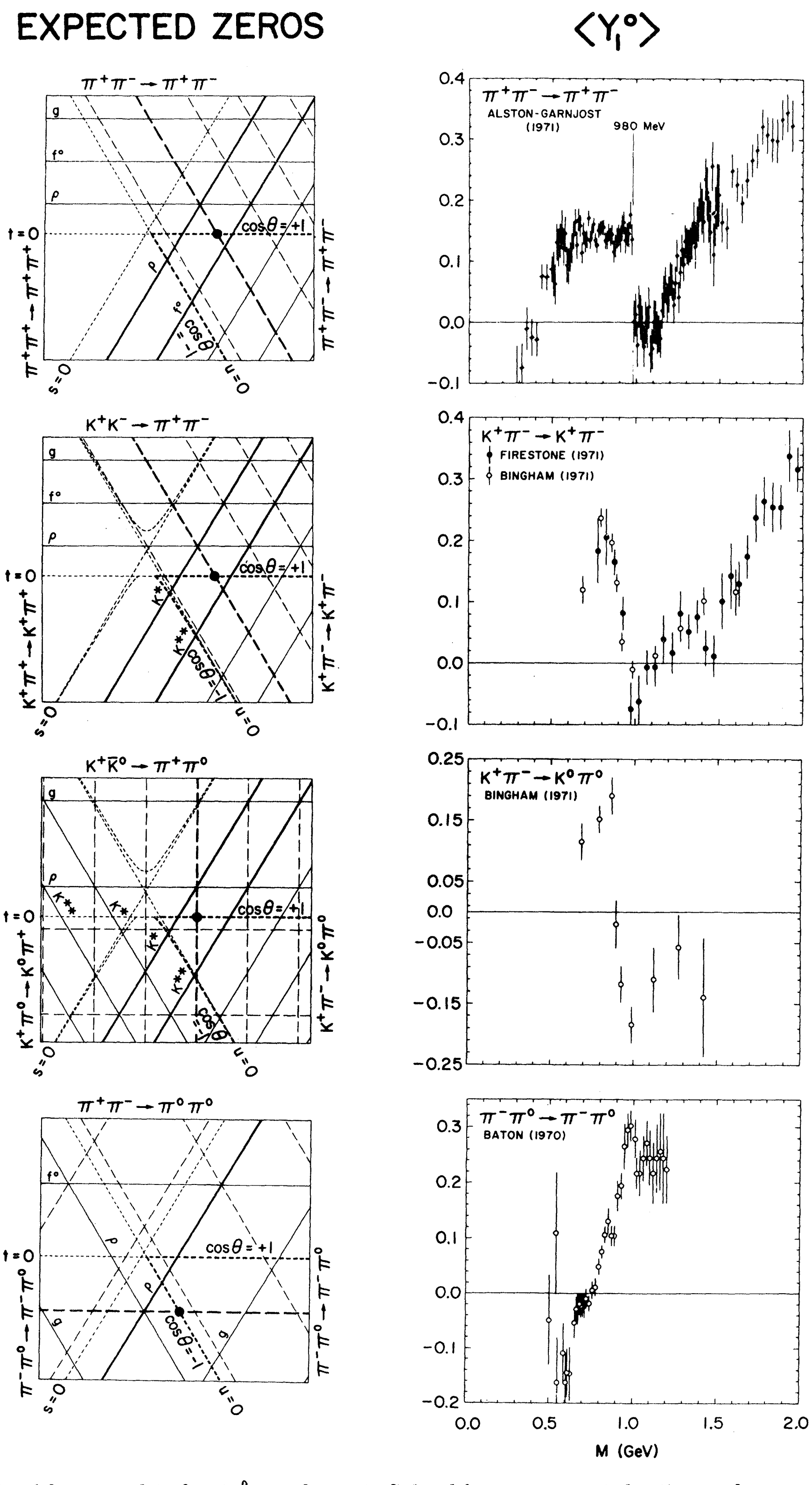

FIG. 8. Right-hand frames: data for $\langle Y_1^0 \rangle$, Ref. 83. Left-hand frames: expected patterns of zeros (dashed lines) in the (s, t, u) plane. The entries of zeros into the physical region (heavy dots) are interpreted as responsible for the steep variations observable in $\langle Y_1^0 \rangle$ near $\simeq 1$ GeV.

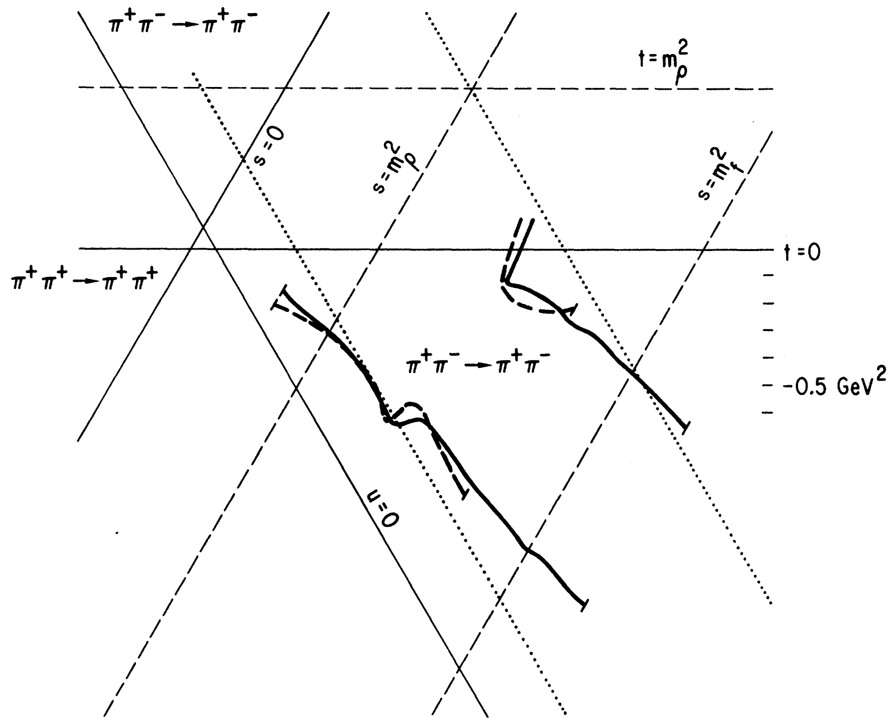

FIG. 9. Plot in the (s,t,u) plane of $\pi^+\pi^- \to \pi^+\pi^-$ of the zeros determined from the phase-shift analyses of Ref. 14 (dashed curve) and Ref. 15 (solid curve). Dotted lines indicate the approximate behavior expected for the zeros if exact linearity is assumed for them.

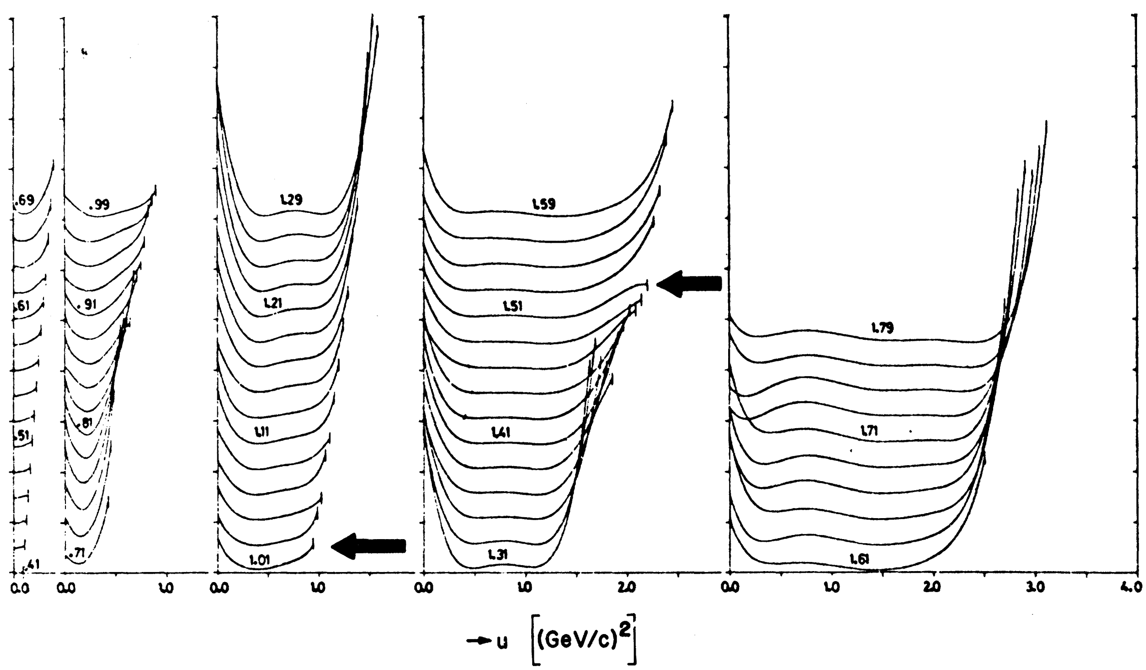

FIG. 10. $d\sigma/du$ for $\pi^+\pi^- \to \pi^+\pi^-$ as a function of u. Each curve represents a $\pi^+\pi^-$ mass bin of 20 MeV (the central value is written in the curve) and is displaced by $\frac{1}{2}$ scale unit. 1 scale unit$\simeq$ 20 000 events. Arrows indicate the flattening of the forward peak at $\simeq$ 1 and $\simeq$ 1.5 GeV (Ref. 17).

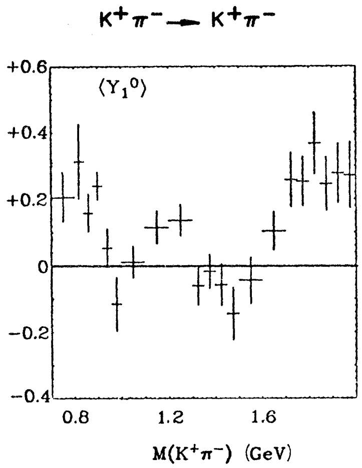

FIG. 11. $\langle Y_1^0\rangle$ for the $K^+\pi^-$ system produced in $K^+n \to K^+\pi^-p$ at 12 GeV/c, after extrapolation to the pion pole (Ref. 19). The figure shows the existence at $\simeq 1.4$ GeV of an effect less pronounced but similar to that observed at $\simeq 0.9$ GeV.

have seen incompatibility between this association and the type of understanding that we are discussing here, but we do not see any reason for that. Of course, the expectation that a certain pattern of resonances and zeros shows up in a process is presumed to be in accord with unitarity, otherwise the expectation would not make sense. Cases, like that in question, in which unitarity constraints have more visible effects, make no exception. There should not be, therefore, any conceptual difficulty in observing a zero, whose entry is expected to occur within a certain mass interval, to enter just at a mass value and in such a way so as to satisfy relevant unitarity constraints. What matters, as far as the present considerations are concerned, is that the type of effect expected does actually show up. Its quantitative details—e.g., its exact location and its sharpness—are beyond the natural scope of these considerations. It is worth mentioning again in this connection that a similar effect appears at ~1.5 GeV as expected, with no reasonable relation this time to threshold effects.[20,21]

C. Final Remarks and Conclusions of This Section

According to the experimental evidence discussed in this section the patterns of Fig. 3 appear able to provide a schematic description of the resonance and zero structure present in the scattering of two pseudoscalar mesons, and in the amplitude $A(s,t)$ of meson-baryon two-body processes. The amplitude $B(s,t)$ of these processes, on the contrary, does not appear to be representable in such a fashion. In the complete expression for the transition amplitude momentum components appear in front of $B(s,t)$, whereas they are not present in front of $A(s,t)$, which corresponds to a simpler spin configuration. We shall comment in Sec. III about the possible relevance of this fact for the zero properties of $B(s,t)$. We wish to mention here, however, that there is evidence to believe that some sort of zero regularities, though not specifically those of Fig. 3, exist also in amplitudes corresponding to complex spin configurations. As an example, Fig. 12 displays the Dalitz plot of $\bar{p}n \to \pi^-\pi^-\pi^+$ in flight ($p_{\text{lab}} \simeq 1.2$ GeV/c) resulting from the data of Ref. 22. In spite of the circumstance that in flight many partial waves contribute, a clean (alternate) hole structure is present in the Dalitz plot, the existence of which would not be possible without some systematic regularity of the zero trajectories of the several amplitudes involved (see Ref. 23).

We make a final comment about the general quality of the experimental effects discussed here. Data able to show these effects in a more dramatic way would surely be desirable. Most of the relevant data existing at present do not have high statistical accuracy—typically, in K^--initiated two-body processes, less than a thousand events per momentum setting. As for $\pi\pi$ and $K\pi$ scatterings, we are just beginning to have data accurate enough to allow this type of effects to emerge. Most of the existing data have been col-

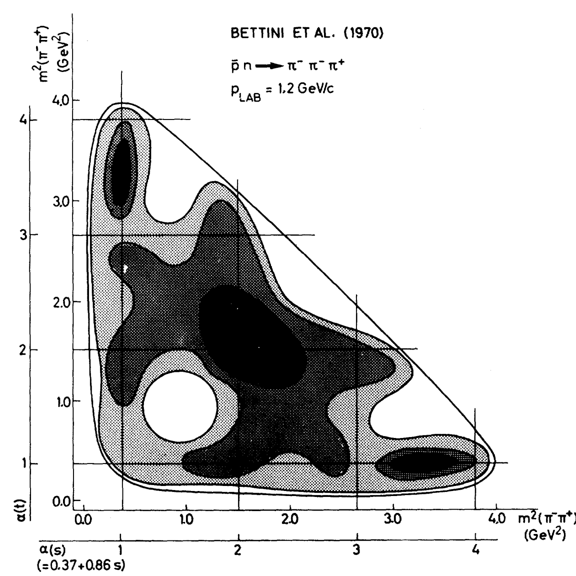

FIG. 12. Dalitz plot for $\bar{p}n \to \pi^-\pi^-\pi^+$ in flight from Ref. 22, exhibiting an alternate presence of holes in the frames formed by resonances.

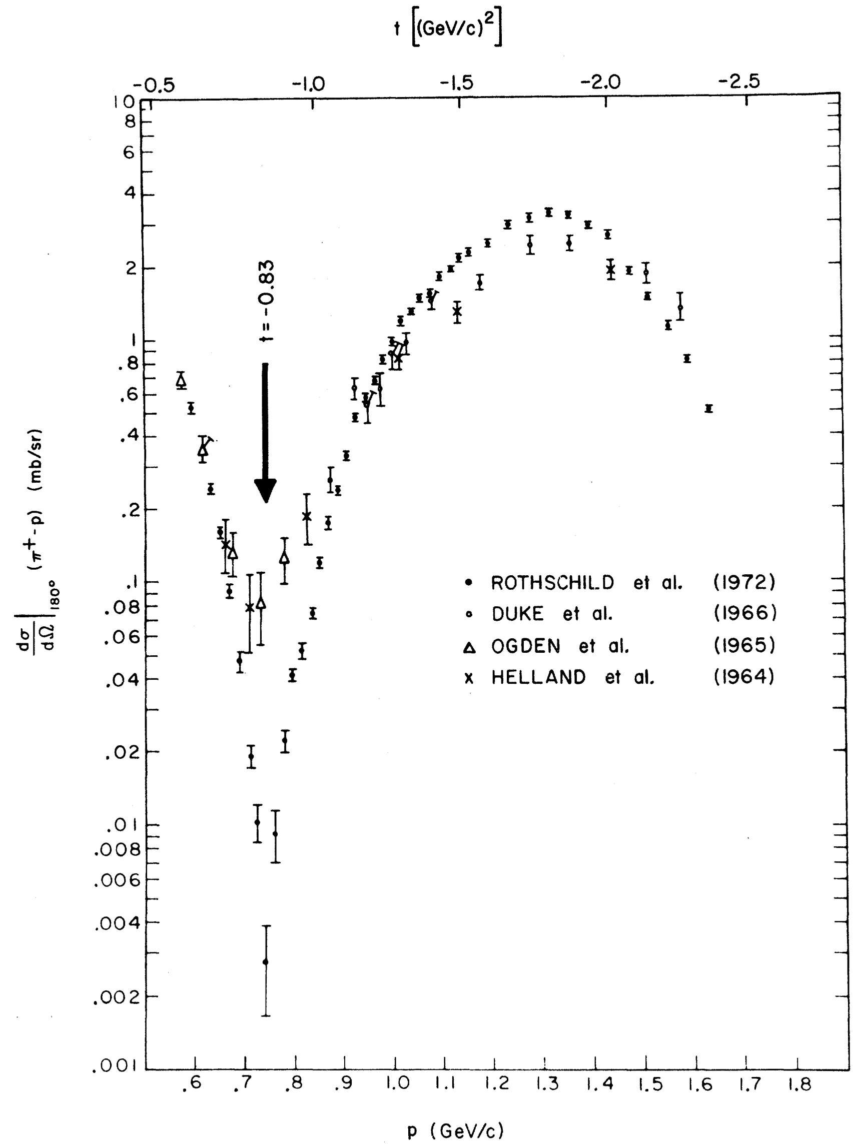

FIG. 13. Elastic differential cross section for $\pi^+p\to\pi^+p$ at 180° plotted versus the laboratory momentum (lower scale) and versus corresponding values of the squared momentum transfer t (upper scale). Data from Ref. 24. The figure shows the existence of a pronounced dip to be associated with the exit from the physical region of the zero at $t\simeq-0.8$ (GeV/c)^2.

lected for the purpose of studying resonance structure. In order to study reliably zeros, which produce lack of events instead of excess of events, higher statistics are obviously needed. An idea of the sharpness with which effects of this type may show up in data with good statistics and high resolution is given by Fig. 13, referring to the energy distribution of $\pi^+p\to\pi^+p$ at 180°. The dip at $p_{\text{lab}}=750$ MeV/c in this distribution corresponds to the exit from the physical region of the zero at $t\simeq -0.8$ (GeV/c)^2 (see Ref. 5). At 180° $d\sigma/d\Omega$ is described by a single amplitude, and therefore the effect of the zero cannot be smoothed here by other contributions. As one can see, the improved accuracy and resolution of the data of Rothschild *et al.*[24] have lowered the bottom of the dip, as observed in previous experiments, by almost two orders of magnitude.[25]

III. COUPLING-CONSTANT RELATIONS

Having linear zeros in an amplitude implies the holding of strict relations among the coupling constants of the resonances which contribute to it. The purpose of this section is to discuss how such coupling-constant relations emerge and what are their general characteristics.

The patterns in the (s,t,u) plane of Regge families of resonances and linear zeros shown in Fig. 3 allow for simple analytic representations of the Veneziano type[26,27,28]:

$$\begin{aligned} &\mathrm{I} = V(s,t)\,, \\ &\mathrm{II}^{\pm} = V(s,t) \pm V(u,t)\,, \\ &\mathrm{III}^{\pm} = V(s,t) + V(u,t) \pm V(s,u)\,, \\ &V(s,t) = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}\,, \\ &\alpha(s)+\alpha(t)+\alpha(u) = 1\,. \end{aligned} \tag{2}$$

As mentioned in Sec. II, the requirement of linearity of zeros is especially significant in processes in which no exotic channel is present. In a generic dual-resonance model there are no specific restrictions for the superpositions of Veneziano terms to use in describing such processes. According to Eq. (2), instead, only superpositions $\mathrm{II}^{\pm}$ and $\mathrm{III}^{\pm}$, in which all coefficients are equal in magnitude, are acceptable in such cases. In the following we shall refer to the model represented by Eq. (2) as the LZ (linear zero) model.

Even small deviations from the conditions of Eq. (2) generally cause qualitative changes in the behavior of zeros. In Fig. 14 is illustrated how zero trajectories in the superposition II^{+} change when one deviates from the supplementary condition $\alpha(s)+\alpha(t)+\alpha(u)=1$. This condition is responsible for the absence of odd-daughter resonances (like ρ').

The specific requirements of Eq. (2) lead to the emergence of definite relations between the couplings of resonances contributing to different channels (s, t, or u). One can realize in a direct way how prescriptions for the behavior of zeros are connected to coupling-constant relations of this type. As discussed in Sec. II, near the intersection in the (s,t,u) plane of two resonances, say, at $s=M_s^2$ and $t=M_t^2$, respectively, the following equation for the amplitude $F(s,t)$ is active:

$$(s-M_s^2)(t-M_t^2)\,F(s,t) = r_s\,(t-M_t^2) + r_t\,(s-M_s^2) + O(\ldots)\,, \tag{3}$$

where $O(\ldots)$ represents higher-order terms in $(s-M_s^2)$ and $(t-M_t^2)$. The quantities r_s and r_t are the numerical values at the intersection point of the residues of the s-channel and t-channel resonances, respectively, as can be seen by dividing the whole equation by $(s-M_s^2)(t-M_t^2)$. The local orientation of the zero trajectory which crosses the resonance intersection determines uniquely the ratio r_s/r_t. If, e.g., it is at constant u, then $r_s/r_t=1$ ($s+t+u=$ sum of the squares of the external masses). Therefore the prescriptions for the behavior of zeros contained in the diagrams of Fig. 3 determine the ratios of resonance residues at any of the resonance intersections appearing in the diagrams. As indicated in Fig. 3, top right, at the resonance intersections denoted by E, O, and H the two resonance residues must be equal, equal and opposite, and one equal to one-half of the other, respectively. It appears clear that in a generic dual model, in which no specific prescriptions for the behavior of zeros are contained, definite dynamical relations of this type are not required to exist, at least systematically.

A feature exhibited by the LZ model of Eq. (2) is that when vector and tensor exchanges coexist in a process, they must stay in a definite exchange-degenerate relationship. The idea of exchange degeneracy was originally motivated by the observed absence of exotic resonances.[29] In the model of Eq. (2) exchange degeneracy is required to hold even when the condition of absence of exotic states is not, at least directly, active. The holding of exchange degeneracy is tightly related to the fact that in the expressions of Eq. (2) the coefficients of the various terms are always

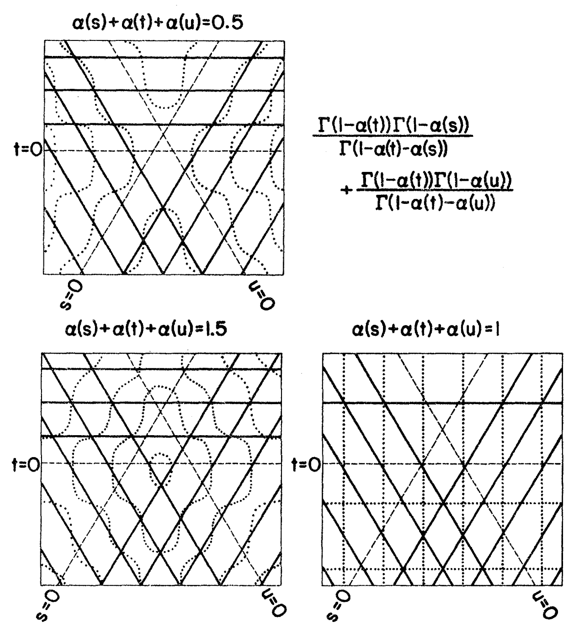

FIG. 14. Zero trajectories in the (s,t,u) plane of the analytic expression in the top-right frame for a number of values of $\alpha(s)+\alpha(t)+\alpha(u)$, illustrating the deviations from linear behavior of the zero trajectories when the supplementary condition is not satisfied.

equal in modulus.

Directly connected with the exchange-degeneracy conditions implicit in Eq. (2) is the basic *nonlinearity* character of the associated system of resonance coupling relations. A linear combination of I, $\mathrm{II}^{\pm}$, and $\mathrm{III}^{\pm}$ is not in general an expression still of this type. Only very specific linear combinations preserve the property of having coefficients all of the same magnitude. Therefore if one thinks of two hypothetical solutions of such a system of relations, each of them materializing into a specific assignment of expressions I, $\mathrm{II}^{\pm}$, and $\mathrm{III}^{\pm}$ to the various transition amplitudes, a linear combination of them is not in general another solution. This nonlinear character is the feature which mostly distinguishes the LZ dual model from conventional dual models. It is also the main feature responsible for such peculiar results as resonance coupling suppressions and local cancellations of resonance contributions, which will be discussed in Sec. IV.

The nonlinearity inherent in the model poses immediately the question of which amplitudes expressions I, $\mathrm{II}^{\pm}$, and $\mathrm{III}^{\pm}$ should apply to. In πN scattering, for instance, one can consider the amplitude $A(I=\frac{1}{2})$ with definite isospin $I=\frac{1}{2}$ in the direct channel. This amplitude can be written as

$$A(I=\tfrac{1}{2})=\tfrac{3}{2}A(\pi^- p\to\pi^- p)-\tfrac{1}{2}A(\pi^+ p\to\pi^+ p).$$

It is clear that if we wish $A(\pi^- p\to\pi^- p)$ and $A(\pi^+ p\to\pi^+ p)$ to be represented by one of the expressions of Eq. (2), $A(I=\frac{1}{2})$ cannot be represented in such a manner. In old phenomenological applications of dual models it was felt that $A(I=\frac{1}{2})$ ought to have a simpler representation than $A(\pi^- p\to\pi^- p)$, which includes both $I=\frac{1}{2}$ and $I=\frac{3}{2}$. The property of linearity of zeros, however, is observed in amplitudes describing *actual physical processes*. According to data, therefore, expressions I, $\mathrm{II}^{\pm}$, and $\mathrm{III}^{\pm}$ should be applied to such amplitudes—like those of $\pi^- p$ and $\pi^+ p$ elastic scatterings—and not to superpositions of them—like $A(I=\frac{1}{2})$—although these might appear simpler from certain theoretical points of view.

Again because of its nonlinearity the consistency of the model with unitary symmetries—either SU(2) or SU(3)—is highly nontrivial. Such symmetries, indeed, imply the existence of linear relations among amplitudes corresponding to different physical processes, and there is no *a priori* reason why it should be possible to satisfy such relations requiring the amplitudes to be represented only by the expressions of Eq. (2). All the results which will be derived in Sec. IV are consequences, basically, of this critical relationship between the model and unitary symmetries. Section V is specifically devoted to a discussion of this point.

The physical interpretation of the LZ model of Eq. (2) deserves a few comments. The model should be considered with a spirit quite different from that present in old-fashioned applications of the Veneziano formula, predominantly concerned with finding a detailed and quantitative description of the data. The LZ model is intended to provide only a *stylized representation* of reality. For example, deviations from full degeneracy of internally exchanged Regge trajectories—like N_α, N_γ, and Δ_δ in πN scattering—are completely disregarded in it. The general motivation of the model is to try to find out what is the meaning of the observed tendency of zero trajectories to follow linear paths. Experimentally, such linearity is verified only to the extent to which resonances satisfy the above mass degeneracies. A specific question that the model is supposed to answer is what would happen to resonance coupling-constants if linearity of zeros and the consequent mass degeneracies held exactly. One can presume, as in the case of SU(3) symmetry, that the results of the model for resonance coupling constants will remain significant also in the presence of experimental breaking of the mass degeneracies implicit in it. This presumption, in the case of couplings of resonances staying on leading trajectories, is supported by the fact that within the model these couplings are not affected by changes in the values of the intercepts of Regge trajectories, and of external masses. Such changes, instead, alter considerably the couplings of daughter resonances, and therefore the predictions of the LZ model for these resonances should be disregarded. The driving idea behind all this is that the partially broken experimental regularity represented by the linearity of zeros may be pointing to some more fundamental and more exact regularity concerning the coupling constants of leading resonances.

Linearity of zeros is experimentally supported only for the scattering of pairs of pseudoscalar mesons and for the invariant amplitude $A(s,t)$ of meson-baryon two-body processes. The other invariant amplitude $B(s,t)$ of these processes $[T=i\bar{u}(-A(s,t)+i\gamma\cdot QB(s,t))u]$ does not appear to share this property. The application of the LZ dual model of Eq. (2), of course, makes sense only for amplitudes in which linearity of zeros is observed to hold, or in which it may at least hold in principle. Although a reliable understanding of what happens in $B(s,t)$ has to wait for more experimental information, it appears natural to think that the presence of momentum components in front of $B(s,t)$ in the expression of the transition amplitude is of some relevance in the context. As originally discussed by De Alfaro *et al.* in their paper on superconvergent sum rules,[30] such a circumstance affects, because of unitarity, the high-energy behavior of the amplitude—in the case of $B(s,t)$ this becomes $\sim s^{\alpha-1}$ instead of $\sim s^{\alpha}$ at fixed t—and this fact because of analyticity leads to the emergence of constraints on the parameters of resonances present at low energy. It is worth recalling that one of the original motivations of the Veneziano model was to find a solution of the superconvergent sum rules for any value of t.[9] In this light the LZ model of Eq. (2) may be regarded as a specific solution of the superconvergent sum rules applying to amplitudes in which no extra constraints caused by the spin structure are present. When such extra constraints are present, different types of solution may be required.

IV. EFFECTS FROM NONLINEARITY

This section deals with a number of specific applications of the LZ dual model presented in Sec. III. The emphasis in these applications is on clarifying characteristic features of the model and checking them with experimental data. In deriving the results extensive use is made of unitary symmetries—SU(2) or SU(3). As mentioned in Sec. III, the compatibility of the model with such symmetries is highly nontrivial. The circumstance that in the applications considered here it does occur is accepted in this section as a simple matter of fact. The significance of such a compatibility from a theoretical point of view will be discussed in Sec. V, which is entirely devoted to this subject.

a. $\pi N\to\pi N$.[31] With the assumption of isospin conservation the amplitudes of these processes must satisfy the relations

$$\begin{aligned} &A(\pi^0 p\to\pi^0 p) = \tfrac{1}{2}[A(\pi^+ p\to\pi^+)+A(\pi^- p\to\pi^- p)],\\ &\frac{1}{\sqrt{2}}A(\pi^- p\to\pi^0 n)=\tfrac{1}{2}[A(\pi^+ p\to\pi^+ p)-A(\pi^- p\to\pi^- p)]\ . \end{aligned} \tag{4}$$

Again because of isospin conservation, any other $\pi N\to\pi N$ amplitude is equal, apart from a factor, to one of the four amplitudes appearing in Eq. (4). This equation, therefore, exhausts the constraints due to isospin conservation which must be satisfied in applying the LZ model. In assigning expressions I, $\text{II}^\pm$, and $\text{III}^\pm$ of Eq. (2) to the amplitudes appearing in Eq. (4), meant to be $A(s,t)$-invariant amplitudes, one must also care about the crossing relations linking them. This implies in particular that $A(\pi^+ p\to\pi^+ p)$ and $A(\pi^- p\to\pi^- p)$ must contain the same $V(s,u)$ term [Eq. (2)]. All these requirements can be satisfied by a small finite number of solutions. In order to have a unique solution we impose the additional requirement, suggested on a general ground by data, that none of the considered amplitudes must have an identically vanishing imaginary part in the direct channel. The only solution left is then

$$A(\pi^+ p\to\pi^+ p)=V(s,u)-V(s,t)+V(u,t)\,, \tag{5a}$$

$$A(\pi^- p\to\pi^- p)=V(s,u)+V(s,t)-V(u,t)\ , \tag{5b}$$

$$A(\pi^0 p\to\pi^0 p)= V(s,u)\ , \tag{5c}$$

$$\frac{1}{\sqrt{2}}A(\pi^- p\to\pi^0 n)= V(s,t)-V(u,t)\,. \tag{5d}$$

The symmetry of $A(\pi^+ p\to\pi^+ p)$ and $A(\pi^- p\to\pi^- p)$ with respect to the isospin and crossing requirements has been resolved, requiring $A(\pi^- p\to\pi^- p)$ to be dominated by N_α resonances (even signature).

Equation (5) provides a complete, although stylized, description of the amplitude $A(s,t)$ in $\pi N\to\pi N$ processes. As discussed in Sec. III, it is expected to provide sensible predictions for the contributions to this amplitude of boson and baryon resonances staying on *leading* trajectories. In the following discussion we shall refer exclusively to this type of resonances, unless otherwise specified.

According to Eq. (5c) the f^0 resonance is predicted not to contribute to $A(s,t)$. Since $A(s,t)$ at high energy and fixed t coincides with $f(\Delta\lambda_s=1)$ —the s-channel helicity-flip amplitude—this implies that f^0 exchange decouples from $\bar{N}N$ in such helicity configuration. As mentioned in Sec. II this is in agreement with experimental indications from data on the polarization and the spin-rotation parameters of $\pi^\pm p$ and pp elastic scattering.[10]

Equation (5b) requires a definite even signature for resonance contributions to $A(s,t)$ in $\pi^- p\to\pi^- p$, because of the positive relative sign of $V(s,u)$ and $V(s,t)$. This means that contributions to $A(s,t)$ from resonances with odd signature ($J=\frac{3}{2},\frac{7}{2}$, etc.) must cancel out for each angular momentum separately in $\pi^- p\to\pi^- p$. When comparing these cancellations with data, the sensible quantity to deal with, in order to avoid problems of contamination from daughter resonances, is the coefficient of the dominant power in t (or equivalently in u) appearing in the $A(s,t)$ resonance residue. This is equivalent to considering such cancellations in the region at large t (or large u) where central waves are unimportant. In so doing one automatically removes p^{2J} factors, where p is the center-of-mass-system momentum, thus making corrections for deviations from mass degeneracy analogous to those considered in SU(3) fits to baryon resonance partial widths. The cancellations are expected to occur between resonances of opposite normality. Such resonances, indeed, give contributions of opposite signs to $f(\Delta\lambda_s = 1)$, and for existing resonances this remains true also for $A(s,t)$. For $J=\frac{3}{2}$ the relevant resonances are $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$; for $J=\frac{7}{2}$, $\Delta_{7/2^+}(1950)$ and $N_{7/2^-}(2190)$. Experimentally the relevant $\Delta_{3/2^+}(1235)/N_{3/2^-}(1520)$ ratio is -1.12 according to Ref. 32, and -1.08 according to Ref. 33. These values are compatible with -1, taking errors into account. The cancellation is also illustrated in Fig. 15(a), showing the imaginary part of $A(s,t)$ in $\pi^-p \to \pi^-p$. This quantity is plotted at a positive value of t in order to enhance peripheral contributions. The relevant $\Delta_{7/2^+}(1950)/N_{7/2^-}(2190)$ ratio is -0.85 according to Ref. 33, which is also compatible with -1, taking errors into account. Similar cancellations, as we shall see, are expected to occur in $K^-p \to \pi^+\Sigma^-$.

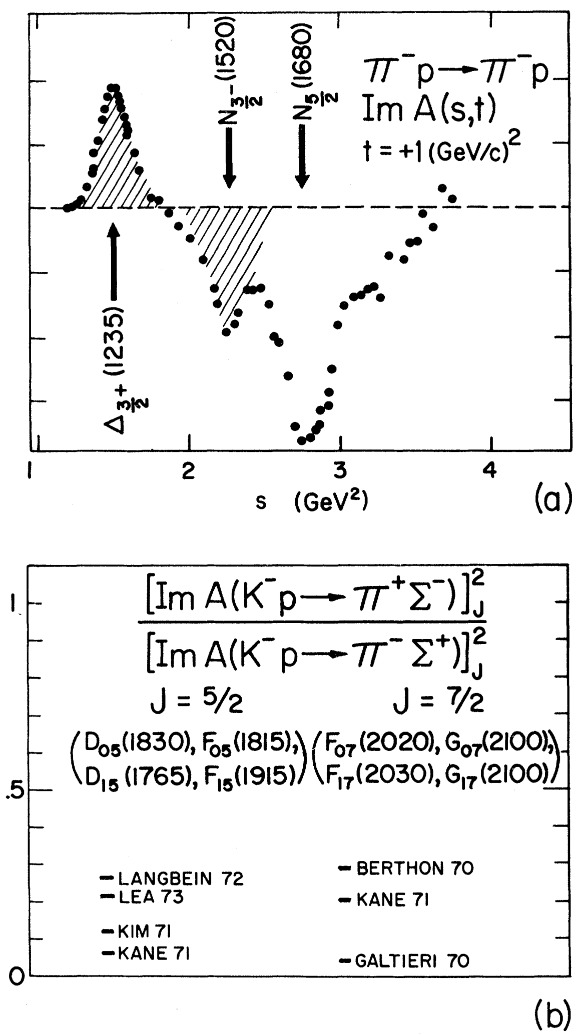

FIG. 15. (a) Cancellation of $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$ contributions to the amplitude $A(s,t)$ in $\pi^-p \to \pi^-p$. PWA (partial-wave analysis) from Ref. 32. (b) Cancellation of Λ and Σ resonance contributions to $A(s,t)$ in $K^-p \to \pi^+\Sigma^-$ for $J=\frac{5}{2}$ and $J=\frac{7}{2}$, respectively. See Ref. 84 for PWA's used.

The decoupling of f^0 and the cancellation between $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$ were already recognized in Sec. II as necessary prerequisites in order to have approximate linear behavior of zeros in the amplitude $A(s,t)$ of $\pi^\pm p$ elastic scattering, once pattern III^- is assigned to this amplitude. We have shown here that the existence of these effects can be expected on a much more general ground.

Besides these characteristic effects, Eq. (5) contains other detailed predictions for leading resonances. In $\pi^+p \to \pi^+p$ it predicts the dominance of resonances with odd signature, because of the relative minus sign of $V(s,t)$ and $V(u,t)$ in Eq. (5a), and negative normality, since the $V(s,t)$ term has in $A(\pi^+p \to \pi^+p)$ a sign opposite to that in $A(\pi^-p \to \pi^-p)$ which is assumed to be dominated by N_α resonances. These quantum numbers are those of the Δ_δ resonances, which are thus predicted to dominate in $\pi^+p \to \pi^+p$. This prediction is genuine and not implicitly contained in the assumptions. The scale of Δ_δ resonance couplings with respect to those of N_α resonances is fixed by the condition that the residues of $\Delta(1235)$ and $\Delta(1950)$ in $\pi^+p \to \pi^+p$ be numerically equal to that of $N(1680)$ (F_{15} and D_{15} combined) in $\pi^-p \to \pi^-p$ at the corresponding intersection points in the (s,t,u) plane (Fig. 7). This follows from the assignment of diagram III^- to $\pi^\pm p$ elastic scattering. These equalities are well verified experimentally, as discussed in Ref. 5.

b. $\bar{K}N \to \pi\Sigma$.[31] The isospin relations for this set of processes have the same structure as in $\pi N \to \pi N$, since $\bar{K}N \to \pi\Sigma$ is connected by line reversal with $\pi N \to K\Sigma$, which involves the same SU(2) representations of $\pi N \to \pi N$. They can be written

$$A(K^-p \to \pi^0\Sigma^0) = \tfrac{1}{2}[A(K^-p \to \pi^-\Sigma^+) + A(K^-p \to \pi^+\Sigma^-)],$$

$$\frac{1}{\sqrt{2}} A(\bar{K}^0 p \to \pi^+\Sigma^0) = \tfrac{1}{2}[A(K^-p \to \pi^-\Sigma^+) - A(K^-p \to \pi^+\Sigma^-)]. \qquad (6)$$

Differently from $\pi N \to \pi N$, there are no crossing relations among the amplitudes appearing in Eq. (6). Therefore a larger number of solutions is allowed. In a solution approaching reality, however, $A(K^-p \to \pi^-\Sigma^+)$ must be represented by II^- $[V(s,u) - V(u,t)]$, since K^* and K^{**} contributions in the t channel are both expected to be finite and $\pi^+p \to K^+\Sigma^+$ exhibits resonance activity with dominance of Δ_δ resonances $[\Delta_{7/2^+}(1950)]$. As to $A(K^-p\to\pi^+\Sigma^-)$, absence of exotics demands that only $V(s,u)$ can be present in it. Equation (6) then requires in practice that the sum and the difference of $A(K^-p\to\pi^-\Sigma^+)$ and $A(K^-p\to\pi^+\Sigma^-)$ be equal either to $V(s,u)\pm V(u,t)$ or to $V(u,t)$, other possibilities being excluded by the absence of $V(s,t)$ terms in these amplitudes. If $A(K^-p\to\pi^-\Sigma^+)$ is finite, this implies then that $A(K^-p\to\pi^+\Sigma^-)$ must vanish identically—i.e., Λ and Σ resonance contributions to the amplitude $A(s,t)$ have to cancel out in this process for each angular momentum separately. This effect is completely analogous to the cancellation of $\Delta_{3/2^+}(1235)$ and $N_{3/2^-}(1520)$ in $\pi^-p\to\pi^-p$. Figure 15(b) shows that such cancellations do actually take place experimentally for $J=\frac{5}{2}$ and $J=\frac{7}{2}$.[34] Im $A(K^-p\to\pi^\pm\Sigma^\mp)$, in the figure, represents the sum of the $A(s,t)$ residues of the pertinent resonances (listed in parenthesis for each angular momentum) evaluated at large t, in accord with the related discussion concerning the cancellations in $\pi^-p\to\pi^-p$. Im $A(K^-p\to\pi^-\Sigma^+)$ (nonexotic in t channel) is used as a convenient scale to gauge the cancellations.[35]

c. η and X^0 couplings. In the limit of exact SU(3), in which mass differences among members of the pseudoscalar nonet are neglected, η and X^0 cannot be distinguished theoretically from any other pair of mutually orthogonal linear combinations of them. This is equivalent to say that there is no restriction on the value that the mixing angle between the pseudoscalar octet and the pseudoscalar singlet can assume. Requiring expressions I, $\text{II}^\pm$, and $\text{III}^\pm$ of Eq. (2) to describe the amplitudes of $P+P\to P+P$ processes, where P stays for any member of the pseudoscalar nonet, this continuous degeneracy is broken. Indeed, once a solution is found for η and X^0 couplings, linear combinations of the two-particle states would correspond in general to transition amplitudes incompatible with Eq. (2). This circumstance is of special interest, since unlike the vector and the tensor meson nonets, conventional SU(3)-breaking schemes do not seem to work for the pseudoscalar nonet. Perturbative approaches like the Gell-Mann-Okubo mass formula, although often considered in the literature, are highly questionable for the pseudoscalar nonet because of the large mass differences involved. Also a quark-model description does not appear appropriate in this case in light of the general picture presented by data.

The requirement that $P+P\to P+P$ amplitudes be represented by expressions I, $\text{II}^\pm$, and $\text{III}^\pm$ happens to be, nontrivially, compatible with the validity of SU(3) symmetry for VPP and TPP couplings, V and T representing vector and tensor mesons, respectively. VPP couplings are uniquely fixed by SU(3), since only $V_8P_8P_8$ can be different from zero. For TPP couplings, on the contrary, there are several ways in which P_8, P_1, T_8, and T_1 can combine, and the corresponding coupling constants are left undetermined by SU(3). Such coupling constants and the η-X^0 mixing angle are constrained by the LZ conditions. By detailed calculation one finds that only two solutions of the LZ model involving these quantities exist. For reasons which will be discussed below we shall call them the quark solution[36] and the nonquark solution,[26] respectively.

Quark solution:

$$\{P_8P_8\}_D T_8 - (4/\sqrt{5})P_8P_8T_1 + (\tfrac{2}{5})^{1/2}P_8P_1T_8 + (\tfrac{2}{5})^{1/2}P_1P_1T_1, \tag{7a}$$

$$\tan\theta = +(1/\sqrt{2}).$$

Nonquark solution:

$$\{P_8P_8\}_D T_8 - (4/\sqrt{5})P_8P_8T_1 + (1/\sqrt{10})P_8P_1T_8 + (1/\sqrt{10})P_1P_1T_1, \tag{7b}$$

$$\tan\theta = -1/\sqrt{2}.$$

θ is the η-X^0 mixing angle defined as $\eta=\eta_8\cos\theta - \eta_1\sin\theta$. The two solutions are determined up to certain ambiguities of sign, which, however, do not affect the magnitude of any coupling and will be disregarded. When comparing with data, moreover, the roles of the here-defined η and X^0 can of course be interchanged.

Both solutions require the vanishing of several vertex couplings, and a simple way of analyzing their content is to focus on such decouplings. In the quark solution the couplings $f'\pi\pi$, $A_2\pi\eta$, $f^0\eta\eta$, $f^0\eta X^0$, $f'X^0X^0$, and $f'\eta X^0$ do vanish (f^0 and f' are defined assuming the conventional canonical value for their mixing angle). This solution corresponds to the conventional decoupling prescriptions of the quark model with $\eta\sim\bar\lambda\lambda$ and $X^0\sim\bar p p+\bar n n$. In the nonquark solution the vanishing couplings are $f'\pi\pi$, $A_2\pi X^0$, $K^{**}K\eta$, $f^0X^0X^0$, and $f^0\eta X^0$ (one decoupling less than in the quark solution). In this solution the decouplings involving η and X^0 cannot be described in a quark-model fashion. In particular, the coupling $K^{**}K\eta$ is required to vanish, whereas $K^*K\eta$ must stay finite. This clearly clashes with quark-model ideas.

Experimental data do provide some striking verifications of the nonquark solution. The vanishing of $K^{**}K\eta$ should lead to a $K^-p\to\eta\Lambda$ angular distribution similar to that of $\pi^-p\to\pi^0n$, with a dip at the WSNS value of the K^* trajectory, and quite different therefore from those of $K^-p\to\pi^0\Lambda$ and $K^-p\to X^0\Lambda$ in which the presence of exchange-degenerate K^* and K^{**} is expected to produce an essentially structureless distribution. The data displayed in Fig. 16, with a sharp dip at $t \simeq -0.4$ $(\mathrm{GeV}/c)^2$ in $K^- p \to \eta\Lambda$, show that this is actually what happens experimentally.[26] The vanishing of $A_2 \pi X^0$ implies a suppression of X^0 production from pions in two-body and quasi-two-body processes. Experimentally $\sigma(\pi^+ p \to X^0 \Delta^{++})/\sigma(\pi^+ p \to \eta \Delta^{++}) \simeq 0.2$.[37–43] The same decoupling also implies a suppression of the strong decay mode $X^0 \to \eta\pi\pi$, since if $A_2 \pi X^0$ is suppressed, the whole amplitude for $\pi X^0 \to \pi\eta$ (diagram III^+) is suppressed. Experimentally the total width of X^0 is in fact abnormally small (< 1.9 MeV).[44] The predicted magnitude of the mixing angle $|\theta| = 35.26°$ is in agreement with the available data concerning the production and the decays of π^0, η, and X^0. Particularly constraining are the data[45] from an high-statistics experiment on $p+d \to \mathrm{He}^3 + \mathrm{MM}$, which point to a magnitude of $|\theta| \simeq 38°$, with an error $\simeq \pm 15°$ mainly reflecting the uncertainty on the F/D ratio for $P\bar{N}N$ couplings, needed as an independent input for the determination of θ.[46] The discrepancy with the value given by the quadratic Gell-Mann–Okubo mass formula ($\simeq 11°$) should not cause worry, since as mentioned above this is a perturbative formula which has no reasons to be valid for the pseudoscalar nonet, where large differences in mass are present.[47–51]

d. $P+P \to P+V$ processes. Capps[36] has also studied the constraints ensuing from Eq. (2) for this type of processes. Although there is no experimental evidence that expressions like those of Eq. (2) should be appropriate for these processes, the corresponding results for VVP and TVP couplings have interesting similarities with those for TPP couplings found in $P+P \to P+P$. Also in this case one finds two types of solutions. In one of them the existing decouplings can be described, attributing, in VVP and TVP couplings, a quark-like structure to vector mesons—e.g., the coupling $\phi\rho\pi$ vanishes in this type of solution. This time the quark-model solution seems to be the sensible one experimentally, in view of the small width of the 3π decay of ϕ[52] and of the observed suppression of ϕ production from pions.[86] Both solutions for VVP and TVP couplings are compatible with either one of the solutions for TPP couplings found in $P+P \to P+P$.

e. Decoupling of exotic resonances. The application of the LZ conditions of Eq. (2) in conjunction with unitary symmetries to $P+P \to P+P$ processes leads to results of theoretical interest concerning the suppression of couplings of exotic resonances. The imposition of the LZ conditions, in general, restricts the acceptable solutions of the duality bootstrap to a small discrete set. Some of these solutions may contain exotics. The fact that such exotic solutions are in a very limited number and are well determined facilitates their rejection on physical grounds. We shall consider [8]+[8]→[8]+[8], where [8] is the pseu-

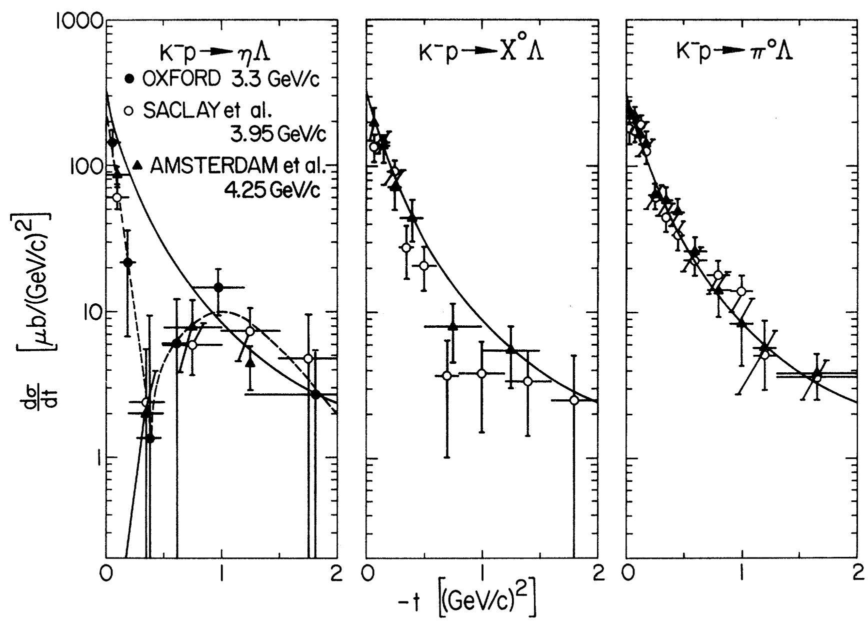

FIG. 16. Angular distributions of $K^- p \to \eta\Lambda$, $X^0\Lambda$, and $\pi^0\Lambda$. The figure shows the existence of a pronounced dip at $t \simeq -0.4$ $(\mathrm{GeV}/c)^2$ in $K^- p \to \eta\Lambda$ contrasted to a structureless behavior in $K^- p \to X^0\Lambda$ and $K^- p \to \pi^0\Lambda$. See Ref. 85 for data.

doscalar-meson octet, in order to investigate "internal" exotics, i.e., solutions in which exotic vector and tensor mesons are internally exchanged. We shall then briefly discuss the question of "external" exotics, i.e., the existence of solutions for the scattering of pseudoscalar mesons with exotic quantum numbers.

In order to illustrate the general procedure for finding the solutions, let us first discuss as a simple example $\pi\pi$ scattering, or in other terms $[1]+[1]\to[1]+[1]$, where the numbers in brackets denote isospin values. Solutions in this case can be described by vectors $(A(0), A(1), A(2))$ defining the contributions of isospins $I=0$, 1, and 2 to the imaginary part of the amplitude. Odd values of I correspond to vector mesons, and even values of I to tensor mesons. Imaginary parts in the s, t, and u channels are related to each other by the duality constraints of Eq. (2). Since s, t, and u channels are identical, the vector describing a solution must be an eigenvector with eigenvalue $+1$ of the SU(2) crossing matrix. Defining channels s, t, and u as

$$\begin{aligned} s&: [1]+[1'] \to [1'']+[1'''] ,\\ t&: [1]+[1''] \to [1']+[1'''] ,\\ u&: [1]+[1'''] \to [1'']+[1'] , \end{aligned} \tag{8}$$

the $t\to s$ crossing matrix X^{st} is[53]

$$\begin{bmatrix} A_s(0) \\ A_s(1) \\ A_s(2) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6} \\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} A_t(0) \\ A_t(1) \\ A_t(2) \end{bmatrix} . \tag{9}$$

The eigenvectors of this matrix are easy to find. The operation of crossing $u\to t$ is equal to that of crossing $u\to s$ times an inversion in the initial order of the final particles [Eq. (8)]. Hence one has the equation $X^{su}=X^{tu}D$, where D is the diagonal matrix

$$[D]_{ij}=\delta_{ij}(-1)^{I_{\max}-I_i} ,$$

$I_{\max}$ being the maximum I that can be exchanged in the process; $I_{\max}=2$ in our case. This implies $X^{st}X^{tu}=X^{su}=X^{tu}D$, or more explicitly

$$[X^{st}]_{ij}[X^{tu}]_{jp}=(-1)^{I_{\max}-I_p}[X^{tu}]_{ip} .$$

In other words the eigenvectors of X^{st} are the columns of the X^{tu} crossing matrix, the pth column corresponding to an eigenvalue $(-1)^{I_{\max}-I_p}$. Again looking to the definitions of Eq. (8) one realizes that $X^{tu}=DX^{st}$, or

$$[X^{tu}]_{ip}=(-1)^{I_{\max}-I_i}[X^{st}]_{ip} ,$$

and therefore the eigenvectors can be obtained from the columns of the very same X^{st} matrix by changing the signs of the elements in the odd rows. One finds in this way two $+1$ eigenvectors for $[1]+[1]\to[1]+[1]$:

$$E_0=(\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{1}{3}),$$

$$E_2=(\tfrac{5}{3}, \tfrac{5}{6}, \tfrac{1}{6}).$$

The solution must therefore be of the form aE_0+E_2. The coefficient a is to be determined requiring that in any "physical" process—i.e., for any definite charge configuration of the four external particles—vector and tensor contributions are either in exchange-degenerate relationship, or at least one of them vanishes. Since we are dealing with an elastic process, the elements of the vector representing the solution must be all nonnegative. At the end two solutions are found to exist for $[1]+[1]\to[1]+[1]$. They are represented by $S_1=(3, 2, 0)$ and $S_2=(5, 0, 2)$. S_1 is the well-known physical solution with no exotic states in which vector and tensor mesons (i.e., ρ and f^0) coexist. S_2, which contains exotic states, is a relatively trivial solution in which only tensor mesons exist, and all amplitudes are assigned the pattern III$^+$.

Let us now consider $[8]+[8]\to[8]+[8]$ assuming that SU(3) symmetry holds. The solutions may be now represented by vectors $(A(1), A(8_{ss}), A(8_{aa}), A(10), A(\overline{10}), A(27))$, the missing components $A(8_{sa})$ and $A(8_{as})$ being identically zero. The eigenvectors of the SU(3) crossing matrix can be obtained from the explicit expression of this matrix[53] as in the case of $[1]+[1]\to[1]+[1]$. D is now a 6×6 diagonal matrix, whose diagonal elements are $+1$ if corresponding to symmetric representations $(1, 8_{ss}, 27)$, and -1 if corresponding to antisymmetric representations $(8_{aa}, 10, \overline{10})$. There are three solutions satisfying the exchange-degeneracy constraints of Eq. (2) and the positivity conditions. They are $S_1=(16, 5, 9, 0, 0, 0)$, $S_2=(5, 1, 0, 0, 0, 1)$, and $S_3=(4, 0, 1, 1, 1, 0)$. S_1 is the well-known physical solution in which vector and tensor mesons coexist. As discussed in Sec. IV c it does not actually satisfy the constraints of Eq. (2) when $I=0$ pseudoscalar mesons are involved. For this purpose a ninth SU(3)-singlet pseudoscalar-meson must be introduced, with suitable interaction constants. Solutions S_2 and S_3 contain exotic states. In S_2 only symmetric representations survive, which means that only tensor mesons are present. All amplitudes are assigned the pattern III$^+$. This solution is analogous to the solution S_2 in $[1]+[1]\to[1]+[1]$. In S_3, except for one SU(3)-singlet tensor meson, only vector mesons are instead present. If one requires that vector- and tensor-meson multiplets be both present, the exotic solutions S_2 and S_3 are then ruled out.

In order to have an idea of what happens when exotic pseudoscalar mesons are introduced, let us consider the scattering of pairs of such mesons with $I=2$, $Y=0$ quantum numbers—$[2]+[2]\to[2]+[2]$. Solutions are now represented by 5-dimensional vectors $(A(0),A(1),A(2),A(3),A(4))$, which must be $+1$ eigenvectors of the SU(2) crossing matrix with non-negative elements, and must satisfy the LZ exchange-degeneracy constraints of Eq. (2). The SU(2) crossing matrix is reported in Appendix A, together with a list of the "physical" processes to be taken into account. Only two solutions exist: $S_2=(7,0,2,0,2)$ and $S_3=(5,2,0,2,0)$. These solutions are completely analogous to solutions S_2 and S_3 in $[8]+[8]\to[8]+[8]$. In one of them, S_2, only tensor mesons are present, and in the other one, S_3, except for an $I=0$ tensor meson only vector mesons are allowed. This time there is no solution analogous to the physical solution S_1 in $[8]+[8]\to[8]+[8]$, that is, there is no solution in which vector and tensor mesons can coexist.

In all the examples considered in this paragraph[54] the holding of the LZ constraints of Eq. (2) makes the *presence of exotic states incompatible with the coexistence of vector- and tensor-meson multiplets*. These examples show well how, in the presence of exotic states, solutions are possible only by evading the exchange-degeneracy requirements of Eq. (2), by eliminating either the vector mesons or the tensor mesons (except for one tensor meson with $I=0$). The possibility of obtaining stronger results is clearly limited by the circumstance that the LZ model of Eq. (2) is applicable only when all external particles are pseudoscalar, and not when they are replaced partly or completely by vector or tensor mesons. In other words, it is limited by the fact that the constraints which we are dealing with do not constitute a complete bootstrap of the involved particles.

V. CONNECTIONS WITH UNITARY SYMMETRIES AND THE QUARK MODEL

The applications discussed in Sec. IV indicate the existence of considerable interrelations between the LZ dual model of Eq. (2) and unitary symmetries. One should be partly prepared for that in light of previous theoretical work concerning the interplay of duality and SU(3). We refer to the results originally obtained by Harari concerning the compatibility of duality with SU(3) and absence of exotics in the scattering of two pseudoscalar mesons.[55] We also refer to the several attempts[56] made to derive SU(n) symmetry (for mesons) from duality constraints plus a minimum of extra input.[57] The basic idea behind this line of research is that unitary symmetries could be possibly derived from some appropriate dynamical bootstrap of hadrons.[59]

In early work concerning the Veneziano model it was already realized how in processes with an exotic channel, like $K^+\pi^-$ and $\bar{K}K$ scatterings, which are describable by a single Veneziano formula [expression I in Eq. (2)], the coupling-constant constraints between the s- and the t-channel resonances arising from the formula do coincide with relations implied by SU(3) and the assumption of quark-model values for the involved mixing parameters.[60] The new fact emerging from the applications of Sec. IV is that the recoupling coefficients of SU(2) and, as far as pseudoscalar-pseudoscalar scattering is concerned, of SU(3) turn out to be naturally suitable for the kind of nonlinear duality embodied by the LZ model of Eq. (2). To appreciate this point further, let us consider the relevant SU(2) relations in $\pi\Sigma\to\pi\Sigma$. Defining

$$\begin{aligned}
A_1&=A(\pi^-\Sigma^+\to\pi^-\Sigma^+),\\
A_2&=A(\pi^-\Sigma^+\to\pi^+\Sigma^-),\\
A_3&=A(\pi^-\Sigma^+\to\pi^0\Sigma^0),\\
A_4&=A(\pi^0\Sigma^+\to\pi^0\Sigma^+),\\
A_5&=A(\pi^0\Sigma^+\to\pi^+\Sigma^0),\\
A_6&=A(\pi^+\Sigma^+\to\pi^+\Sigma^+),\\
A_7&=A(\pi^0\Sigma^0\to\pi^0\Sigma^0),
\end{aligned}\tag{10}$$

the involved isospin relations are

$$\begin{aligned}
A_1-A_2&=A_4-A_5,\\
A_6&=A_4+A_5,\\
A_1+A_2&=A_7+A_3,\\
A_6&=A_7-A_3.
\end{aligned}\tag{11}$$

The structure of this system of equations is clearly ideal for a solution in terms of the expressions I, II$^\pm$, and III$^\pm$ of Eq. (2), although there is no *a priori* reason for it to be so.[61]

SU(3) relations are naturally compatible with the LZ model in pseudoscalar-pseudoscalar scattering (Secs. IV c and IV d). It is not so for the invariant amplitude $A(s,t)$ in meson-baryon scattering. For this amplitude the LZ model turns out to be incompatible with the simultaneous validity of SU(3) in the s and t channels, unless physically unacceptable possibilities for the baryon spectrum are considered, like, e.g., the suppression of the $J^P=\frac{3}{2}^+$ baryon-resonance decuplet. This is shown in detail in Appendix B. Again, in light of previous theoretical work concerning the interplay of duality and SU(3), the emergence of complications when baryons are involved is not surprising. Besides the well-known difficulties pointed out by Rosner[62] concerning baryon-antibaryon scattering, it has been remarked by several authors that even in meson-baryon processes in order to satisfy duality, SU(3), and absence of exotics one needs the introduction of an octet of $J^P=\frac{3}{2}^+$ resonances never observed experimentally.[63] With the tighter duality constraints of Eq. (2) the "difficulties," not unexpectedly, become more acute.

Experimentally there are some fair indications from high-energy data that, at least as far as $\bar{N}N$, $\bar{N}\Lambda$, and $\bar{N}\Sigma$ couplings are concerned, vector- and tensor-meson exchanges do obey SU(3).[64] The experimental state of SU(3) for baryon resonance decay rates is more confused. There is a non-negligible amount of model-dependence in the extraction of the relevant coupling constants from partial-wave analyses[65,66] (assumptions about the background—crucially important for total widths, choice of the centrifugal-barrier factors, etc.). The data, especially those for hyperon resonances, are not very accurate statistically, and it is generally agreed that several of the resonances considered as established have a status inferior to that of the much debated Z^*'s in K^+N scattering.[67]

The application of the LZ model to the amplitude $A(s,t)$ is compatible with the holding of SU(3) in the t channel only. The coupling $f^0\bar{N}N$ appearing in $A(s,t)$ is required to vanish by the model (Sec. IV*a*). Absence of exotic states in the direct channel demands in general f' and ϕ (Ref. 68) to decouple from $\bar{N}N$ (Ref. 69). These decouplings fix completely the SU(3) parameters in the t channel[70] and the resulting predictions for vector and tensor exchanges are verified to satisfy the LZ exchange-degeneracy conditions in any meson-baryon process.[71] It makes therefore sense, both experimentally and theoretically, to consider the possibility that SU(3) for baryon resonances be broken, and that the predictions of the *LZ model in conjunction with SU(3) symmetry in t channel* be physically meaningful [$P\bar{B}B$ couplings, where P is the pseudoscalar octet and B the stable baryon octet, are not involved since they do not appear in the amplitude $A(s,t)$]. An experimental test of this possibility is offered by the ensuing predictions for baryon-resonance decay rates. In $\pi N \to \pi N$, $\bar{K}N \to \bar{K}N$, $\bar{K}N \to \pi\Lambda$, and $\bar{K}N \to \pi\Sigma$, besides the $V(s,t)$ and $V(u,t)$ terms [Eq. (2)] determined by t-channel SU(3), also the $V(s,u)$ terms are known. In $\pi N \to \pi N$ they are determined by the LZ model (Sec. IV a). In $\bar{K}N \to \bar{K}N$ they must be absent in order to avoid exotic states. In $\bar{K}N \to \pi\Lambda$ and $\bar{K}N \to \pi\Sigma$ patterns $\text{II}^\pm$ must apply (except for $K^-p \to \pi^+\Sigma^-$, whose amplitude $A(s,t)$ is expected to vanish identically, Sec. IV b) in order to comply with t-channel SU(3), which requires K^* and K^{**} exchanges to stay in anti-exchange-degenerate relationship, and to have at the same time finite hyperon-resonance contributions to $A(s,t)$ as observed experimentally. Thus contributions of hyperon resonances to the amplitude $A(s,t)$ in $\bar{K}N \to \bar{K}N$, $\pi\Lambda$, and $\pi\Sigma$ can be predicted with *no free parameters* from the corresponding contributions of nonstrange baryon resonances in $\pi N \to \pi N$, assuming the validity of the LZ model and of SU(3) in t channel. As discussed in Sec. III, the predictions concern only resonances staying on leading trajectories, and involve the over-all contributions of groups of such resonances having the same angular momentum J. According to the discussion of Sec. IV a, because of experimental mass shifts the sensible quantities to deal with are the $A(s,t)$ resonance residues at large values of t, or in other words the coefficients of the dominant powers in t of such residues. This method is essentially equivalent to removing conventional centrifugal-barrier factors. The predictions of Fig. 17 refer to such quantities, evaluated for the leading hyperon resonances in $\bar{K}N \to \bar{K}N$, $\pi\Lambda$, and $\pi\Sigma$, and normalized to the corresponding quantities in $\pi^\pm p \to \pi^\pm p$ of leading resonances having the same angular momentum.[72] As one can see, the predictions compare quite well with the results from partial-wave analyses. The experimental pattern of signs is exactly reproduced. In spite of the large uncertainties the over-all agreement in magnitude is also significant. In the case of $J=\frac{5}{2}$ in $\bar{K}N \to \pi\Lambda$, the contributions of $D_{15}(1765)$ and $F_{15}(1915)$ partially cancel each other, and consequently the result is strongly affected by errors. We report in this case the interval of values allowed by the errors quoted in Ref. 73. This no-free-parameter test of the LZ model in conjunction with SU(3) symmetry in t channel is to be compared with conventional SU(3) fits[65] to the same data, which, although equally acceptable quantitatively, do involve several free parameters (F/D ratios and mixing parameters). Existing data clearly do not discriminate between the two types of prediction.

A detailed study of the breaking of s-channel SU(3) implied by the LZ model plus t-channel SU(3) is hindered by the fact that the model is not able to separate resonances of opposite parities, and by the general lack of knowledge about $V(s,u)$ terms in processes which are not accessible experimentally. Only in a very few cases, thanks to the circumstance that in elastic processes parity determines the signs of resonance contributions to $A(s,t)$, is it possible to obtain some indications. There are definite predictions, for example, for

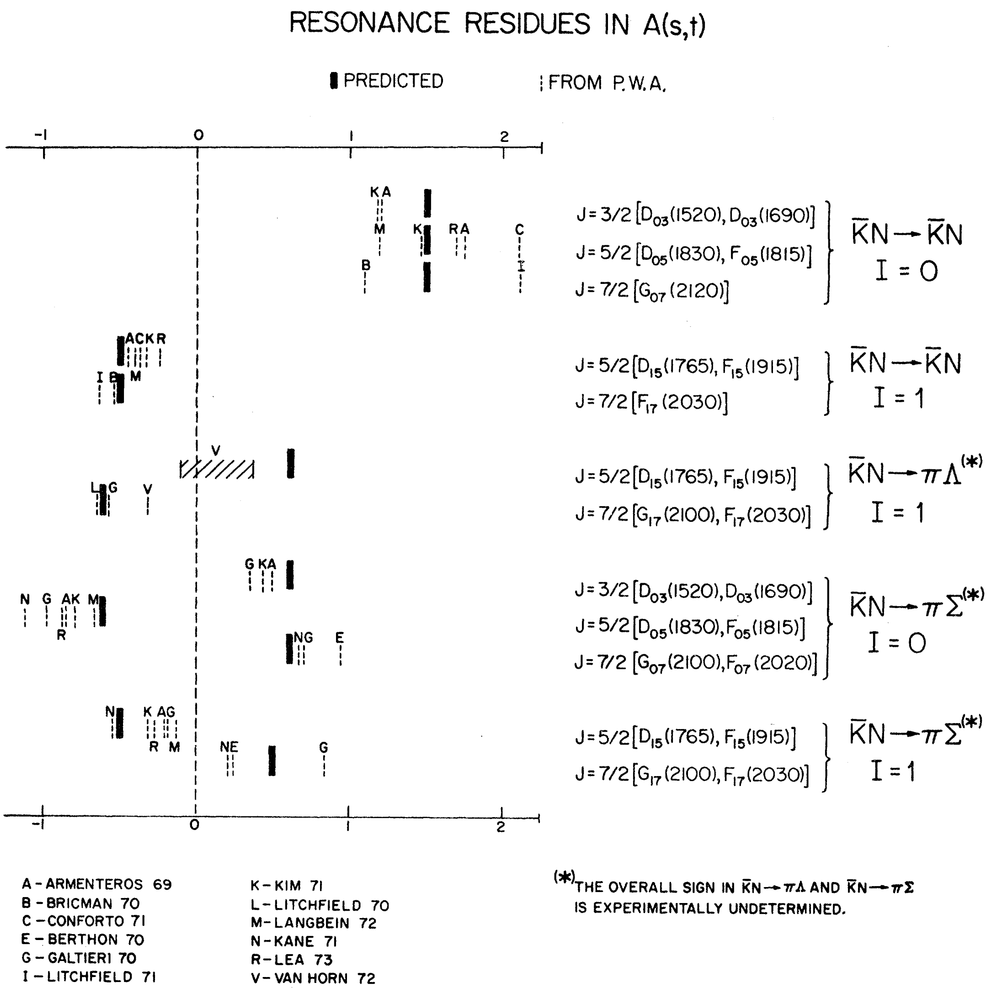

FIG. 17. Comparison with the results from PWA's of the predicted resonance contributions to $A(s,t)$ in $\bar{K}N\to\bar{K}N$, $\pi\Lambda$, and $\pi\Sigma$. Quantities are normalized to contributions to $A(s,t)$ in $\pi^{\pm}p\to\pi^{\pm}p$ of nonstrange baryon resonances with the same angular momentum J. $\bar{K}N$ and πN PWA's from Ref. 84 and Ref. 32, respectively.

the couplings of $\Delta(1235)$ to πN and $K\Sigma$, and of $\Omega(1672)$ to $K\Xi$, provided that apart from Regge recurrences no other leading resonances with the same SU(3) quantum numbers exist. These predictions are found to coincide with those of SU(3). The coupling of $\Sigma_{3/2^+}(1385)$ to $\bar{K}N$, on the contrary, is predicted to be higher than what is expected from SU(3). Since this coupling is not known experimentally, for a check with data one has to consider the $J^P=\frac{7}{2}^+$ resonance decuplet, for which of course the same predictions hold. The LZ model [plus t-channel SU(3)] then predicts $\Gamma_{\bar{K}N}^{\text{exp}}/\Gamma_{\bar{K}N}^{\text{SU(3)}} \geq 1.5$ for $\Sigma_{7/2^+}(2030)$. The equality sign holds if contributions from $I=1$ resonances with $J^P=\frac{7}{2}^-$ are negligible. This appears to be the case experimentally,[52] and therefore the ratio should be expected to be essentially equal to 1.5. Existing analyses of the data report this ratio as 1.42 ± 0.25 (Ref. 65) and 1.5 ± 0.45.[66] These numbers, if anything, do not disfavor the LZ-model prediction.

Definite interrelations between the quark model and the LZ dual model of Eq. (2) have been brought into evidence by some of the applications of Sec. IV. One of the solutions for η and X^0 couplings in $P+P\to P+P$ complies exactly with the prescriptions of the quark model (Sec. IV c), and the same is true of one of the solutions for VVP and TVP couplings in $P+P\to P+V$ (Sec. IV d). Also the absence of exotic mesons, typically associated with the quark model, can be partially accounted for by applying the LZ model to $P+P\to P+P$ processes (Sec. IV e). However, these interrelations, as in the case of SU(3), do not go beyond a certain point. The solution of the LZ model for η and X^0 couplings which agrees with data definitely clashes with quark-model ideas, since it requires the $K^{**}K\eta$ coupling to vanish and the $K^*K\eta$ coupling to stay finite.

VI. GAUGELIKE DECOUPLINGS OF RESONANCES

The applications of Sec. IV show that the LZ dual model of Eq. (2) requires in a natural way the vanishing of a number of resonance couplings, which are found to correspond to dynamical suppressions actually observed experimentally.

The experimental phenomenon of coupling suppressions is interesting in itself. More and more instances of it keep appearing as data accumulate. Table I contains a list of the effects of this type so far established experimentally. Conventionally, some of these decouplings—$f'\pi\pi$, $\phi\rho\pi$, $f'\bar{N}N$, $\phi\bar{N}N$, and the E_2 $\Delta(1235)\to\gamma N$ transition—have been understood by means of the quark model. With the introduction of orbital excitation the quark model is able to account also for a few of the observed suppressions of photon couplings of baryon resonances (see Table I). However, this kind of explanation does not cover many of these effects, as indicated in Table I,[74,75,76] and at least in one case so far—$K^{**}K\eta$—it is in clear contradiction with data. As discussed in Sec. IV, moreover, strictly associated with these decouplings there appear to be certain effects of local cancellation between leading resonances, also

TABLE I. Suppressions of resonance couplings and local cancellations of resonance contributions observed experimentally (see Sec. VI).

Coupling suppressions	(a)	(b)	Experimental source
$f'\pi\pi$	QM	LZ	decay suppr. [c]
$f'\bar{N}N$, $\phi\bar{N}N$	QM	LZ [d]	absence of backward peaks in $K^-N\to f'\Lambda/\Sigma$, $\phi\Lambda/\Sigma$ [e]
$\phi\rho\pi$	QM	LZ	suppr. of decay and of $\sigma(\pi N\to\phi N/\Delta)$ [f]
$f^0\bar{N}N$, $\omega\bar{N}N$, $\Delta\lambda_s=1$		LZ	$\pi^\pm p, pp$ pol. and R/A [f]
$\rho\bar{N}N$, $A_2\bar{N}N$, $\Delta\lambda_s=0$			$d\sigma/dt$ of πp, KN CEX [g]
$K^{**}(1420)K\eta$		LZ	dip in $K^-p\to\eta\Lambda$ $d\sigma/dt$ [f]
$A_2\pi X^0$		LZ	suppr. of $\sigma(\pi N\to X^0N/\Delta)$ and of $X^0\to\eta\pi\pi$ [f]
$\Delta(1235)\gamma N$, E_2 transition	QM		$\gamma N\to\pi N$ multipole analysis [i]
$N(1520)\gamma N$, N =proton and $\lambda_R=\frac{1}{2}$			(same)
$N(1520)\gamma N$, isoscalar γ and $\lambda_R=\frac{3}{2}$	QM [h]		(same)
$N(1690)\gamma N$, $\lambda_R=\frac{1}{2}$			(same)
$N(1690)\gamma N$, N =neutron and $\lambda_R=\frac{3}{2}$	QM [h]		(same)
$\Delta(1950)\Delta(1235)\pi$, $\lambda_R=\frac{1}{2}$			$\pi^+p\to\pi^0\Delta^{++}$ PWA (partial-wave analysis) [j]
πf^0A_2, $\lambda_s=0$ of produced A_2			$\pi N\to A_2N$ $d\sigma/dt$ and $\rho_{mm'}$ [k]
$\pi\omega B$, $\lambda_s=0$ of produced B (?)			$\pi N\to BN$ $\rho_{mm'}$ [k]
Local cancellations			
$-\Delta_\delta$ and N_γ resonances in the ampl. $A(s,t)$ of $\pi^-p\to\pi^-p$		LZ	$\pi N\to\pi N$ PWA [f]
$-\Lambda$ and Σ resonances in the ampl. $A(s,t)$ of $K^-p\to\pi^+\Sigma^-$		LZ	$\bar{K}N\to\pi\Sigma$ PWA [f]

[a] Accounted for by the quark model (QM).
[b] Accounted for by linear-zero conditions (LZ).
[c] Particle Data Group, Ref. 52.
[d] Derivable from duality, SU(3) for vector and tensor exchanges, and absence of exotics in $P+B\to P+B$, Ref. 69.
[e] See, e.g., Ref. 78.
[f] See Sec. IV.
[g] See, e.g., Ref. 10.
[h] Introducing orbital excitation, see Sec. VI.
[i] Reference 79.
[j] Reference 80.
[k] See, e.g., the discussions in Refs. 39 and 81.

listed in Table I, whose existence is totally unexpected in the framework of the quark model.

The LZ dual model of Eq. (2) provides a natural understanding of the suppressions listed in Table I for all those couplings which are accessible through $P+P\to P+P$ and $P+P\to P+V$ processes and the invariant amplitude $A(s,t)$ of $P+B\to P+B$ processes. Furthermore, just in the same way in which it explains these suppressions and the local resonance cancellations reported in Table I, the model accounts—partially—for the absence of exotic mesons. The fact that some of the observed decouplings can be explained at the same time by the quark model is not disturbing since, as it is clear from the results of Sec. IV and the discussion of Sec. V, there are several apparent interconnections between the two models. The specific ingredient of the LZ dual model responsible for the decouplings is its nonlinearity. A conventional dual model without the tight constraints of Eq. (2) would not be able to produce such effects.[77] Since suppressions of this type are observed also for couplings appearing in amplitudes corresponding to spin configurations more complex than those to which the specific model of Eq. (2) applies, one is led to think that dual constraints of the same nature of those of Eq. (2), although of course of more general form, are active in any hadronic amplitude.

The central fact emerging from the applications of Sec. IV is that definite interchannel relations between resonance-pole residues are tied up with the vanishing of some couplings. We wish to point out the *structural similarity* of such an interconnection with something better known happening in electromagnetic interactions. Let us consider, e.g., $\gamma p\to\pi^+ n$ treating strong interactions perturbatively and in first-order approximation. Two graphs contribute to this process (Fig. 18), one representing the s-channel nucleon pole and the other the t-channel pion pole. The residues of the two poles are proportional to $e_p\cdot g_{\pi N}$ and $e_\pi\cdot g_{\pi N}$, respectively, where e_p is the electric charge of the proton, e_π the electric charge of the pion, and $g_{\pi N}$ the strong πNN coupling constant. If e_p and e_π, and thus the s-channel pole and the t-channel pole residues, were unrelated, all three helicity-states of the photon would have finite couplings. But if the two residues are related to each other requiring $e_p=e_\pi$, i.e., if the electric charge is required to be conserved, then longitudinally polarized photons have to decouple. From this example it appears that the connection between the LZ dual constraints of Eq. (2) and the observed hadronic decouplings bear some resemblance to the interrelation between charge conservation and gauge invariance in electromagnetic interactions. Such a resemblance is clearly limited by several qualitative differences. A fundamental one is that, in contrast with the exact holding of gauge invariance, the dynamical suppressions listed in Table I are of more approximate nature. We believe, though, that the phenomenological facts presented here, their crudity notwithstanding, point to some sort of qualitative similarity between the general structures of hadronic and electromagnetic interactions.

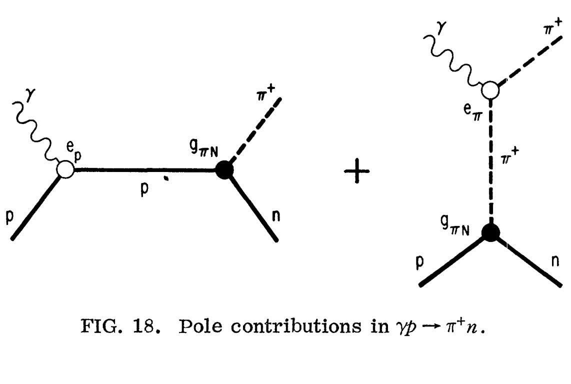

FIG. 18. Pole contributions in $\gamma p\to\pi^+ n$.

VII. SUMMARY AND CONCLUSIONS

The main result obtained in the paper is that a specific form of dual resonance model is able to predict in a natural way a number of suppressions of resonance couplings, all of which are actually observed in the data. Besides these effects, the model does also account partially for the decoupling of exotic resonances. The interconnection between the dual constraints of the model and the resonance decouplings exhibits a structural similarity with the interconnection existing in electromagnetic interactions between the coupling-constant relations implied by the conservation of the electric charge and the decoupling of longitudinal photons associated with gauge invariance.

The dual resonance model presented is directly suggested by the approximate linearity of zero trajectories experimentally observed to hold in the scattering of two pseudoscalar mesons and in the invariant amplitude $A(s,t)$ of meson-baryon two-body processes. The model is expected to provide a sensible, although approximate, representation of the couplings of leading resonances appearing in such amplitudes. In order to derive the resonance decouplings the model has to be used in conjunction with either SU(2) or SU(3) symmetry. It is apparent, however, that there are strong interconnections between the model and such symmetries, which cannot be fully appreciated within the limited group of amplitudes to which the model applies in its present form.

Similar interconnections exist with the quark model, which also accounts for some of the observed decoupling effects. As far as SU(3) and the quark model are concerned, however, such interconnections do not go beyond a certain point. The dual-resonance model discussed is incompatible with full SU(3) symmetry in meson-baryon two-body processes—although it provides a correct and non-free-parameter description of all the relevant data—and explains experimental effects unexpected from the quark model or even in conflict with it [like the suppression of the $K^{**}(1420)K\eta$ coupling]. Although the model in its present form applies only to certain amplitudes with simple spin configurations, there are reasons to believe that dual constraints of the same nature are active in any hadronic amplitude. For example, decoupling effects are observed also in amplitudes with more complex spin configurations, and regularities in the behavior of zeros manifest themselves also in processes with a nonsimple amplitude structure like $\bar{p}n \to \pi^-\pi^-\pi^+$ in flight.

The obtained results may be considered of general relevance for the theory of dual models. Besides supporting the significance of these models for the interpretation of the observed low-energy structure (dips and resonance couplings), the results presented here provide specific indications for the theory. The interplay of dual-resonance models and unitary symmetries appears to be much more intriguing and rich in consequences than the conventional handling of such symmetries through Chan-Paton[6] factors would allow one to suspect. A whole potential yet to be developed appears to exist in dual models in this connection. Concerning the treatment of the spin of external particles, while simple dual constraints are observed to be active in amplitudes in which spin wave-functions do not couple with momentum components, less simple dual constraints seem to hold in amplitudes in which such a coupling does occur. To have bosons or fermions as external particles appears to make no difference in this connection. Unfortunately, existing data cover only few classes of amplitudes which can be studied from this point of view. What is observable at present, though, can be interpreted as indicating that superconvergence constraints, whose importance was stressed some time ago by De Alfaro *et al.*,[30] may play a role in determining the type of dual relations holding in a specific amplitude.

APPENDIX A: ELASTIC SCATTERING OF TWO $I=2$ PSEUDOSCALAR MESONS

The SU(2) crossing matrix for $[2]+[2]\to[2]+[2]$ is (see Sec. IV e for the notations):

$$\begin{bmatrix} A_s(0) \\ A_s(1) \\ A_s(2) \\ A_s(3) \\ A_s(4) \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} & 1 & \frac{7}{5} & \frac{9}{5} \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{6}{5} \\ \frac{1}{5} & \frac{3}{10} & -\frac{3}{14} & -\frac{4}{5} & \frac{18}{35} \\ \frac{1}{5} & 0 & -\frac{4}{7} & \frac{1}{2} & -\frac{9}{70} \\ \frac{1}{5} & -\frac{2}{5} & \frac{2}{7} & -\frac{1}{10} & \frac{1}{70} \end{bmatrix} \begin{bmatrix} A_t(0) \\ A_t(1) \\ A_t(2) \\ A_t(3) \\ A_t(4) \end{bmatrix} \tag{A1}$$

There are fourteen $[2]+[2]\to[2]+[2]$ processes which differ among themselves in isospin composition. They are listed in Table II.

APPENDIX B: PHYSICAL INCOMPATIBILITY OF THE LZ MODEL WITH FULL SU(3) SYMMETRY IN MESON-BARYON TWO-BODY PROCESSES

As stated in Sec. V, the LZ dual model of Eq. (2), when applied to the invariant amplitude $A(s,t)$ of $P+B\to P+B$ processes, is not compatible with the simultaneous holding of SU(3) symmetry in the direct (s and u) and exchange (t) channels, unless unphysical possibilities for baryon-resonance couplings are considered. We prove this statement here.

Using the notations of Eq. (2), we can write the amplitude $A(s,t)$ in an SU(3)-invariant way:

$$A(s,t)=F_{tu}\cdot V(t,u)+F_{us}\cdot V(u,s)+F_{st}\cdot V(s,t), \tag{B1}$$

where F_{tu}, F_{us}, and F_{st} are vectors $(1, 8_{as}, 8_{sa}, 8_{ss}, 8_{aa}, 10, \overline{10}, 27)$ describing the content in terms of SU(3) representations in the s channel. Because of the symmetry of the s and u channels, F_{us} must be an eigenvector of the SU(3) crossing matrix X^{us} having eigenvalue $+1$, and F_{tu} must be the transformed of F_{st} by the same matrix. As discussed in Sec. V the model, together with the requirement of absence of exotic states, fixes completely F_{st}, which takes the form

$$F_{st}=(2, (\tfrac{5}{2})^{1/2}, (\tfrac{5}{2})^{1/2}, 0, 1, -1, 0, 0). \tag{B2}$$

As to F_{us}, of the four eigenvectors of X^{us} with eigenvalue $+1$ (see Sec. IV e) only two are left after exotics are eliminated. They can be written (Auvil *et al.*, Ref. 63):

$$E_1=(4, 0, 0, \tfrac{5}{4}, -\tfrac{9}{4}, 0, 0, 0),$$
$$E_2=(-2, (\tfrac{5}{4})^{1/2}, (\tfrac{5}{4})^{1/2}, 0, \tfrac{1}{2}, 1, 0, 0). \tag{B3}$$

Writing $F_{us}=c_1E_1+c_2E_2$, c_1 and c_2 have to be such

TABLE II. Isospin coefficients for the scattering of two identical mesons with isospin $I=2$. The 14 charge configurations listed all differ in isospin composition, and those not included have isospin composition identical to one in the list (see Appendix A).

								Tensor contribs.			Vector contribs.	
	(I_3)	+	(I_3')	→	(I_3'')	+	(I_3''')	$I=0$	$I=2$	$I=4$	$I=1$	$I=3$
1	(−2)	+	(−2)	→	(−2)	+	(−2)	...	...	1	...	...
2	(−2)	+	(−1)	→	(−2)	+	(−1)	...	...	$\frac{1}{2}$	...	$\frac{1}{2}$
3	(−2)	+	(0)	→	(−2)	+	(0)	...	$\frac{2}{7}$	$\frac{3}{14}$	...	$\frac{1}{2}$
4	(−2)	+	(0)	→	(−1)	+	(−1)	...	$-(\frac{6}{7})^{1/2}$	$(\frac{6}{7})^{1/2}$	...	...
5	(−2)	+	(1)	→	(−2)	+	(1)	...	$\frac{3}{7}$	$\frac{1}{14}$	$\frac{1}{5}$	$\frac{3}{10}$
6	(−2)	+	(1)	→	(−1)	+	(0)	...	$(\frac{6}{14})^{1/2}$	$-(\frac{6}{14})^{1/2}$	$(\frac{6}{10})^{1/2}$	$-(\frac{6}{10})^{1/2}$
7	(−2)	+	(2)	→	(−2)	+	(2)	$\frac{1}{5}$	$\frac{2}{7}$	$\frac{1}{70}$	$\frac{2}{5}$	$\frac{1}{10}$
8	(−2)	+	(2)	→	(−1)	+	(1)	$\frac{1}{5}$	$-\frac{1}{7}$	$-\frac{2}{35}$	$\frac{1}{5}$	$-\frac{1}{5}$
9	(−2)	+	(2)	→	(0)	+	(0)	$\frac{1}{5}$	$-\frac{2}{7}$	$\frac{3}{35}$	...	...
10	(−1)	+	(−1)	→	(−1)	+	(−1)	...	$\frac{3}{7}$	$\frac{4}{7}$	...	...
11	(−1)	+	(0)	→	(−1)	+	(0)	...	$\frac{1}{14}$	$\frac{3}{7}$	$\frac{3}{10}$	$\frac{1}{5}$
12	(−1)	+	(1)	→	(−1)	+	(1)	$\frac{1}{5}$	$\frac{1}{14}$	$\frac{8}{35}$	$\frac{1}{10}$	$\frac{2}{5}$
13	(−1)	+	(1)	→	(0)	+	(0)	$\frac{1}{5}$	$\frac{1}{7}$	$-\frac{12}{35}$	...	...
14	(0)	+	(0)	→	(0)	+	(0)	$\frac{1}{5}$	$\frac{2}{7}$	$\frac{18}{35}$	...	...

that in any $P+B\to P+B$ process $A(s,t)$ takes one of the forms allowed by Eq. (2). Let us now require that, in $\pi^+p\to\pi^+p$, Δ_δ resonances dominate, or, in other words, that the resonance spectrum in this process has a definite odd signature. This constrains $c_2=1$ (E_1 does not contribute). One can then verify, for example, that whatever resonance spectrum is chosen for $\pi^-p\to K^0\Lambda$ (i.e., even signature, odd signature, exchange degenerate), thus fixing c_1 (equal to −1, 3, and 1, respectively), it is always impossible to have all $\pi N\to K\Sigma$ processes complying with Eq. (2).

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(see Ref. 28),

$$\mathrm{II}^- = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))\sin\frac{1}{2}\pi(\alpha(t)-\alpha(s))}{\Gamma(1-\alpha(s)-\alpha(t))\sin\frac{1}{2}\pi(\alpha(t)+\alpha(s))}$$

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