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Two-Particle Azimuthal Correlations in High-Energy Collisions

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The angular correlations between the transverse momenta of particles produced in high-energy collisions are studied. An explicit expression is derived for the so-called minimal azimuthal correlations, in terms of the total cross section and the single-particle distribution function. A simple model is also considered for illustration. Experimental distributions for several π^+p reactions at 8.5 GeV/c and at 20 GeV/c are presented and compared with the minimal correlations.

I. INTRODUCTION

Recently the study of two-particle distributions has received considerable attention, on both theoretical and experimental grounds. Such a distribution can be invariantly defined by

$$\rho^{(2)}(s, q_1, q_2) = \frac{4\omega_1\omega_2}{\sigma_T}\, \frac{d\sigma}{dq_1 dq_2},$$

where (ω_i, q_i), $i=1,2$, are the four-momenta of the observed particles. Since it is seen experimentally that transverse momenta of the collision particles are bounded to relatively low values in any frame collinear with the collision axis, a natural choice of the independent variables in $\rho^{(2)}$ is the following: the magnitudes q_{1t}, q_{2t} of the transverse momenta, the azimuthal angle ϕ between them, and the longitudinal momenta q_{1l}, q_{2l} in the c.m. frame.

In this paper we concentrate on the azimuthal distributions, i.e.,

$$\frac{d\sigma^{(n)}}{d\phi} = \sum_{i\neq j} \frac{d\sigma^{(n)}}{d\phi_{ij}},$$

where ϕ_{ij}, as discussed before, is the angle between the transverse momenta $\vec{q}_{1t}$, $\vec{q}_{2t}$ of the two final-state particles. If there should be no correlation, then this distribution will be isotropic in ϕ. The theoretical significance of this distribution to discriminate between different models of particle production has been discussed by several groups.[1]

An important question in analyzing this and the related distributions is the separation of kinematical and dynamical effects. By kinematic correlations we mean those arising simply from energy-momentum conservation and the observed damping of the transverse momenta. For example, momentum conservation requires the azimuthal distribution to be nonisotropic and to peak near 180°, the peak becoming less pronounced as the number of particles increases. A convenient way to effect the separation of so-called dynamics may be to study the Fourier analysis of the distribution, i.e., expand the distribution in its partial waves:

$$\frac{d\sigma^{(n)}}{d\phi} = \sum_{l=0}^{\infty} C_l \cos l\phi\ ,$$

C_l being the coefficient of the lth partial wave. Energy-momentum conservation constrains these coefficients to obey certain sum rules. We find that these sum rules constrain the coefficients of only s and p (i.e., $l=0$ and $l=1$) waves to be nonzero. From lack of further information on the other partial waves, we assume that the higher partial waves simply do not contribute to the expansion. This is what we call "minimal correlation," i.e., that just required by kinematics alone.

In Sec. II we develop the above approach and obtain an explicit expression for the so-called minimal azimuthal distribution function. In Sec. III, we compare our results with the experimental distributions. The following fitted reactions were used: $\pi^+p \to (5\pi)^+p$ at 20 GeV/c,[2] and $\pi^+p \to (3\pi)^+p$, $(5\pi)^+p$, $(7\pi)^+p$ at 8.5 GeV/c.[3] The minimal correlations compare rather well with the data, the agreement becoming better as the number of particles in the final state increases. For the reaction $\pi^+p \to (5\pi)^+p$ (for which we have data at two energies), we find that $(1/\sigma^{(n)})(d\sigma^{(n)}/d\phi)$ is very nearly the same at 8.5 and 20 GeV/c. It seems, therefore, that the over-all azimuthal distribution (summed over all pairs of particles) does not display significant dynamics. We then turn to the distributions $d\sigma^{(n)}/d\phi_{ij}$, where i and j are the leading pion and the proton, respectively. These deviate significantly from minimal correlations and may provide interesting and sensitive tests for various theoretical models.[1]

In Sec. IV, we solve the so-called random-walk model. This may be considered as a generalization of the protostatistical model.[1] In this model, the coefficient of the lth partial wave decreases rapidly with l. As the number of particles in the final state becomes large, this coefficient behaves essentially as

$$\left(\frac{1}{l\,!}\frac{1}{2^{2l}}\right)\frac{1}{(n-2)^{l+1}}\ .$$

Therefore, keeping only s and p waves in the expansion should be a good approximation. Thus this model illustrates to some extent the validity of the assumption of neglecting higher partial-wave contributions to the expansion.

II. PARTIAL-WAVE ANALYSIS

Let us consider the reaction

$$A+B\rightarrow C_1+C_2+\cdots+C_n\ .$$

Let $\vec{k}_1, \ldots, \vec{k}_n$ be the transverse momenta of $C_1, \ldots, C_n$, respectively. We are interested in describing the azimuthal correlations between two outgoing particles, i.e., the distribution

$$\frac{d\sigma^{(n)}}{d\phi}=\sum_{i\neq j}\frac{d\sigma^{(n)}}{d\phi_{ij}}\,,$$

where ϕ_{ij} is the angle between $\vec{k}_i$ and $\vec{k}_j$.

Let

$$\lambda(\vec{k},\vec{k}')\equiv\frac{d\sigma^{(n)}}{d^2k\,d^2k'}$$

and

$$\tilde{\lambda}(\vec{k})\equiv\frac{d\sigma^{(n)}}{d^2k}$$

be the two-particle and single-particle transverse-momentum distribution functions, respectively. By definition, the following relation holds:

$$\int d^2k'\,\lambda(\vec{k},\vec{k}')=(n-1)\,\tilde{\lambda}(\vec{k})\ .$$

Now assuming that $\lambda(\vec{k},\vec{k}')$ is an analytic function of $\vec{k}\cdot\vec{k}'=|\vec{k}|\,|\vec{k}'|\cos\phi$, it is natural to expand it in the following form:

$$\lambda(\vec{k},\vec{k}')\equiv\lambda(k,k',\cos\phi)=\sum_{m=0}^{\infty}a_m(k,k')(k\ k')^m\cos m\phi.$$

Here ϕ is the angle between $\vec{k}$ and $\vec{k}'$ and k, k' are their magnitudes.

The coefficient of the mth partial wave is given by

$$a_m(k,k')=\frac{1}{\pi}\,\frac{1}{(k\ k')^m}\int_0^{2\pi}\lambda(\vec{k},\vec{k}')\cos m\phi\,d\phi,\quad m>0$$

and

$$a_0(k,k')=\frac{1}{2\pi}\int_0^{2\pi}\lambda(\vec{k},\vec{k}')\,d\phi\ .$$

We have two sum rules which the distribution function $\lambda(k,k')$ must satisfy. The first sum rule just gives the over-all normalization of the distribution

$$\int\lambda(\vec{k},\vec{k}')\,d^2k\,d^2k'=n(n-1)\,\sigma_n\ ,\tag{1}$$

where σ_n is the total cross section for the reaction under consideration.

The second sum rule expresses transverse-momentum conservation

$$\int\lambda(\vec{k},\vec{k}')\,\vec{k}'\,d^2k'=-\vec{k}\,\tilde{\lambda}(k)\ .\tag{2}$$

From Eq. (1), we get

$$2\pi\int\left[\sum_{m=0}^{\infty}a_m(k,k')\,(k\ k')^m\cos m\phi\right]k\,k'\,d\phi\,dk\,dk'=n(n-1)\,\sigma_n\ ,$$

which gives the following relation for the s-wave coefficient:

$$(2\pi)^2\int a_0(k,k')\,(k\ k')\,dk\,dk'=n(n-1)\,\sigma_n\ .\tag{3}$$

Similarly from the sum rule (2) we get the following relation for the p-wave coefficient:

$$\pi\int k'^3\,a_1(k,k')\,dk'=-\tilde{\lambda}(k)\ .\tag{4}$$

To make further progress with the above formalism, we assume that the p-wave coefficient a_1 factorizes in k and k':

$$a_1(k,k')=\alpha\phi(k)\,\phi(k')\ .$$

It follows then that $\phi(k)$ is simply proportional to $\tilde{\lambda}(k)$, the single-particle distribution function. Relation (4) then gives a_1 in terms of single-particle distribution function $\lambda(k)$:

$$a_1(k,k')=-\left[\frac{1}{\pi\int dk'k'^3\tilde{\lambda}(k')}\right]\tilde{\lambda}(k)\,\tilde{\lambda}(k')\ .\tag{5}$$

There is no further information on the other partial waves. Let us assume then that the coefficients $a_m(k,k')=0$ for $m>2$, i.e., there is minimal correlation, just that arising from kinematic constraints. The azimuthal distribution is then given by

$$\frac{d\sigma}{d\phi}\equiv\frac{2\pi}{n(n-1)}\int\lambda(\vec{k},\vec{k}')\ k\,dk\,k'\,dk'$$

$$=\frac{2\pi}{n(n-1)}\left\{\left[\int k\,k'\,a_0(k,k')\,dk\,dk'\right]+\left[\int(k\ k')^2\,a_1(k,k')\,dk\,dk'\right]\cos\phi\right\}\ .$$

Using Eqs. (3) and (5), we finally get

$$\frac{d\sigma^{(n)}}{d\phi}=C_0+C_1\cos\phi\ , \tag{6}$$

where

$$C_0=\frac{\sigma^{(n)}}{2\pi},$$

$$C_1=-\frac{2}{n(n-1)\int k'^3\tilde{\lambda}(k')\,dk'}\left[\int k'^2\tilde{\lambda}(k')\,dk'\right]^2 .$$

Equation (6) gives the minimal correlation function in terms of the single-particle distribution $\tilde{\lambda}(k)$ and the cross section $\sigma^{(n)}$.

By minimal correlation we mean that in the Fourier expansion of the two-particle distribution we have retained only those coefficients which are required to be nonzero in order to satisfy momentum conservation. The terms with $m\geq 2$ may not add constructively and thus Eq. (6) does not give the theoretically minimum correlation function.

A few remarks about the input in deriving Eq. (6) are in order:

(a) Over-all normalization of the two-particle distribution function.

(b) Momentum conservation. It is to be noted that Eq. (6) does not contain all the kinematical information. Only momentum conservation has been used in deriving this, while energy conservation has been left out. But this hopefully has very little effect on the azimuthal distributions because the large transverse momenta (which are most affected by this constraint) are strongly damped experimentally.

(c) Factorization assumption. We have assumed that the p-wave coefficient of the Fourier expansion of $\lambda(\vec{k},\vec{k}')$ factorizes in k and k':

$$a_1(k,k')=\alpha\phi(k)\,\phi(k')\ .$$

Clearly this assumption has no theoretical basis and has been made for calculational convenience only. It turns out, however, that the ratio C_1/C_0 is not very sensitive to the assumption made above. We shall see in Sec. IV, for example, that the random-walk model (which does not satisfy factorization) gives essentially the same result for this ratio, provided n is large.

We now make an estimate for the coefficient C_1. Let us assume that the single-particle distribution is given by

$$\tilde{\lambda}(k)=a\,e^{-bk^2}\ .$$

Then

$$C_1=-\frac{\pi}{4}\frac{a}{b}\frac{1}{n(n-1)}=-\frac{\sigma^{(n)}}{4(n-1)}\ ,$$

since

$$\sigma^{(n)}=\frac{1}{n}\int\tilde{\lambda}(k)\,d^2k=\frac{\pi a}{nb}\ .$$

Thus

$$\frac{d\sigma^{(n)}}{d\phi}\sim\frac{\sigma^{(n)}}{2\pi}-\frac{\sigma^{(n)}}{4(n-1)}\cos\phi. \tag{7}$$

According to Eq. (7), the azimuthal distribution should peak near 180°, the peak becoming less pronounced as the number of particles in the final state increases.

III. EXPERIMENTAL DISTRIBUTIONS

In Fig. 1 we present the azimuthal angular distributions for several π^+p reactions. The following fitted reactions were used: $\pi^+p\to(3\pi)^+p$, $(5\pi)^+p$,

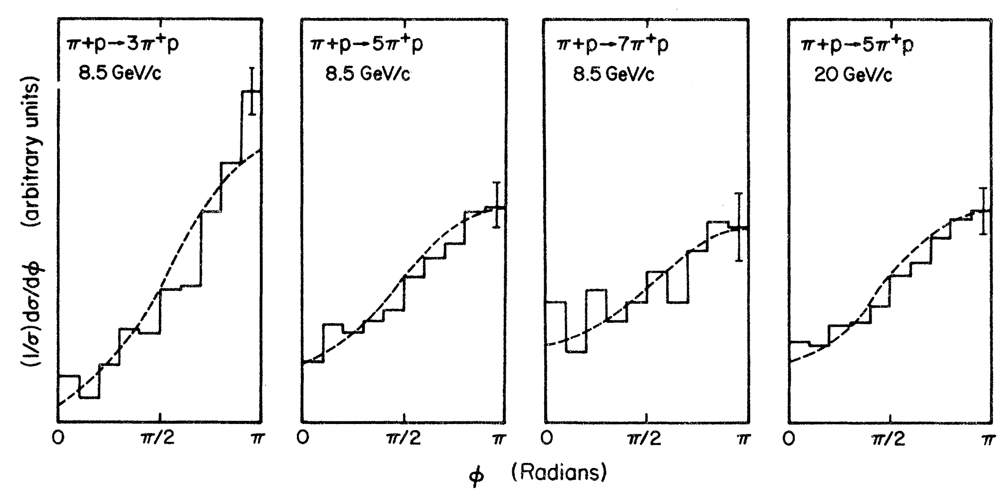

FIG. 1. Normalized $d\sigma^{(n)}/d\phi$ distributions for several π^+p fitted reactions. The dashed curves show the minimal correlation function described in the text.

$(7\pi)^+p$ at 8.5 GeV/c and $\pi^+p\to(5\pi)^+p$ at 20 GeV/c. We have normalized the distribution in each case so that

$$\int_0^\pi \frac{d\sigma^{(n)}}{d\phi}\,d\phi = 1\ .$$

In the figure the dashed lines show the minimal correlations described in Sec. II. The p-wave contribution has been calculated using the experimental single-particle distributions in each case. It is found that the agreement between the calculated and the experimental curves becomes better as n increases.

Figure 2 shows the azimuthal correlations between the leading particles, i.e., between the proton and the fastest forward-going pion in the c.m. system. One may show that the energy-momentum conservation does not require this distribution to be nonisotropic.[4] We note that the correlations between the leading particles are larger than the correlation in the over-all distribution. This observation is quite interesting and may play an important part in our understanding of multiparticle production.

In Table I we give experimental values of the backward-forward asymmetry parameter R, defined by

$$R=\left(\int_{\pi/2}^{\pi}\frac{d\sigma}{d\phi}\,d\phi-\int_0^{\pi/2}\frac{d\sigma}{d\phi}\,d\phi\right)\Big/\int_0^{\pi}\frac{d\sigma}{d\phi}\,d\phi\ .$$

In particular, for the reaction $\pi^+p\to(5\pi)^+p$ (for which we have the distributions at two energies), we find the asymmetry parameter to be $\approx 19.3\%$ at 8.5 GeV/c and $\approx 18.2\%$ at 20 GeV/c lab momenta. (See Table I.) Also the distribution $(1/\sigma^{(n)})(d\sigma^{(n)}/d\phi)$ for this reaction is very nearly the same at the two energies considered.

TABLE I. The asymmetry parameter R for various π^+p fitted reactions. Errors quoted are statistical.

Reaction	Particle combination	R	Minimal value of R
$\pi^+p\to(3\pi)^+p$	All pairs $\left(\frac{d\sigma^{(n)}}{d\phi}\right)$	0.317 ± 0.020	0.33
	Leading particles	$0.41\ \pm0.05$	
$\pi^+p\to(5\pi)^+p$ (8.5 GeV/c)	All pairs	0.193 ± 0.015	0.20
	Leading particles	$0.28\ \pm0.04$	
$\pi^+p\to(7\pi)^+p$	All pairs	0.093 ± 0.025	0.14
$\pi^+p\to(5\pi)^+p$	All pairs	0.182 ± 0.015	0.20
	Leading particles	$0.25\ \pm0.04$	

IV. RANDOM-WALK MODEL

The random-walk model is defined by

$$\frac{d\sigma^{(n)}}{d\vec{k}_1\cdots d\vec{k}_n}=\prod_{i=1}^{n}\phi(k_i)\,\delta^2\left(\sum_{i=1}^{n}\vec{k}_i\right).\tag{8}$$

$\phi(k_i)$ is obviously related to the single-particle spectrum. Again energy conservation has been left out from this model.

From (8)

$$\frac{d\sigma^{(n)}}{d\vec{k}\,d\vec{k}'}=\frac{1}{(n-2)!}\,\phi(k)\,\phi(k')\int\prod_{i=1}^{n-2}d^2\vec{k}_i\,\phi(k_i)\times\delta^2\left(\sum_{i=1}^{n-2}\vec{k}_i+\vec{k}+\vec{k}'\right).$$

Using a Fourier representation of the $\delta^{(2)}$ function, we get

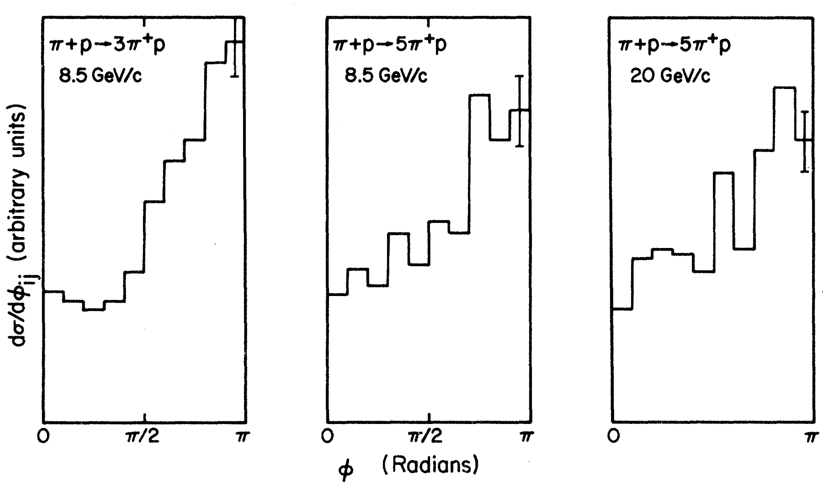

FIG. 2. Normalized $d\sigma^{(n)}/d\phi_{ij}$ distributions, where i and j are, respectively, the proton and the fastest forward-going pion in the c.m. system. The distribution for $\pi^+p\to(7\pi)^+p$ has been left out for reasons of inadequate statistics for this reaction.

$$\frac{d\sigma^{(n)}}{d\vec{k}\,d\vec{k}'}=\left(\frac{1}{2\pi}\right)^2\frac{1}{(n-2)!}\,\phi(k)\,\phi(k')\int d^2x\,e^{i(\vec{k}+\vec{k}')\cdot\vec{x}}\times[\psi(x)]^{n-2},$$

where

$$\psi(x)=\int d^2l\,e^{i\vec{l}\cdot\vec{x}}\phi(\vec{l})$$

so that

$$\lambda(\vec{k},\vec{k}')=\left(\frac{1}{2\pi}\right)^2\frac{1}{(n-2)!}\phi(k)\,\phi(k')\times\int_0^\infty x\,dx[\psi(x)]^{n-2}\int_0^{2\pi}d\theta\,e^{i|\vec{k}+\vec{k}'|x\cos\theta}$$
$$=\frac{1}{(n-2)!}\phi(k)\,\phi(k')\times\int_0^\infty x\,dx[\psi(x)]^{n-2}\frac{1}{2\pi}J_0(|\vec{k}+\vec{k}'|x)\,.$$

Now the Bessel function $J_0(|\vec{k}+\vec{k}'|x)$ can be expanded in the following form:

$$J_0(|\vec{k}+\vec{k}'|x)=J_0(kx)J_0(k'x)+2\sum_{l=1}^{\infty}(-1)^l J_l(kx)J_l(k'x)\cos l\phi\,,$$

where ϕ is the angle between $\vec{k}$ and $\vec{k}'$. Accordingly we can write

$$\lambda(\vec{k},\vec{k}')=f_0(k,k')+\sum_{l=1}^{\infty}f_l(k,k')\cos l\phi\,,$$

where

$$f_0(k,k')=\frac{1}{(n-2)!}\phi(k)\,\phi(k')\times\frac{1}{2\pi}\int_0^{\pi}x\,dx[\psi(x)]^{n-2}J_0(kx)J_0(k'x)$$

and

$$f_l(k,k')=\frac{(-1)^l}{(n-2)!}\phi(k)\,\phi(k')\times\frac{1}{\pi}\int_0^\infty x\,dx[\psi(x)]^{n-2}J_l(kx)J_l(k'x)\,.$$

Therefore

$$\frac{d\sigma^{(n)}}{d\phi}\equiv\frac{2\pi}{n(n-1)}\int_0^\infty k\,dk\int_0^\infty k'\,dk'\,\lambda(\vec{k},\vec{k}')=C_0+\sum_{l=1}^{\infty}C_l\cos l\phi\,.$$

We next investigate the asymptotic behavior of C_l as the number of particles produced gets large. We do this by the method of steepest descent:

$$C_l=\frac{2(-1)^l}{n!}\int_0^\infty k\,dk\int_0^\infty k'\,dk'\,\phi(k)\,\phi(k')\times\int_0^\infty x\,dx[\psi(x)]^{n-2}J_l(kx)J_l(k'x)$$
$$=\frac{2(-1)^l}{n!}\int_0^\infty x\,dx[\psi(x)]^{n-2}[\alpha_l(x)]^2,\quad l>0 \tag{9}$$

$$C_0=\frac{1}{n!}\int_0^\infty k\,dk\int_0^\infty k'\,dk'\,\phi(k)\,\phi(k')\times\int_0^\infty x\,dx[\psi(x)]^{n-2}J_0(kx)J_0(k'x)\,,$$

where

$$\alpha_l(x)=\int_0^\infty k\,dk\,J_l(kx)\,\phi(k)\,.$$

Let us first examine $\psi(x)$:

$$\psi(x)=2\pi\int_0^\infty k\,dk\,\phi(k)J_0(kx)\,.$$

Its first derivative is given by

$$\psi'(x)=-2\pi\int_0^\infty k^2\,dk\,\phi(k)J_1(kx)\,.$$

Since $\phi(k)$ represents essentially the single-particle momentum distribution function, it is a positive definite function of k with maximum near $k\sim 0$. Therefore

$$\psi'(x)<0,\quad x\neq 0$$

and

$$\psi'(0)=0.$$

Further, the second derivative of $\psi(x)$ at $x=0$ is

$$\psi''(0)=-\pi\int_0^\infty k^3\,dk\,\phi(k)<0,$$

so that $\psi(x)$ is a monotonic function of x with maximum at $x=0$. Therefore

$$[\psi(x)]^{n-2}\equiv\exp[(n-2)\ln\psi(x)]$$
$$=\exp\{(n-2)\ln[\psi(0)+\tfrac{1}{2}\psi''(0)x^2+\cdots]\}$$
$$\approx[\psi(0)]^{n-2}\exp\left[\frac{n-2}{2}\frac{\psi''(0)}{\psi(0)}x^2\right].$$

Let us next see how $\alpha_l(x)$ behaves near $x\sim 0$. Using

$$J_l(ax)\sim\left(\frac{ax}{2}\right)^l\frac{1}{l!}\left[1-\frac{1}{l+1}\left(\frac{ax}{2}\right)^2+O(x^4)\right],\quad x\to 0$$

we obtain

$$\alpha_l(x)\underset{x\sim 0}{\sim}x^l\frac{1}{2^l}\frac{1}{l!}\int_0^\infty k^{l+1}\,dk\,\phi(k)\,.$$

Substituting these in Eq. (9), we find that

$$C_l \sim \frac{(-1)^l}{n!} 2 \frac{1}{2^{2l}} \frac{1}{(l!)^2} \left[\int_0^\infty k^{2l+1} dk\, \phi(k) \right]^2 [\psi(0)]^{n-2}$$

$$\times \int_0^\infty x^{2l+1} dx \exp\left[\frac{n-2}{2} \frac{\psi''(0)}{\psi(0)} x^2 \right]$$

$$= \frac{(-1)^l}{n!} \frac{1}{2^{2l}} \frac{1}{l!} \frac{1}{(n-2)^{l+1}} \frac{[\psi(0)]^{n-2}}{\left[-\frac{1}{2} \frac{\psi''(0)}{\psi(0)} \right]^{l+1}}$$

$$\times \left[\int_0^\infty k^{l+1} dk\, \phi(k) \right]^2 . \qquad (10)$$

From (10) it is clear that if n is large, C_l decreases rapidly with l. C_l behaves essentially as $(1/2^{2l})\,(1/l!)\,[1/(n-2)^{l+1}]$. We remark that the assumption of "minimal correlation" described in Sec. II, i.e., s- and p-wave domination of the distribution, is clearly illustrated by this model.

We have

$$\frac{d\sigma^{(n)}}{d\phi} = C_0 + C_1 \cos\phi + \cdots .$$

The first two terms in this model are given by

$$C_0 \sim \frac{1}{n!} \frac{1}{2} \frac{1}{n-2} \frac{[\psi(0)]^{n-2}}{\left[-\frac{1}{2} \frac{\psi''(0)}{\psi(0)} \right]} \left[\int_0^\infty k\, dk\, \phi(k) \right]^2 ,$$

$$C_1 \sim \frac{-1}{n!} \frac{1}{2^2} \frac{1}{(n-2)^2} \frac{[\psi(0)]^{n-2}}{[-\frac{1}{2}\psi''(0)/\psi(0)]^2} \left[\int_0^\infty k^2\, dk\, \phi(k) \right]^2 .$$

Clearly C_0 is related to the total cross section for this reaction.

Also C_1 can be expressed in terms of the single-particle distribution:

$$\tilde{\lambda}(k) = \frac{1}{(n-1)!} \phi(k) \int \prod_{i=1}^{n-1} dk_i\, \phi(k_i)\, \delta^2\left(\sum_{i=1}^{n-1} \vec{k}_i + \vec{k} \right)$$

$$= \frac{1}{2\pi} \frac{1}{(n-1)!} \phi(k) \int_0^\infty x [\psi(x)]^{n-1} J_0(kx)\, dx$$

$$\simeq \frac{1}{2\pi} \frac{1}{(n-1)!} \phi(k) \int_0^\infty x\, dx [\psi(0)]^{n-1}$$

$$\times \exp\left[\frac{n-1}{2} \frac{\psi''(0)}{\psi(0)} x^2 \right] J_0(kx)$$

$$= -\frac{1}{2\pi} \frac{1}{(n-1)!} \frac{[\psi(0)]^n}{\psi''(0)}$$

$$\times \exp\left[-\frac{k^2}{2(n-1)\,\psi''(0)/\psi(0)} \right] \phi(k) \left(\frac{1}{n-1} \right)$$

$$\simeq \frac{-1}{2\pi} \frac{1}{(n-1)!} \frac{[\psi(0)]^n}{\psi''(0)} \phi(k) \left(\frac{1}{n-1} \right).$$

The cross section is then given by

$$\sigma^{(n)} \sim \frac{1}{2\pi} \frac{1}{(n-1)!} \frac{[\psi(0)]^n}{n-1} \left(-\frac{\psi(0)}{\psi''(0)} \right).$$

Keeping the first two terms,

$$\frac{d\sigma^{(n)}}{d\phi} = \frac{2\pi}{(n-2)^2} \left\{ \frac{n-2}{2} \frac{1}{\sigma^{(n)}} \left[\int_0^\infty k\, \tilde{\lambda}(k)\, dk \right]^2 - \frac{[\int_0^\infty k^2\, \tilde{\lambda}(k)\, dk]^2}{\pi \int_0^\infty k^3\, \tilde{\lambda}(k)\, dk} \cos\phi \right\}$$

$$= \frac{n}{n-2} \frac{\sigma^{(n)}}{2\pi} - \frac{2}{(n-2)^2} \frac{[\int_0^\infty k^2\, \tilde{\lambda}(k)\, dk]^2}{\int_0^\infty k^3\, \tilde{\lambda}(k)\, dk} \cos\phi .$$

We note that the ratio

$$\frac{C_1}{C_0} \approx -\frac{[\int k^2\, \tilde{\lambda}(k)\, dk]^2}{\int k^3\, \tilde{\lambda}(k)\, dk} \frac{2}{n(n-2)} \frac{2\pi}{\sigma^{(n)}}$$

in this model is essentially the same as obtained in Sec. II. This ratio is thus not very sensitive to the assumptions involved.

V. CONCLUSIONS

We have studied the angular correlations between the transverse momenta of particles produced in high-energy collisions. An attempt is made to identify the correlations which arise simply from momentum conservation and the experimentally observed damping of the transverse momenta. An explicit expression is derived for the so-called minimal azimuthal correlations in terms of the total cross section and the single-particle transverse-momentum distribution function. We find that the shape and multiplicity dependence of the over-all azimuthal distribution function $d\sigma^{(n)}/d\phi$ for several π^+p reactions considered here compares rather well with the minimal correlations, the agreement becoming better as the number of particles in the final state increases. We conclude therefore that $d\sigma^{(n)}/d\phi$ does not display significant dynamical effects.

The azimuthal distributions $d\sigma^{(n)}/d\phi_{ij}$ of specific pairs of particles, however, show significant deviations from the minimal correlations. In particular, if i and j are in the opposite hemisphere in the c.m. system, kinematics does not restrict this distribution. We have presented the experimental distributions for the leading particles for some π^+p reactions. It is observed that the correlations between the leading particles are larger than the correlation in the over-all distribution. (See Table I.)

Finally we note that for the reaction $\pi^+p \to (5\pi)^+p$, for which we have the distributions at two energies, the distribution $(1/\sigma^{(n)})\ d\sigma^{(n)}/d\phi$ is roughly independent of the energy.

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Production and Decay of Heavy Vector Mesons in a Statistical Model*

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The assumption is made that there are many heavy vector mesons with density given by the statistical bootstrap. Using statistical methods and generalized vector dominance we are then able to calculate angle-integrated cross sections for the reactions $e^+e^- \to \pi^+\pi^-$, $\gamma p \to (\rho\pi\pi)p$, and γ nucleus $\to \pi^+\pi^-$nucleus. We also calculate $\pi p \to (\rho\pi\pi)\Delta$ which does not involve vector dominance. The results compare well with experiment and, along with results previously obtained for other reactions, demonstrate a method for calculating spectral shapes for many processes.

I. INTRODUCTION

The experimental search for vector mesons more massive than the ρ, ω, and ϕ has produced a variety of results.

(1) There are no discrete bumps in the reaction $e^+e^- \to \pi^+\pi^-$ that could be interpreted as high-mass vector mesons.[1] Rather, the cross section falls smoothly and rapidly with increasing energy.

(2) The reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ shows a broad enhancement at 1.6 GeV on the order of 400 MeV wide[2] predominantly in the $\rho\pi\pi$ channel.[3]

(3) The $\mu^+\mu^-$ spectrum from the reaction $pp \to \mu^+\mu^- +$ anything shows no pronounced structure for the $\mu^+\mu^-$ invariant mass varying from 1 to 6 GeV. The spectrum is rather smooth[4] with a slight shoulder and can be parameterized by the approximate proportionality $d\sigma/dm_{\mu^+\mu^-} \propto 1/m_{\mu^+\mu^-}{}^5$.

(4) A study of the reaction $\gamma p \to 2\pi^+2\pi^-p$ using 9.3-GeV/c linearly polarized photons gives evidence for a broad, peripherally produced four-pion enhancement with mass ~1.5 GeV.[5]

(5) Contrasting with the γp study, an experiment on the reaction $\pi^+p \to \rho^0\pi^+\pi^-\Delta^{++}$ at 5 GeV shows no evidence for an enhancement in the $\rho^0\pi^+\pi^-$ spectrum from 1 to 2 GeV.[6]

(6) The reactions $\gamma\,\mathrm{Be} \to \pi^+\pi^-\mathrm{Be}$ (Ref. 7) and $\gamma\,\mathrm{C} \to \pi^+\pi^-\mathrm{C}$ (Ref. 8) show broad enhancements around 1.5 GeV in the $\pi^+\pi^-$ invariant mass.

We describe the results of these experiments here making use of the following.

(a) The density of hadron levels or resonances rises exponentially times some power with mass. Such an exponential rise was first proposed by Hagedorn[9] and has been realized in the statistical bootstrap model of Frautschi[10] as well as in other models.

(b) The coupling of a resonance to a given channel (real or virtual) is inversely proportional to the density of levels at that mass, and coupling constants have more or less random or rapidly oscil-