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Statistical Approach to Parton Models: Application to 90° Cross Sections*

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We present a parton statistical model in which the pairing interaction has been explicitly taken into account. Using a generalized equidistant model for the density of states we calculate the 90° differential elastic two-body cross section in terms of an incoherent sum of direct-channel resonances. Encouraging agreement with the data for $\pi^{\pm}p$ and $p\bar{p}$ elastic reaction is obtained. Two main results are deduced from our analysis: (a) an approximate exponential decrease of $d\sigma/d\Omega|_{90^\circ}$; (b) a break structure in $d\sigma/d\Omega|_{90^\circ}$. The position of the break is fixed at the energy where the temperature of the system reaches its critical value, i.e., a phase transition occurs.

I. INTRODUCTION

The description of interacting hadrons as a many-body system has been advanced recently in two different directions: (1) the statistical bootstrap model (SBM) of Hagedorn and Frautschi[1,2]; (2) the parton picture suggested by Feynman.[3]

The main assumptions of the statistical bootstrap model are as follows.

(a) Resonances rise indefinitely with mass.

(b) The bootstrap hypothesis: Resonances are built from each other, i.e., the constituents are the hadrons themselves.

(c) The mutual interaction of hadrons can be completely represented by resonance formation.

(d) The only other effect of interactions is to confine the constituents within a characteristic volume.

In the framework of this model the density of hadron states with mass m is predicted to be[4,5]

$$\rho(m)=\frac{a}{m^3}e^{m/T_0}, \tag{1}$$

where a is a constant and T_0 is the so-called critical temperature and is given approximately by the mass of the pion. Since $\rho(m)$ is connected to various measurable quantities, relation (1) can be tested. Predictions based on the above level density have been given recently,[6,7] indicating the usefulness of a statistical approach. The physical picture, as stated in assumptions (a)–(d), is not a simple one and a more satisfactory description of the interactions is needed.

The parton model, where one assumes that a hadron is built from fundamental constituents, may be easier to grasp intuitively and has given many gratifying results in various branches of high-energy physics.[8] Since the full understanding of interacting systems in this model amounts to the solution of a many-body problem, one is only able to apply approximation schemes in testing the model.

A natural approach to try when dealing with a many-body problem is to use statistical methods for the description of the various properties of the relevant system. Although one cannot expect to describe the fine details of the data in the framework of a statistical model, one may hope to gain some understanding of the gross features of the interacting system. The fruitfulness of such an approach was clearly demonstrated by statistical models of solid state and nuclear physics, which are generally accepted as the best available explanation of large bodies of data. Encouraged by the results of nuclear statistical models, and the statistical bootstrap model, but aware of the differences between a nucleus and a hadron, we attempt to formulate in this paper a statistical model of the parton picture.

In the parton model the hadron is visualized as a composite system built up of pointlike constituents possibly tightly bound together in the hadron rest frame. Since it is plausible that the current-carrying partons have spin $\frac{1}{2}$, we are dealing here with a many-fermion system resembling, in some aspects, the nucleus. Hence we can try to utilize successful nuclear models to help our intuition regarding hadron-hadron interactions.

In describing the interaction of heavy nuclei one usually distinguishes between two type of reactions: (a) direct reaction, and (b) compound nucleus reactions. The former is what is known as an exchange reaction in hadron physics; the latter is a two-stage reaction where the decay stage is independent of the formation and is described by statistical methods. From another point of view, we may characterize the reactions by the way the resonances add to contribute to the scattering cross section; for type-(a) reactions many direct-channel resonances add coherently, whereas they add incoherently in type-(b) reactions.

In analogy with nuclear physics we propose a two-component interaction picture of the parton model. We would like to distinguish between (a) exchange reactions which are parametrized in terms of meson, nucleon, photon, etc. exchanges and (b) fireball creation and decay interactions which are best described by statistical methods. As in nuclear physics, we expect to find for some reactions quantities which are best described by one of the above-mentioned components, for others quantities best described by the other, and for some reactions quantities best described by a mixture of the two. We are interested here mainly in the nonexchange component; thus we are looking for measurable quantities for which exchanges are not the dominating contribution. As was emphasized by Gunion *et al.*[9] one might have just such a quantity in the differential cross section at 90°, since due to the large momentum transfer and small distances involved the coherent Regge-like states' contribution is expected to be relatively small.

A convenient way of translating the parton fireball hypothesis to testable predictions is to use the density-of-states formalism of statistical mechanics. This formalism was used quite extensively in nuclear physics, and also recently in hadron physics in the framework of the statistical bootstrap model. An analytical expression for the differential cross section in terms of the density-of-states function was recently derived,[7] and will be discussed along with the general density formalism in Sec. II.

As is emphasized in Sec. II as well as in all the following sections, we believe that, similar to other many-fermion systems, the pairing interaction plays an important role in the parton picture of hadrons. The properties and effects of this pairing force in the case of electrons in metals and alloys, and of nucleons in nuclei, have been extensively studied[10] and tested, and they seem to be quite well understood.

In those systems it was found that as a result of the large overlap in the matrix elements of the interaction between fermions with opposite momenta and spin, the ground-state energy is lowered from the lowest possible energy without pairing. Moreover, it was found that the pairing interaction implies a large energy gap between the superconducting ground state and the first excited state. In a previous publication[11] we considered the possibility that pairing between partons causes a large gap in the meson spectrum through a pair-breaking mechanism in which the first excited state is due to breaking a pair for which a minimal energy (denoted 2Δ) is required. Identifying the pion as the superconducting ground state of the meson system, we suggested that the pair-breaking excitation of the pion leads to its corresponding member in the next SU(3) multiplet, the ρ meson. From the BCS theory of superconductivity, a universal relation emerges between the zero-temperature gap parameter $\Delta(0)$ and the critical temperature T_0 (see Refs. 10 and 12):

$$2\Delta(0)=3.52T_0. \tag{2}$$

At T_0 all the pairs are broken, and a phase transition occurs. From Eq. (2) we concluded that a critical temperature of $T_0\simeq 177$ MeV is expected to be of considerable significance in hadron physics. Estimates of this quantity given in the framework of other models[1,4,13,14] agree quite well with our estimate. Whether at T_0 there is a one-side phase transition, the other side being forever inaccessible (i.e., T_0 is a limiting temperature as was suggested by Hagedorn[1]), or whether $T>T_0$ is possible as was claimed by other authors,[15] was not resolved by our considerations. We will comment about this point here.

It is only reasonable to assume that if a phase transition in hadronic matter does exist it should lead to an experimentally detectable phenomenon. In Sec. III we discuss the possibility that the manifestation of such a phase transition may be a break in the 90° differential cross section and show experimental evidence for such a break at center-of-mass energies corresponding to the critical temperature evaluated in our previous publication.[11] It was also noticed in Ref. 11 that due to the presence of an odd number of relevant partons in the baryon spectrum ground state (the nucleon), one does not expect to see a large gap between the ground state and first excited state [$\Delta(1236)$] of the baryon spectrum, a phenomenon similar to the even-odd effect of nuclear physics.[16] Now, since more states are accessible for an odd system than for an even system, due to the possibility of more single-particle excitation levels one expects also to find an even-odd effect in the density of states; this effect is well known in nuclear physics, where experimentally it was found that[17]

$$\tfrac{1}{2}\rho_{\text{odd-odd}}\simeq\rho_{\text{even-odd}}\simeq 5\rho_{\text{even-even}} \tag{3}$$

A similar effect may exist in hadron physics where there are more baryonic than mesonic states.[18]

II. DENSITY OF STATES AND STATISTICAL CROSS SECTIONS

As was explained in the Introduction we consider a fireball-creating interaction, where the resonance levels are due to parton excitations of the interacting system. The scattering amplitude will then be written as a sum over uncorrelated direct-channel resonances, and the differential cross section at 90° is given in terms of an incoherent superposition of a large number of resonances. (The interference terms are assumed to vanish near 90° because of the large number of resonances of many angular momenta.) Using this assumption one finds (for details see Ref. 7) that to a good approximation the differential cross section for the general reaction

$$a+b\to c+d \tag{4}$$

is given by

$$\frac{d\sigma}{d\Omega}\propto\frac{\Gamma_{ab}(\sqrt{s})\Gamma_{cd}(\sqrt{s})\rho(\sqrt{s})}{s\Gamma(\sqrt{s})}\,\frac{\lambda(s,m_c^2,m_d^2)}{\lambda(s,m_a^2,m_b^2)}\,\xi(s,\theta), \tag{5}$$

where the masses of particle a,b,c,d are m_a, m_b, m_c, m_d, respectively, s is the center-of-mass energy squared, Ω represents center-of-mass scattering angle for particle c,

$$\lambda(s,m_a^2,m_b^2)=(s^2+m_a^4+m_b^4-2m_a^2s-2m_b^2s-2m_a^2m_b^2)^{1/2},$$

$\Gamma(\sqrt{s})$ is the total width of the resonance, $\Gamma_{ab}(\sqrt{s})$ and $\Gamma_{cd}(\sqrt{s})$ are the decay widths of the relevant resonances to channel ab and cd, respectively, and

$$\xi(s,\theta)=\frac{\sum_{l=0}^{kR}(2l+1)^2P_l^2(\cos\theta)}{\sum_{l=0}^{kR}(2l+1)}, \tag{6}$$

where R is the hadronic radius and θ is the c.m. scattering angle.

Following nuclear statistical models[19] and the statistical bootstrap model[20] we assume that the decay width into different final states is given in terms of the density of states in the form

$$\Gamma_{ab}(m,m_a,m_b)=\frac{AF(m,m_a,m_b)}{\rho(m)}, \tag{7}$$

where A is a constant and

$$F(m,m_a,m_b)=\frac{m^4-(m_a^2-m_b^2)^2}{m^2} \tag{8}$$

is the ratio of noninvariant two-body phase space to invariant phase space evaluated in the rest frame of the massive state. The factor $F(m,m_a,m_b)$ was introduced in relation (7) in order to compensate for the Lorentz-invariant phase-space element in the expression for the resonance decay width,

$$\Gamma(d\to ab)\propto\int df_{\text{Lips}}(s_d,p_a,p_b)\,|T(d\to ab)|^2, \tag{9}$$

where $df_{\text{Lips}}(s_d,p_a,p_b)$ is the Lorentz-invariant phase-space element and $T(d\to ab)$ is the decay amplitude (see, for example, Ref. 21). Using (7) and (8) we then find

$$\frac{d\sigma}{d\Omega} \propto \frac{\lambda(s, m_c^2, m_d^2)}{\lambda(s, m_a^2, m_b^2)} \left[\frac{s^2 - (m_b^2 - m_a^2)^2}{s}\right]$$

$$\times \left[\frac{s^2 - (m_d^2 - m_c^2)^2}{s}\right] \frac{\xi(s,\theta)}{\Gamma(\sqrt{s})\rho(\sqrt{s})s}. \qquad (10)$$

Let us now calculate the density of states for a fireball composed of two systems of fermions in thermal equilibrium with each other. As stated above we assume that pairing forces exist between fermions of each system. Each system may have either an odd or an even number of fermions leading to even-even, even-odd, or odd-odd fireballs. Note that the effect of pairing should be most significant in an even-even fireball due to the nonexistence of quasiparticles in the ground states (i.e., all the relevant fermions are paired).

Using the well-known saddle-point method the density of states is given by[22]

$$\rho(U') = \frac{e^S}{(2\pi)^{3/2}(\det A)^{1/2}}, \qquad (11)$$

where S denotes the entropy:

$$S = S_1 + S_2 \qquad (12)$$

(S_i, $i = 1, 2$, are the entropies of the two systems), and the excitation energy U' is given in an obvious notation by

$$U' = U_1' + U_2'. \qquad (13)$$

Furthermore,

$$\det A = \begin{vmatrix} \frac{\partial^2(f_1+f_2)}{\partial\beta^2} & \frac{\partial^2 f_1}{\partial\beta\partial\alpha_1} & \frac{\partial^2 f_2}{\partial\beta\partial\alpha_2} \\ \frac{\partial^2 f_1}{\partial\alpha_1\partial\beta} & \frac{\partial^2 f_1}{\partial\alpha_1^2} & 0 \\ \frac{\partial^2 f_2}{\partial\alpha_2\partial\beta} & 0 & \frac{\partial^2 f_2}{\partial\alpha_2^2} \end{vmatrix}, \qquad (14)$$

where the logarithm of the partition function for each system is f_i, $\beta = 1/T$ (T denotes temperature), and $\alpha_i = \beta E_{F_i}$ (E_{F_i} denotes the Fermi level).

Following Brovetto and Canuto (BC)[23] we proceed to calculate

$$f = f_1 + f_2,$$

which is the crucial quantity from which ρ is derived. In BC ρ was calculated for an assembly of particles of one type, and we generalize here to an assembly of two types of particles in thermal equilibrium by using Eqs. (11)–(14). Using the pairing Hamiltonian and an equidistant model for the density of single-particle states BC used the quasiparticle method to obtain

$$f_i = \frac{g}{\beta}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}(e^{\alpha_i n} + e^{-\alpha_i n})(\beta\Delta n)K_1(\beta\Delta n). \quad (15)$$

g is the single-particle level density (assumed to be equal for both systems, which is self evident if one system is composed of antiparticles of the other), Δ is the energy gap (again assumed to be equal for both systems), and $K_l(x)$ will represent a modified Bessel function of order l.

In order to calculate the entropy one uses the well-known relations

$$S_i = f_i + \beta U_i - \alpha_i N_i, \qquad (16)$$

where the energy is given by

$$U_i = -\frac{\partial f_i}{\partial\beta} \qquad (17)$$

and the number of quasiparticles in the ground state is

$$N_i = \frac{\partial f_i}{\partial\alpha_i}. \qquad (18)$$

Explicitly

$$U_i = \frac{g}{\beta^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}(e^{\alpha_i n} + e^{-\alpha_i n})$$

$$\times[(\beta\Delta n)K_1(\beta\Delta n) + (\beta\Delta n)^2K_0(\beta\Delta n)] \quad (19)$$

and

$$N_i = \frac{g}{\beta}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(e^{\alpha_i n} - e^{-\alpha_i n})(\beta\Delta n)K_1(\beta\Delta n). \qquad (20)$$

For an even system there are no quasiparticles in the ground state; thus

$$N_i = 0, \quad \alpha_i = 0 \quad \text{(even)}; \qquad (21)$$

while for the odd system there is one quasiparticle in the ground state;

$$N_i = 1, \quad \alpha_i \neq 0 \quad \text{(odd)}. \qquad (22)$$

Furthermore, the excitation energy U_i' above the normal ground state (which is above the superconducting ground state; see below) satisfies

$$U_i' = U_i \quad \text{(even)}, \qquad (23)$$

and

$$U_i' = U_i - U_i(0) \quad \text{(odd)}, \qquad (24)$$

where $U_i(0) = \Delta$ (see the Appendix) is the energy at $T = 0$, i.e., the energy of the quasiparticle which belongs to the ground state. For completeness let us write also the other derivatives of f_i which appear in $\det A$ [see Eq. (14)]:

$$\frac{\partial^2 f_i}{\partial \alpha_i^2} = \frac{g}{\beta} \sum_{n=1}^{\infty} (-1)^{n-1}(e^{\alpha_i n} + e^{-\alpha_i n})(\beta\Delta n)K_1(\beta\Delta n) ,$$

$$\frac{\partial^2 f_i}{\partial \beta^2} = g\Delta^3 \sum_{n=1}^{\infty} n(-1)^{n-1}(e^{\alpha_i n} + e^{-\alpha_i n}) \times \left[\left(\frac{2}{(\beta\Delta n)^2} + 1\right) K_1(\beta\Delta n) + \frac{1}{\beta\Delta n} K_0(\beta\Delta n)\right] , \tag{25}$$

$$\frac{\partial^2 f_i}{\partial \alpha_i \partial \beta} = -\frac{g}{\beta^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(e^{\alpha_i n} - e^{-\alpha_i n}) \times [(\beta\Delta n)^2 K_0(\beta\Delta n) + (\beta\Delta n)K_1(\beta\Delta n)] .$$

Thus, using the previous equations, ρ can be calculated for all possible assemblies of two types of particles. For $T \to T_0$ our result becomes the level density for an assembly of two systems without pairing. Indeed, since $\Delta \to 0$ (for $T \to T_0$), we may use a similar proof to the one used by BC for an assembly of one type of particle and find after a straightforward calculation that

$$\rho[U'(T \to T_0)] \to \frac{1}{6^{3/4}} g(2gU')^{-5/4} e^{2(\pi^2 gU'/3)^{1/2}} \tag{26}$$

in agreement with the well-known result in nuclear physics.[24]

Let us make a few remarks concerning the numerical calculations leading to ρ. We start by choosing T, then $\Delta = \Delta(T)$ is found by solving numerically the gap equation for each $0 \le T \le T_0$.[10] For an odd system E_{F_i} is calculated solving the implicit Eq. (20) (note that $N_i = 1$), where the limits on E_{F_i} are given in the Appendix; for an even system $E_{F_i} = 0$. Then U' is calculated using Eqs. (19), (23), and (24), and the other quantities needed for ρ are calculated in a similar way, using their respective expressions.

In order to calculate the c.m. energy $E_{\text{c.m.}}$ corresponding to a particular temperature we note that there exists condensation energy (i.e., the energy difference between the normal ground state, which is the ground state in the absence of pairing, and the superconducting ground state[10,25]) and thus

$$E_{\text{c.m.}} = U' + \tfrac{1}{2} g\Delta^2(0) + m , \tag{27}$$

which for $T = T_0$ becomes[23]

$$E_{\text{c.m.}}(T_0) = \tfrac{1}{3}\pi^2 g T_0^2 + \tfrac{1}{2} g\Delta^2(0) + m - U(0) , \tag{28}$$

where m stands for a ground-state mass, and

$$U(0) = \begin{cases} 2\Delta & \text{(even-even)}, \\ 0 & \text{(odd-odd)}, \\ \Delta & \text{(even-odd)}. \end{cases} \tag{29}$$

III. DATA ANALYSIS

We will apply formula (10) to fit $\pi^{\pm}p$ and $\bar{p}p$ elastic scattering data at 90°, assuming that the total width Γ is constant and that $\rho(\sqrt{s})$ is given by Eq. (11), where quantities appearing in Eq. (11) are calculated as explained in Sec. II. Since we are unable in the framework of this model to predict the single-particle level density g, common to all three reactions, we leave it as a free parameter to be fitted to the data. It should be noted that excluding the arbitrary scale factors which are used to normalize each set of data, this is the only free parameter in the model. When g is fixed, one is in position to calculate the center-of-mass energy at which the temperature of the system will reach its critical value T_0, where all the parton pairs are broken and the hadronic matter is passing a phase transition from its superconducting phase to its normal phase.

As to the mechanism governing beyond the critical energy (the energy for which the system reaches its critical temperature), things are not clear; however, we will discuss an interesting possibility later on.

In Figs. 1–3 we give the results of our model (setting $g = 0.0143$ MeV^{-1}) together with the 90° dif-

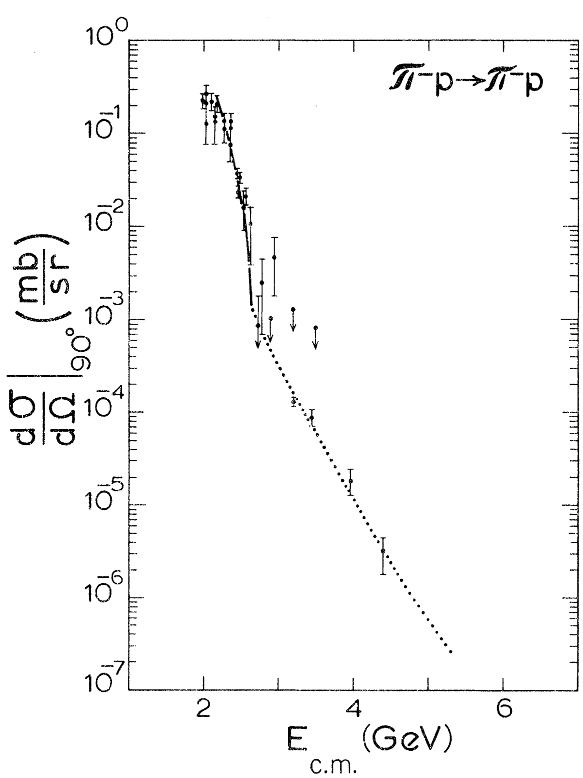

FIG. 1. $\pi^- p$ elastic scattering data as compiled in Ref. 7. The full line is a prediction of our model; the dotted line corresponds to values predicted by our model assuming $T > T_0$; see text. $g = 0.0143$ MeV^{-1}.

ferential cross-section data compiled in Ref. 7. On the same figure we also show (dotted lines) the predictions of the equidistant model assuming that $T > T_0$ is possible. In calculating $d\sigma/d\Omega|_{90°}$ we assume that the πN fireball, being a nonstrange baryon system, is an "odd-odd" assembly like the nucleon,[26] and has a ground-state mass equal to $m_\pi + m_N$. The $\bar{p}p$ fireball as a nonstrange meson system is a "even-even" system like the pion and has a ground-state mass equal to $2m_N$. Using Eqs. (27)–(29) it is then clear that if one starts to calculate $d\sigma/d\Omega|_{90°}$ for $\pi N \rightarrow \pi N$ at a certain $E_{\text{c.m.}} = \sqrt{s}$, then our formulas apply to $\bar{p}p \rightarrow \bar{p}p$ from a higher $E_{\text{c.m.}}$. Indeed, the formulas are applicable to πN for $E_{\text{c.m.}} > 2$ GeV, while for $\bar{p}p$ they are applicable for $E_{\text{c.m.}} > 2.6$ GeV.

The main conclusions to be deduced are the following:

(a) The equidistant model predicts an approximate exponential decrease of the differential cross section at 90° down to the critical energy.

(b) The model predicts a break in $d\sigma/d\Omega|_{90°}$ at $E_{\text{c.m.}} \simeq 2.7$ GeV for the πp system and at $E_{\text{c.m.}} \simeq 4.1$ GeV for the $\bar{p}p$ system, a prediction which seems to be compatible with the data.

It is quite surprising to see that if $T > T_0$ is possible, this model gives a good fit to the data beyond the critical energy. Since we are unable to decide in the framework of our model whether $T > T_0$ is possible or not, we give this result here only as an interesting possibility.

Although we do not analyze inelastic reactions here we would like to point out that in the few-GeV region near 90° they behave in a manner very similar to elastic scattering both in energy dependence and in absolute value.[27,28] Especially, we would like to draw attention to the data of Ref. 27, which may also exhibit a break pattern structure in $d\sigma/d\Omega|_{90°}$. It should be emphasized that we expect to see a $d\sigma/d\Omega|_{90°}$ break structure whenever hadronic matter passes a phase transition, including exotic systems for which our formalism does not apply. It is maybe worthwhile to note that applying our formalism to the pp system does not predict $d\sigma/d\Omega|_{90°}$ compatible with the data; however, it does predict correctly the position of the break at $E_{\text{c.m.}} \simeq 4.1$.[29]

IV. CONCLUSIONS

We have presented a parton statistical model in which the pairing forces have been explicitly taken

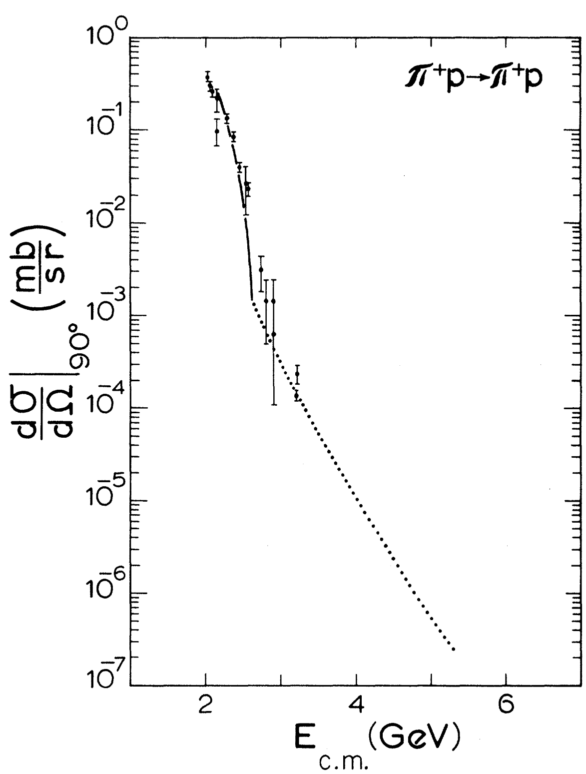

FIG. 2. π^+p elastic scattering data as compiled in Ref. 7. Curves as in Fig. 1.

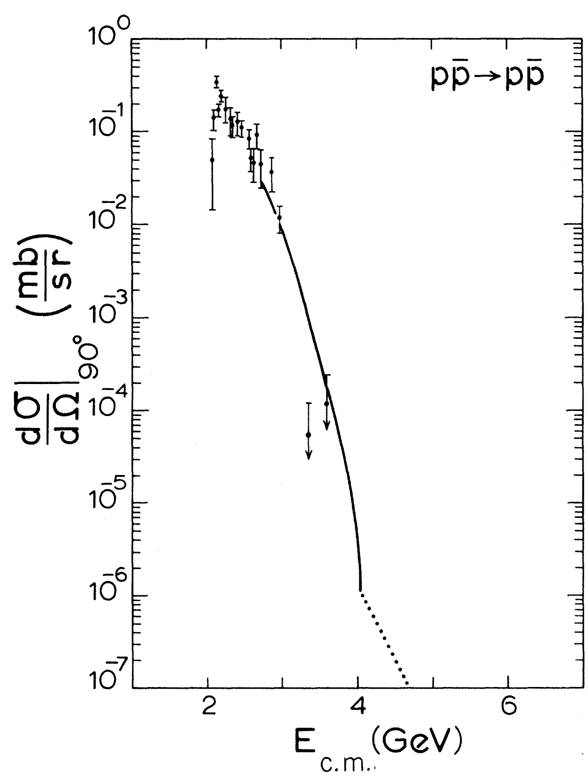

FIG. 3. $\bar{p}p$ elastic scattering data as compiled in Ref. 7. Curves as in Fig. 1.

into account. Generalizing Brovetto and Canuto's[23] equidistant model for the density of states we calculate the 90° differential elastic two-body cross section for the $\pi^{\pm}p$ and $\bar{p}p$ reactions. Although the comparison of our results with the data is quite gratifying, one can expect deviations to some extent from the statistical cross-section formula used here, since at relatively low energies where there are not too many resonances coherent effects may be important even at 90°. The gross features of the model, however, are clear and seem to be compatible with the present data. They can be summarized as follows:

(a) The model predicts an approximate exponential decrease of the differential two-body elastic cross section at 90° down to the critical energy.

(b) A break is predicted in $d\sigma/d\Omega|_{90°}$ at the critical energy of the system, where the system passes a phase transition.

A conclusion similar to (a) was also arrived at in the framework of the statistical bootstrap model which predicts[7]

$$d\sigma/d\Omega|_{90°} \sim s e^{-\sqrt{s}/T_0}. \tag{30}$$

T_0, however, in this model is not only a critical temperature but also a limiting one (i.e., $T > T_0$ is *not* possible), while in our model the limiting nature of T_0 is not settled. The possibility of $T > T_0$ is examined in our model and a surprisingly good agreement with the data has been found. To this end let us conclude that even though the agreement with the available data is quite good, much more accurate data are needed, especially for the $\bar{p}p$ system, before the model can be seriously tested.

Finally we would like to comment about the limitations of our model: A complete theory of the BCS type includes parameters such as the strength of the interaction G, the Fermi energy ϵ_F, etc., besides the single-particle level density g, which is phenomenologically determined by our analysis. These other parameters are not determined by our analysis; consequently the "weak coupling" approximation $gG \ll 1$ which leads to Eq. (2) remains an assumption[11] [note that for real superconductors $0.2 \lesssim gG \lesssim 0.4$ (see Ref 10)]. Furthermore, let us note that since ϵ_F is not determined here it is impossible to estimate, using our value for g, the number of partons inside a hadron.

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APPENDIX: THE FERMI LEVEL AND THE ENERGY FOR $T \to 0$

In the first part of this appendix we calculate $E_F(0)$, the Fermi level for the quasiparticles for $T \to 0$, and in the second part $U(0)$ is calculated (U denotes the energy). Since for $T \to T_0$, where T_0 is the critical temperature, $E_F \to 1/g$, where g is the single-particle level density, and $E_F(T)$ is bounded between $E_F(T_0)$ and $E_F(0)$.

Let us consider an odd system [for an even system $E_F(T) = 0$] of one type of particles. Since the number of quasiparticles in the ground state is given by

$$\begin{aligned} 1 &= N \\ &= \frac{g}{\beta} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (e^{\alpha n} - e^{-\alpha n})(\beta\Delta n)(K_1(\beta\Delta n), \end{aligned} \tag{A1}$$

where $\beta = 1/T$, $\alpha = \beta E_F$, Δ is the energy gap, and K_1 is a modified Bessel function, the above sum diverges for $\beta \to \infty$. Using

$$K_1(x) \to (\tfrac{1}{2}\pi)^{1/2} \frac{e^{-x}}{\sqrt{x}} \quad (x \to \infty), \tag{A2}$$

we have from (A1) that for large β

$$F(Z, \tfrac{1}{2}) \equiv \sum_{n=1}^{\infty} \frac{Z^n}{\sqrt{n}} = -\left(\frac{2\beta}{\pi\Delta}\right)^{1/2} \frac{1}{g}, \tag{A3}$$

where $Z = -\exp[\beta(E_F - \Delta)]$ ($|Z| > 1$). $F(Z, S)$ is the Jonquière function which satisfies the relation[30]

$$F(Z, s) + e^{i\pi s} F\left(\frac{1}{Z}, s\right) = \frac{(2\pi)^s}{\Gamma(s)} e^{i\pi s/2} \zeta\left(1 - s, \frac{\ln z}{2\pi i}\right), \tag{A4}$$

where

$$\zeta(r, y) = \sum_{n=0}^{\infty} \frac{1}{(y+n)^r} \tag{A5}$$

is the generalized ζ function. Equation (A4) provides the needed analytical continuation of (A3), so that for $|Z| > 1$ one obtains, after neglecting $F(1/Z, \tfrac{1}{2}) \to 1/Z$ (for $Z \to \infty$),

$$\begin{aligned} F[-e^{-\beta(E_F - \Delta)}, \tfrac{1}{2}] \underset{\beta\to\infty}{\sim} -\frac{2}{\sqrt{\pi}} (E_F - \Delta)^{1/2} \sqrt{\beta} \\ \times [1 + O(1/\beta)], \end{aligned} \tag{A6}$$

where we have used the asymptotic expansion of ζ,

$$\zeta(r, y) = \frac{1}{\Gamma(r)} [y^{1-r}\Gamma(r-1) + \tfrac{1}{2}\Gamma(r) y^{-r} + \cdots], \tag{A7}$$

valid for large $|y|$ and $|\arg y| < \pi$. Therefore, using (A3) and (A6), the final result is

$$E_F(0) = \frac{1}{2g^2\Delta} + \Delta \,. \tag{A8}$$

Let us now find $U(0)$, again for an odd system [for an even system $U(0)=0$] of one type of particles. Since

$$U = \frac{g}{\beta^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} (e^{\alpha n} + e^{-\alpha n}) \times [(\beta\Delta n)K_1(\beta\Delta n) + (\beta\Delta n)^2 K_0(\beta\Delta n)] \,, \tag{A9}$$

we obtain using (A1) and (A2) that

$$U(0) = \Delta \,. \tag{A10}$$

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