$\omega \to \pi\gamma$, $\pi^0 \to \gamma\gamma$, and $\eta \to \gamma\gamma$). These results are not very sensitive to the well-width parameter α^2. Calculations for $l=1$ resonances decaying via radiative emission *are* quite sensitive to the value of α^2 in general. The prediction for $A_2^\pm \to \pi^\pm + \gamma$ [Γ(predicted) = 0.4–0.6 MeV] is in good agreement with the experimental value Γ(experimental) ~ 0.5 MeV.

On the other hand, our calculations give a prediction for the decay width of $f(1260) \to \gamma\gamma$ which is lower than the value predicted by Kunzst *et al.* on the basis of a phenomenological Lagrangian by at least a factor of three. It will, therefore, be interesting to obtain experimental information on this decay width as a test of models. We conclude with the remark that, although the quark model gives good agreement with experiment for radiative decays of low-lying meson states, more experimental information is needed to test the validity of the quark-oscillator model for the decays of more highly excited mesonic states such as the tensor mesons.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

Perturbative Treatment of Threshold Contributions to a Rising *pp* Total Cross Section*

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We analyze the relation between increasing $N\bar{N}$ production and increasing σ_{tot} in a two-component picture. The possible increasing contribution to the cross section of diffractive dissociation into high-mass states is taken into account. We conclude that it is likely that both effects are important at CERN ISR energies. We also find that the short-range correlation part of the inelastic cross section without $N\bar{N}$ production must decrease above $s \sim 150$ GeV^2. The decrease is consistent with the form $\sigma^{(0)} \propto s^{\alpha_0 - 1}$ with $\alpha_0 = 0.92 \pm 0.04$.

I. INTRODUCTION

A possible explanation of the rise in the *pp* total cross section over the CERN ISR energy range[1,2] ($500 \leq s \leq 3000$ GeV^2) is the presence of threshold effects. For example, within the context of a two-component model, Chew and others have shown that the contribution to σ_{tot} of single diffractive dissociation into high missing mass (M) exhibits a logarithmic threshold increase with energy.[3-5]

The delayed threshold for this effect arises from the simultaneous constraints M^2, s/M^2 both large, and probably corresponds to a threshold energy somewhat below $s \simeq 200$ GeV^2. If the "single-fireball" component[6] and the low-mass diffractive dissociation component[7] *both* remain constant or nearly so over the same energy range, this threshold mechanism can account for the net increase of 3–4 mb.

Recent experiments at the CERN ISR[1,8] as well as earlier analyses of cosmic-ray data[2,9] have also shown that antiproton production in pp collisions increases markedly over the same energy range in question. In view of the relations provided by various sum rules (e.g., energy, multiplicity) between σ_{tot} and inclusive cross sections it is interesting to ask how much of the increase in σ_{pp}^{tot} is due to this nucleon-antinucleon ($N\bar{N}$) production.

Below $s \sim 200$ GeV^2, where both the $\bar{p}$ production and the diffractive dissociation into high-mass states are suppressed, σ_{pp}^{tot} is smoothly behaved and is nearly constant. Therefore, through unitarity, an isolated pole near $J=1$ is obtained by the production of pions and kaons alone, i.e., at $s \sim 200$ GeV^2, we have already passed the threshold regime of the pion and kaon production. However, both theoretical arguments and realistic numerical estimates indicate that this "bare" Pomeron has to be below $J=1$, leading to a slight decrease for this component.[10] Since we have already passed its production threshold we expect it to continue decreasing as we move into the ISR energies. For the SRC (short-range correlated[6]) cross section to remain nearly constant over the ISR range, a different threshold effect is required. This, we suggest, is due to $N\bar{N}$ production.

The threshold effect associated with $N\bar{N}$ production has been noted previously in connection with the $\bar{p}$ inclusive distribution. Whereas both the pion and the proton inclusive cross sections are describable at present machine energies by a Mueller-Regge behavior, the $\bar{p}$ cross section cannot be explained in such simple terms.[11] This is generally thought to be due to the smallness of the number of $N\bar{N}$ pairs produced; and the $\bar{p}$ cross section is expected to follow the Mueller-Regge behavior only when this number becomes large. This threshold effect can lead to a rising contribution to σ_{tot}.

In this paper we consider both $N\bar{N}$ production and diffractive dissociation in a multiperipheral picture in which they are treated as threshold, rather than asymptotic, phenomena at current machine energies. We find from an analysis of the data that, while the increase in σ_{tot} over the ISR energy range may be due to $N\bar{N}$ production alone, it is unlikely to be due solely to increasing diffractive dissociation. Probably both contribute and in any case the bare Pomeron generated with pion and kaon production alone is found to have an intercept $\alpha_0 = 0.92 \pm 0.04$. This is a consequence of the fact that the combined cross sections for $N\bar{N}$ production and for diffractive dissociation into high-mass states increase by more than 3–4 mb from $s \cong 150$ GeV^2 to $s = 2800$ GeV^2.

II. ESTIMATE OF $N\bar{N}$ CONTRIBUTION

Assuming that the $N\bar{N}$ contribution and the diffractive-dissociation contribution are both small at ISR energies so that their interference can be ignored, we consider the following decomposition of the pp total cross section:

$$\sigma_{\text{tot}} = \sigma^{(0)} + \sigma' + \sigma_D \ . \tag{1}$$

σ_D is the contribution of diffractive dissociation into both low-mass (including elastic) and high-mass states, and it has been discussed previously in a perturbative manner by Frazer, Snider, and Tan.[12] The first two terms on the right-hand side of Eq. (1) belong to the SRC component of σ_{tot}, and

$$\sigma' \equiv \sum_{i=1}^{\infty} \sigma^{(i)} \ , \tag{2}$$

where the superscript denotes the number of antinucleons present in the final state. As we have already mentioned, the component $\sigma^{(0)}$ without $N\bar{N}$ pairs is assumed to have Regge behavior with a pole $\alpha_0 < 1$.

The average antinucleon multiplicity

$$\langle n \rangle_{\bar{N}} \equiv \left(\sum_{i=1}^{\infty} i\sigma^{(i)} \right) \Big/ \sigma_{\text{tot}} \tag{3}$$

can be obtained from the multiplicity sum rule

$$\langle n \rangle_{\bar{N}} \sigma_{\text{tot}} = \int dp \left(\frac{d\sigma_{\bar{p}}}{dp} + \frac{d\sigma_{\bar{n}}}{dp} \right) , \tag{4}$$

where $d\sigma_{\bar{p}}/dp$ and $d\sigma_{\bar{n}}/dp$ are antiproton and antineutron inclusive cross sections, respectively. Assuming that they are nearly equal an upper limit of $\langle n \rangle_{\bar{N}}$ over the entire accelerator energy range is found to be 0.3.[8] The smallness of $\langle n \rangle_{\bar{N}}$ has contributed to the belief that antinucleon production does not play an important role in the unitarity sum for the forward absorptive amplitude.

To estimate the effect of $\bar{N}$ production let us define a different average multiplicity by normalizing with respect to $\sigma'(s)$

$$\langle N\rangle \equiv \sum_{i=1}^{\infty} (i\sigma^{(i)}/\sigma') = \langle n\rangle_{\overline{N}} (\sigma_{\text{tot}}/\sigma') , \quad (5)$$

so that the multiplicity sum rule can be written as

$$\langle N\rangle \sigma'(s) \cong 2 \int dp (d\sigma_{\overline{p}}/dp) . \quad (6)$$

The quantity $\langle N\rangle$ is the average number of $N\overline{N}$ pairs produced in collisions in which at least one pair is produced; hence $\langle N\rangle \geq 1$. The smallness of the integral in (6) is now a statement on the magnitude of $\sigma'(s)$. In particular we have

$$\sigma'(s) \leq 2 \int dp (d\sigma_{\overline{p}}/dp) . \quad (7)$$

Using the analysis of Ref. 8 we find that for $s \leq 500$ GeV^2 $\sigma'(s)$ is less than 4.5 mb, and for $s \cong 3000$ GeV^2 it can be as large as 10.2 mb. Thus the increase $\Delta\sigma'(s)$ in $\sigma'(s)$ over the ISR energies may be as large as 5–6 mb.[13] However, $\Delta\sigma'(s)$ will be less if $\langle N\rangle$ increases above unity.

III. PERTURBATIVE TREATMENT OF $N\overline{N}$ CONTRIBUTION

To investigate the increase in $\sigma'(s)$ quantitatively we next calculate $\langle N\rangle$ in the context of a simple multiperipheral model where $N\overline{N}$ production is treated perturbatively. From the experimental value of $\langle N\rangle\sigma'$ (taken from Ref. 8) it is then possible to compute $\sigma'(s)$. We will then be able to discuss the decrease in $\sigma^{(0)}$ that follows from the assumption that it has Regge behavior and hence be able to estimate the additional cross section required from σ_D or from other sources.[14]

Using quite general arguments, involving damping in transverse momentum of produced particle combined with a kinematic $t_{\min}$, one can show that the minimum separation between peripherally produced particles in the rapidity space is of order $\Delta = \ln(M^2/|t_0|)$, where M is the mass of the produced particle and t_0 a transverse-momentum-squared cutoff.[15] Since the effective $N\overline{N}$ mass is so much greater than that of a pion (corresponding to $\Delta \sim 2$–3) and since 2Δ is required for the production of the first pair, it is perhaps not surprising that $\langle N\rangle$ turns out to be near unity. Since Regge behavior comes from a summation over many-particle intermediate-state contributions one therefore does not expect cross sections involving $\overline{N}$ production to follow a simple Regge prescription at the present machine energies. Rather, $N\overline{N}$ production exhibits threshold behavior in this energy range, and a perturbative treatment of it is appropriate.

To illustrate the perturbative approach we now discuss a highly simplified multiperipheral model where the threshold phenomenon is built in by a hard-core approximation. The model is characterized by the following assumptions[16]:

(1) $N\overline{N}$ are always produced in pairs and can be treated as a single particle with a minimum separation Δ in the rapidity space from the end of the multiperipheral chain and a separation Δ_0 from adjacent $N\overline{N}$ pairs.[17]

(2) Aside from the two leading protons, all other particles produced are pions.[18] For simplicity we assume that the minimum-gap effect is absent for pions and that each leading proton occupies an additional length δ in the rapidity space.

(3) Consider one type of exchange only and assume that the component of σ_{pp}^{tot} without $N\overline{N}$ production alone leads to Regge behavior:

$$\sigma^{(0)} \sim G^4 e^{-Y} e^{\alpha_0 (Y-2\delta)} \theta(Y-2\delta) , \quad (8)$$

where $0 < 1-\alpha_0 \ll 1$, $Y \simeq \ln s$ and G is the coupling of the proton to the exchanged object.

Using the simplified phase space of DeTar,[19] the cross section $\sigma^{(n)}$ can now be obtained. The phase space is typically of the form

$$\sigma^{(n)} \propto e^{-2Y} \int_0^Y dz_1 dz_2 \ldots dz_{n+1} \times \delta\left(Y - \sum_{i=1}^{n+1} z_i - (n-1)\Delta_0 - 2\Delta - 2\delta\right), \quad (9)$$

where Δ_0 and Δ are, respectively, the minimum rapidity intervals between adjacent $N\overline{N}$ pairs and between an $N\overline{N}$ pair and an adjacent leading proton. In writing Eq. (9) we have already performed the pion phase-space integrations, and summation over the number of pions produced leads to a factor $e^{(\alpha_0+1)z_i}$ associated with the ith interval, for $i = 1, \ldots, n+1$. Integration over $N\overline{N}$ phase space then produces the result

$$\sigma^{(n)} \sim g\sigma^{(0)}(Y) \frac{(\overline{\lambda}^2)^n}{n!} [Y-(n-1)\Delta_0-2\Delta-2\delta]^n \times \theta(Y-(n-1)\Delta_0-2\Delta-2\delta) , \quad (10)$$

where

$$g = \exp[-(2\Delta-\Delta_0)(\alpha_0+1)] ,$$

$$\overline{\lambda}^2 = \lambda^2 \exp[-\Delta_0(\alpha_0+1)] ,$$

and λ^2 is the coupling strength for producing an additional $N\overline{N}$ pair.

In order to obtain $\langle N\rangle$, we first determine the parameters in Eq. (10) by fitting the expression for $\langle N\rangle\sigma'$ that resulted from combining Eqs. (2), (5), and (10) to the data of Ref. 8. Because of the θ function in Eq. (10), the sum in Eq. (2) is always truncated for any finite value of Y. For example, if only $n=1$ and 2 terms contribute, we then have

$$\sigma'(s) = g\sigma^{(0)}(s)(\bar{\lambda}^2) \times [(Y-2\Delta-2\delta) + \tfrac{1}{2}\bar{\lambda}^2\,(Y-\Delta_0-2\Delta-2\delta)^2] \qquad (11)$$

and

$$\langle N\rangle\sigma'(s) = g\sigma^{(0)}(s)(\bar{\lambda}^2) \times [(Y-2\Delta-2\delta) + \bar{\lambda}^2\,(Y-\Delta_0-2\Delta-2\delta)^2]\,. \qquad (12)$$

Physical arguments place some constraints on these parameters: $\Delta \sim 2-3$, $\Delta_0 \geqslant \Delta$, $\sigma^{(0)} \sim 30$ mb, $\delta < 0.6$, and $\alpha_0 \lesssim 1$. The data, as displayed in Fig. 1, clearly show a break, corresponding to the onset of some new phenomenon, in the neighborhood of $s \simeq 200-400$ GeV^2. There are two ways to interpret this break:

(I) It is due to the onset of two-$N\bar{N}$-pair production.

(II) The break is due to the threshold for the first pair of $N\bar{N}$ production.

For model I, we use the data below $s=150$ GeV^2 to determine $\Delta+\delta=2$ and $g\sigma^{(0)}\bar{\lambda}^2 \simeq 1.2$ according to the $n=1$ term in Eq. (12). To get agreement with the ISR data it is then necessary to maximize the contribution from $n=2$, so we take $\delta=0$ and $\Delta_0 \simeq \Delta$. This gives $\bar{\lambda}^2 \simeq 2$, $g \simeq \frac{1}{50}$ for $\sigma^{(0)} \simeq 30$ mb. For model II, the nonzero value below threshold is due to penetration of the Δ barrier, and the parameters are to be determined by fitting the $n=1$ term of Eq. (12) to the ISR data. In this way we find $\Delta+\delta=2.5$ and $g\sigma^{(0)}\bar{\lambda}^2 \simeq 3.5$. The (approximately) logarithmic rise of $\langle N\rangle\,\sigma'$ over the ISR energy range suggests taking $\Delta_0 \gtrsim 3$ so that the double-pair-production threshold is beyond or near the end of the ISR range. To minimize the value of $\bar{\lambda}^2$ needed we choose $\delta=0.5$, $\Delta=2$, $\Delta_0 \sim 4$, $\sigma_0 \sim 30$ mb so that $g \sim 1$ and $\bar{\lambda}^2 \sim 0.15$. The fits are shown as solid curves in Fig. 1. They seem to support that model II is probably more nearly correct. Since $\langle N\rangle$ remains at one for model II, this will lead to an increase of 5–6 mb for $\sigma'(s)$ over the ISR energies.

IV. DISCUSSION

Independent of the interpretation given to the rapid increase of $N\bar{N}$ production starting at a few hundred GeV^2, we find that the quantity $\sigma_{\text{tot}}-\sigma'-\sigma_{\text{el}}$ is approximately constant at 30.5 ± 0.4 mb up to $s=130$ GeV^2 and then drops by about 2 mb between $s=130$ GeV^2 and $s=550$ GeV^2.[20] This is because $\langle N\rangle$ remains near unity below $s=550$ GeV^2 in both cases. Since

$$\sigma_{\text{tot}}-\sigma'-\sigma_{\text{el}} = \sigma^{(0)} + (\sigma_D - \sigma_{\text{el}})$$

and $(\sigma_D-\sigma_{\text{el}})$ is either increasing or nearly constant

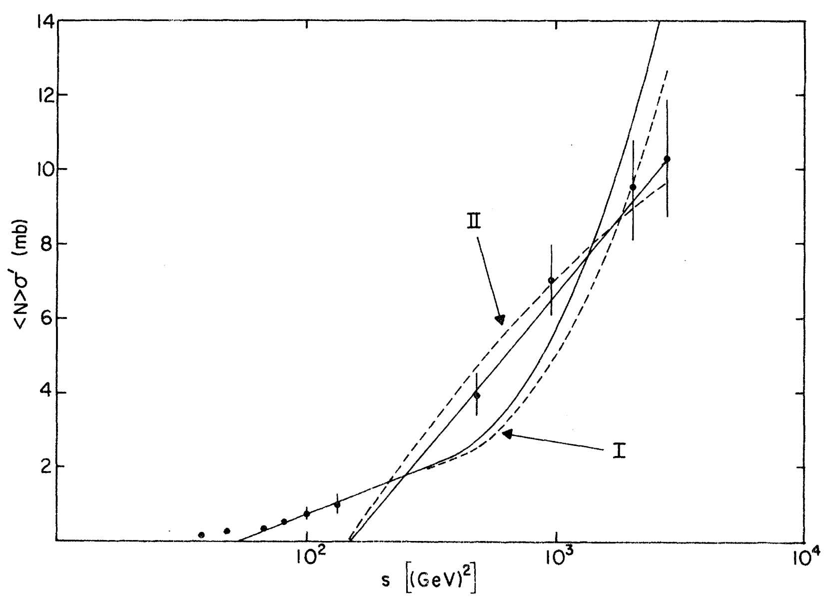

FIG. 1. Integrated inclusive $N\bar{N}$ cross section as a function of s. Experimental data are taken from Ref. 8. Solid curves show predictions for the two models discussed in the text with $\sigma^{(0)}$ = const. Dashed lines show results for the same models when $\sigma^{(0)}$ decreases with energy.

in this range,[7,12] we conclude that a decrease in $\sigma^{(0)}$ is definitely required. Furthermore, since $\sigma^{(0)} \propto s^{\alpha_0 - 1}$ with $\alpha_0 < 1$, this decrease will continue into the ISR energies.

As we move into the ISR energy range the pictures presented by these two models are quite different. In model I, $\langle N \rangle$ increases over the ISR energies with the result that $\sigma_{tot} - \sigma' - \sigma_{el}$ remains roughly constant: 28.2 ± 1 mb at 550 GeV^2 to 29.6 ± 1.7 mb at 2800 GeV^2 (alternatively for $\sigma_{tot} - \sigma' - \sigma_{D,M<M_0}$, with $\sigma_{D,M<M_0} = 10$ mb for diffractive dissociation into low-mass states,[12] we find 25 ±1 mb at 550 GeV^2 and 26.2 ± 1.7 mb at 2800 GeV^2). Since $\sigma^{(0)}$ is decreasing, an increasing contribution from diffractive dissociation into high-mass states, $\sigma_{D,M>M_0}$, is required in this case. An acceptable fit is found to be

$$\sigma^{(0)} \sim (34.8 \text{ mb}) s^{\alpha_0 - 1} ,$$

$$\alpha_0 = 0.95 \pm 0.01 \text{ for } s \gtrsim 150 \text{ GeV}^2 ,$$

and

$$\sigma_{D,M>M_0} \sim (1 \text{ mb}) \ln(s/400 \text{ GeV}^2) .$$

This is shown in Fig. 2(a).

In model II the data permit, but do not require, an increasing σ_D. Since $\langle N \rangle$ is unity and σ' continues to increase rapidly over the ISR energy range, $\sigma_{tot} - \sigma' - \sigma_{el}$ is found to decrease further by another 3 mb: 28 ± 1 mb at $s = 550$ GeV^2, 25.2 ± 1.7 mb at $s = 2800$ GeV^2. With $\sigma_{D,M<M_0} \approx 10$ mb, it is possible to have

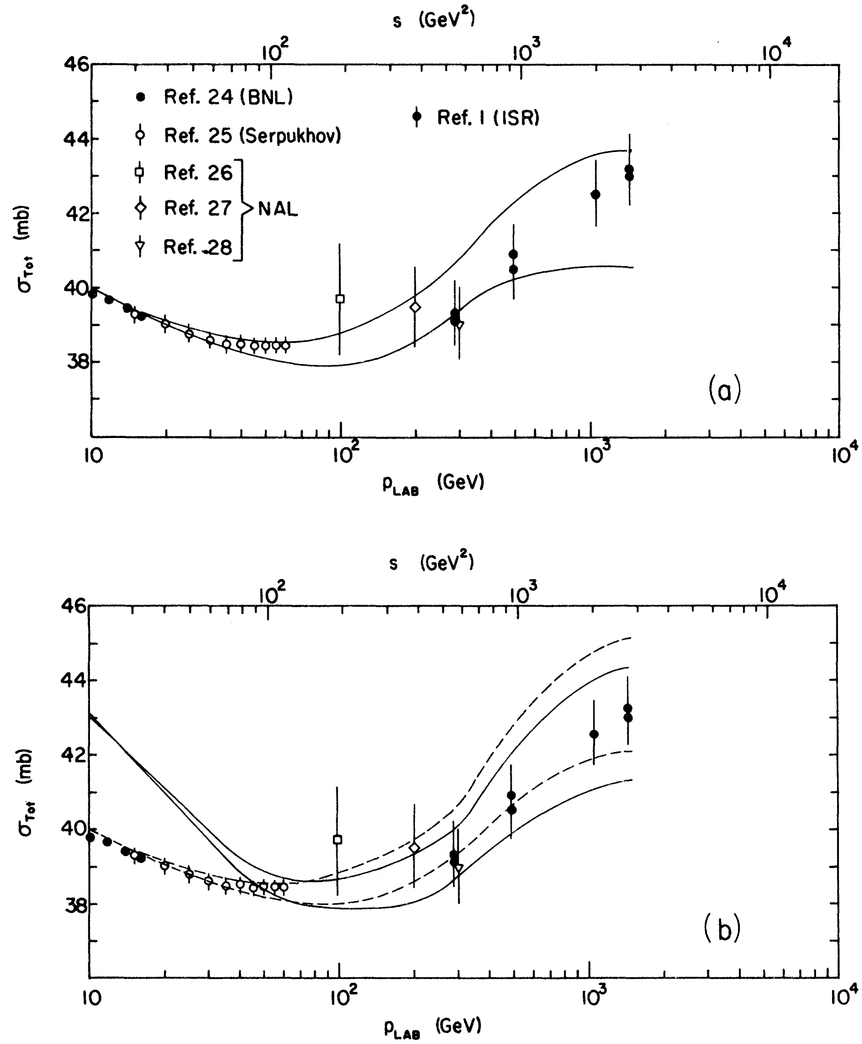

FIG. 2. (a) A fit to σ_{pp}^{tot} using model I. (b) Fits to σ_{pp}^{tot} using model II. The dashed curves correspond to σ_D = const., and the solid curves contain the contribution from $\sigma_{D,M>M_0} \sim \ln(s/100 \text{ GeV}^2)$. In both 2(a) and 2(b) the width of the model result for σ_{tot} is due to the experimental uncertainty in the measurement of $\langle N \rangle \sigma'$.

$$\sigma_{D,M>M_0} \simeq 0$$

and

$$\sigma^{(0)} \simeq (34.8 \text{ mb}) s^{\alpha_0 - 1} ,$$

$$\alpha_0 \simeq 0.95 \pm 0.01$$

[see the dashed curves in Fig. (2b)]. However, if we use the analysis of Refs. 5 and 21, where

$$\sigma_{D,M>M_0} \simeq (1 \text{ mb}) \ln(s/100 \text{ GeV}^2) ,$$

an equally acceptable fit is found with

$$\sigma^{(0)} \simeq (46.2 \text{ mb}) s^{\alpha_0 - 1} ,$$

$$\alpha_0 \simeq 0.89 \pm 0.01$$

[see the solid curves in Fig. (2b)].

Our analysis suggests that the increase in the cross section for diffractive dissociation into high-mass states from $s \simeq 550$ GeV^2 to 2800 GeV^2 is at most 2 mb. Since the total cross section increases by 3–4 mb in the same energy range it is unlikely that this increase is completely due to the diffractive effect. At any rate, we find that the bare Pomeron generated with pion and kaon production alone has an intercept $\alpha_0 = 0.92 \pm 0.04$. This is a consequence of the fact that the combined cross sections for $N\bar{N}$ and for diffractive dissociation into high-mass states increase by more than 3–4 mb from $s \simeq 150$ GeV^2 to 2800 GeV^2.

The J-plane structure resulting from an "infinite" iteration of $N\bar{N}$ production is complicated. Equation (10) will lead to at least two real poles and infinitely many complex-conjugate pairs of Regge poles.[16,22] In particular, our bare pole α_0 will be "renormalized" upward. This displacement, in our model, is of order $\bar{\lambda}^2$, and a value of 0.15 for model II is perfectly acceptable. The larger value of $\bar{\lambda}^2$ found for model I may simply be due to the inadequacy of our approximation, and is certainly not a sufficient reason for us to reject the model for the description of threshold phenomena. As a further consistency check, we show by the dashed curves in Fig. 1 the model prediction for $\langle N \rangle \sigma'(s)$ when the variation with energy of $\sigma^{(0)}$ is taken into account. (For model II, we use $\sigma^{(0)} = 46.2\, s^{-0.11}$.) Model II appears to give the best picture of the energy dependence of $N\bar{N}$ production; the threshold for production of a single $N\bar{N}$ pair is in the neighborhood of $s \sim 200$ GeV^2. Below this energy the small $N\bar{N}$ production is due to penetration of the barrier in rapidity between the $N\bar{N}$ pair and the end of the multiperipheral chain.

This picture of $N\bar{N}$ production as a threshold phenomenon with $s \sim 200$ GeV^2 is also supported by an analysis of the contribution of $N\bar{N}$ production to the energy-conservation inclusive sum rule.[13] Both models discussed above give satisfactory agreement with the energy dependence of this contribution obtained from the parametrization of the inclusive $\bar{p}$ data described in Ref. 23. It is also interesting to note that $\sigma^{(0)}$ does not start to decrease until $s \sim 150$ GeV^2. [In particular, see the solid curves in Fig. 2(b).] This may suggest that for lower energies kaon production has not yet become asymptotic and should be treated as a threshold effect in a way similar to the treatment of $N\bar{N}$ production here.

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[6]This will also be called the short-range correlated (SRC) component.

[7]The low-mass diffractive dissociation component, which includes the elastic cross section, is expected to follow a $(\ln s)^{-1}$ decrease at sufficiently high energies. However, they seem to remain fairly constant over both NAL and ISR energies, indicating that the slope of the Pomeron at $t = 0$ is small.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 11 1 DECEMBER 1973

Radiative Corrections to the Muon Polarization in $K^0_{\mu 3}$ Decays*

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Radiative corrections to the muon polarization vector in $K^0_{\mu 3}$ decays have been calculated, based on a phenomenological weak K-π vertex and perturbation theory. All terms which contribute to order α have been retained, with no approximations concerning the smallness of the muon mass or the "real" inner bremsstrahlung. The results depend logarithmically on a cutoff, but are not particularly sensitive to variations between one and two proton masses. The radiative corrections to the three components of the polarization vector (longitudinal, transverse, and perpendicular) have been evaluated numerically at 10-MeV intervals throughout the Dalitz plot, for various complex values of ξ. In addition, the integrated radiative correction to the degree of polarization as a function of muon energy has also been computed. It is found that these corrections are generally small, less than 1% degree of polarization in magnitude.

I. INTRODUCTION

Measurements of the polarization of the muons from $K^0_{\mu 3}$ decays have been the subject of recent high-statistics experiments[1,2] for two reasons. First, such measurements provide a sensitive and independent method for determining the form factor ratio[3] $\xi(q^2)$, as suggested by Cabibbo and Maksymowicz.[4] In $K_{\mu 3}$ decays, the muon is 100% polarized at each point in the Dalitz plot along a