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Multiparticle Variables for High-Energy Reactions*

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A new approach to the analysis of detailed high-energy experiments leading to multiparticle final states is suggested. We construct several multiparticle variables, i.e., variables whose value depends in each event on all or most of the particles produced in this event. The one-dimensional distributions of such variables offer a clearer distinction between the various models and pictures for multiparticle processes and could conceivably facilitate the discovery of new patterns.

I. INTRODUCTION

The main effort in the field of multiparticle reactions has been directed towards inclusive cross sections and, more recently, two-body correlations.[1] The advent of the fragmentation[2] and scaling[3] hypothesis, and Mueller's optical formula,[4] which facilitated the application of Regge pole and duality techniques,[5] contributed to the interest in the subject. The scope of this approach—particularly of that focusing on inclusive cross sections alone—is however quite limited.[6] Several, rather different, models seem to satisfactorily account[7] for the inclusive data, a success which might be largely due to the transverse momenta damping and relativistic phase space incorporated in these models.[8]

In the following we adopt the basic assumption that there is more to multiparticle reactions than just these features and the effects due to low-lying resonances, and address ourselves to the problem of optimal ways to find genuine multiparticle patterns.

The most obvious approach is to study directly on an event-by-event basis the three-dimensional picture of the momenta of outgoing particles, or appropriate two-dimensional projections thereof. (A particular version of such event diagrams which uses the c.m. rapidity

$$Y \equiv \log_{10}\left(\frac{E+P^L}{E-P^L}\right),$$

or

$$R \equiv \log_{10}\left(\frac{E+P^L}{2\text{ GeV}}\right),$$

has been recently advocated by Bjorken.[9])

We would like to suggest an alternative approach[10] in which the one-dimensional distributions $f_\alpha(V)$ of many multiparticle variables $[V_\alpha(P_1\xi_1, \ldots, P_n\xi_n)$ for a reaction in which n particles of types ξ_i and momenta P_i are produced$]$ will be studied. Obviously if any pattern were to be found directly in the Bjorken plots it would correspond to a peculiar distribution of an appropriately defined V_α. By appropriately choosing the V_α we could check for the existence of a particular pattern and also obtain much more decisive tests of a particular model than those offered by the inclusive distributions.

The experimental distribution $f_\alpha(V)$ is compared in each case with a background distribution of the same variable for a set of Monte Carlo–generated events. Ideally the Monte Carlo program is designed to reproduce the semi-inclusive distribution[11] $d\sigma^n/dP_i$ of a particle of type ξ_i in the abovementioned n-particle reaction. In practice, we used a program incorporating the transverse momentum cutoff and also (via cutoffs on invariant momentum transfers) the leading-particles effect. The peculiarity of the distribution $f_\alpha(V)$ is measured by its deviation from the background distribution. A large deviation would indicate a genuine new dynamical effect which has not been built into the background.

An "ideal" $f_\alpha(V)$ has δ-function distribution. This corresponds to a discovery of a new "integral of motion" and is quite unlikely. The more peaked the distribution $f_\alpha(V)$ is, the more significant it will be. A general ambitious program would attempt to find better and better $f_\alpha(V)$ by successive trials.

In the present work, we will limit ourselves to specific variables suggested by models and pictures of the collision process and by considerations of simplicity and symmetry. We next list the variables and their motivation and then proceed to a preliminary investigation of their distribution for a sample of four- and six-prong K^-p and π^+p data.

II. THE VARIABLES

The variables to be considered divide naturally into two classes: (a) variables (V_1, V_2, and V_3) which depend on the momenta of all the particles produced so that their distribution can be plotted only for completely fitted events, and (b) variables (V_4, V_5, V_6, and V_7) involving only the charged particles.

$$\text{(i)}\quad V_1 = \left| \sum_{i=1}^{n} Y_i \vec{P}_i \right| , \tag{1}$$

where $\vec{P}_i$ are the transverse momenta and Y_i the rapidities. This variable is motivated by the idea of short-range correlations.[12] Specifically, in a simplified multiperipheral model the produced particles are ordered along the chain according to their rapidities (Fig. 1). Assuming an average separation Δ in rapidities, we see that a correlation between $\vec{P}_i$ and $\vec{P}_j$ requires transfer of transverse momentum across

$$n \propto |Y_i - Y_j| / \Delta$$

links of the chain, which leads naturally to an exponential $e^{-\alpha|Y_i - Y_j|}$ decrease of transverse correlations, a common feature of many theoretical models.[13]

The purpose of our variable V_1, which is effectively a "torque" with the $\vec{P}_i$ playing the role of forces and Y_i corresponding to the lever arms, is precisely to isolate and measure this effect in a given event. The question we address ourselves to is how local (in rapidity) is transverse momentum conservation ($\sum \vec{P}_i = 0$). A large V_1 means a flow across a large rapidity interval of large transverse momenta. The strong damping of V_1 (beyond the damping due to the $e^{-\alpha \vec{P}_i^2}$ factor damping the transverse momenta themselves—and perhaps also the milder $e^{-\beta Y_i^2}$ damping the rapidities) is therefore a critical test of any of these multiperipheral-type physical pictures.

Note that $\sum \vec{P}_i = 0$ implies that V_1 is invariant under longitudinal boosts $Y_i \to Y_i + C$. This property will obviously not hold for variables designed to test models like the fragmentation picture in which particular frames, the target and projectile frames, play a special role.

$$\text{(ii)}\quad V_2 = \sum (P_{ix} P_{iy}) \Big/ \left(\sum P_{ix}^2\right)^{1/2} \left(\sum P_{iy}^2\right)^{1/2} . \tag{2}$$

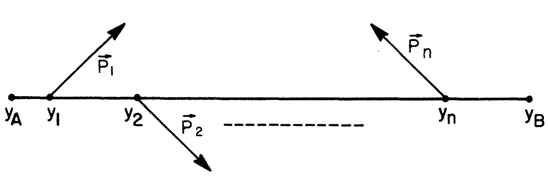

FIG. 1. Labeling of particles according to their rapidities.

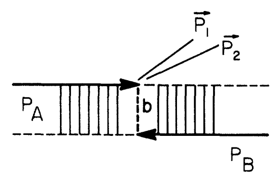

FIG. 2. The definition of the transverse impact plane.

This variable is motivated by considering the semiclassical concept of an impact plane defined by P_{initial}, the initial c.m. momentum, and $\vec{b}$, the impact vector (Fig. 2).

While there should be no preferred direction in the transverse plane, once we have averaged over many events, the particles emitted in each individual collision may tend to move in this plane—or in a plane orthogonal to it. The more aligned the final-particle emission is, the closer V_2 is to ± 1.

A nice feature of this variable is that, given only the $e^{-\beta \vec{P}_i^2}$ distribution and the $\delta(\sum \vec{P}_i)$ constraint, the expected background distribution can be readily obtained from purely geometric considerations (see Appendix A). V_2 is distributed just like the polar cosine on the $(n-2)$-dimensional sphere, where n is the number of particles produced in the class of events considered. Thus

$$f^{(2)}_{\text{background}}(V_2) = (1 - V_2^2)^{(n-4)/2} . \tag{2'}$$

Enhancement at $V_2 \approx \pm 1$ relative to the above background distribution would indicate a tendency for alignment.

A similar variable was suggested earlier in connection with tests in e^+e^- collisions[14] and the importance of study of distributions of momenta conjugate to transverse configuration vectors was recently emphasized.[15] The V_2 variable however offers a direct test of the persistence of the impact-plane direction in the final multiparticle state and has a particular simple background distribution.[16]

$$\text{(iii)}\quad V_3 = \sum_{i=1}^{n} |\vec{P}_i|^2 . \tag{3}$$

In two-body processes there exists a simple reciprocity between the momentum transfer ($\approx$ transverse momentum of final particle for the case of small angles) and the impact parameter, collisions with small impact parameter corresponding to large momentum transfer.

It has been conjectured that a similar reciprocity is obtained also in multiparticle processes.[15] In particular, central collisions with small impact parameters would correspond to reactions in which many of the produced particles have large transverse momenta. Our variable V_3 can therefore be thought of as a general measure of the "centrality" of the collision.

Since a reaction for which several of the $\vec{P}_i$ are large is likely to be central, we expect a tail of $f_3(V_3)$ extending beyond the background distribution

$$f_3(V_3) = C_n (V_3)^{n-1} e^{-\alpha V_3} \qquad (3')$$

which is again readily obtained for $e^{-\alpha \vec{P}_i^2}$ transverse distributions and the transverse $[\delta(\sum \vec{P}_i)]$ momentum conservation constraint.[16]

$$\text{(iv)} \quad V_4 = \sum Q_i Y_i \,, \qquad (4)$$

where Q_i equals the charge of ith particle.

A well-known feature of high-energy collisions is the leading-particle phenomenon, i.e., the existence of particles with large forward (or backward) momenta in the c.m. system and with the quantum numbers of the projectile (or target).

Let us first consider reactions in which the incident particles A and B have opposite charges so that $\sum Q_i = 0$. In this case V_4 is, like V_1, boost-invariant. If exact charge retention occurs, there will be a forward leading final-state particle P_1 of type A and rapidity $\approx Y_A$, and a backward leading particle P_n of type B and rapidity $\approx Y_B$. We would then expect V_4 to peak around the value $Y_A - Y_B$, with the width of the distribution reflecting the effect of the emission of the other particle to the extent that it is not "locally (in rapidity) charge-conserving."

In reality the situation is obviously more complicated: In a K^-p reaction it may quite often happen that a leading K^{*-} emerges which decays into a $\bar{K}^0$ and a much slower π^-. V_4, being the first moment of the charge distribution in rapidity, gives us in general a direct measurement of the effect of charge migrations.

In the case when $Q_A + Q_B = 2$, the situation is different and, in particular, V_4 is frame-dependent. We picked the c.m. frame since $f_4(V_4)$ is then expected to peak around $V_4 = 0$.

$$\text{(v)} \quad V_5 = \left| \sum Y_i Q_i \vec{P}_i \right| \,. \qquad (5)$$

V_5 is a hybrid of the variables V_1 and V_4. As mentioned earlier, we expect the V_1 distribution to be somewhat narrower than the background distribution because of the tendency of a large transverse momentum to be balanced by the momenta of particles nearby in rapidity. If, however, there is also some tendency for charge alternation along the rapidity axis, this balancing is expected to be achieved mostly by particles of opposite charges, so that the distribution of V_5 relative to its background should be broad.

$$\text{(vi)} \quad V_6 = \sum_{P_i^L > 0} Q_i \,, \qquad (6)$$

where P_i^L are the c.m. longitudinal momenta, i.e., V_6 is the total charge of the forward-moving particles in the c.m. system. This variable is most clearly suggested by fragmentation or diffractive-dissociation models in which the final-state particles are expected to be composed of two groups—the right-moving fragments of the projectile, and the left-moving fragments of the target.

Such reactions are assumed to involve no quantum number exchange. In the extreme limit of complete dominance of diffraction production, we expect, therefore,

$$f_6(V_6) = \delta(V_6 - Q_A) \,. \qquad (6')$$

The distribution may be broadened somewhat because of the possibility that some of the products of the forward-moving cluster may be finally emitted in the backward direction in the c.m. system, and the existence of a finite fraction of charge-exchange reactions (which should yield, neglecting double charge exchange, $V_6 = Q_A \pm 1$).

On purely statistical grounds, we expect a much broader distribution. A crude approximation envisions a particle production reaction with total charge $Q = Q_A + Q_B$, i.e., with $(n-Q)/2$ negative and $(n+Q)/2$ positive particles, as a random choice of n_f^- forward-going negative particles out of $(n-Q)/2$ and n_f^+ out of the $(n+Q)/2$ positive particles. The binomial-type distribution for the difference $V_6 = n_f^+ - n_f^-$ is quite similar to the one obtained by the Monte Carlo procedure taking into account energy-momentum conservation.

$$\text{(vii)} \quad V_7 = n_F - n_B \,, \qquad (7)$$

where n_F (n_B) refer to the total number of forward- (backward-) moving prongs (in the c.m. system), and we consider reactions where the total number of charged particles produced, $n = n_F + n_B$, is fixed. The distribution of this variable can be easily obtained from experiment with 4π counting arrays around the ISR (CERN Intersecting Storage Rings) and possibly similar future colliding-beam devices.

The motivation for considering V_7 is the very different qualitative behavior predicted for it by multiperipheral-like models on the one hand, and fragmentation or diffractive-dissociation (diff) models on the other.

Recently many authors[17] discussed special cases of the second class of models where the partial cross sections for n-particle production approach at high energies

$$\sigma_n^{\text{diff}} \approx \text{const}/n^2 \,. \qquad (8)$$

This implies a very slow falloff of the cross section of large multiplicity, in sharp distinction to the multiperipheral (mp) model, which predicts a

Poisson distribution for the number of particles in any fixed rapidity interval ΔY:

$$\sigma^{\mathrm{mp}}_{n,\,\Delta Y} \approx \frac{(C\Delta Y)^n}{n!} e^{-\Delta Y}. \tag{9}$$

Nonetheless, both models give [with $C=1$ in Eq. (9) or with appropriate assumptions about the n-particle phase-space distribution for the case of diffractive models] constant cross sections, logarithmic increase of multiplicities, and a dx/x spectrum for $x \equiv 2P^L/\sqrt{s} \approx 0$.

To simplify the ensuing discussion, we will neglect the effect of production of neutrals and assume that (8) and (9) hold for all n. We can furthermore utilize the factorizability of the production process of the forward- and backward-moving particles[18] which holds for both models to write

$$f_n^{\mathrm{diff}}(V_7) \approx \frac{C}{(n+V_7)^2(n-V_7)^2}, \tag{7$'$}$$

$$f_n^{\mathrm{mp}}(V_7) \approx \frac{C}{[(n-V_7)/2]!\,[(n+V_7)/2]!} \tag{7$''$}$$

as the predicted distributions of V_7 for given n. Obviously the two distributions have very different shapes: The first has a sharp minimum at the symmetry point $V_7=0$; the second attains there a strong maximum. For $n \approx 16$ we have

$$\frac{f^{\mathrm{mp\ diff}}(V_7=n-2)}{f^{\mathrm{diff}}(V_7=0)} \approx 20,$$

whereas

$$\frac{f^{\mathrm{mp}}(V_7=n-2)}{f^{\mathrm{mp}}(V_7=0)} = \frac{1}{800}.$$

While the above discussion is rather oversimplified,[19] it still illustrates the large qualitative difference expected between the two distributions.

III. COMPARISON WITH 9-GeV/c K^-p AND π^+p DATA

In this section we present the experimental distributions of V_1 to V_7 for 4- and 6-prong K^-p and π^+p collisions at 9 GeV/c. While this energy is rather low, this comparison may be a useful first step in a more extensive study of multiparticle variables at higher energies and for other processes. In contrast with inclusive cross sections, it would be rather difficult to study most of the multiparticle variables at the highest available energies (e.g., at the ISR). A lot can be learned however by a systematic study of available bubble-chamber data in the 7–25-GeV/c range.

The K^-p data at 9 GeV/c come from an exposure taken at the 80-in. bubble chamber at BNL.[20] The events in the sample reported on here were measured on a precision film-plane digitizer and processed through the reconstruction and fitting programs TVGP and SQUAW. For the events which had acceptable 4-constraint fits for more than one mass interpretation, that interpretation with the lowest χ^2 was chosen.

Monte Carlo calculations were performed using the matrix element[21]

$$\exp\left[-\left(a_K t'_K + a_p t'_p + \sum_i b_i \vec{\mathrm{P}}_i{}^2\right)\right], \tag{10}$$

where the parameters a_K, a_p, and b_i were determined from fits to the data. In Eq. (10) t'_K (t'_p) are the values of $t - t_{\min}$ for the kaon (proton) and $\sum_i$ extends over the remaining (2 or 4) pions. Note that different slopes were allowed for π^+ and π^-, as well as for the four- and six-prong reactions.

This parametrization of the single-particle spectra reproduces well the $\vec{\mathrm{P}}^2$ distributions for the leading particles and for the pions and it gives a reasonable qualitative description for the longitudinal variable distribution. Figure 3 shows the comparison between the fit of Eq. (10) and the four-prong K^-p inclusive data. The only discrepancy is an overemphasis in the Monte Carlo program of the leading-particle effect.[22] Similar fits were obtained for the six-prong K^-p reactions.

The π^+p data[23] were measured on film-plane digitizers at Columbia University, and slow protons were identified by their ionization. This sample of events with no missing neutrals was selected by using cuts on missing energy and missing momentum.

The matrix element (10) was used. The t' cutoff was now done for the "leading π" identified in each event as the fastest (lab) pion. The fits obtained for p^2 and Y distributions were good. The experimental Y distribution of the "leading π" was somewhat more peaked than the one generated by the t' exponential damping. This is indeed expected because of our choice of the leading pion to be the fastest in each case.

The experimental and background distributions $f_1, \ldots, f_7$ are shown in Figs. 4 and 5 for the four- and six-prong K^-p data and in Figs. 6 and 7 for the four- and six-prong π^+p data.

The following comments can be made about the various distributions:

V_1. In both K^- and π^+ data the calculated $f_1(V_1)$ is not sensitive to the parameters in Eq. (10). In all cases there seemed to be a certain, rather weak indication for experimental distributions more peaked at $V_1 \simeq 0$ than $f_1(V_1)_{\mathrm{backg}}$, i.e., some evidence for "local $\vec{\mathrm{P}}$ conservation."

V_2. $f_2(V_2)_{\mathrm{backg}}$, the calculated distribution, is not very sensitive to the parameters. There seems to be a weak indication for alignment via a slight excess of data events with $V_2 \simeq 1$, especially in the four-prong $\pi^+ p$ reaction

V_3. The experimental distributions $f_3(V_3)$ show strong deviations from the calculated $f_3(V_3)_{\text{backg}}$ in all four cases. This is quite encouraging and suggests that the extension of the concept of peripheral and central collisions to multiparticle reactions is very useful.

V_4. As might be expected from our discussion in part (iv) of Sec. II the leading-particle effect is very important in calculating $f_4(V_4)_{\text{backg}}$. The calculations are not too sensitive to the specific choice of parameters which roughly reproduce the one-particle distributions via Eq. (10). Except for the smaller width of the experimental distributions, no significant differences between the experimental and Monte Carlo-generated distributions were observed.

V_5. The Monte Carlo calculation was again not too sensitive to the specific choice of parameters, and no effect was seen.

V_6. The four-prong $K^- p$ data peak at $V_6 = 1$, the value expected from the diffractive picture. That peaking is significantly stronger than the peaking of the background distribution at the same point which to a large extent resulted from the leading-particle effect in our Monte Carlo matrix element, Eq. (10). A weaker effect in the same direction is evident also in the four-prong $\pi^+ p$ data. At this point it is worthwhile to emphasize that even a stronger effect could be expected for V_6 and V_7 from a more realistic background distribution which does not overemphasize the leading-particle effect like our own program.

No enhancement over the background distribution at $V_6 = +1$ was observed in the six-prong data.

V_7. The experimental distribution of V_7 from the four-prong data shows a considerable excess over the corresponding background distribution at $V_7 = 2$ and a considerable depletion at $V_7 = 0$. This deviation from the background distribution follows the pattern suggested in part (vii) of Sec. II above by the diffractive-type model. No significant effect was seen in the six-prong $K^- p$ and $\pi^+ p$ data.

Thus excepting a few cases—mainly in four-prong data—and most notably in V_3 and V_6 and V_7, no significant deviation of the data from the background distribution was observed. The inclusion of the leading-particle effect in this background was in many cases very crucial, and indeed much stronger effects were suggested by an earlier version which included the transverse momentum damping only.

The pattern of whatever deviations were observed did however follow remarkably well what is expected from a diffractive picture. This is particularly significant in cases (like V_7) when the diffractive and multiperipheral pictures suggest opposite trends. It would be very interesting to see if data from other higher-energy bubble-chamber experiments would corroborate this trend.

IV. SUMMARY AND CONCLUSIONS

The main proposition of the present work is that if some feature is sought in the data it is more profitable to investigate directly the corresponding multiparticle variables rather than to look for its—often weak—reflection in inclusive or even 2-body-correlation data.

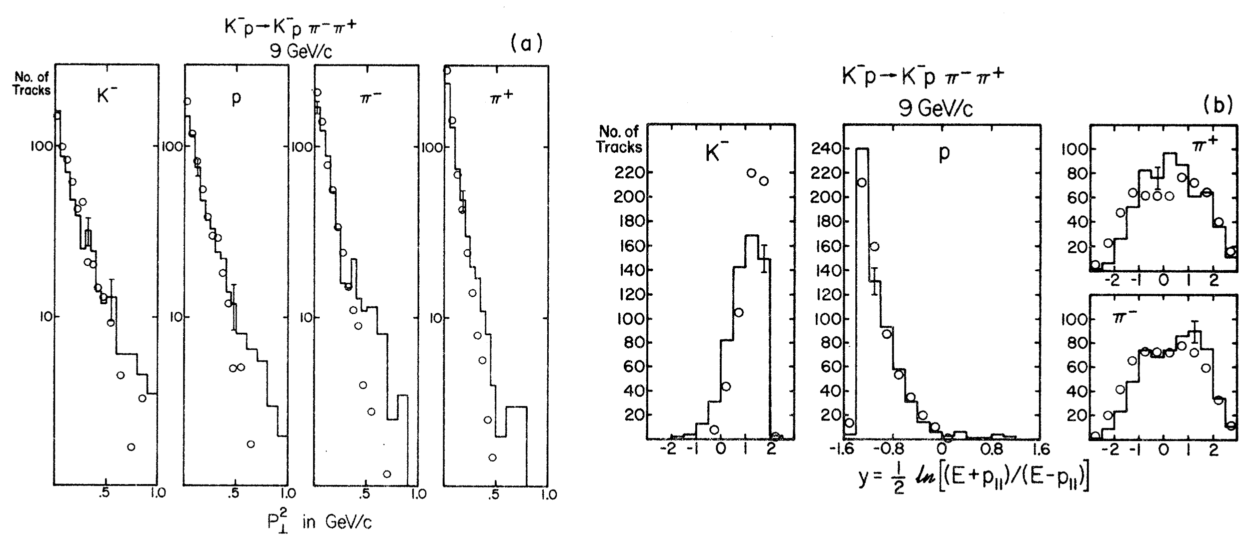

FIG. 3. The experimental distributions (histograms) for various particles produced in the 4-prong $K^- p$ reactions compared with the Monte Carlo computations (open circles). (a) Transverse momenta. (b) Rapidities.

There are essentially three different motivations for using multiparticle variables.

(a) At the most basic phenomenological level of searching for patterns in the data it may be a useful complement to direct searches in the raw data. Once we specify what pattern are we looking for, it is much easier to use the corresponding variable to check its existence.

(b) A few suitable multiparticle variables may serve as a much more critical test for choosing between different models—or rather between different classes of models such as the diffractive-type and multiperipheral-type models—than the inclusive distributions.

(c) The final and admittedly most speculative and interesting facet of multiparticle variables is that some variables will prove to have an intrinsic physical significance.

What we have in mind is best illustrated by analogy with nuclear physics. A well-known approximation often employed there is to concentrate on single-particle motion—lumping the effect of the complicated pairwise forces between nucleons into

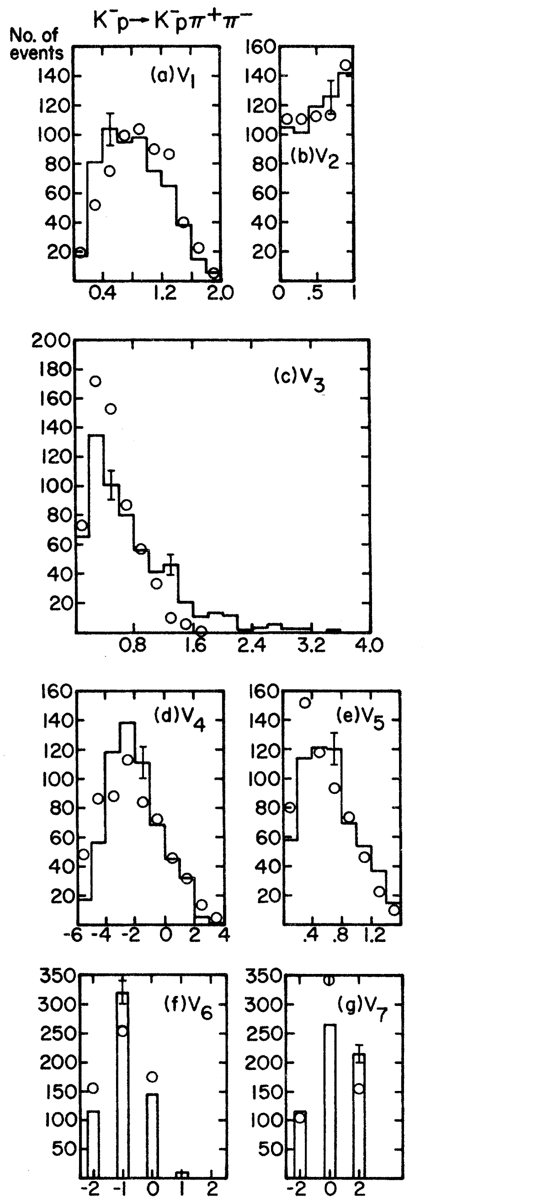

FIG. 4. The experimental distributions of the $V_1, \ldots, V_7$ (histograms) for the 4-prong K^-p reaction compared with the Monte Carlo computation. For V_1, V_3, and V_5 the abscissa unit is GeV/c.

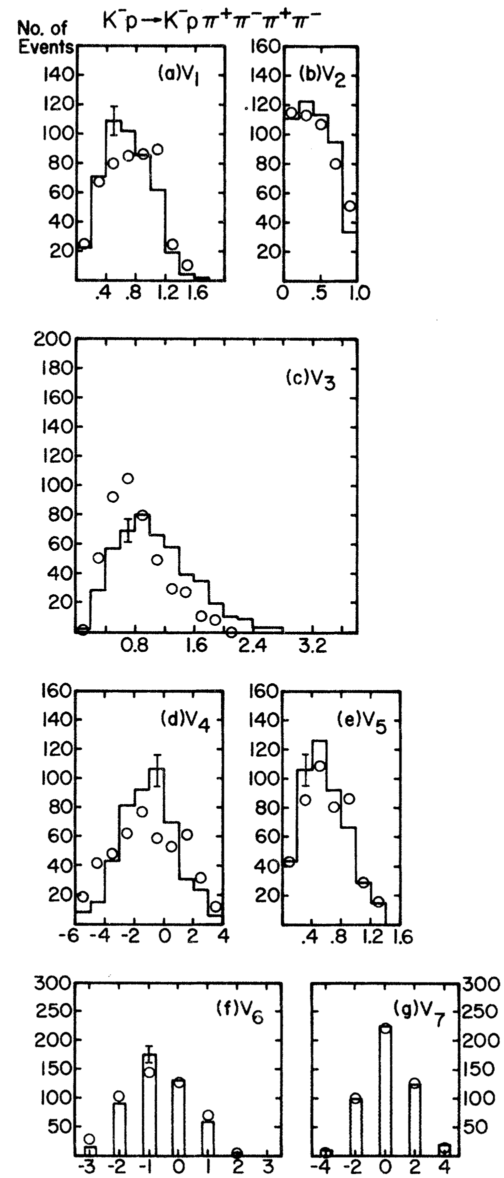

FIG. 5. The comparison for the 6-prong K^-p reaction.

some effective simple central potential. The resulting shell structure is somewhat reminiscent of the simple Regge-model picture which seems to hold for inclusive cross sections.

The shell picture does not however exhaust completely the dynamics of nuclei. For some purposes it seems more profitable to use "collective variables" involving all the nucleons.

We would like to hope that the high-energy analog of such collective variables would eventually emerge from extensive studies of multiparticle variables.

The present work is rather limited in its scope. Relatively few multiparticle variables were considered, and a rather limited amount of experimental information was analyzed. In particular the data were at a relatively low momentum, 9 GeV/c, where two- and three-body resonance effects are very important.[24]

However, just as in the case of the Horn-Schmid duality where the low-lying resonances conspired to give effects typical of the asymptotic behavior, also in this case the observed dynamic effects in the various $f_\alpha(V_\alpha)$ may be indicative of genuine high-energy multiparticle dynamics whether or not

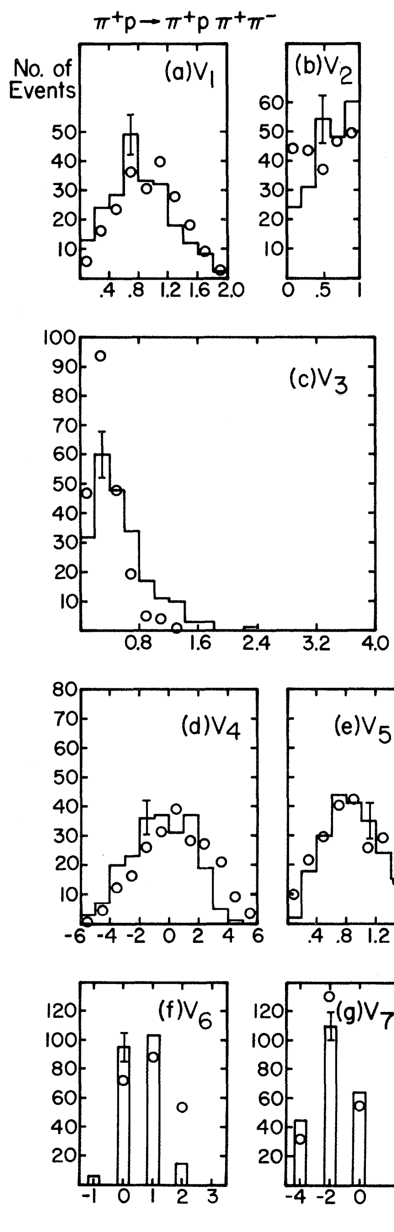

FIG. 6. The comparison for the 4-prong π^+p reaction.

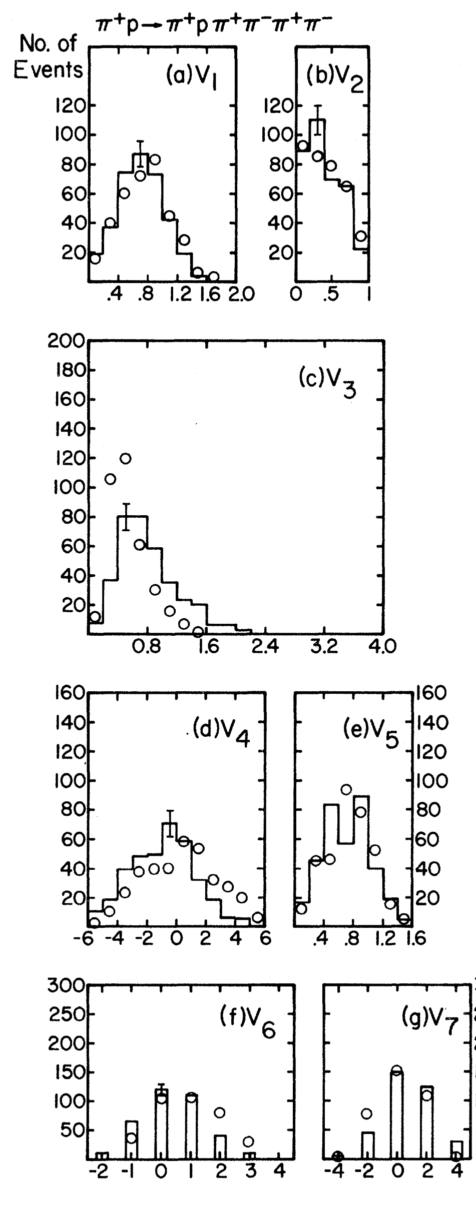

FIG. 7. The comparison for the 6-prong π^+p reaction.

an alternative explanation due to resonances is possible.

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APPENDIX

In order to prove the statement in Sec. II that the expected background distribution can be obtained from purely geometric considerations, let us consider the n-dimensional vectors defined by

$$X=(P_{1x},\ldots,P_{nx}),$$

$$Y=(P_{1y},\ldots,P_{ny}).$$

Since the distribution function involves only $e^{-|X|^2}e^{-|Y|^2}$, we conclude that

$$\hat{X}\equiv\frac{X}{|X|}=\left(\frac{P_{1x}}{(\sum P_{ix}{}^2)^{1/2}},\ldots,\frac{P_{nx}}{(\sum P_{ix}{}^2)^{1/2}}\right)$$

are uniformly distributed on an n-dimensional hypersphere $|X|=1$, and that the same is true for $\hat{Y}=Y/|Y|$. The energy-momentum conservation restricts X (and $\hat{X}$) to the plane $X\cdot\hat{n}=0$, where $\hat{n}=(1,1,\ldots 1)$, and similarly $\hat{Y}$ is restricted to $\hat{Y}\cdot\hat{n}=0$. This implies that in effect $\hat{X}$ and $\hat{Y}$ are uniformly distributed over $(n-1)$-dimensional spheres. Taking $\hat{X}$ as the polar axis $(1,0,0,0\ldots)$ on the sphere, we readily realize that $V_2\equiv\hat{X}\cdot\hat{Y}=\cos\theta$, where θ is the polar angle on that sphere. The unit area, integrated over all other spherical coordinates except θ, has the form

$$d\Omega_{n-1}=(\sin\theta)^{(n-1)-2}d\theta$$

$$=(\sin\theta)^{n-4}d\cos\theta.$$

The uniform distribution over the sphere means that $d\Omega_{n-1}$ (with no extra factors) gives the correct weight for the distribution, which in turn implies Eq. (2′).

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Theoretical Analysis of the Frascati Data on the Single-Bremsstrahlung Process $e^+e^- \to e^+e^-\gamma$

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The angular distributions of the process $e^+e^- \to e^+e^-$ measured by the Bologna-CERN-Frascati group at the electron-positron colliding-beam facility Adone are analyzed in terms of standard quantum electrodynamics (QED) to order α^3 including soft- and hard-photon emission. An analytical expression is given for the cross section of the process $e^+e^- \to e^+e^-\gamma$ for hard photons. Detailed numerical results for that specific experimental setup are obtained by a Monte Carlo program. It is found that this bremsstrahlung process is responsible for the noncollinear and noncoplanar events observed at Frascati. Therefore, these data, together with the present calculation, provide a test of QED for this particular physical situation involving high energies and large momentum transfers.

I. INTRODUCTION

There exists presently a considerable interest in the study of processes which involve the emission of hard photons in electron-positron collision processes. This aspect of e^+e^- interactions in high-energy physics is expected to become more and more important in the near future because of the increasing number of colliding-beam facilities at various laboratories either already working or under construction.

In this paper we shall discuss this theoretical aspect of e^+e^- interactions and compare our results with recent experimental data from Frascati. In fact, quite recently the angular distributions of two of the more typical processes of quantum electrodynamics (QED), namely

$$e^+e^- \to e^+e^-, \qquad (1)$$

$$e^+e^- \to \mu^+\mu^-, \qquad (2)$$

were measured by a Bologna-CERN-Frascati collaboration[1] (BCF group) at the electron-positron colliding-beam facility Adone in the total center-of-mass energy range from 1.6 GeV to 2.0 GeV and in the squared-momentum-transfer interval from -0.38 GeV^2 to -3.4 GeV^2. The outgoing charged particles were detected at large scattering angles (45°–135°); the total sensitive angle spanned was about 2.6 sr.

The above processes do not involve the structure of other particles whose effects are often difficult to distinguish from the effect due to the breakdown of QED, and therefore they make possible a test of the theory in physical situations involving large momentum transfers and high energies.

The BCF group has provided a considerable amount of experimental information on reaction (1): the absolute value of the cross section and its energy dependence, the production angular distribution, and the noncollinearity and noncoplanarity distributions. In the context of this experiment these angles are defined as follows:

(i) The noncollinearity angle θ_{12} is the supplement of the angle between the outgoing electron and positron [Fig. 1(a)].

(ii) The noncoplanarity angle ϕ is the angle between the two planes containing the beam axis and the outgoing electron or positron, respectively [Fig. 1(b)]. For experimental reasons those events which had a ϕ larger than 5° were called noncoplanar events.

The interest attached to the former two angular distributions stems from the fact that in the ab-