PHYSICAL REVIEW D VOLUME 8, NUMBER 10 15 NOVEMBER 1973

Self-Consistent Regge-Pole and -Cut Trajectories and the Reggeon-Reggeon-Pomeron Coupling

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(Received 23 April 1973)

Using the expression for the form of a partial-wave amplitude near a double-Regge branch point as obtained from t-channel unitarity equations, we investigate the conditions required for self-consistency of a Regge pole and its associated Reggeon-Pomeron branch point near $t=0$ where the two singularities collide. The self-consistent solution we obtain implies the vanishing of the Reggeon-Reggeon-Pomeron coupling at $t=0$.

INTRODUCTION

At high energies, it is thought that two-body scattering amplitudes may be described in terms of the exchange of t-channel Regge poles together with cut contributions arising from multiple-Reggeon exchange. Where the t-channel quantum numbers are such that Pomeron exchange is not possible, the exchanges of other Reggeons together with their associated Reggeon-Pomeron (RP) cuts provide the dominant contributions to the amplitude. At sufficiently small t and high enough energies the other multi-Reggeon exchange contributions (RPP, $RPPP$, RR, etc.) will be unimportant since they will fall off as a power of s or $\ln s$ more rapidly than the Reggeon-Pomeron cut contribution. If the Pomeron is assumed to be a Regge pole of intercept 1, we find that at $t=0$ a Regge pole R and the $RP, RPP, \ldots$ branch points collide.

By using t-channel unitarity equations Gribov *et al.*[1] have shown that, near a double-Regge branch point at $j=\alpha_c(t)$, the t-channel partial-wave amplitude for a process in which the t channel is elastic has the form

$$f(j,t) \underset{j\to\alpha_c}{\approx} \frac{A(j,t)}{B(j,t)+\nu(j,t)\ln[j-\alpha_c(t)]}\,, \qquad (1)$$

where A and B are undetermined functions of j and t. ν is a known function depending on the Regge trajectories being exchanged. The function B will contain weak singularities associated with multi-Regge exchange cuts, and may also contain fixed or moving poles or fixed j-plane cuts without violating any general principles. However, B cannot contain any t-plane cuts other than the multi-Regge cuts, as this would be inconsistent with Mandelstam analyticity. From the above discussion, we see that in the case of the Reggeon-Pomeron cut, near $t=0$, $f(j,t)$ should also contain the Regge pole. It is now well known[2] that if the pole is also to occur in the Reggeon scattering amplitude it must occur as a zero in the denominator in Eq. (1). Therefore, if

$$D(j,t)\equiv B(j,t)+\nu(j,t)\ln(j-\alpha_c) \qquad (2)$$

we must have

$$D(\alpha(t),t)=0\,, \qquad (3)$$

where $\alpha(t)$ is the trajectory of the Regge pole. In an investigation[2] of the special case of the Pomeron-Pomeron cut, Bronzan has obtained solutions to Eq. (3) in terms of consistency equations relating coefficients in an expansion of B about $j=1$, $t=0$, and the parameters defining the Pomeron trajectory. Assuming that any fixed j-plane cuts in B are weak, Bronzan finds that if the double-Pomeron exchange cut is to have the Mandelstam sign it is necessary for A and B to share a second-order Castillejo-Dalitz-Dyson (CDD) pole which moves through $j=1$ at $t=0$. A consequence of this is that the triple-Pomeron coupling vanishes at $t=0$. In this paper we extend Bronzan's analysis to the general Reggeon-Pomeron cut, and again find it is necessary for A and B to contain a second-order CDD pole. This results in a vanishing of the Reggeon-Reggeon-Pomeron coupling at $t=0$.

II. THE SELF-CONSISTENCY EQUATIONS

First of all we consider the function $\nu(j,t)$ in Eq. (1). In Ref. 1 it is shown that for the exchange of identical poles, $\beta(t)$, ν is given by

$$\nu=\{\beta'(\tfrac{1}{4}t)[\beta'(\tfrac{1}{4}t)+\tfrac{1}{2}t\beta''(\tfrac{1}{4}t)]\}^{-1/2}. \qquad (4)$$

It is not too difficult to show that for nonidentical trajectories, α and β, this becomes

$$\nu=2\left([\alpha'(t_1^c)+\beta'(t_2^c)]^2+\frac{2t\{[\alpha'(t_1^c)]^2\beta''(t_2^c)+[\beta'(t_2^c)]^2\alpha''(t_1^c)\}}{\alpha'(t_1^c)+\beta'(t_2^c)}\right)^{-1/2}, \qquad (5)$$

where the derivatives are evaluated at the branch point, i.e., at the points $t_1=t_1^c$, $t_2=t_2^c$ for which j

$=\alpha(t_1)+\beta(t_2)-1$ is a maximum subject to the constraint $t^{1/2}=t_1^{1/2}+t_2^{1/2}$. t_1^c and t_2^c may be expressed as functions of t.

We note that in the special case we are going to consider where one of the trajectories has intercept 1 and positive signature, Eq. (5) is unaltered if instead of using the analysis of Ref. 1 we use White's expression[3] for the discontinuity across the double-Regge cut where signatured amplitudes are used.

Using Eq. (5) we may write D, for small t, as

$$D=B(j,t)+\left[\frac{2}{\alpha'(t_1^c)+\beta'(t_2^c)}+O(t)\right]\ln(j-\alpha_c)\,. \tag{6}$$

It is obvious that Eq. (3) cannot be satisfied unless either B, α, or β contains a singularity at $t=0$ or $j=\alpha_c(0)$, otherwise the logarithm in Eq. (6) will lead to D diverging at $t=0$. Solutions to Eq. (3) may be found by introducing a fixed j-plane branch point at $j=\alpha_c(0)$ in B. However, we find that in such solutions the fixed cut will dominate over the moving cut for $t<0$ and at $t=0$ will completely cancel the double-Regge cut contribution so that there is no cut contribution to the amplitude in the forward direction. We feel that such a strong fixed-cut contribution is unacceptable, and also, when $\alpha=\beta=P$, we should like our solution to Eq. (3) to reduce to that of Ref. 2. Therefore we shall follow Bronzan in assuming that A and B have a second-order CDD pole and that the trajectories may have a left-hand cut starting at $t=0$. In order that this t-plane cut in the trajectory functions should not result in a corresponding branch point in the partial-wave amplitude, it is convenient, as in Ref. 2, to use the inverse functions T_1, T_2, and T_c defined by

$$\begin{aligned} t_1&=T_1(\alpha(t_1))\,,\\ t_2&=T_2(\beta(t_2))\,,\\ t&=T_c(\alpha_c(t))\,. \end{aligned} \tag{7}$$

A branch point at $t=0$ in the trajectory functions corresponds to fixed j-plane branch points in the T_i. By expressing Eq. (1) in terms of the inverse trajectory functions we can ensure that $f(j,t)$ will only contain fixed j-plane cuts but no fixed t-plane cuts even though the trajectory functions have such cuts.

Taking β to be the Pomeron and α to be an arbitrary Regge pole trajectory having intercept α_0, we assume that the T_i have expansions of the form

$$\begin{aligned} T_1(j_1)&=d_0(j_1-\alpha_0)+d_1(j_1-\alpha_0)^2+d_2(j_1-\alpha_0)^2\ln(j_1-\alpha_0)+\cdots\,,\\ T_2(j_2)&=e_0(j_2-1)+e_1(j_2-1)^2+e_2(j_2-1)^2\ln(j_2-1)+\cdots\,,\\ T_c(j)&=f_0(j-\alpha_0)+f_1(j-\alpha_0)^2+f_2(j-\alpha_0)^2\ln(j-\alpha_0)+\cdots\,. \end{aligned} \tag{8}$$

In terms of the functions T_i the Reggeon-Pomeron branch point is given by

$$j=j_c\equiv\max(j_1+j_2-1)\,, \tag{9}$$

subject to $t^{1/2}=T_1^{1/2}(j_1)+T_2^{1/2}(j_2)$. The position $j_1=j_1^c$, $j_2=j_2^c$ of this maximum is given by

$$\frac{[T_1'(j_1)]^2}{T_1(j_1)}=\frac{[T_2'(j_2)]^2}{T_2(j_2)}\,. \tag{10}$$

Using Eq. (10) we may relate j_1^c and j_2^c, and near $t=0$ we may expand them in terms of $j_c-\alpha_0$, obtaining

$$\begin{aligned} j_1^c-\alpha_0&=\frac{d_0}{d_0+e_0}(j_c-\alpha_0)+\frac{d_0e_0}{(d_0+e_0)^3}\left[3\left(d_2\ln\frac{d_0}{d_0+e_0}-e_2\ln\frac{e_0}{d_0+e_0}+d_1-e_1\right)+2(d_2-e_2)\right](j_c-\alpha_0)^2\\ &\quad+\frac{3d_0e_0(d_2-e_2)}{(d_0+e_0)^3}(j_c-\alpha_0)^2\ln(j_c-\alpha_0)+\cdots\,,\\ j_2^c-1&=\frac{e_0}{d_0+e_0}(j_c-\alpha_0)-\frac{d_0e_0}{(d_0+e_0)^3}\left[3\left(d_2\ln\frac{d_0}{d_0+e_0}-e_2\ln\frac{e_0}{d_0+e_0}+d_1-e_1\right)+2(d_2-e_2)\right](j_c-\alpha_0)^2\\ &\quad-\frac{3d_0e_0(d_2-e_2)}{(d_0+e_0)^3}(j_c-\alpha_0)^2\ln(j_c-\alpha_0)+\cdots\,. \end{aligned} \tag{11}$$

Using Eq. (11) together with the equation

$$T_c^{1/2}(j_1^c+j_2^c-1)=T_1^{1/2}(j_1^c)+T_2^{1/2}(j_2^c)\,, \tag{12}$$

we may obtain

$$T_c(j)=(d_0+e_0)(j-\alpha_0)+\frac{1}{d_0+e_0}\left(d_0d_2\ln\frac{d_0}{d_0+e_0}+e_0e_2\ln\frac{e_0}{d_0+e_0}+d_0d_1+e_0e_1\right)(j-\alpha_0)^2$$

$$+\frac{(d_0d_2+e_0e_2)}{(d_0+e_0)}(j-\alpha_0)^2\ln(j-\alpha_0)+\cdots\,. \tag{13}$$

If α and β are given by

$$\alpha(t)=\alpha_0+\alpha_1t+\alpha_2t^2+\alpha_3t^2\ln t+\cdots\,,$$
$$\beta(t)=1+\beta_1t+\beta_2t^2+\beta_3t^2\ln t+\cdots\,, \tag{14}$$

then from Eq. (13) we find that

$$\alpha_c(t)=\alpha_0+\frac{\alpha_1\beta_1}{\alpha_1+\beta_1}t+\frac{1}{(\alpha_1+\beta_1)^4}\left(2\alpha_3\beta_1{}^4\ln\frac{\beta_1}{\alpha_1+\beta_1}+2\beta_3\alpha_1{}^4\ln\frac{\alpha_1}{\alpha_1+\beta_1}+\alpha_2\beta_1{}^4+\beta_2\alpha_1{}^4\right)t^2+\frac{(\beta_3\alpha_1{}^4+\alpha_3\beta_1{}^4)}{(\alpha_1+\beta_1)^4}t^2\ln t+\cdots\,. \tag{15}$$

We can now consider the problem of reexpressing Eq. (1) in terms of the functions T_i. At small t we find that

$$\frac{2}{\alpha'(t_1^c)+\beta'(t_2^c)}\to\frac{2T_1'(j_1^c)T_2'(j_2^c)}{T_1'(j_1^c)+T_2'(j_2^c)}+O(t)=\frac{2}{\alpha_1+\beta_1}+O(t)\,. \tag{16}$$

We also find that

$$j-\alpha_c(t)=\frac{T_c(j)-t}{T_c'(j)}+O(t,j-\alpha_0)\,. \tag{17}$$

Therefore, for small t, we may write Eq. (1) as

$$f(j,t)\underset{j\to\alpha_c}{\approx}\frac{A(j,t)}{B(j,t)+\dfrac{2T_1'T_2'}{T_1'+T_2'}\ln\left[\dfrac{T_c(j)-t}{T_c'(j)}\right]+O(t,j-\alpha_0)}\,. \tag{18}$$

Bronzan regards this form for $f(j,t)$ as more fundamental than that of Eq. (1) since it is still valid (in the sense that no undesirable t-plane cuts occur in it) if the trajectory functions contain branch points at $t=0$.

In order to obtain a self-consistent description of the Regge pole α and the Regge-Pomeron cut near $t=0$ we now have to determine the constraints imposed by Eq. (3). As described above, it is necessary to assume that A and B contain a double CDD pole. We assume that any cuts in A and B are weak, and that near $t=0$, $j=\alpha_0$ the functions take the form

$$B=\frac{b_0+b_1t+b_2(j-\alpha_0)+b_3t^2+b_4(j-\alpha_0)^2+b_5(j-\alpha_0)t}{[c_1t+c_2(j-\alpha_0)]^2}\,,$$
$$A=\frac{a^2(j,t)}{[c_1t+c_2(j-\alpha_0)]^2}\,, \tag{19}$$

where $a(j,t)$ is analytic at $t=0$, $j=\alpha_0$. To second order in t and $(j-\alpha_0)$ we may write Eq. (1) as

$$f(j,t)\approx\frac{a^2(j,t)}{b_0+b_1t+b_2(j-\alpha_0)+b_3t^2+b_4(j-\alpha_0)^2+b_5(j-\alpha_0)t+\cdots+\dfrac{2[c_1t+c_2(j-\alpha_0)]^2}{\alpha_1+\beta_1}\ln\left[\dfrac{T_c(j)-t}{T_c'(j)}\right]}\,. \tag{20}$$

In order that this expression should contain the Regge pole at $j=\alpha(t)$, we must have

$$b_0+b_1T_1(j)+b_2(j-\alpha_0)+b_3[T_1(j)]^2+b_4(j-\alpha_0)^2+b_5(j-\alpha_0)T_1(j)+\frac{2[c_1T_1(j)+c_2(j-\alpha_0)]^2}{\alpha_1+\beta_1}\ln\left[\frac{T_c(j)-t}{T_c'(j)}\right]+\cdots=0\,. \tag{21}$$

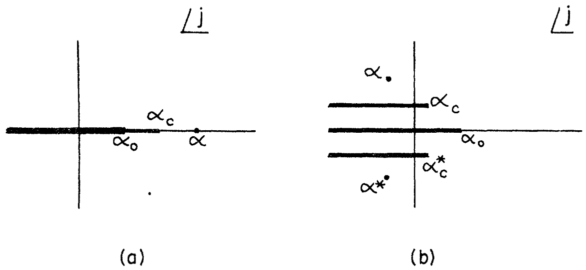

FIG. 1. j-plane singularities of $f(j,t)$ for (a) $t>0$; (b) $t<0$.

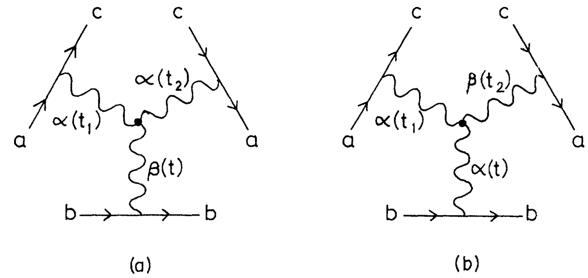

FIG. 2. Reggeon-Reggeon-Pomeron vertices.

Substituting for T_1 and equating coefficients of powers of $(j-\alpha_0)$ to zero, we obtain [working only to second order in $(j-\alpha_0)$]

$$b_0=0\,,\quad b_1=\frac{2\alpha_1}{\alpha_3(\alpha_1+\beta_1)}(c_1+c_2\alpha_1)^2\,,\quad b_2=-\frac{b_1}{\alpha_1}\,,$$
$$b_3=-b_4\alpha_1{}^2-b_5\alpha_1+\frac{2\alpha_2}{\alpha_3(\alpha_1+\beta_1)}(c_1+c_2\alpha_1)^2-\frac{2(c_1+c_2\alpha_1)^2}{\alpha_1+\beta_1}\ln\frac{\alpha_1{}^2}{\alpha_1+\beta_1}\,. \tag{22}$$

Thus the consistency requirement provides relations between the b_i, c_i, α_i, and β_i which reduce the number of undetermined parameters in Eq. (20). If we were to take higher terms in our expansions we should find that B has to contain a weak fixed cut at $j=\alpha_0$. Looking at the j-plane structure of $f(j,t)$, we find that for $t>0$ there is a pole at $j=\alpha(t)$, the Regge-Pomeron branch point at $j=\alpha_c(t)$, where α_c is given by Eq. (15) and a fixed cut with branch point at $j=\alpha_0$ [see Fig. 1(a)]. As t is decreased the pole and cuts converge on one another and collide at $t=0$. For $t<0$ we find that $f(j,t)$ contains complex conjugate pairs of poles and moving cuts such that the real analyticity of the amplitude is maintained.

DISCUSSION

Theoretically, the form for $f(j,t)$ given by Eqs. (20) and (22) would provide a more satisfactory means of parametrizing s-channel inelastic scattering amplitudes near $t=0$ than is obtained using the absorption model. However, unless further constraints are imposed on the large number of undetermined parameters, the resulting amplitude will have very little predictive power.

The most interesting point arising out of the above analysis is in connection with the Reggeon-Reggeon-Pomeron (RRP) coupling. In Ref. 1 it is shown that if the particle scattering partial-wave amplitude is given by Eq. (1), then the Reggeon production amplitude has the form

$$N_j(t)\underset{j\to\alpha_c}{\simeq}\frac{\sqrt{A}}{B+\nu\ln(j-\alpha_c)}\,. \tag{23}$$

Therefore the expression corresponding to Eq. (20) is given by

$$N_j(t)\underset{j\to\alpha_c}{\simeq}\frac{a(j,t)[c_1t+c_2(j-\alpha_0)]}{b_1t+b_2(j-\alpha_0)+\cdots+\dfrac{2[c_1t+c_2(j-\alpha_0)]^2}{\alpha_1+\beta_1}\ln\left[\dfrac{T_c(j)-t}{T_c'(j)}\right]}\,. \tag{24}$$

Looking at the Regge-pole contributions we find that in the particle scattering amplitude the residue of the pole is finite at $t=0$. However, in the Reggeon production amplitude the residue vanishes at $t=0$ because of the factor $c_1t+c_2(j-\alpha_0)$ in the numerator of Eq. (24). This implies that the RRP coupling, $n_{RRP}(t,t_1,t_2)$, vanishes when $t=t_1=t_2=0$. By carrying out a calculation similar to that in Ref. 4, we may deduce that the coupling $n_{RRP}(0,t,t)$ vanishes linearly as $t\to 0$. Thus if Bronzan's analysis of the Pomeron-Pomeron cut is extended to the Reggeon-Pomeron cut, we find that in addition to the triple-Pomeron coupling the RRP coupling must also vanish at $t=0$. At first sight it would appear that this prediction could easily be tested by looking at inclusive cross sections of the type $a+b\to c+(\text{anything})$, where $a\bar{c}$ does not have vacuum quantum numbers. However, when the helicity dependence of the Reggeon production amplitude is taken into account (see Refs. 3 and 5), we find that the coupling which we predict to vanish corresponds, at $t=0$, to the inclusive coupling shown in Fig. 2(b), but is not the same as that shown in Fig. 2(a).[6] Thus it becomes necessary to look at inclusive reactions in which $a\bar{c}$ has vacuum quantum numbers, but even then our prediction can be tested experimentally only if the triple-Pomeron coupling and the two RPP couplings all vanish at $t=0$. Although a number[7,8] of the proofs of the vanishing of the triple-Pomeron coupling at $t=0$ are in doubt,[9,4] the derivations of Abarbanel and Green,[10] that of Ref. 2, and the arguments[11] based on Gribov's Reggeon calculus still lead us to believe that this coupling vanishes. Experimentally there is evidence[12] that it is small, but the data are not good enough to confirm that it vanishes. When more is known both theoretically and experimentally about the RPP couplings, the vanishing of the RRP coupling at $t=0$ may be open to experimental verification.

ACKNOWLEDGMENTS

We should like to thank Dr. I. G. Halliday for many useful discussions. One of us (P.A.C.) also wishes to thank the Science Research Council for a Postdoctoral Fellowship.

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