Vector-Meson Dominance in Gauge Theories

Kazuo Fujikawa*
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England
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The off-shell behavior of the current amplitude is studied in the framework of a field-theoretical model of vector-meson dominance based on a spontaneously broken gauge theory. We discuss mainly the effects of the Higgs scalar mesons, in particular the implications of a straightforward modification of the ordinary algebra of fields. The Schwinger term, which depends on the Higgs scalars, and some of the Schwinger-term sum rules are briefly discussed. The ratio R in the deep-inelastic process and the Callan-Gross relation are discussed in detail based on a simple model. Our result suggests that the longitudinal component is still logarithmically enhanced relative to the transverse component. The renormalizability and the enhancement of the longitudinal component are therefore independent properties. It is also emphasized that the ratio R, given by the dispersive approach of vector dominance, is logically independent of the large ratio R required by the algebra of fields. The effects of the Higgs scalars on electron-positron annihilation are also compared with those of the dilaton; the annihilation cross section scales up to logarithmic factors. We note that the naive scale dimension of the current and Bjorken scaling (or scaling in the annihilation process) have very little to do with each other in the present approach. We also note that in the ordinary perturbative sense the lepton current couples to a nonconserved piece of the hadron source current. The implication of this property for the construction of the dual current amplitude is discussed. The extension of the model to include many vector mesons is also commented on.

I. INTRODUCTION

The notion of vector-meson dominance[1] and its field-theoretical implementation via the current field identity[2] and field algebra[3,4] has provided a convenient means to study low-energy electromagnetic and weak phenomena. A recent development in the gauge theory[5-7] allows us to construct a renormalizable (and therefore more well-defined) theory of massive vector particles. The gauge theory of vector dominance was first discussed by 't Hooft[7] and later generalized by Bardakci and Halpern.[8,9] In the unified model of strong and weak interactions by Bars, Halpern, and Yoshimura,[10] the idea of vector dominance was utilized to resolve some of the "mismatches" of hadronic and leptonic symmetries. The existence of the so-called Higgs scalars plays an essential role in the ordinary treatment of the spontaneously broken gauge theory. These Higgs scalars somehow suppress the longitudinal component of the massive vector mesons. They also modify the ordinary algebra of fields.[9,10]

In this paper we study the effects of these new scalar mesons. In particular we are interested in the off-shell behavior of the current amplitude. This off-shell behavior is important even though these models are primarily intended for use in the low-energy region. One such example where the off-shell behavior has some relevance is the electromagnetic mass difference. The knowledge of the off-shell behavior is essential when one estimates the physical plausibility of the finite mass difference[9,11] given by the gauge theory. Also, a recent study[12] indicates that the low-lying scalar and vector mesons may be important in determining the asymptotic behavior of the scaling functions in the deep-inelastic process. It is therefore desirable to know the detailed properties of the low-lying vector mesons and their interactions with currents. In the model which we discuss below, the form factor of the vector-dominant type is already built in; it could therefore be valid up to fairly large momentum transfers.

To study the dynamical aspects of the gauge theory we use mainly the simplest model possible, namely, an Abelian model. This model is certainly too simple to be realistic. However one can learn certain characteristic aspects of the gauge theory based on this model. We learn, for example, that the renormalizability of the underlying Lagrangian as such does not imply the finiteness of the ratio R in deep-inelastic electroproduction. Some other aspects are more dependent on the detailed algebraic structure of the model. In such cases we use a simple non-Abelian model to discuss the necessary modifications.

The recent discovery[13] that the zero-slope limit of the dual resonance model corresponds to a spontaneously broken gauge theory also makes the dynamical study of the gauge theory more appropriate. We try to compare some of the relevant properties of the gauge theory with those of the dual resonance model.

II. ABELIAN MODEL

We now turn to a simple Abelian model to study the effects of the Higgs scalars in the deep-inelastic process[14] and related processes. The ordinary vector-gluon model has been widely used to investigate the scaling property in the framework of a Lagrangian field theory.[15] The Lagrangian we discuss below is very close to the conventional one, but it also incorporates the idea of vector-meson dominance. The major difference between the ordinary vector-gluon model and the present vector-dominant one is the following: The non-Abelian extension of the former is rather straightforward, and it does not alter the basic structure of the currents as far as one includes only the neutral vector gluons. On the other hand, the extension of the latter to the non-Abelian case substantially modifies the algebraic structure of the currents.

Some of the algebraic properties of the non-Abelian model are summarized in the Appendix. In spite of this limitation of the Abelian version of vector-meson dominance, we use it mainly in the following discussions because it is simple and also allows us to see what is going on in a more transparent way. When we come to properties which depend in an essential way on the Abelian nature of the model, we discuss the necessary modifications based on the non-Abelian model given in the Appendix.

A. Algebra

We start with the Lagrangian:

$$\begin{aligned}\mathcal{L} &= \bar{\mu} i\gamma^\mu(\partial_\mu - ieA_\mu)\mu - m_\mu \bar{\mu}\mu \\ &+ \bar{\psi} i\gamma^\mu(\partial_\mu - igB_\mu)\psi - m\bar{\psi}\psi \\ &- \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\ &+ |(\partial_\mu - igB_\mu - ieA_\mu)\Phi|^2 - \lambda(|\Phi|^2 - \tfrac{1}{2}v^2)^2. \end{aligned} \tag{2.1}$$

After spontaneous breaking we have

$$\Phi(x) = \frac{1}{\sqrt{2}}[\varphi(x) + i\chi(x) + v]. \tag{2.2}$$

The unitary gauge indicates (the gauge-compensating effective action should also be added; see Appendix) that

$$\chi(x) \equiv 0. \tag{2.3}$$

We also define

$$V_\mu(x) \equiv B_\mu(x) + \frac{e}{g}A_\mu(x). \tag{2.4}$$

The Lagrangian can now be written as

$$\begin{aligned}\mathcal{L} &= \bar{\mu} i\gamma^\mu(\partial_\mu - ieA_\mu)\mu - m_\mu\bar{\mu}\mu \\ &+ \bar{\psi} i\gamma^\mu(\partial_\mu - igV_\mu + ieA_\mu)\psi - m\bar{\psi}\psi \\ &- \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\ &+ \tfrac{1}{2}(\partial_\mu\phi(x))^2 + \tfrac{1}{2}g^2\phi(x)^2 V_\mu V^\mu - \tfrac{1}{4}\lambda(\phi^2 - v^2)^2, \end{aligned} \tag{2.5}$$

where

$$\phi(x) \equiv v + \varphi(x). \tag{2.6}$$

In this Lagrangian the mass of the "photon" field A_μ is not quite zero because of the mixing of fields in the term $(\partial_\mu B_\nu - \partial_\nu B_\mu)^2$. We can now write the equations of motion from Eq. (2.5):

$$\partial_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) = e\bar{\mu}\gamma^\mu\mu + eJ^\mu(x) \tag{2.7}$$

and

$$g^2\phi^2 V^\mu(x) - gJ^\mu(x) = 0, \tag{2.8}$$

where the hadron current is given by

$$J^\mu(x) = \frac{1}{g}\partial_\nu(\partial^\mu B^\nu - \partial^\nu B^\mu) - \bar{\psi}\gamma^\mu\psi. \tag{2.9}$$

Equation (2.8) also gives (current-field identity)

$$J_\mu(x) = \frac{1}{g}[g\phi(x)]^2 V_\mu(x). \tag{2.10}$$

The quantization proceeds as usual; in particular,

$$[\Pi_\varphi(x),\ \varphi(y)]_{x^0=y^0} = -i\delta^3(\vec{x} - \vec{y}) \tag{2.11}$$

and

$$[\Pi_j(t,\vec{x}),\ B^j(t,\vec{y})] = -i\delta^3(\vec{x} - \vec{y}), \tag{2.12}$$

where

$$\Pi_\varphi(x) \equiv \dot{\varphi}(x) \tag{2.13}$$

and

$$\Pi_j(x) \equiv F_{j0}(x) = \partial_j B_0(x) - \partial_0 B_j(x). \tag{2.14}$$

The quantization of the electromagnetic field is the same as in the ordinary treatment. A rather detailed discussion of this together with a general discussion of the renormalization program is found in Ref. 4. See also Ref. 7.

The algebraic properties are given by

$$[J^0(t,\vec{x}),\ J^0(t,\vec{y})] = 0, \tag{2.15}$$

$$[J^0(t,\vec{x}),\ J^j(t,\vec{y})] = i\frac{[g\phi(y)]^2}{g^2}\partial^j\delta^3(\vec{x} - \vec{y}), \tag{2.16}$$

and

$$[\partial_0 J^k(t,\vec{x}),\ J^j(t,\vec{y})] = -4i\frac{1}{[\phi(x)]^2} J^k(t,\vec{x}) J^j(t,\vec{x})\,\delta^3(\vec{x}-\vec{y}) - ig^2[\phi(x)\phi(y)]^2 \delta^{kj}\delta^3(\vec{x}-\vec{y}) - i\left[\frac{\phi(y)}{\phi(x)}\right]^2 \partial_x^k [\phi(x)]^2 \partial^j \delta^3(\vec{x}-\vec{y}) + i[\phi(y)]^2 \partial^k\partial^j\delta^3(\vec{x}-\vec{y}) + \text{quantum-electrodynamic corrections.} \quad (2.17)$$

It should be noted that the effects of the Higgs scalar field on the algebraic properties are not sensitive to the group structure of the model (see Appendix). Equation (2.16) shows that the Schwinger term[16] depends on the Higgs scalars. In the limit of large λ, Eq. (2.17) becomes

$$[\partial_0 J^k(t,\vec{x}),\ J^j(t,\vec{y})] = -4i\frac{1}{v^2} J^k(t,\vec{x}) J^j(t,\vec{y})\,\delta^3(\vec{x}-\vec{y}) - ig^2 v^2 \delta^{kj}\delta^3(\vec{x}-\vec{y}) + iv^2\partial^k\partial^j\delta^3(\vec{x}-\vec{y}) + \text{QED corrections.} \quad (2.18)$$

Actually this is valid for $m_\varphi^2 = 2\lambda v^2 >> m_V^2 = g^2v^2$. If one starts with the limit $\lambda=\infty$, we obtain instead

$$[\partial_0 J^k(t,\vec{x}),\ J^j(t,\vec{y})] = iv^2\partial^k\partial^j\delta^3(\vec{x}-\vec{y}) - ig^2v^2\delta^{kj}\delta^3(\vec{x}-\vec{y}) + \text{QED corrections.} \quad (2.19)$$

and

$$[J^0(t,\vec{x}),\ J^j(t,\vec{y})] = i\frac{(gv)^2}{g^2}\partial^j\delta^3(\vec{x}-\vec{y}). \quad (2.20)$$

Equations (2.19) and (2.20) correspond to the ordinary algebra of fields.[3,4] The existence of the Higgs scalar gives rise to the difference between Eq. (2.18) and Eq. (2.19); these equations also show that the limit $\lambda\to\infty$ and the algebraic manipulation do not commute.

B. Callan-Gross Relation

At this stage we quote the main result of Callan and Gross.[17] They derived the following relation (assuming scaling):

$$\int_0^4 \frac{d\xi^2}{\xi}[F_T(\xi)\,(\delta_{ij}-\hat{p}_i\hat{p}_j) - F_L(\xi)\hat{p}_i\hat{p}_j] = \lim_{p_0\to\infty}\frac{m}{p_0^{\,2}} C_{ij}(p), \quad (2.21)$$

where

$$C_{ij}(p) \equiv -\int d^4x\,\delta(x^0)\langle p|[\partial_0 J_i(x),\ J_j(0)]|p\rangle \quad (2.22)$$

and

$$\xi \equiv (-q^2)/(qp) \equiv 2/\omega. \quad (2.23)$$

The structure functions are defined by[14]

$$F_T(\xi) = \xi F_1(\xi) \quad (2.24)$$

and

$$F_L(\xi) = F_2(\xi) - \xi F_1(\xi). \quad (2.25)$$

Equation (2.17) [or (2.18)] gives rise to

$$\int dx\,\delta(x^0)\langle p|[\partial_0 J_i(x),\ J_j(0)]|p\rangle = A(p^2)p_ip_j + B(p^2)\delta_{ij}. \quad (2.26)$$

On the other hand, Eq. (2.19) gives

$$\int dx\,\delta(x^0)\langle p|[\partial_0 J_i(x),\ J_j(0)]|p\rangle = 0. \quad (2.27)$$

Therefore Eq. (2.17) suggests that

$$F_T(\xi) = 0 \quad (2.28)$$

and

$$F_L(\xi) \text{ is unspecified.} \quad (2.29)$$

Equation (2.19) similarly suggests

$$F_T(\xi) = F_L(\xi) = 0. \quad (2.30)$$

C. Perturbation Calculation

In the following we examine Eq. (2.28)–Eq. (2.30) based on perturbation calculations. Our approach is akin to the effective-Lagrangian approach[18]; we limit ourselves to tree diagrams. For this purpose we further rewrite the Lagrangian in Eq. (2.5) as

$$\mathfrak{L} = \bar{\mu}i\gamma^\mu\left[\partial_\mu - i\hat{e}\hat{A}_\mu - i\left(\frac{\hat{e}^2}{G}\right)\hat{B}_\mu\right]\mu - m_\mu\bar{\mu}\mu + \bar{\psi}i\gamma^\mu(\partial_\mu - iG\hat{B}_\mu + i\hat{e}\hat{A}_\mu)\psi - m\bar{\psi}\psi - \tfrac{1}{4}(\partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu)^2 - \tfrac{1}{4}(\partial_\mu\hat{B}_\nu - \partial_\nu\hat{B}_\mu)^2 + \tfrac{1}{2}M^2\hat{B}_\mu\hat{B}^\mu + \tfrac{1}{2}(\partial_\mu\varphi)^2 + \tfrac{1}{2}\hat{g}^2\hat{B}_\mu\hat{B}^\mu\varphi^2 + M\hat{g}\hat{B}_\mu\hat{B}^\mu\varphi - \tfrac{1}{4}\lambda(\varphi^2+2v\varphi)^2, \quad (2.31)$$

where we redefined the fields by

$$\hat{A}_\mu = (gA_\mu - eB_\mu)/\hat{g} \tag{2.32}$$

and

$$\hat{B}_\mu = (gB_\mu + eA_\mu)/\hat{g}. \tag{2.33}$$

The photon field $\hat{A}_\mu$ in (2.32) is massless in contrast to A_μ in Eq. (2.5).

The coupling constants are defined by

$$\begin{aligned} \hat{g} &\equiv (g^2+e^2)^{1/2}, \\ G &= g^2/\hat{g}\ , \\ \hat{e} &= ge/\hat{g}\ , \end{aligned} \tag{2.34}$$

and

$$M = \hat{g}v. \tag{2.35}$$

This Lagrangian gives rise to the equations of motion:

$$\partial_\nu(\partial^\mu \hat{A}^\nu - \partial^\nu \hat{A}^\mu) = \hat{e}\bar{\mu}\gamma^\mu\mu - \hat{e}\bar{\psi}\gamma^\mu\psi, \tag{2.36}$$

$$\partial_\nu(\partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu) = G\bar{\psi}\gamma^\mu\psi + (\hat{e}^2/G)\bar{\mu}\gamma^\mu\mu + \hat{g}^2[\varphi(x)+v]^2\hat{B}^\mu(x). \tag{2.37}$$

Note that Eq. (2.36) and Eq. (2.37) do not satisfy the current field identity. This kind of arbitrariness in defining the fields is well known.[4] Equation (2.37) can also be written as

$$\partial_\nu(\partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu) - M^2\hat{B}^\mu = G\bar{\psi}\gamma^\mu\psi + (\hat{e}^2/G)\bar{\mu}\gamma^\mu\mu + \hat{g}^2[\varphi(x)^2 + 2v\varphi(x)]\,\hat{B}^\mu(x). \tag{2.38}$$

This suggests that the hadron source current to which the massive vector field couples is not conserved:

$$\partial_\mu[\,\hat{g}^2(\varphi^2+2v\varphi)\hat{B}^\mu(x)] \neq 0. \tag{2.39}$$

Equation (2.39) vanishes only for the Landau gauge[7]:

$$\partial_\mu \hat{B}^\mu(x) = 0. \tag{2.40}$$

In the Landau gauge, however, the Goldstone excitation remains in the Lagrangian. This is one of the aspects of the Nambu-Goldstone mechanism; nonconserved currents can be made conserved if one adds Goldstone bosons to the Lagrangian. Equation (2.39) and its implications will be discussed later.

We now discuss two distinct cases depending on the mass of the Higgs scalar meson.

1. $m_\varphi{}^2 = 2\lambda v^2 = \infty$

In our simple Abelian model renormalizability is still maintained in this limit. In this limit the Lagrangian in Eq. (2.31) becomes identical to that of the ordinary vector-gluon model except for the interaction

$$(\hat{e}^2/G)\bar{\mu}\gamma^\mu \hat{B}_\mu \mu. \tag{2.41}$$

This term simulates vector-meson dominance.[10] This can be seen by considering the lowest-order process shown in Fig. 1. The amplitude for Fig. 1. is given by

$$-i\hat{e}^2 j_\mu\left(\frac{g^{\mu\nu}}{q^2} - \frac{g^{\mu\nu} - q^\mu q^\nu/M^2}{q^2-M^2}\right)J_\nu = i\hat{e}^2 j_\mu \frac{1}{q^2}\frac{M^2}{q^2-M^2}J^\mu, \tag{2.42}$$

where

$$\begin{aligned} &j_\mu = \bar{\mu}\gamma_\mu\mu, \\ &J_\mu = \text{hadron current in the ordinary vector-gluon model.} \end{aligned} \tag{2.43}$$

It is also easy to see that Eq. (2.42) is exact up to all orders in G and to second order in $\hat{e}$. Therefore the "scaling function" in the present model is given by

$$\begin{aligned} W_1(\omega) &= \left(\frac{M^2}{q^2-M^2}\right)^2 \tilde{W}_1(\omega), \\ \nu W_2(\omega) &= \left(\frac{M^2}{q^2-M^2}\right)^2 \nu\tilde{W}_2(\omega), \end{aligned} \tag{2.44}$$

where $\tilde{W}_1$ and $\nu\tilde{W}_2$ are given by the ordinary vector-gluon model. A perturbative study of the vector-gluon model indicates that $\tilde{W}_1$ and $\nu\tilde{W}_2$ scales up to logarithmic factors.[15] Equation (2.44) therefore indicates that W_1 and νW_2 vanish in the scaling

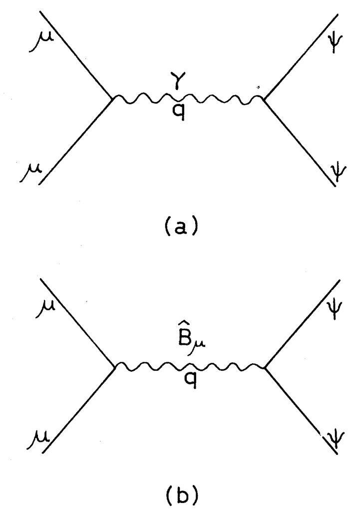

FIG. 1. The lowest-order diagrams for lepton-hadron scattering.

limit, in accordance with the Callan-Gross relation. These equations also suggest that we can obtain scaling laws if we have vector mesons with mass (a sort of Yukawa relation):

$$M_V{}^2 \sim (-q^2) \sim \frac{1}{\vec{r}^2}\,. \tag{2.45}$$

This is the basic assumption of generalized vector-meson dominance.[19] The physical meaning of this is that the hadronic system has no scale by itself; its scale is determined by the scale of the detector ($\sim q^2$). In this case the ratio R is similar to that in the ordinary vector-gluon model. It is therefore expected to be rather small. This conclusion is not, however, maintained if one goes to the non-Abelian model. Because of the highly nonlinear self-coupling of the Yang-Mills fields, the algebraic structure is greatly modified. The longitudinal component of the scaling functions is enhanced by this nonlinear self-coupling. In this case the ratio R becomes large. This can be remedied to some extent by incorporating more vector fields with large masses. In any case, the infinitely many vector-meson excitation operators in the dual resonance model[20] indicate that we may, after all, have to introduce infinitely many "elementary" vector excitation operators into the model. This procedure seems to be necessary even if the single vector field already incorporates certain effects of higher vector states, as one can easily see by writing a spectral representation for the vector field.

Equations (2.44) also suggests that the large ratio R is not an inherent property of the vector-meson dominance model. It is rather the property of the detailed coupling scheme in the field-theoretical model (see, e.g., Ref. 21), and it is also the property of analyticity in the virtual photon amplitude in the dispersive approach. This analyticity assumption in the dispersive approach combined with the simplest implementation of the gauge condition gives rise to the large ratio R independently of the algebraic structure of the model.[19] A different analyticity assumption therefore gives rise to a different ratio R. This problem of the gauge condition also appears explicitly when we include the effects of the Higgs scalar mesons. We now turn to this problem.

2. *Finite* $m_\varphi{}^2$

For finite $m_\varphi{}^2$, Eq. (2.29) indicates that the ratio R is expected to be large if $F_L(\omega)$ scales. The direct exchange of the vector meson between the quark and the lepton is still suppressed by a factor $M^2/(q^2-M^2)$ as in Eq. (2.44). The lowest-order term which survives in the scaling limit is

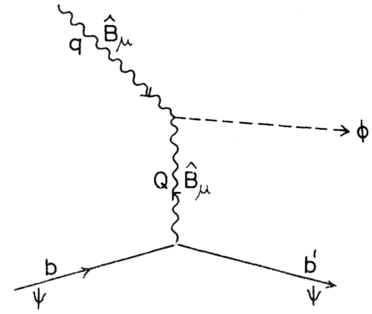

FIG. 2. The lowest-order nonvanishing diagram for the deep-inelastic process.

given by the diagram in Fig. 2. The amplitude for this diagram is given by

$$T_\mu = iG\bar{u}(b')\gamma_\alpha u(b)(-i)\frac{g^{\alpha\beta}-Q^\alpha Q^\beta/M^2}{Q^2-M^2}\,(2i\hat{g}M)g_{\beta\mu}\,. \tag{2.46}$$

After a simple calculation we get

$$W_1 = 0,$$
$$\nu W_2 \approx \frac{(2G\hat{g})^2}{4\pi}\,\frac{\omega-1}{\omega}\,. \tag{2.47}$$

The lowest-order contribution thus exactly satisfies the Callan-Gross relation. It should be noted that we have no logarithmic factors which violate scaling laws. This is due to the fact that the current operator which appears in Fig. 2 corresponds to the second term in [see Eq. (2.10)]

$$J_\mu = \frac{1}{\hat{g}}\,[M^2 + 2\hat{g}M\varphi(x) + \hat{g}^2\varphi(x)^2]\,V_\mu(x). \tag{2.48}$$

The second term has scale dimension 2 instead of 3, which is necessary to get scaling based on a naive dimensional argument. (We use the term "scale dimension" in a very naive way. The dimension[22] of the time component is actually more complicated.) More about this will be discussed later.

We next examine the contribution from the last term in Eq. (2.48), which has scale dimension 3. The lowest-order process which depends on this coupling is represented by diagrams in Fig. 3. Figures 3(a)–3(c) are similar to those of pion-pair creation via two photon exchange in electron-electron scattering. The diagram in Fig. 3(d) is a special diagram for the Higgs mechanism. The entire amplitude for Figs. 3(a)–3(d) is given by

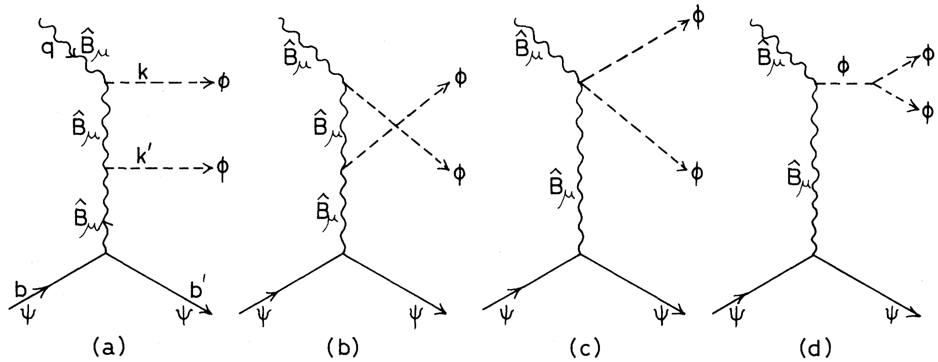

FIG. 3. The lowest-order process which depends on the current with scale dimension 3.

$$T_\mu \equiv (2i\hat{g}^2 G)\bar{u}(b')\gamma^\beta u(b)\frac{1}{Q^2-M^2}\left\{\frac{2M^2}{(k'-Q)^2-M^2}\left[g_{\mu\beta}-\frac{(k'-Q)_\mu(k'-Q)_\beta}{M^2}\right] + \frac{2M^2}{(k-Q)^2-M^2}\left[g_{\mu\beta}-\frac{(k-Q)_\mu(k-Q)_\beta}{M^2}\right]+\left[1+\frac{3m_\varphi^2}{(k+k')^2-m_\varphi^2}\right]g_{\mu\beta}\right\}. \tag{2.49}$$

This amplitude is considerably simplified if one takes the special choice $m_\varphi^2 = M^2$, the vector-meson mass. We perform the calculation for this choice in the following. In this case T_μ is written as

$$T_\mu = (-i\hat{g}^2 G)\bar{u}(b')\gamma^\beta u(b)\frac{1}{Q^2-M^2}\left\{\left[\frac{(q-2k)_\mu(2k'-Q)_\beta}{(k'-Q)^2-M^2}+\frac{(q-2k')_\mu(2k-Q)_\beta}{(k-Q)^2-M^2}-2g_{\mu\beta}\right] -2g_{\mu\beta}\left[\frac{3M^2}{(k+k')^2-M^2}+\frac{2M^2}{(k'-Q)^2-M^2}+\frac{2M^2}{(k-Q)^2-M^2}\right] +q_\mu\left[\frac{(2k'-Q)_\beta}{(k'-Q)^2-M^2}+\frac{(2k-Q)_\beta}{(k-Q)^2-M^2}\right]\right\}. \tag{2.50}$$

The first group in Eq. (2.50) is the same as the charged pion-pair creation amplitude. The remaining is the extra contribution. The last term in Eq. (2.50) gives a vanishing contribution to the scattering process when combined with the conserved leptonic current. It is, however, interesting to see that T_μ in Eq. (2.50) [and also T_μ in Eq. (2.46)] explicitly violates the gauge condition in the sense that

$$q^\mu T_\mu \neq 0. \tag{2.51}$$

However the amplitude T_μ lacks the photon pole when combined with the lepton current. Therefore the entire amplitude contains the following factor:

$$\frac{1}{q^2-M^2}=\frac{1}{q^2}\,\frac{q^2}{q^2-M^2}\,. \tag{2.52}$$

The extra factor $q^2/(q^2-M^2)$ can be included in the hadronic amplitude when we define structure functions. Thus the hadronic amplitude vanishes at $q^2=0$. In this sense the amplitude satisfies the gauge condition. In fact this is the way the gauge condition is implemented for the longitudinal component of the amplitude in the analytic approach of vector-meson dominance. In the present model Eq. (2.50) shows that a part of the transverse component also picks up this extra kinematical zero. The amplitude therefore shows rather rapidly changing behavior around $-q^2 \approx m_\varphi^2$. This effect may be observable above the threshold of Higgs scalar-meson pair creation.

The second group in Eq. (2.50) gives rise to the structure functions in the diffractive region ($\omega \gg 1$):

$$W_1 \approx 0,\qquad \nu W_2 \approx 4\,\frac{(\hat{g}^2 G)^2}{(2\pi)^3}\,\frac{\omega-1}{\omega}\,(\ln\omega+\text{const}). \tag{2.53}$$

The interference between the first group and the second group in Eq. (2.50) gives

$$W_1 \approx 0,\qquad \nu W_2 \approx (-)\,\frac{(\hat{g}^2 G)^2}{(2\pi)^3}\,\frac{\omega-1}{\omega}\,. \tag{2.54}$$

The Callan-Gross relation is satisfied by those parts of the amplitude. Finally the first group in Eq. (2.50) gives rise to (for $\omega >> 1$)

$$W_1 \approx \frac{1}{3} \frac{(\hat{g}^2 G)^2}{(2\pi)^3} \omega \ln\left(\frac{-q^2}{M^2}\right),$$
$$\nu W_2 \approx \frac{1}{6} \frac{(\hat{g}^2 G)^2}{(2\pi)^3} \left[\ln\left(\frac{-q^2}{M^2}\right)\right]^2. \tag{2.55}$$

This result shows that the ratio R becomes large at least logarithmically. In this sense the Callan-Gross relation is still satisfied. Equation (2.55) can be compared with the scaling functions of muon-pair creation in the $\gamma - \gamma$ collision.[23]

When m_φ^2 is large compared with M^2, our conclusion is still expected to be valid. Equation (2.53)–Eq. (2.55) thus can be regarded as a check of the Callan-Gross relation for the field algebra and the current-field identity. The perturbative treatment of the ordinary vector-gluon model[15] suggests that the property of the lower-order diagrams is maintained when one sums the leading terms up to all orders. Therefore the ratio R is still expected to be large in the present simple vector-dominance model. At this point we emphasize that the main content of field algebra would have been missed if one had looked at only the lowest-order diagram.

The present calculation shows that renormalizability and the enhancement of R are independent properties. It also suggests that the cancellation of the longitudinal component of the scaling functions in the spontaneously broken gauge theory is highly unlikely. In the following we discuss briefly this problem based on the non-Abelian model given in the Appendix. It is not difficult to see that the diagrams in Fig. 2 and Fig. 3 are also present in this non-Abelian model. The different final states in the deep-inelastic process do not interfere with each other. Certain final states with Higgs scalars already strongly enhance the longitudinal component relative to the transverse component as we see in Eq. (2.47) and Eq. (2.55). In particular, the enhancement of the longitudinal component by $[\ln(-q^2/M^2)]^2$ factor as indicated in Eq. (2.55) is the maximum enhancement we expect from the diagrams with the structure in Fig. 3. It is, therefore, not sufficient to suppress the longitudinal component in the non-Abelian model to get the finite R, but rather we have to enhance the transverse component. It is, however, rather unlikely that we can enhance the transverse component by just adding scalar mesons to the Lagrangian, although it is known that certain longitudinal components are suppressed by the scalar mesons. A naive algebraic manipulation indicates this also [see Eq. (A12)]. The ratio R therefore seems to stay large in the spontaneously broken gauge theory at least in perturbative calculations, where renormalizability is best defined. We also note that Eq. (2.55) can be removed if the mass m_φ^2 is taken at sufficiently large values. But we cannot regulate the enhanced longitudinal component in the "bare" non-Abelian Yang-Mills coupling in this limit of large m_φ^2.

D. Electron-Positron Annihilation

Another characteristic prediction of the ordinary field algebra is found in the electron-positron annihilation process. As is well known, the ordinary field algebra predicts[24]

$$\sigma < \frac{1}{(q^2)^2}\,. \tag{2.56}$$

This is in sharp contrast to the parton-model prediction[25,26]

$$\sigma \sim \frac{1}{q^2}\,. \tag{2.57}$$

The prediction (2.53) is greatly modified if one includes the scalar-meson "dressing" of the vector current. This has been discussed by Friedman *et al.*[21,27] They used the following hadronic current:

$$J_\mu = g_b [1 + b\varphi(x)]^2 V_\mu(x), \tag{2.58}$$

where $\varphi(x)$ is the dilaton field. It is amusing to see that the gauge theory automatically gives rise to the form indicated in Eq. (2.58) [see Eq. (2.48)]. Therefore the discussion given by Friedman *et al.* can also be applied to the present case. In particular, the term $2M\varphi(x)V_\mu(x)$ in Eq. (2.48) may be important in the medium energy if the Higg scalar has a rather small mass [this is the case if one identifies $\varphi(x)$ with the ϵ meson]. The contribution from this term to the annihilation process is given in Fig. 2 if one looks at the diagram from the t-channel direction (with some adjustment of the coupling constants). It is easy to see that this contribution again scales

$$\sigma \sim \frac{1}{q^2} \quad \text{for } e^+ + e^- \to B_\mu + \varphi. \tag{2.59}$$

The first term in Eq. (2.48) gives the t-channel version of Fig. 1 and it satisfies Eq. (2.56); $\sigma \sim 1/q^6$. The most singular term in Eq. (2.48) gives rise to the t-channel version of Fig. 3(a)–Fig. 3(d) in the annihilation process. After some calculation, one can confirm that these diagrams violate scaling and give the result

$$\sigma \sim \frac{1}{q^2}\left[\ln\left(\frac{q^2}{M^2}\right)\right]^2 \quad \text{for } e^+ + e^- \to B_\mu + \varphi + \varphi. \tag{2.60}$$

We note that there are only two parameters among three terms in Eq. (2.48) and Eq. (2.58); this is necessary to preserve "unitarity," namely, to prevent the total annihilation cross section from approaching a constant. Incidentally Eq. (2.50) exemplifies how unitarity is preserved in the spontaneously broken gauge theory. The amplitude can be split into two parts; one of them is conserved and the other nonconserved piece picks up a mass factor in the numerator. Thus the "bad" behavior of the vector-meson projection operator is regulated. A similar example has been recently discussed by Schechter and Ueda.[28] Their example, however, depends only on the Yang-Mills coupling. The present example is more intrinsic to the spontaneously broken gauge theory.

In the present approach the scaling behavior and the scale dimension of current operators are virtually independent. The current operator with scale dimension 2 is sufficient to ensure scaling. In the ordinary parton model the cutoff procedure (or softening)[26] is required to avoid logarithmic factors. This is also true in the present case if one uses the current with scale dimension 3. The only difference is that the scaling law in the annihilation process is already violated at the lowest-order tree level in the present approach if one uses the current with scale dimension 3; in the naive muon-parton analogy,[25] scaling in the annihilation process is exact. It is however interesting to observe that the softening procedure in the parton model[26] could also be understood as a reduction of the scale dimension of the currents involved.

We also note that the correspondence between the deep-inelastic process and the *total* annihilation cross sections in Eqs. (2.47), (2.55), (2.59), and (2.60) is one of the characteristic features of the vector-dominance approach.[19]

E. Schwinger-Term Sum Rules

The scaling behavior in Eq. (2.57) is fairly independent of details of the underlying dynamics. In fact it can be regarded as a consequence of the structure of the Schwinger term. It is well known that the annihilation cross section is related to the Schwinger term.[25,29,30] More specifically,

$$\langle 0|[J^0(0,\vec{x}),J^i(0)]|0\rangle = i\partial^i\delta(\vec{x})\frac{1}{32\pi^2\alpha^2}\times\int_0^\infty dq^2\, q^2\sigma(q^2). \qquad (2.61)$$

Unfortunately both sides of Eq. (2.61) diverge. Nevertheless a plausible argument was given for the use of Eq. (2.61) to infer the asymptotic behavior of $\sigma(q^2)$. An interesting argument is found in Gribov *et al.*[30] Their conclusion is that the elementary charged fermion or scalar field gives rise to the scaling law in Eq. (2.57). The vector-dominance model based on gauge theories gives rise to the Schwinger term in (2.16); this still remains true for more complicated realistic models. A detailed discussion of this is found in Bars *et al.*[10] (See also the Appendix.) Equation (2.16) is almost identical to the Schwinger term based on the elementary scalar fields. Following the argument given by Gribov *et al.*[30] we therefore expect the relation (2.57). On the other hand, the ordinary field algebra gives a c-number Schwinger term and we get Eq. (2.56). This can be also seen by taking the limit $m_\varphi^2\to\infty$ in our simple model. The annihilation process is a t-channel version of Eq. (2.42). It should be noted, however, that a perturbative treatment of the ordinary massive non-Abelian Yang-Mills theory does not necessarily give rise to Eq. (2.56) because unitarity is not guaranteed.

We next briefly comment on the Jackiw–Van Royen–West[31] sum rule in the deep-inelastic process. They suggest the following sum rule:

$$\langle p|[J^0(0,\vec{x}),J^i(0)]|p\rangle = \frac{i}{2\pi}\partial^i\delta(\vec{x})\times\int_1^\infty d\omega F_L(\omega). \qquad (2.62)$$

Unfortunately this result is also highly divergent in the gauge theory. Our conclusion in Eq. (2.55) suggests that the right-hand side diverges linearly in ω. This is, in some sense, consistent with the argument similar to that given by Gribov *et al.* in the case of electron-positron annihilation. In any case, Eq. (2.62) is not valid if the ratio R stays constant as the presently available data suggest.[32] It is interesting to recall that the c-number Schwinger term given by the ordinary algebra of field gives rise to a vanishing result for the left-hand side of Eq. (2.62). On the other hand, the scalar part of the structure function in this case is most likely constant (or divergent). The renormalizability of the massive vector-meson theory based on the Higgs mechanism therefore makes this Schwinger-term sum rule more sensible. [Or it might be that Eq. (2.62) loses its meaning when the right-hand side diverges, as suggested by Jackiw *et al.*[31]]

III. DISCUSSION AND CONCLUSION

A. Gauge Condition

In Section II A, we found that the gauge condition in the ordinary algebraic sense,

$$q_\mu J^\mu = 0, \qquad (3.1)$$

is violated in the perturbative calculation. However, we found that the amplitude which violates Eq. (3.1) has a kinematical zero at $q^2=0$ to ensure the absence of the scalar ghost. This property combined with the zero-slope limit of the dual resonance model[13] suggests that the gauge condition in the dual resonance model might be implemented by a kinematical zero rather than by strictly imposing the condition in Eq. (3.1). Although this is just a wild speculation, it would be useful to keep this alternative possibility of the gauge condition in mind when one discusses the dual current amplitude.

In the following we briefly note that the current operator given by (2.10) finds a very close counterpart in the dual current amplitude discussed by Nambu[33] and also by Manassah and Matsuda.[34] The Nambu current operator is in its simplest form

$$\langle m|J_\mu|n\rangle = \left\langle m\left|\tfrac{1}{2}\left\{\frac{d\overline{Z}_\mu}{d\tau}, e^{iq\cdot\overline{Z}}\right\}\right|n\right\rangle, \tag{3.2}$$

where

$$Z_\mu(\tau,\sigma)=Z_{\mu,0}(\tau)+\sum_{n=1}^{\infty}2Z_{\mu,n}(\tau)\cos n\sigma \tag{3.3}$$

and

$$Z_{\mu,n}(\tau)=\left(\frac{1}{2n}\right)^{1/2}(a_{\mu,n}e^{-in\tau}+a_{\mu,n}^{\dagger}e^{in\tau}). \tag{3.4}$$

$|m\rangle$ and $|n\rangle$ are arbitrary states in the dual model.[20] The operators $a_{\mu,n}$ are the ordinary vector excitation operators in the dual model. The bar over Z_μ in Eq. (3.2) indicates that we regulate the divergence induced by the straightforward application of Eq. (3.3). Equation (3.2) is essentially the ρ-meson dominance of the hadron current as one can see it by looking at the symmetric Sciuto vertex[20] at the ρ-meson pole position. Manassah and Matsuda[34] modified Eq. (3.2) as follows:

$$\begin{aligned}\overline{Z}_\mu &\equiv \int_0^\pi \rho(\sigma,\zeta)Z_\mu(\tau,\sigma)\,d\sigma\\ &=Z_{\mu,0}(\tau)+2\sum_{n=1}^{\infty}\rho_n(\zeta)Z_{\mu,n}(\tau),\end{aligned} \tag{3.5}$$

with

$$\rho_n(\zeta)=e^{-n\zeta^2/2},\quad n=0,1,2\ldots,$$

where $\zeta(\tau)$ stands for the coordinate of the scalar harmonic excitations. Equation (3.5) is very close to the structure of the current in the gauge theory. It is actually closer to the dilaton form discussed by Friedman *et al.*[21] The deep-inelastic process based on Eq. (3.5) has been worked out by Manassah and Matsuda. They found a scaling of νW_2, but the ratio R turned out to be large. It is interesting to see that a scalar meson dressing of the vector-field operator in the dual amplitude gives rise to a result very similar to ours. On the other hand, Friedman *et al.*[21] derived the same result as that of the ordinary quark algebra[25,26] by modifying the interactions between the vector mesons and scalar mesons. (An arbitrary modification of interactions does not guarantee unitarity, even in the tree approximation, for all physical processes, however.) This may indicate that the dual dynamics is closer to the Yang-Mills coupling.

B. Extension of the Model

We would like to briefly discuss the extension of our model in Eq. (2.1) to include more than one vector meson. One of the simplest generalizations is to take the following form:

$$\begin{aligned}\mathcal{L}=&\bar\mu i\gamma^\mu(\partial_\mu-ieA_\mu)\mu-m_\mu\bar\mu\mu+\bar\psi i\gamma^\mu(\partial_\mu-igB_\mu)\psi-m\bar\psi\psi\\ &+|[\partial_\mu-iG_1(\sin\theta\cos\varphi A_\mu+\cos\theta\cos\varphi B_\mu-\sin\varphi V_\mu)]\phi_1(x)|^2-\lambda_1(|\phi_1|^2-\tfrac{1}{2}v_1^2)^2\\ &+|[\partial_\mu-iG_2(\sin\theta\sin\varphi A_\mu+\cos\theta\sin\varphi B_\mu+\cos\varphi V_\mu)]\phi_2(x)|^2-\lambda_2(|\phi_2|^2-\tfrac{1}{2}v_2^2)^2\\ &-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\tfrac{1}{4}(\partial_\mu V_\nu-\partial_\nu V_\mu)^2\end{aligned} \tag{3.6}$$

and

$$G_1\sin\theta\cos\varphi=G_2\sin\theta\sin\varphi\equiv e,$$
$$G_1\cos\theta\cos\varphi=G_2\cos\theta\sin\varphi\equiv g,$$

where θ and φ are the "mixing angles." To lowest order in the coupling constants, Eq. (3.6) gives rise to an expression similar to the diagonal approximation in the generalized vector-dominance model.[19] Although we can satisfy a "current field identity" and modify the high-energy behavior of the current by this Lagrangian, the lack of the universality of the V-meson charge in this model makes it rather *ad hoc* at best. In any case, it is very difficult to incorporate more than one vector meson, in particular, in non-Abelian models.

C. Conclusion

In this note we discussed some of the high-energy properties of the field theoretical version of the vector-dominance model. We found that the renormalizable field theory gives rise to results more consistent with naive algebraic manipulations. The ratio R in the deep-inelastic process, in particular, seems to stay large at least in perturbative calculations. On the other hand, the total electron-positron annihilation cross section becomes more in line with the ordinary parton-model predictions. The difficulty related to the ratio R can be resolved if one succeeds in replacing the Higgs scalar field with a bilinear combination of fermion fields.

The electromagnetic mass difference is another interesting example of the test of algebraic relations.[9,25] One can, however, easily see that the lowest-order calculation of the mass difference does not constitute a complete test of the underlying algebra. A more detailed discussion of this problem will be given elsewhere. We, however, note that the finite mass difference given by the gauge theory may not be reliable if the off-shell behavior of the current amplitude contradicts experimental indications.

Another problem of the vector-dominance approach is that of whether we should attach the "correct" vector-dominant form factor to the quarks. The model by Bars *et al.*[10] should also be examined from this point. Unless one is sure that the binding force of the physical proton preserves the electromagnetic properties of each individual quark, the form factor of the physical nucleon might well be quite different from that of constituent quarks.

In conclusion the virtue of the renormalizability of the hadronic Lagrangian is still to be examined, especially when one does not trust the perturbative approach to hadron physics.

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APPENDIX

In this Appendix we briefly discuss the algebra of fields based on the simplest non-Abelian model discussed by 't Hooft.[7] This Appendix exemplifies what kind of properties of the Abelian model are modified and which aspects of the Abelian model are preserved in the non-Abelian model. See also Ref. 9 and Ref. 10.

We start with the Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu - g\vec{B}_\mu \times \vec{B}_\nu)^2 \\ &- \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ &+ (\nabla_\mu K)(\nabla^\mu K)^* - \tfrac{1}{2}\lambda(K^*K - \tfrac{1}{2}v^2)^2 \\ &+ \bar{\mu} i\gamma^\mu(\partial_\mu - ieA_\mu)\mu - m_\mu \bar{\mu}\mu \\ &+ \bar{\psi} i\gamma^\mu(\partial_\mu - ig\vec{B}_\mu \cdot \vec{T})\psi - m\bar{\psi}\psi, \end{aligned} \tag{A1}$$

where we added a lepton singlet and a fermion triplet to the Lagrangian:

$$\psi = \begin{pmatrix} p_1(x) \\ p_2(x) \\ p_3(x) \end{pmatrix} \text{ and } T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \text{ etc.} \tag{A2}$$

Note that ψ has nothing to do with the real hadrons. (We can add a fermion doublet as well if we allow nonintegral charges.)

We also defined

$$\begin{aligned}&\nabla_\mu = \partial_\mu - ig\vec{B}_\mu \cdot \vec{\sigma} - i\tfrac{1}{2}eA_\mu, \\ &K = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix},\end{aligned} \tag{A3}$$

with K_1 and K_2 complex scalar fields.

In the following we work in the unitary gauge

$$K(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi(x) \\ 0 \end{pmatrix}, \tag{A4}$$

where $\phi(x)$ is a real scalar field.

After the spontaneous breakdown of the vacuum symmetry, we have

$$\phi(x) = v + \varphi(x), \tag{A5}$$

where v is a constant. The gauge-compensating term is given by

$$S_{\text{g.c.}} = (-i)\, 2\delta^4(0) \int d^4x \ln\left[1 + \frac{\varphi(x)}{v}\right]. \tag{A6}$$

Actually the unitary gauge is only well defined after the spontaneous breaking; the gauge compensating term is ill defined for $v=0$. The advantage of the unitary gauge lies in the fact that the gauge compensating term does not depend on gauge fields nor on the time derivative of the Higgs scalar field. Because of this property we work out the algebraic relations in the unitary gauge, although the Green's functions are not defined in this gauge.

The hadron currents are now given by

$$J^a_\mu(x) = -\frac{1}{g}(\delta^{ac}\partial^\rho - g\epsilon^{abc}B^{b\rho})(\partial_\rho B^c_\mu - \partial_\mu B^c_\rho - g\epsilon^{cb'c'}B^{b'}_\rho B^{c'}_\mu) - \bar{\psi}\gamma_\mu T^a\psi \tag{A7}$$

$$= \frac{1}{g}\left[\frac{g\phi(x)}{2}\right]^2 V^a_\mu(x). \tag{A8}$$

Algebraic relations are given by

$$[J_0^a(x),\ J_0^b(y)]_{x^0=y^0}=i\epsilon^{abc}J_0^c(x)\,\delta^3(\vec{x}-\vec{y}), \tag{A9}$$

$$[J_0^a(x),\ J_j^b(y)]_{x^0=y^0}=i\epsilon^{abc}J_j^c(x)\,\delta^3(\vec{x}-\vec{y})+i\frac{1}{g^2}\left[\frac{g\phi(y)}{2}\right]^2\delta_{ab}\partial_j\delta^3(\vec{x}-\vec{y})+\text{QED corrections.} \tag{A10}$$

The Schwinger term depends on the Higgs scalar field.[10] The space-space commutator with a time derivative becomes rather complicated:

$$\begin{aligned}[\partial_0 J_j^a(x),\ J_k^b(y)]\,\delta(x^0-y^0)=&-iM(x)V_j^a(x)V_k^b(y)\delta(x-y)-\frac{i}{g^2}M(x)M(y)\,\delta_{ij}\,\delta_{ab}\,\delta(x-y)\\ &+\frac{i}{g^2}\frac{M(y)}{M(x)}\partial_j M(x)[g\epsilon^{abc}V_k^c(x)\,\delta(x-y)+\delta_{ab}\,\partial_k\,\delta(x-y)]\\ &+\frac{i}{g^2}M(y)[-g^2\epsilon^{ab'c}\epsilon^{bc'c}V_j^{b'}(x)V_k^{c'}(x)\,\delta(x-y)\\ &\quad+\delta_{ab}\,\partial_j\,\partial_k\,\delta(x-y)+g\epsilon^{abc}\partial_j V_k^c(x)\,\delta(x-y)\\ &\quad+g\epsilon^{abc}V_k^c(x)\,\partial_j\,\delta(x-y)+g\epsilon^{abc}V_j^c(x)\,\partial_k\,\delta(x-y)]+\text{QED corrections,}\end{aligned} \tag{A11}$$

with $M(x)\equiv[g\phi(x)/2]^2$. Finally we would like to make a comment on the algebraic relations, Eq. (2.17) and Eq. (A11). These equations have the form

$$[\partial_0 J_j(x),\ J_k(y)]\,\delta(x^0-y^0)=-i[2g\phi(x)]^2V_j(x)V_k(x)\,\delta^4(x-y)-i[g\phi(x)^2]^2\,\delta_{kj}\,\delta^4(x-y)+\text{Schwinger terms} \tag{2.17$'$}$$

and

$$\begin{aligned}[\partial_0 J_j{}^3(x),\ J_k{}^3(y)]\,\delta(x^0-y^0)=&-i\left[\frac{g\phi(x)}{2}\right]^2\sum_a V_j^a(x)V_k^a(x)\,\delta^4(x-y)\\ &-i\left[\frac{g\phi(x)^2}{4}\right]^2\delta_{kj}\,\delta^4(x-y)+\text{Schwinger terms.}\end{aligned} \tag{A12}$$

If one looks at the component $j=k$, the first two terms in these equations are, in a naive sense, positive semidefinite (at least for compact groups). Therefore if we can neglect the Schwinger terms or if they are c-numbers, these equations show that the Higgs scalar meson and the vector mesons do not cancel each other. They rather add up to give an isoscalar contribution.[9]

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[1]Y. Nambu, Phys. Rev. 106, 1366 (1957); W. R. Frazer and J. R. Fulco, *ibid.* 117, 1603 (1960); J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

[2]N. M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. 157, 1376 (1967). This article contains a detailed list of references.

[3]T. D. Lee, S. Weinberg, and Bruno Zumino, Phys. Rev. Lett. 18, 1029 (1967).

[4]T. D. Lee and Bruno Zumino, Phys. Rev. 163, 1667 (1967).

[5]P. Higgs, Phys. Rev. 145, 1156 (1966); T. W. B. Kibble, *ibid.* 155, 1554 (1967).

[6]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967) and 27, 1688 (1971). A. Salam, *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8); edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[7]G.'t Hooft, Nucl. Phys. B35, 167 (1971); B. W. Lee, Phys. Rev. D 5, 823 (1972); B. W. Lee and J. Zinn-Justin, *ibid.* 5, 3121 (1972); 5, 3137 (1972); 5, 3155 (1972).

[8]K. Bardakci and M. B. Halpern, Phys. Rev. D 6, 696 (1972).

[9]K. Bardakci, Nucl. Phys. B51, 174 (1972).

[10]I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. Lett. 29, 969 (1972); Phys. Rev. D 7, 1233 (1973).

[11]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972). See also 't Hooft, Ref. 7.

[12]P. Langacker and M. Suzuki, Phys. Lett. 40B, 561 (1972); Phys. Rev. D 7, 273 (1973); K. Fujikawa and P. J. O'Donnell, Phys. Rev. D 8, 3200 (1973).

[13]J.-L. Gervais and A. Neveu, Nucl. Phys. B46, 381 (1972), and references therein.

[14]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-

796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U. S. S. R., 1970.

[15]V. N. Gribov and L. N. Lipatov, Phys. Lett. 37B, 78 (1971); P. M. Fishbane and J. D. Sullivan, Phys. Rev. D 4, 251 (1971); 6, 645 (1972), and references therein; C. Nash, Nucl. Phys. (to be published). See also S. Adler and W.-K. Tung, Phys. Rev. Lett. 22, 978 (1969); R. Jackiw and G. Preparata, *ibid.* 22, 975 (1969). Cf. D. Capper, Phys. Rev. D 4, 3777 (1971).

[16]T. Goto and T. Imamura, Prog. Theor. Phys. (Kyoto) 14, 396 (1955); J. Schwinger, Phys. Rev. Lett. 3, 296 (1959).

[17]C. G. Callan, Jr. and David J. Gross, Phys. Rev. Lett. 22, 156 (1969).

[18]See, e.g., S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[19]K. Fujikawa, Phys. Rev. D 4, 2794 (1971); J. J. Sakurai and D. Schildknecht, Phys. Lett. 40B, 121 (1972); 41B, 489 (1972). A. Bramon, E. Etim, and M. Greco, *ibid.* 41B, 609 (1972).

[20]A review of the dual resonance model is found in S. Mandelstam, *Lectures on Elementary Particles and Quantum Field Theory,* edited by S. Deser *et al.* (M.I.T., Cambridge, 1970).

[21]M. H. Friedman, P. Nath, and R. Arnowitt, Phys. Lett. 42B, 361 (1972); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. D 7, 1197 (1973).

[22]M. A. B. Bég, J. Bernstein, David J. Gross, R. Jackiw, and A. Sirlin, Phys. Rev. Lett. 25, 1231 (1970). I am grateful to Professor P. J. O'Donnell for a helpful discussion on this problem.

[23]K. Fujikawa, Nuovo Cimento 12A, 83 (1972), and references therein.

[24]J. Dooher, Phys. Rev. Lett. 19, 600 (1967).

[25]J. D. Bjorken, Phys. Rev. 148, 1467 (1966); J. D. Bjorken and E. Paschos, *ibid.* 185, 1975 (1969).

[26]A review of the parton model is found in P. V. Landshoff and J. C. Polkinghorne, Phys. Rep. 5C, 1 (1972).

[27]See also, J. Ellis, Nucl. Phys. B22, 478 (1970).

[28]J. Schechter and Y. Ueda, Phys. Rev. D 7, 3119 (1973).

[29]K. Johnson, Nucl. Phys. 25, 431 (1961); S. Okubo, Nuovo Cimento 44A, 1015 (1966).

[30]V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Phys. Lett. 24B, 554 (1967).

[31]Roman Jackiw, Roger Van Royen, and Geoffrey B. West, Phys. Rev. D 2, 2473 (1970).

[32]J. J. Sakurai, Phys. Rev. Lett. 30, 244 (1973).

[33]Y. Nambu, Phys. Rev. D 4, 1193 (1971).

[34]Jamal T. Manassah and Satoshi Matsuda, Phys. Rev. D 4, 3062 (1971).