Nonleptonic Weak Interactions in Unified Gauge Theories*

V. S. Mathur and H. C. Yen
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 19 July 1973; revised manuscript received 30 August 1973)

Weinberg has shown in general unified gauge theories that if (1) the strong interactions are described by a neutral-vector-gluon model, and (2) all quark masses are much smaller than all intermediate-massive-vector-boson masses, then the order-α effects of weak and electromagnetic corrections to the strong-interaction symmetries are just the conventional electromagnetic corrections plus corrections to the quark mass matrix which preserve parity, strangeness, charm, etc. In this paper we use his method to further show that if quark masses are also much smaller than Higgs scalar-boson masses, and some technical conditions stated in the text are satisfied, then to order $G_F m^2$ (m = a typical quark mass), only a certain part of vector-boson exchanges induces the dominant contribution to "proper" nonleptonic weak interactions which violate parity or strangeness or charm, etc., while the remaining part of vector-boson exchanges, all of scalar-boson exchanges, and all of tadpole diagrams can only produce corrections to the quark mass matrix, which preserve the quantum numbers of strong interactions. We also offer speculation on possible mechanisms to obtain the $\Delta I = \frac{1}{2}$ rule of nonleptonic decays.

I. INTRODUCTION

The problem of nonleptonic weak interactions in renormalizable gauge theories has been discussed by many people,[1] most of whom have tried to obtain or incorporate the $\Delta I = \frac{1}{2}$ rule into various specific models without, however, taking the strong-interaction dynamics into consideration. In this paper we use Weinberg's recent method[2] to study this problem, assuming for convenience the strong interactions to be described by a neutral-vector-gluon model. Our purpose is to investigate nonleptonic weak interactions in a gauge-model-independent way, and with the strong interactions fully taken into account. We are able to show that if all quark masses are much smaller than all intermediate-heavy-vector-boson masses and all Higgs scalar-boson masses and certain other conditions discussed in Sec. II are satisfied, then the lowest-order "proper" nonleptonic weak interactions can be induced to a reasonable approximation only from vector-boson exchanges in unified gauge theories. In such a case vector-boson exchanges would, of course, be primarily responsible for the $\Delta I = \frac{1}{2}$ property of nonleptonic decays.

However, if quark masses are much smaller than intermediate-vector-boson masses, but comparable to or even larger than scalar-boson masses, then both vector-boson exchanges and scalar-boson exchanges can contribute to the lowest-order nonleptonic decays, and in general it is not clear that either of the two would dominate over the other. In this connection, Lee and Treiman[1] have given in the Georgi-Glashow 8-quark model a possible explanation of the $\Delta I = \frac{1}{2}$ rule, based on the assumption that scalar-boson exchanges dominate over vector-boson exchanges for nonleptonic decays. They have argued that this assumption might be valid provided the Higgs scalar mass is small compared to some quark masses. Our analysis sheds no light on the validity of their argument, and the situation for this case seems rather inconclusive to us.

II. WEAK CORRECTIONS OF ORDER $G_F m^2$

Weinberg[2] has shown that the weak and electromagnetic corrections to the S matrix for a transition from a general hadron "in" state I to a general hadron "out" state F, to all orders in the gluon coupling but only to second order in the gauge coupling e, can be expressed as

$$\delta S_{FI} = \delta_{\rm em} S_{FI} + \delta_{A1'} S_{FI} + \delta_{\phi 1} S_{FI} + \delta_{A\phi} S_{FI} + \delta_{AT} S_{FI} + \delta_{T1} S_{FI} , \tag{1}$$

where

$$\delta_{\rm em} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \int d^4k\, F^{FI}_{\rm em}(k) [(k^2)^{-1} - (k^2+\Lambda^2)^{-1}] , \tag{2}$$

$$\delta_{A1'} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \int d^4k\, \eta^{\mu\nu} F^{FI}_{\alpha\mu,\beta\nu}(k) (k^2 + \mu'^2)^{-1}{}_{\alpha\beta} , \tag{3}$$

$$\delta_{\phi 1} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \int d^4k\, F^{FI}_{ij}(k) (k^2 + M^2)^{-1}{}_{ij} , \tag{4}$$

$$\delta_{A\phi}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j\int d^4k\,F^{FI}_{ij}(k)(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}\,, \tag{5}$$

$$\delta_{AT}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\,F^{FI}_i(\theta_\beta\theta_\alpha\lambda)_i\int d^4k\,(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}\,, \tag{6}$$

$$\delta_{TI}S_{FI}=-2i\,(2\pi)^8\delta^4(P_F-P_I)F^{FI}_iM^{-2}{}_{ij}\,\frac{\partial V_1(\lambda)}{\partial\lambda_j}\quad. \tag{7}$$

Here $F^{FI}_{\text{em}}(k)$ is the matrix element of the time-ordered product of two electromagnetic currents; $F^{FI}_{\alpha\mu,\beta\nu}(k)$ and $F^{FI}_{ij}(k)$ are the corresponding quantities for the hadronic vector and scalar currents, respectively; F^{FI}_i is the matrix element of a single scalar current; $V_1(\lambda)$ is the "one-loop potential" of the tadpole diagrams. (See Ref. 2 for their exact definitions and for other notations appearing in the above equations.)

We note that among the six terms in Eq. (1), $\delta_{\text{em}}S_{FI}$ and $\delta_{A\phi}S_{FI}$ are finite by themselves, but the rest are all divergent. However, Weinberg[2] has shown that the divergent part of each of the latter is of the same form as would be produced by adding G-invariant, gluon-gauge-invariant, Lorentz-invariant, Hermitian, and renormalizable terms to the original Lagrangian, and therefore such divergences can always be absorbed by a renormalization of the parameters in the original Lagrangian; they have no contribution to the corrections to "natural" zeroth-order symmetries of strong interactions. Since we are interested in only the corrections to such symmetries, we may remove the divergent part from each of the above terms in Eq. (1), so that each of them becomes a finite quantity of order e^2.

Now we will assume that in the present theory all quark masses[3] are much smaller than all intermediate-massive-vector-boson masses, so that we have another small parameter of the theory $(m/\mu_W)^2$ in addition to e^2. To analyze the contents of δS_{FI} we will dissect each term of Eq. (1) into pieces with different power dependence on $(m/\mu_W)^2$. Generally each piece would behave like $(m/\mu_W)^{2n}$ times a certain power of $\ln(m/\mu_W)^2$. We will in the following refer to such a piece simply as of nth order in $(m/\mu_W)^2$, with the understanding that it may also contain a factor of powers of $\ln(m/\mu_W)^2$. We proceed to classify all terms in Eq. (1) into those of zeroth order in $(m/\mu_W)^2$, those of first order in $(m/\mu_W)^2$, and those of higher orders in $(m/\mu_W)^2$. Terms of order $e^2(m/\mu_W)^0$ have the order of magnitude of electromagnetic corrections and will be denoted as "of order α" in the following; terms of order $e^2(m/\mu_W)^2$ have the order of magnitude of the conventional weak corrections and will be denoted as "of order G_Fm^2"; terms of higher orders in $(m/\mu_W)^2$ are much weaker than the conventional weak corrections and will be neglected because we are looking for the lowest-order weak interactions only. So we would like to pick out terms of order α and G_Fm^2 in each of the terms in Eq. (1).

Before we set out to do this term by term, we wish to make some general observations regarding the relevance of the asymptotic behavior of the matrix elements involved particularly in Eqs. (3), (4), and (5), which provide a useful way of sorting out terms of order α and of order G_Fm^2. For this purpose, consider as an example the type of integral occurring in the $A1'$ term, which we may write as

$$I=\int d^4k\,F(k^2)(k^2+\mu_W{}^2)^{-1}$$

with $F(k^2)$ behaving asymptotically as

$$F(k^2)\sim\frac{a(k)}{k^2}+\frac{b(k)}{k^4}+\frac{c(k)}{k^6}+\cdots,$$

where the numerators $a(k), b(k), c(k),\ldots$ are either constants or at most powers of $\ln k^2$. From Weinberg's "bridge" theorem,[2] the matrix element in Eq. (3) indeed possesses such an asymptotic behavior. Because of the leading term $a(k)/k^2$, the integral I has an ultraviolet divergence. As mentioned before the divergent part can be isolated and absorbed by suitable renormalization procecure, and shall not concern us here. The remaining finite part of I can be dissected into pieces with different power dependence on $(m/\mu_W)^2$:

$$I-I_\infty\equiv I_{\text{finite}}=I_0+I_1+I_2+\cdots,$$

where I_n contains all terms of order $(m/\mu_W)^{2n}$ times any power of $\ln(m^2/\mu_W{}^2)$. Thus I_0 contributes to the order-α term and I_1 to the order-G_Fm^2.

Now, it can easily be shown that I_0 receives contributions solely from the leading term $a(k)/k^2$ in the asymptotic expansion of $F(k^2)$. Therefore the order-α term can be extracted from the leading term in the asymptotic expansion of the matrix element. This is an important feature, since only the simplest of the bridges contribute to I_0 and therefore to the order-α term. As shown by Weinberg, these simple bridge contributions provide only a correction to the fermion mass matrix, and for the neutral-vector-gluon theory of strong interactions an immediate consequence is that parity, strangeness, charm, etc. are automatically conserved in order α. In deriving this result we would like to emphasize the role of the assumption $\mu_W >> m$, which leads to the fact that [apart from the em term in Eq. (2)] the order-α contributions arise only from the leading term in the asymptotic expansion of the matrix elements.

One might expect that to order $G_F m^2$, it would be sufficient to consider the next leading term in the asymptotic expansion of $F(k^2)$. However, as shown in the Appendix, I_1 receives contributions from not only the next leading term $b(k)/k^4$, but also all the other terms in the expansion.[4] It would then appear that the order-$G_F m^2$ term cannot be extracted by merely looking at the $b(k)/k^4$ term in the asymptotic expansion of $F(k^2)$. Fortunately, however, there exists a circumstance under which the relevance of the $b(k)/k^4$ term can be reasserted. The analysis given in the Appendix shows that I_1 consists of two parts:

$$I_1 = (m/\mu_W)^2[\text{powers of } \ln(m/\mu_W)^2] + (m/\mu_W)^2(\text{constant}),$$

where the first part receives contributions solely from the $b(k)/k^4$ term. The highest power of the logarithm in this term is $l+1$ if $b(k) \sim (\ln k^2)^l$. Now if $l \geq 0$ and if we assume that $(\mu_W/m)^2$ is sufficiently large so that powers of $\ln(m/\mu_W)^2$ in the first part of I_1 overwhelm the constant of the second part, I_1 may be well approximated by the contributions from the $b(k)/k^4$ term alone.[5] In principle, of course, l may be negative. We consider this unlikely, since specific calculations based on perturbation theory do not bear this out. Furthermore, arguments based on scaling suggest that $b(k)$ may in fact be a constant.[2] Above all, we feel there is a great deal of simplicity and appeal in the fact that the dominant contribution to order $G_F m^2$ is controlled by the next to leading asymptotic term in the expansion of $F(k^2)$, since as we shall see, the bridges that contribute to $b(k)$ are also reasonably simple and may well account for the $\Delta I = \frac{1}{2}$ rule. Accordingly, we will keep only the $1/k^4$ term in the asymptotic expansion of $F(k^2)$ to obtain terms of order $G_F m^2$. This we will do specifically for the $A1'$ term. The $A\phi$ term also requires a similar analysis. It is easy to see that in the rest of the terms, however, this approximation is not required. Note, in particular, that the $\phi 1$ term is already of order $G_F m^2$ and can be evaluated to this order by the leading asymptotic term in $F_{ij}^{FI}(k)$, assuming only that the ϕ mass M is much larger than the quark mass m, and not requiring the stronger assumption discussed above. For purposes of clarity, however, by order $G_F m^2$, we will hereafter refer to terms of this order containing only the *dominant* logarithmic factor.

We shall now discuss the various terms in Eq. (1) separately.

(a) em term. $\delta_{\text{em}} S_{FI}$ is finite and of order α. This is the conventional lowest-order electromagnetic corrections but with an ultraviolet cutoff Λ.

(b) A1′ term. It is easy to see from Eq. (3) that the terms of order α in the finite part of $\delta_{A1'} S_{FI}$ are determined by the leading asymptotic terms of $\eta^{\mu\nu} F_{\alpha\mu,\beta\nu}^{FI}(k)$ in the limit of large k. According to Weinberg's "bridge" theorem,[2] we have an asymptotic expansion of the following form[6]:

$$\int d\Omega_k \eta^{\mu\nu} F_{\alpha\mu,\beta\nu}^{FI}(k) \underset{K\to\infty}{\sim} \frac{1}{K^2}\langle F|Q_{\alpha\beta}^1(K)|I\rangle + \frac{1}{K^4}\langle F|Q_{\alpha\beta}^2(K)|I\rangle + O(1/K^6), \tag{8}$$

where K is the Euclidean magnitude of the four-vector k, and $Q_{\alpha\beta}^N(K)$ behave asymptotically at most like powers of $\ln K$. Weinberg[2] has found the leading asymptotic term to be

$$\frac{1}{K^2}\langle F|Q_{\alpha\beta}^1(K)|I\rangle = \langle F|[-\bar\psi U_{\alpha\beta}^{(1)}(K)\psi - \bar\psi\gamma^\mu U_{\alpha\beta}^{(2)}(K)(\partial_\mu + ig_B B_\mu)\psi - \tfrac{1}{4}U_{\alpha\beta}^{(3)}(K)G_{\mu\nu}G^{\mu\nu}]|I\rangle . \tag{9}$$

Thus the term of zeroth order in $(m/\mu_W)^2$ in the finite part of $\delta_{A1'} S_{FI}$ is given by

$$(2\pi)^4\delta^4(P_F - P_I)\int_0^\infty K\,dK\langle F|Q_{\alpha\beta}^1(K)|I\rangle \times[(K^2+\mu'^2)^{-1}{}_{\alpha\beta} - (K^2+\Lambda^2)^{-1}\delta_{\alpha\beta}], \tag{10}$$

where the divergent part of $\delta_{A1'} S_{FI}$ has been explicitly subtracted out (see Ref. 2 for details). The term in Eq. (10) is of order α, and as shown by Weinberg is just the same as a term in the Lagrangian of the form

$$\Delta_{A1'}\mathfrak{L} = -\bar\psi\,\delta_{A1} m\psi .$$

That is, the $A1'$ term gives rise to a correction $\delta_{A1} m$ of order α in the quark mass matrix m. Since one can always use a unitary transformation[7] on the quark fields to reduce the total quark mass matrix $m + \delta_{A1} m$ to a diagonal matrix free of γ_5 terms, without changing the remainder of the strong-interaction Lagrangian (which is described in terms of a neutral vector gluon in the present theory), the net effect of this order-α correction is just a pure quark mass shift—it preserves parity, strangeness, charm, etc. This is a key point of the analysis: In the present theory whenever a correction of the form $\Delta\mathcal{L} = -\overline{\psi}\delta m\psi$ arises, its only net effect on strong-interaction symmetries is a pure quark mass shift, which would not induce any violation of parity, strangeness, charm, etc.

Now consider the next leading term in $\delta_{A1'}S_{FI}$, which is given by

$$(2\pi)^4\delta^4(P_F-P_I)\int K^{-1}dK\langle F|Q^2_{\alpha\beta}(K)|I\rangle \times (K^2+\mu'^2)^{-1}{}_{\alpha\beta}\,. \qquad (11)$$

As discussed before this gives the dominant contribution to order $G_F m^2$, and can be shown *not* to be equivalent to a term of the form $\Delta\mathcal{L} = -\overline{\psi}\,\delta m\,\psi$ in the Lagrangian. To see this we observe that by Weinberg's "bridge" theorem $Q_{\alpha\beta}{}^2(K)$ must contain four-fermion interaction terms, among others, which clearly are not of the mass correction type. Thus Eq. (11) will give rise to some "proper" hadronic weak interactions which would in general violate parity, strangeness, and other quantum numbers of strong interactions. It turns out that this is the dominant term in Eq. (1) that can produce "proper" hadronic weak interactions to order $G_F m^2$.

(c) ϕ 1 term. The basic structure of the ϕ 1 term is quite similar to that of the $A1'$ term. However, since $F^{FI}_{ij}(k)$ of Eq. (4) contains the factor $\Gamma_i\Gamma_j$ and since $\Gamma \sim e(m/\mu_W)$ in magnitude,[2] it is easy to see that $F^{FI}_{ij}(k)$ is intrinsically smaller in magnitude than $F^{FI}_{\alpha\mu,\beta\nu}$ by a factor of $(m/\mu_W)^2$. This makes $\delta_{\phi 1}S_{FI}$ smaller than $\delta_{A1'}S_{FI}$ by the same factor. Again Weinberg's "bridge" theorem leads to the asymptotic expansion

$$\int d\Omega_k F^{FI}_{ij}(k) \underset{K\to\infty}{\sim} \frac{1}{K^2}\langle F|R^1_{ij}(K)|I\rangle + \frac{1}{K^4}\langle F|R^2_{ij}(K)|I\rangle + O(1/K^6), \qquad (12)$$

where $R^N_{ij}(K)$ behave asymptotically at most like powers of $\ln K$, and the leading term is given by[2]

$$\frac{1}{K^2}\langle F|R^1_{ij}(K)|I\rangle = \langle F|[-\overline{\psi}V^{(1)}_{ij}(K)\psi - \overline{\psi}\gamma^\mu V^{(2)}_{ij}(K)(\partial_\mu + ig_B B_\mu)\psi - \tfrac{1}{4}V^{(3)}_{ij}(K)G_{\mu\nu}G^{\mu\nu}]\,|I\rangle. \qquad (13)$$

Assuming $M >> m$, the term of the lowest order in $(m/M)^2$ in the finite part of $\delta_{\phi 1}S_{FI}$ is given by

$$(2\pi)^4\delta^4(P_F-P_I)\int_0^\infty K\,dK\langle F|R^1_{ij}(K)|I\rangle \times[(K^2+M^2)^{-1}{}_{ij} - (K^2)^{-1}\delta_{ij}]\,. \qquad (14)$$

Notice that we are taking here only the leading term in the expansion (12) because the contributions from the higher terms would be smaller than (14) by at least a factor of $(m/M)^2$, which is assumed to be very small in the present discussion. Without this last assumption (14) would not be a sensible approximation to the finite part of $\delta_{\phi 1}S_{FI}$. Equation (14) in itself is of order $G_F m^2$ because $R^1_{ij}(K)$ contains the factor $\Gamma_i\Gamma_j$. Identical arguments used by Weinberg in the discussion of the order-α term in $\delta_{A1'}S_{FI}$ can be used here to show that Eq. (14) gives rise to only a correction $\delta_{\phi 1}m$ of order $G_F m^2$ to the quark mass matrix. This means that to order $G_F m^2$ the $\phi 1$ term does not lead to any hadronic weak interactions that violate parity or strangeness conservation. As mentioned before, in the analysis of the $\phi 1$ term, we do not need the stronger assumption $[\ln (M/m)^2]^n >> 1$ $(n \geq 1)$.

(d) $A\phi$ term. This term is very similar to the ϕ 1 term, and a similar analysis shows that the leading term in $\delta_{A\phi}S_{FI}$ is given by

$$(2\pi)^4\delta^4(P_F-P_I)(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j \times\int_0^\infty K\,dK\langle F|R^1_{ij}(K)|I\rangle(K^2)^{-1}(K^2+\mu^2)^{-1}{}_{\alpha\beta}\,, \qquad (15)$$

which gives the dominant contribution to order $G_F m^2$ and leads only to a quark mass correction $\delta_{A\phi}m$.

(e) AT term. It is obvious from Eq. (6) that the finite part of the AT term gives rise to only a quark-mass correction $\delta_{AT}m$:

$$\delta_{AT}m = (16\pi^2)^{-1}\Gamma_i(\theta_\beta\theta_\alpha\lambda)_i \times\int_0^\infty K\,dK[(K^2+\mu^2)^{-1}{}_{\alpha\beta} - (K^2+\Lambda^2)^{-1}\delta_{\alpha\beta}]\,. \qquad (16)$$

Notice that this term is of order α exactly.

(f) $T1$ term. To all orders of $(m/\mu_W)^2$, the finite part of the $T1$ term gives rise to only a quark mass correction $\delta_{T1}m$:

$$\delta_{T1}m = -\Gamma_i M^{-2}{}_{ij}\frac{\partial}{\partial\lambda_j}[V_1(\lambda) - V_{1\infty}(\lambda)]\,. \qquad (17)$$

This term actually contains a piece of order α and a piece of order $G_F m^2$, but in any case the net effect of all these pieces is only a quark mass shift.

In summary, we see that the weak and electromagnetic corrections to S_{FI}, to second order in the gauge coupling e, are given by the following:

(1) To order α, δS_{FI} consists of the conventional electromagnetic corrections $\delta_{em}S_{FI}$ and weak corrections to the quark mass matrix resulting from the $A1'$, AT, and $T1$ terms, which preserve parity, strangeness, charm, etc.—this was the main result of Weinberg's paper.[2]

(2) To order $G_F m^2$, δS_{FI} consists of weak corrections to the quark mass matrix resulting from the $\phi 1$, $A\phi$, and $T1$ terms which preserve parity, strangeness, charm, etc., and some "proper" hadronic weak interactions resulting from the $A1'$ term which violate these quantum numbers.

Thus our conclusion is that if $[\ln(\mu_W/m)^2]^n >> 1$ for some $n \geq 1$, and $m << M$, then to order $G_F m^2$ "proper" nonleptonic weak interactions arise only from vector-boson exchanges (the $A1'$ term), and furthermore, they are given by the second leading term in the asymptotic expansion of $\eta^{\mu\nu} F^{FI}_{\alpha\mu,\beta\nu}(k)$ [see Eqs. (8) and (11)], provided $Q_{\alpha\beta}{}^2(K)$ behaves asymptotically as a non-negative power of $\ln K$.

III. POSSIBLE MECHANISMS FOR $\Delta I = \frac{1}{2}$ RULE

To discuss problems like the $\Delta I = \frac{1}{2}$ rule or the octet dominance of nonleptonic weak decays, we would have to study the SU(2) or SU(3) structure of the matrix element $\langle F | Q_{\alpha\beta}{}^2(K) | I \rangle$ in the theory. This requires the use of Weinberg's bridge theorem and the study of bridge graphs of asymptotic power $\alpha_N = -3$ or -4. With Wilson's operator-product expansion, one can show that $Q_{\alpha\beta}{}^2(K)$ contains three different types of terms[8]: (1) 4-fermion bridges, (2) 2-fermion and gluon bridges, and (3) pure gluon bridges. Terms of the third type can be ignored for our discussion, since they do not involve any quark fields and hence are not relevant to the conventional strangeness-changing decays of hadrons. Terms of the second type have the natural octet SU(3) structure or the $I = \frac{1}{2}$ isospin structure for the strangeness-changing decays of the ordinary hadrons, since they involve only bilinear products of the quark fields. The first-type terms, however, contain in general the 27-plet as well as the octet components. Thus we would be able to obtain the octet dominance rule or the $\Delta 1 = \frac{1}{2}$ rule if there is a mechanism that would suppress the first-type against the second-type terms.

We will now discuss two possible mechanisms that can lead to the validity of the $\Delta I = \frac{1}{2}$ rule.

(1) It may happen that the contribution to $\langle F | Q_{\alpha\beta}{}^2(K) | I \rangle$ from the 4-fermion bridges has a weaker logarithmic K dependence than the corresponding contribution from the 2-fermion bridges. In such a case, the dominant contribution to terms of order $G_F m^2$ will satisfy the $\Delta I = \frac{1}{2}$ rule, with $\Delta I \neq \frac{1}{2}$ contribution suppressed by some power of the logarithmic factor $\ln(\mu_W/m)^2$.

(2) A second possibility is based on the following considerations. So far we have used a neutral-vector-gluon model to describe the strong interactions. However, if we use instead a non-Abelian gauge model for the strong interactions, as recently discussed,[9] we expect the main results presented above in this paper to remain true. Now Gross and Wilczek,[10] and Politzer[10] have indicated that non-Abelian gauge theories may be asymptotically free. But in the free-field limit the first-type terms can be easily shown to vanish, while some of the second-type terms would survive in this limit.[11] Therefore, if the limit of vanishing strong-interaction coupling constants is relevant to us here, then the second-type terms will dominate and we would have the octet rule or the $\Delta I = \frac{1}{2}$ rule for nonleptonic decays.

IV. THE CASE $m \gtrsim M$

Finally, we briefly discuss the case in which quark masses are not necessarily small as compared to scalar-boson masses but are small compared to the vector-boson masses. In this case the asymptotic expansion of $F^{FI}_{ij}(k)$, Eq. (12), is useless in the discussion of the $\phi 1$ term. Although $\delta_{\phi 1} S_{FI}$ is still of order $G_F m^2$, its effects will not be just a correction to the quark mass matrix, but also contain some proper nonleptonic weak interactions. Therefore, both $\delta_{A1'} S_{FI}$ and $\delta_{\phi 1} S_{FI}$ can induce nonleptonic decays in order $G_F m^2$ if quark masses are not all much smaller than scalar-boson masses. In general we would not be able to say anything about the relative importance of these two sources of nonleptonic weak interactions. Furthermore, the possible mechanisms described above for obtaining the octet-dominance rule would not apply to $\delta_{\phi 1} S_{FI}$ here, because these mechanisms would only deal with the asymptotic behavior of $F^{FI}_{ij}(k)$, which however is no longer relevant in the case of small scalar-boson masses.

ACKNOWLEDGMENTS

We thank R. N. Mohapatra for useful discussions, and Professor S. Weinberg for pointing out a mistake in the earlier version of this paper.

APPENDIX

In this appendix we want to analyze integrals of the form

$$I = \int d^4k\, G(k)(k^2 + \mu_W{}^2)^{-1},$$

where $G(k)$ behaves asymptotically at most like $1/k^2$ times non-negative powers of $\ln k^2$. First we do a Wick rotation in the k^0 plane and call the angular integration of $G(k)$ as $-iF(k^2)$, where k is from now on the Euclidean momentum. Then

$$I=\int_0^\infty k^3 dk F(k^2)(k^2+\mu_W{}^2)^{-1} .$$

Suppose the asymptotic expansion of $F(k^2)$ is given by

$$F(k^2)\sim \frac{a(k)}{k^2}+\frac{b(k)}{k^4}+\frac{c(k)}{k^6}+\cdots ,$$

where $a(k)$, $b(k)$, $c(k)$, ... may be constants or powers of $\ln k^2$. Subtracting out the divergence that arises from the leading asymptotic term, we may write

$$I-I_\infty \equiv I_{\text{finite}}=I_0+I_1+I_2+\cdots ,$$

where I_n contains all terms of order $(1/\mu_W{}^2)^n$ times power of $\ln\mu_W{}^2$. Only I_0 and I_1 are relevant to our discussion here. Now the leading term $a(k)/k^2$ in the expansion of $F(k^2)$ will in general contribute to all I_n $(n=0,1,2,\ldots)$, but fortunately in the present theory the contributions from this leading term always correspond only to quark-mass correcting effects, which have nothing to do with the "proper" nonleptonic weak interactions. (See Ref. 2 and Sec. II of this paper.) Therefore, insofar as we are interested only in obtaining the "proper" nonleptonic weak interactions, we can always ignore the contributions to I from the $a(k)/k^2$ term.

Now we want to show that (1) the $b(k)/k^4$ term or any higher term does not contribute to I_0, and (2) I_1 receives contributions from the $b(k)/k^4$ term as well as all the higher terms in the expansion, but the contributions from the $b(k)/k^4$ term are larger than those from the higher terms by at least a factor of $\ln\mu_W{}^2$.

To see this consider the integral

$$J_n^l=\int_\lambda^\infty k^3 dk[\ln(k^2/\lambda^2)]^l (k^2+\mu_W{}^2)^{-1}(k^2)^{-n} ,$$

where $n\geq 2$ is an integer, λ is an arbitrary infrared cutoff, and we have assumed that $l\geq 0$. J_n^l represents the contribution to I from a typical piece of the $1/(k^2)^n$ term in the expansion of $F(k^2)$. Introducing new variables $v=\lambda^2/k^2$ and $\beta=\lambda^2/\mu_W{}^2$, we rewrite J_n^l as[12]

$$J_n^l=\frac{\beta}{2(\lambda^2)^{n-1}}\int_0^1 dv\,\frac{v^{n-2}}{v+\beta}[\ln(1/v)]^l .$$

Using the identity

$$\frac{v^{n-2}}{v+\beta}\equiv\frac{(-1)^n\beta^{n-2}}{v+\beta}+v^{n-3}-\beta v^{n-4}+\cdots+(-\beta)^{n-3},$$

we obtain

$$J_n^l=\frac{(-1)^n}{2}\left(\frac{\beta}{\lambda^2}\right)^{n-1}\int_0^1 dv\,\frac{[\ln(1/v)]^l}{v+\beta}+\frac{\beta}{2(\lambda^2)^{n-1}}\sum_{k=0}^{n-3}(-\beta)^k\int_0^1 dv\,v^{n-3-k}[\ln(1/v)]^l .$$

In the limit of large $\mu_W{}^2$, or $\beta\to 0$, we can prove that[12]

$$\int_0^1 dv\,\frac{[\ln(1/v)]^l}{v+\beta}\sim\frac{1}{l+1}[\ln(1/\beta)]^{l+1} .$$

Then J_n^l can be written as

$$J_n^l\approx\frac{(-1)^n}{2}\left(\frac{\beta}{\lambda^2}\right)^{n-1}\left(\frac{1}{l+1}[\ln(1/\beta)]^{l+1}+\cdots\right)+\frac{\beta}{2(\lambda^2)^{n-1}}\sum_{k=0}^{n-3}(-\beta)^k\frac{\Gamma(l+1)}{(n-2-k)^{l+1}} .$$

From this equation we see immediately that for $n\geq 2$, J_n^l is at most of order $(\ln\mu_W{}^2)^{l+1}/\mu_W{}^2$. Therefore the $b(k)/k^4$ term and all the higher terms in the expansion of $F(k^2)$ cannot contribute to I_0. Furthermore, the contribution of J_n^l to I_1 is given by

$$\sim\frac{1}{\mu_W{}^2}\,\frac{1}{2(l+1)}\left(\ln\frac{\mu_W{}^2}{\lambda^2}\right)^{l+1}\quad\text{for } n=2,$$

and by

$$\frac{1}{2\mu_W{}^2(\lambda^2)^{n-2}}\,\frac{\Gamma(l+1)}{(n-2)^{l+1}}\quad\text{for } n\geq 3 .$$

Since $l\geq 0$, the contribution from the $n=2$ term is at least of order $(\ln\mu_W{}^2)/\mu_W{}^2$. From this we conclude that I_1 can be divided into two parts:

$$I_1=\frac{1}{\mu_W{}^2}(\text{powers of }\ln\mu_W{}^2)+\frac{1}{\mu_W{}^2}(\text{constant}),$$

where the first part (which receives contributions solely from the $n=2$ term) is larger by at least a factor of $\ln\mu_W^2$ than the second part. If we assume the first part of I_1 dominates over the second part, then we can approximate I_1 by the contributions from the $b(k)/k^4$ term only. We have used this approximation in the analysis of the $A1'$ term and the $A\phi$ term in Sec. II.

---

*Research supported by the U. S. Atomic Energy Commission.

[1]B. W. Lee and S. B. Treiman, Phys. Rev. D 7, 1211 (1973); L. P. Yu, UCSD Report No. 10P10-119, 1972 (unpublished); M. A. B. Bég, Phys. Rev. D 8, 664 (1973); A. Pais, Phys. Rev. Lett. 29, 1712 (1972); T. P. Cheng, Phys. Rev. D 8, 496 (1973); D. Bailin, A. Love, D. V. Nanopoulos, and G. G. Ross, Rutherford Lab. Report No. RPP/T/50, 1973 (unpublished); R. N. Mohapatra and P. Vinciarelli, Phys. Rev. Lett. 30, 804 (1973); Phys. Rev. D 8, 481 (1973); T. C. Yang, Nucl. Phys. B58, 283 (1973).

[2]S. Weinberg, Phys. Rev. D 8, 605 (1973).

[3]We will use m to denote the quark mass matrix, μ the vector-boson mass matrix, and M the scalar-boson mass matrix.

[4]We would like to thank Professor S. Weinberg for bringing this to our attention.

[5]For a crude estimate let us take the quark mass to be ~5 GeV and the W-boson mass to be ~100 GeV. Then $\ln(\mu_W^2/m^2)\approx 6$. Thus if the ratio of the first part to the second part of I_1 is only a single power of $\ln(\mu_W^2/m^2)$, then our approximation is probably only good to within about 20%. However, this ratio may well in fact be a higher power of $\ln(\mu_W^2/m^2)$. For a ratio of $[\ln(\mu_W^2/m^2)]^2$, our approximation could be correct to within 5%.

[6]Odd-power terms are left out in the expansion because of symmetric integration over k. This is also the reason for the nonappearance in δS_{FI} of terms of order $e^2(m/\mu_W)^n$ with odd n.

[7]See footnote 8 of Weinberg's paper, Ref. 2.

[8]The details of these terms are not relevant to us here.

[9]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); R. N. Mohapatra, J. C. Pati, and P. Vinciarelli, Phys. Rev. D 8, 3652 (1973).

[10]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, *ibid.* 30, 1346 (1973).

[11]This can be verified also by using the Bjorken-Johnson-Low expansion of $\eta^{\mu\nu}F^{FI}_{\alpha\mu,\,\beta\nu}(k)$ to calculate $Q^2_{\alpha\beta}(k)$, which is essentially given by

$$\eta^{\mu\nu}\int d^3x\,[J_{\alpha\mu}(x),[[[J_{\beta\nu}(0),H],H],H]];$$

cf. Mohapatra *et al.*, Ref. 9.

[12]J_n^l is essentially the generalized zeta function $\Phi(-1/\beta, l+1, n-1)$. See K. Mitchell, Phil. Mag. 40, 351 (1949); *Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. 1, p. 27.