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Spontaneous Symmetry Breaking Without Scalar Mesons*

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By combining the ideas of Nambu in his study of superconductivity and of Johnson, Baker, and Willey in their approach to electrodynamics we construct a gauge theory of spontaneous symmetry breaking which is free of elementary spin-zero fields. The theory contains two fermions and two vector mesons, one of which acquires a mass via the Higgs mechanism. A formula for this vector-meson mass is derived which becomes exact, and nonzero, in the limit as the strength of interaction is appropriately scaled to zero. The vacuum energy is also discussed.

I. INTRODUCTION

The observation that a symmetry-violating solution of a Yang-Mills theory may lead via the Higgs mechanism to a renormalizable field theory of massive vector mesons coupled to nonconserved currents has aroused a flurry of efforts to apply these ideas to construct a unified theory of the weak and electromagnetic interactions.[1] To our knowledge, essentially all such efforts in this regard have utilized field theories with elementary scalar fields whose nonzero vacuum expectation values are the source of the symmetry breaking. In this sense, these theories are analogous to the many-body theory of Bose systems in the superfluid phase. However, in constrast with the situation in nonrelativistic many-body theory, where the existence of an expectation value for the Bose field follows from the repulsive nature of the potential between the particles,[2] the vacuum expectation value of the scalar field in the proposed field-theoretical models follows only after it is essentially put in by hand—by giving the mass term in the free-particle Lagrangian the wrong sign. Furthermore, since there is no suggestion from observation that scalar mesons play a significant role in weak or electromagnetic phenomena, it is generally necessary in constructing theories with scalar mesons to arrange that their observable effects are sufficiently small, i.e., that their masses are sufficiently large.

In this paper[3] we present a simple model field theory in which the spontaneous symmetry breaking and the consequent massiveness of a vector meson occur in a manner similar to the violation of electric current conservation and the consequent Meissner effect in the theory of superconductivity. Indeed, our efforts to construct a theory of this kind were inspired by the work of Nambu[4] concerning the gauge invariance of the BCS theory of superconductivity.

The specific model we study is presented in Sec. II. It will be evident that this model is not intended as a realistic theory of weak or electromagnetic interactions. Rather, it is only an example of what we feel is probably a large class of theories in which the spontaneous symmetry breaking derives from general features of an apparently symmetric interaction, without the necessity of a mass term with the wrong sign and without an elementary scalar field having a vacuum expectation value.

The model we discuss has two spin-$\frac{1}{2}$ fermions of equal bare mass and two massless vector mesons. The vector mesons are coupled to different currents, each of which generates a separate O(2) invariance of the theory. The first symmetry is an invariance with respect to rotations of the two fermion fields into each other, and it is this symmetry which is spontaneously violated, giving a mass to the associated vector meson. The other vector meson is coupled to the fermion number current, whose conservation is unbroken. This second vector meson plays the role in our model which the phonon plays in the theory of superconductivity; it provides the force which allows a nontrivial (i.e., nonzero) solution to the homogeneous Dyson equation for the symmetry-violating part of the fermion propagator.

In Sec. II we argue that the theory admits asymmetric solutions. The character of the symmetry-violating part of the fermion Green's function is similar to the chiral-symmetry-breaking part (i.e., mass term) of the electron Green's function in the Baker-Johnson[5] approach to electrodynamics. In particular, the scale of the symmetry-violating part of the propagator at asymptotically large momentum squared appears to be a free parameter, whereas the rate of decrease of this quantity as a function of momentum squared is an explicit function of the coupling constants.

In Sec. III we impose the Ward identity on the vector-meson–fermion vertex function and show that this function develops a pole at zero q^2. This pole generates a mass for the vector meson coupled to the violated current. A formula for the self-energy of this meson at zero momentum is derived.

Section IV is devoted to demonstrating that the Goldstone excitation which accompanies the symmetry breaking does not occur as a pole in the physical scattering amplitude. More specifically, we exhibit a cancellation between a pole at $q^2=0$ in the vector-meson exchange part of the scattering amplitude and a corresponding pole in the vector-meson irreducible part of this amplitude.[6,7]

In Sec. V we emphasize that the spontaneous symmetry breaking induced by the interaction leaves a finite, residual effect even when the coupling constants of this interaction are appropriately scaled to zero. The approximation of retaining only these zero-order effects of the interaction is termed the "platform approximation" to convey our impression (as yet not fully substantiated) that from this platform the remaining effects of the interaction can be calculated, in principle, as a power series in the coupling constants. The content of the theory in this approximation is essentially a free-field theory, but with the masses altered from their original symmetric values. In particular, the originally massless vector meson coupled to the violated current has a mass which, as we demonstrate, can be calculated exactly in the platform approximation.

In Sec. VI we confront the question as to which of the various solutions to the theory is the preferred one in the sense that it yields the lowest vacuum energy—which in relativistic field theory is the zero-temperature, zero-chemical-potential limit of the thermodynamic potential. Somewhat to our surprise, we are led to conclude that all the solutions, including the symmetric solution, have equal vacuum energy. We argue that this conclusion is implied whenever there is a continuous range of solutions characterized by a continuous parameter. In the present model, this parameter can be taken to be the apparently arbitrary scale (zero for the symmetric solution) of the symmetry-violating part of the fermion propagator for asymptotically large momentum. The situation is thus quite different from usual Higgs theories with scalar fields, where the difference in vacuum energy between the normal and spontaneously broken solutions is nontrivial, both at the tree level and when radiative corrections are included.[8]

II. THE MODEL

The model we study has the Lagrangian density

$$\mathcal{L}=-\bar{\psi}\left(\frac{1}{i}\gamma_\mu\partial_\mu+m_0\right)\psi-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{4}G_{\mu\nu}{}^2+g\bar{\psi}\gamma_\mu\psi A_\mu+g'\bar{\psi}\gamma_\mu\tau_2\psi B_\mu\,, \tag{2.1a}$$

where ψ represents the two fields ψ_1 and ψ_2, where

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\,, \tag{2.1b}$$

$$G_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu\,, \tag{2.1c}$$

and where τ_2 is the 2×2 matrix

$$\tau_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

connecting ψ_1 and ψ_2. The Lagrangian density is locally invariant under rotations through an angle θ in the plane of ψ_1 and ψ_2,

$$\psi\to e^{i\tau_2\theta}\psi\,, \tag{2.2a}$$

if simultaneously the vector field B undergoes the gauge transformation

$$B_\mu\to B_\mu+\frac{1}{g'}\partial_\mu\theta\,. \tag{2.2b}$$

We will look for solutions which violate this symmetry. The $\mathcal{L}$ in (2.1a) is also invariant under $\psi\to e^{i\alpha}\psi$, with a corresponding gauge translation of A_μ; we do not break this symmetry, except that it is always possible to add a mass term for A_μ without affecting either renormalizability or number current conservation.

The Dyson equation for the fermion propagator is

$$S^{-1}(p)=\not{p}+m_0+ig^2\int\frac{d^4k}{(2\pi)^4}\Delta^A_{\mu\nu}(k)\gamma_\mu S(p-k)\Gamma^A_\nu(p-k,p)+ig'^2\int\frac{d^4k}{(2\pi)^4}\Delta^B_{\mu\nu}(k)\gamma_\mu\tau_2 S(p-k)\Gamma^B_\nu(p-k,p)\,, \tag{2.3}$$

where the last two terms on the right refer, respectively, to the two graphs in Fig. 1, and where $\Delta^{A,B}_{\mu\nu}$ and $\Gamma^{A,B}_\nu$ are the propagators and vertex functions of the indicated mesons. We want to know if there exists a solution to this equation in which $S^{-1}(p)$ has a part proportional to the matrix

$$\tau_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{2.4}$$

and which therefore violates the O(2) symmetry of Eq. (2.2).

Let us begin by considering Eq. (2.3) to first order in g^2 and g'^2. If we write

$$S^{-1}(p)-\not{p}-m_0=\Sigma(p)\equiv\Sigma_s(p)+\tau_3\Sigma_v(p) \tag{2.5}$$

and take $\Delta_{\mu\nu}^{A,B}$ to be in the Landau gauge, the order g^2, g'^2 version of Eq. (2.3) becomes

$$\Sigma_s(p) = i(g^2+g'^2)\int \frac{d^4k}{(2\pi)^4}\,\frac{\delta_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\,\gamma_\mu\,\frac{\not{p}-\not{k}+m_0+\Sigma_s(p-k)}{[\not{p}-\not{k}+m_0+\Sigma_s(p-k)]^2-\Sigma_v{}^2(p-k)}\gamma_\nu \tag{2.6a}$$

and

$$\Sigma_v(p) = i(g'^2-g^2)\int \frac{d^4k}{(2\pi)^4}\,\frac{\delta_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\,\gamma_\mu\,\frac{\Sigma_v(p-k)}{[\not{p}-\not{k}+m_0+\Sigma_s(p-k)]^2-\Sigma_v{}^2(p-k)}\gamma_\nu\,, \tag{2.6b}$$

where we have projected out separately the part of Σ proportional to the unit matrix and the part proportional to τ_3.

Equation (2.6b) for the symmetry-violating part of the fermion self-energy is similar in structure to the equation for the chirally asymmetric part of the inverse electron propagator in the Baker-Johnson-Willey[5] approach to electrodynamics. Guided by the work of these authors, we ask if there is a solution to (2.6b) which for asymptotically large p^2 behaves like

$$\Sigma_v(p^2) \underset{p^2\to\infty}{\sim} \delta m\left(\frac{p^2}{m^2}\right)^{-\epsilon}, \tag{2.7}$$

where ϵ is positive, and m is some mass. Since the large-p^2 behavior of the integral in (2.6b) comes from large $(p-k)^2$, we check the consistency of the ansatz (2.7) by substituting (2.7) into the integrand in (2.6b), ignoring all but the $(\not{p}-\not{k})^2$ in the denominator, and performing the integration. The result is

$$\Sigma_v(p^2) \underset{p^2\to\infty}{\sim} \frac{3(g^2-g'^2)}{16\pi^2\epsilon(1-\epsilon)}\,\delta m\left(\frac{p^2}{m^2}\right)^{-\epsilon}. \tag{2.8}$$

Equations (2.7) and (2.8) can be made consistent for ϵ in the range $0<\epsilon<1$ providing

$$0<g^2-g'^2<\frac{4\pi^2}{3}, \tag{2.9}$$

which is not much of a restriction, except that it requires g^2 to be larger than g'^2. In particular, the choice $g=0$ in Eq. (2.1a) would not have allowed a symmetry-violating solution of the kind envisaged in Eq. (2.7).

As mentioned in Sec. I, for the asymptotic solution (2.7) the scale δm is arbitrary, and it is this feature which leads to our conclusion in Sec. VI that the vacuum energy is independent of δm. It is natural at this point to wonder if this feature is also true for the exact solution to the nonlinear equation (2.6b) and, for that matter, for the solution for the symmetry-violating part of the full Dyson equation in (2.3). We have convinced ourselves, albeit nonrigorously, that exact solutions to (2.8b) exist with the asymptotic behavior in (2.7) for a continuous range of δm including $\delta m = 0$. Concerning the full Dyson equation we have little to say in this paper, except to mention that we have given preliminary consideration to the question of how the solution of the simple equation (2.6b) could be extended to include corrections to arbitrary order in g^2 and g'^2, and that it is our current impression that such an extension can be realized with the asymptotic form for Σ_v in (2.7) and the arbitrary scale for δm persisting to all orders. Further comments in this regard are included in Sec. V.

The Dyson equation in (2.6a) for the symmetric self-energy Σ_s can presumably be solved iteratively with standard techniques.[9] We do not pursue this question further here, except to observe that there appears the attractive alternative that m_0 could be chosen to be zero, and that the chirally noninvariant part of Σ_s could be generated spontaneously in a manner similar to that which we propose for Σ_v. This idea, which of course is not original here,[5] leads to a spontaneous breakdown of both chiral and scale invariance. Since the currents which generate these symmetries are generally associated with anomalies, one would not expect a zero-mass boson to accompany their violation.[10] By contrast, we pursue in the subsequent sections the point of view that the breakdown of the O(2) symmetry in Eq. (2.2) implied by a nonzero Σ_v is associated with a Goldstone excitation which via the Higgs mechanism gets decoupled from the S matrix by giving a mass to the B vector meson of Eq. (2.1).

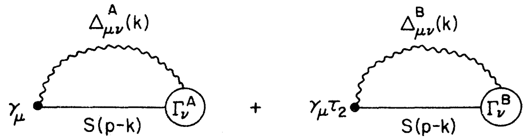

FIG. 1. The two contributions to the fermion self-energy.

III. WARD IDENTITY AND B-MESON MASS

Stimulated by the existence of a Ward identity[4] for the electric current vertex in the nonrelativistic theory of superconductivity, and also by the fact that in relativistic field theory there are no anomalies to destroy the conservation of vector[11] (as opposed to axial-vector) currents, we assume that the vertex function Γ_μ^B satisfies the conventional Ward identity,

$$k_\mu \Gamma_\mu^B(p-k,p) = \tau_2 S^{-1}(p) - S^{-1}(p-k)\tau_2\,, \tag{3.1}$$

even when Σ_v is nonzero. It is evident that the right-hand side of (3.1) is not zero at $k=0$, if Σ_v in (2.5) does not vanish. This observation forces us to conclude that Γ_μ^B is singular at $k=0$ such that

$$k_\mu \Gamma_\mu^B(p-k,p)\big|_{k=0} = 2i\tau_1 \Sigma_v(p)\,, \tag{3.2}$$

where τ_1 is the 2×2 matrix

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3.3}$$

The vertex Γ_μ^B must essentially have the form

$$\Gamma_\mu^B(p-k,p) = \overline{\Gamma}_\mu^B(p-k,p) + i\tau_1 \frac{k_\mu}{k^2}\Gamma_v^B(p-k,p)\,, \tag{3.4}$$

where $\overline{\Gamma}_\mu^B$ is regular at $k^2=0$, and Γ_v^B satisfies

$$\Gamma_v^B(p-k,p)\big|_{k=0} = 2\Sigma_v(p)\,. \tag{3.5}$$

The B-meson self-energy function $\Pi_{\mu\nu}$ is shown in Fig. 2. It is given by

$$\Pi_{\mu\nu}^B(k) = -ig'^2 \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\Gamma_\mu^B(p-k,p)S(p)\gamma_\nu \tau_2 S(p-k)] + \text{contact terms}, \tag{3.6}$$

where the trace is over the eight-dimensional direct-product space of the τ and Dirac matrices. The contact terms in (3.6) play no essential role in the following except to guarantee that as a consequence of (3.1)

$$k_\mu \Pi_{\mu\nu}^B = k_\nu \Pi_{\mu\nu}^B = 0\,. \tag{3.7}$$

That is, $\Pi_{\mu\nu}^B$ must be of the form

$$\Pi_{\mu\nu}^B(k) = \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]\Pi_B(k^2)\,, \tag{3.8}$$

where the vector-meson propagator is given in terms of Π_B by

$$\Delta_{\mu\nu}^B(k) = \frac{\delta_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 + \Pi_B(k^2)} - \lambda\frac{k_\mu k_\nu}{k^4}\,. \tag{3.9}$$

Here λ is a gauge parameter which is zero in the Landau gauge.

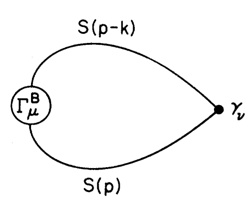

FIG. 2. The B-vector-meson self-energy $\Pi_{\mu\nu}$.

It is evident from (3.9) that the B meson acquires a mass if $\Pi_B(0)$ is unequal to zero; that is, if the coefficient of $k_\mu k_\nu$ in (3.8) has a pole at $k^2=0$. But such a pole is implied by the presence of the pole in Γ_μ^B indicated in (3.4). By inserting (3.4) into (3.6) and isolating the pole in $\Pi_{\mu\nu}$, we obtain

$$\Pi_B(0) = \lim_{k^2\to 0} -\frac{1}{k^2}g'^2 \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\tau_1 \Gamma_v^B(p-k,p) \times S(p)\not{k}\tau_2 S(p-k)]\,. \tag{3.10}$$

A formula for $\Gamma_v^B(p-k,p)$ valid to first order in k is obtained by differentiating (3.1) with respect to k with Γ_μ^B given by (3.4),

$$\tau_1 \Gamma_v^B(p-k,p) = i[S^{-1}(p),\tau_2] - ik_\lambda \partial_\lambda S^{-1}(p)\tau_2 + ik_\lambda \overline{\Gamma}_\lambda^B(p,p) + O(k^2)\,. \tag{3.11}$$

Using this relation in (3.10) we obtain after some algebra the equality

$$\Pi_B(0) = -\frac{ig'^2}{4}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma_\lambda \partial_\lambda S(p) + \gamma_\lambda S(p)\overline{\Gamma}_\lambda^B(p,p)S(p)\tau_2]\,, \tag{3.12}$$

which is applied in Sec. V to obtain an expression for the B-meson mass. Remarkably this does not vanish in the limit where the coupling constants g^2 and g'^2 are scaled to zero [with $g^2 > g'^2$ to satisfy (2.9)], as we shall see in Sec. V.

IV. CANCELLATION OF THE GOLDSTONE EXCITATION

The assumption that the Ward identity (3.1) is valid, despite the apparent violation of the $\bar\psi\gamma_\mu\tau_2\psi$ current arising from a nonzero Σ_v, has been shown in Sec. III to imply a pole at $k^2=0$ in the vertex function Γ_μ^B. This assumed validity of the Ward identity is equivalent to the assumption that the existence of a solution to the homogeneous Bethe-Salpeter equation for the single-B-meson irreducible part of the fermion-antifermion scattering amplitude, which follows from the existence of a solution to the homogeneous Dyson equation for the symmetry-violating part of the fermion propagator,[10] reflects a pole in the solution of the corresponding inhomogeneous Bethe-Salpeter equation. Conversely, this pole in Γ_μ^B indicates a solution to the homogeneous Dyson equation for this vertex function. In this section we show that this pole does not occur in the "physical" fermion-fermion scattering amplitude. Specifically, we demonstrate that the pole in the single-B-meson irreducible part of the scattering amplitude cancels against a pole in the B-meson-exchange part of this amplitude. This cancellation is similar to the cancellation between a bound-state pole and the pole due to an elementary particle with the same quantum numbers.[7]

We denote by R and R', respectively, the coupling strengths of the $\bar{\psi}\gamma_\mu\tau_2\psi$ current and of the fermion-antifermion state to this pole. As indicated in Fig. 3, the single-fermion matrix element of the current $\bar{\psi}\gamma_\mu\tau_2\psi$, which is related to Γ_μ^B by[12]

$$\langle p'|\bar{\psi}\gamma_\mu\tau_2\psi|p\rangle = \bar{u}(p')\Gamma_\mu^B(p',p)u(p), \tag{4.1}$$

thus has a pole which in terms of R and R' is given by

$$\bar{u}(p')\Gamma_\mu^B(p',p)u(p)|_{\text{pole}} = iRR'\frac{k_\mu}{k^2}\bar{u}(p')\tau_1 u(p). \tag{4.2}$$

Here $k=p-p'$ and the $u(p)$ are the spinors for the "physical" fermions,

$$S^{-1}(p)u(p)=\bar{u}(p)S^{-1}(p)=0. \tag{4.3}$$

The R and R' shown in Fig. 3 can be understood in the sense of an effective Lagrangian by thinking of the current $\bar{\psi}\gamma_\mu\tau_2\psi$ as containing a piece proportional to $\partial_\mu\phi$,

$$\bar{\psi}\gamma_\mu\tau_2\psi = R\partial_\mu\phi+\cdots, \tag{4.4}$$

with ϕ the effective field for a massless scalar meson coupled to the fermions by a term in the effective Lagrangian

$$\mathcal{L}'_{\text{eff}} = R'\bar{\psi}\tau_1\psi\phi. \tag{4.5}$$

The effective interaction in (4.5) generates a pole at zero momentum transfer squared in the single-B-vector-meson irreducible part of the fermion-fermion scattering amplitude T_1. This pole term

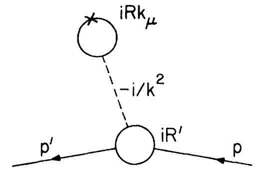

FIG. 3. Diagrammatic representation of the pole in the matrix element of Eqs. (4.1) and (4.2).

indicated in Fig. 4 is

$$\langle p_1'p_2'|T_1|p_1p_2\rangle|_{\text{pole}} = -\frac{R'^2}{k^2}[\bar{u}(p_1')\tau_1 u(p_1)\bar{u}(p_2')\tau_1 u(p_2)-(p_1'\leftrightarrow p_2')]. \tag{4.6}$$

The B-vector-meson-exchange part of the fermion-fermion scattering amplitude, which we call T_2, is shown in Fig. 5 and is equal to

$$\langle p_1'p_2'|T_2|p_1p_2\rangle = -\Delta_{\mu\nu}^B(k)[\bar{u}(p_2')\Gamma_\mu^B(p_2',p_2)u(p_2)\bar{u}(p_1')\Gamma_\nu^B(p_1',p_1)u(p_1) - (p_1'\leftrightarrow p_2')], \tag{4.7}$$

where the B-meson propagator $\Delta_{\mu\nu}^B$ is given in Eq. (3.9).

We wish to show that the pole at $k^2=0$ in (4.6) cancels against a pole in (4.7) which exists because of the pole in Γ_μ^B indicated in (4.2). As a first step in isolating the pole part of (4.7) note that the $k_\mu k_\nu$ terms in $\Delta_{\mu\nu}^B$ contribute nothing because of the Ward identity (3.1) and the equalities in (4.3). But then the pole part of $\Gamma_\mu^B(p_2',p_2)$, which is proportional to k_μ, also gives nothing for the same reasons. The pole part of (4.7) then arises only from the pole part of $\Gamma_\mu^B(p_1',p_1)$, and by appealing to (4.2) we have $[k=p_1-p_1'=-(p_2-p_2')]$

$$\langle p_1'p_2'|T_2|p_1p_2\rangle|_{\text{pole}} = -\Pi_B^{-1}(0)\left[\bar{u}(p_2')\left(\Gamma_\mu^B(p_2',p_2)+iRR'\frac{k_\mu}{k^2}\tau_1\right)u(p_2)\bar{u}(p_1')iRR'\frac{k_\mu}{k^2}\tau_1 u(p_1)-(p_1'\leftrightarrow p_2')\right], \tag{4.8}$$

with Π_B as given in (3.9). Finally, an application of the Ward identity and (4.3) once again eliminates $\Gamma_\mu^B(p_2',p_2)$ to leave

$$\langle p_1'p_2'|T_2|p_1p_2\rangle|_{\text{pole}} = \frac{R^2R'^2}{\Pi_B(0)k^2}[\bar{u}(p_2')\tau_1 u(p_2)\bar{u}(p_1')\tau_1 u(p_1)-(p_1'\leftrightarrow p_2')], \tag{4.9}$$

which shows, by comparing with (4.6), that the pole parts of T_1 and T_2 cancel, providing

$$R^2=\Pi_B(0). \tag{4.10}$$

But according to Eq. (3.8) $\Pi_B(0)$ is the coefficient of $-k_\mu k_\nu$ in the residue of the pole at $k^2=0$ in $\Pi_{\mu\nu}$. In Fig. 6 the pole part of $\Pi_{\mu\nu}(k)$ is shown in terms of R defined in (4.4), and it is easy to check that

$$\Pi_{\mu\nu}(k)|_{\text{pole}} = -\frac{k_\mu k_\nu}{k^2}R^2. \tag{4.11}$$

V. THE PLATFORM APPROXIMATION

In this section we apply Eq. (3.12) to obtain a formula for the B-vector-meson mass which becomes exact in the limit as the coupling constants g^2 and g'^2 approach zero in a manner that allows

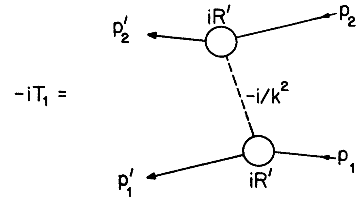

FIG. 4. The pole in the single-B-meson irreducible part of the fermion-fermion scattering amplitude.

the consistency of (2.7) and (2.8) to be maintained, that is, as $g^2 - g'^2$ approaches zero from the positive side. It is evident from Eq. (2.7) that in this limit Σ_v goes to a constant δm, and—so to speak—the only effect of the vanishing interaction is to leave the fermion masses split by an arbitrary amount $2\delta m$ and, as we shall see in Eq. (5.6), to leave the B vector meson with a finite mass whose value, for any nonzero choice of δm, can be arranged completely arbitrarily, since [see Eq. (5.6)] the ratio $g'^2(g^2 - g'^2)^{-1}$ is unrestricted in the limit as g^2 and g'^2 approach zero.

We shall refer to the "platform approximation" as the approximation of retaining only those features of the coupling between the particles which persist when g^2 and g'^2 go to zero in the sense described. This terminology reflects our impression that these zero-order effects of the interaction can serve as a platform from which the remaining effects of the coupling between the particles can be computed in a perturbation expansion in g^2 and g'^2. For example, after choosing a δm to determine the zero-order Σ_v in Eq. (2.7), one can substitute this Σ_v into the right-hand side of (2.6b) to generate the zero-order part of the left-hand side identically and, in addition, a first-order contribution to Σ_v arising from the nonasymptotic region of the integration domain. This first-order Σ_v then generates a second-order contribution when it is inserted into the right-hand side of (2.6b), etc. Although no pitfalls have yet appeared to us which would prohibit the implementation of this program to arbitrary order (i.e., in principle), we should emphasize that we have

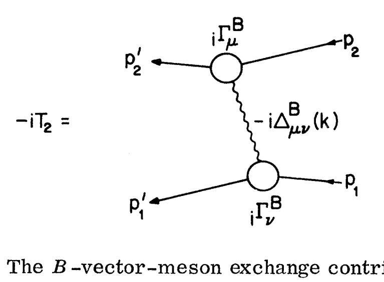

FIG. 5. The B-vector-meson exchange contribution to the fermion-fermion scattering amplitude.

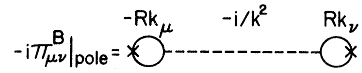

FIG. 6. The pole in the B-meson self-energy $\Pi_{\mu\nu}$.

not yet studied this problem in depth, and we cannot at present assert that the asymmetric theory discussed here is amenable to a finite perturbation calculation to all orders in g^2 and g'^2. This question is of significance, since the theory it describes—a massive vector meson coupled to an apparently nonconserved current—is certainly not renormalizable as conventionally formulated.

We now apply (3.12) to calculate $\Pi_B(0)$ in the platform approximation. Because of the explicit g'^2 in (3.12) we need only retain those parts of $S(p)$ and $\overline{\Gamma}_\lambda^B$ which are of zero order in the coupling constants. Thus, referring to (2.5), we take

$$S(p) = \left[m_0 + \not p + \tau_3 \delta m \left(\frac{p^2}{m^2}\right)^{-\epsilon}\right]^{-1}, \tag{5.1a}$$

$$\overline{\Gamma}_\lambda^B(p,p) = \gamma_\lambda \tau_2 \tag{5.1b}$$

in the integrand of (3.12). Since we can also ignore the derivative of the τ_3 term in $\partial_\lambda S(p)$, we have

$$\begin{aligned}\mathrm{Tr}[\gamma_\lambda \partial_\lambda S(p) + \gamma_\lambda S(p)\overline{\Gamma}_\lambda^B(p,p)S(p)\tau_2] &= \mathrm{Tr}[-\gamma_\lambda S(p)\gamma_\lambda S(p) + \gamma_\lambda S(p)\gamma_\lambda \tau_2 S(p)\tau_2] \qquad (5.2a)\\ &= -2\,\mathrm{Tr}\left[\gamma_\lambda\gamma_\lambda \tau_3{}^2(\delta m)^2\left(\frac{p^2}{m^2}\right)^{-2\epsilon}(p^2 + m_0{}^2)^{-2}\right], \qquad (5.2b)\end{aligned}$$

where in (5.2b) we have kept only the leading dependence of $[S(p)]^4$ for large p (i.e., p^{-4}) together with the term $m_0{}^2$ to avoid a spurious infrared divergence. Performing the trace in (5.2b) and substituting the result into (3.12) leads to

$$\Pi_B(0) = -16ig'^2(\delta m)^2(m^2)^{2\epsilon} \times \int \frac{d^4p}{(2\pi)^4}\,\frac{1}{(p^2 + m_0{}^2)^2(p^2)^{2\epsilon}}, \tag{5.3}$$

which upon integration gives

$$\Pi_B(0) = \frac{g'^2(1-\epsilon)}{\pi \sin 2\pi\epsilon}\left(\frac{m^2}{m_0{}^2}\right)^{2\epsilon}(\delta m)^2. \tag{5.4}$$

In the platform approximation g^2, g'^2, and ϵ all go to zero such that

$$\frac{\epsilon}{g^2 - g'^2} \to \frac{3}{16\pi^2} \tag{5.5}$$

as required for the consistency of (2.4) and (2.8). Thus, since $\Pi_B(0)$ is the square of the B-meson mass M_B in this limit, and since δm is half the fermion mass splitting $m_1 - m_2$, Eq. (5.5) becomes in the platform approximation

$$M_B{}^2 = \frac{2g'^2}{3(g^2 - g'^2)}(m_1 - m_2)^2 . \tag{5.6}$$

Observe that no such result could be obtained in a conventional Higgs theory with scalar particles, because of the arbitrariness of the coupling of the scalar to the fermion fields. Indeed, one of the major advances which follows from elimination of the scalars is the large reduction in independent parameters which can occur in the Lagrangian.

VI. THE VACUUM ENERGY

Both in many-body systems and in Higgs theories with scalars, one argues that the preferred solution (either normal or spontaneously broken) is the one with the lowest ground-state energy. The potential function which is minimized by an appropriate choice of the vacuum expectation value of a Higgs scalar is essentially the vacuum energy difference for the normal and spontaneously broken vacuum; it was used by Jona-Lasinio[13] for the study of Goldstone theories, and the single-loop approximation to it was studied by Coleman and Weinberg for Higgs theories.[8]

Denote by Ω_s, Ω_v the sums of all connected vacuum graphs for the symmetry-preserving and symmetry-violating theories, respectively, divided by a four-dimensional normalization volume, and let $\Omega = \Omega_s - \Omega_v$. Then Ω is the energy difference (per unit three-dimensional volume) between the normal and symmetry-violating vacuums. Both Ω_s and Ω_v are quartically divergent; our first task is to argue that Ω is finite; otherwise, it makes no sense to compare the vacuum energy of the systems.

Let us consider Ω_s or Ω_v as a functional set of all irreducible Green's functions, that is, the proper self-energies, proper vertices, one-particle-irreducible four-point functions, etc. (as well as the vacuum expectation value of scalar fields, in the general case). It can be shown[14] that the stationarity of Ω_s or Ω_v to arbitrary variations of these Green's functions yields the complete set of Dyson equations for the normal theory or the symmetry-violating theory, respectively. There is only one symmetry-breaking parameter in our theory, namely the scale factor δm introduced in Eq. (2.7) into the symmetry-violating self-energy Σ_v. When $\delta m = 0$, we recover the normal theory. Indicating explicitly the dependence of Ω_v on δm we have

$$\Omega_s = \Omega_v(0); \;\; \Omega = \Omega_v(0) - \Omega_v(\delta m). \tag{6.1}$$

It is easy to see (essentially because $\mathrm{Tr}\,\tau_3 = 0$) that Ω_v is an even function of δm; we now argue that there is no infinite part to the $(\delta m)^2$ term in Ω.

The coefficient of $(\delta m)^2$ can be found by varying Ω_v with respect to S_v, Γ_v, etc., and invoking stationarity:

$$\delta\Omega_v = \frac{i}{(2\pi)^4}\,\mathrm{Tr}\int d^4p\,\delta S_v(p)[\text{LHS of Eq. (2.3)} - \text{RHS of Eq. (2.3)}] + \int \delta\Gamma_v \cdots \tag{6.2}$$

[recall that (2.3) is the Dyson equation for Σ_v]. For sufficiently small δm, we have $-\delta S_v = S_s \Sigma_v S_s$; moreover, at large p, the term in square brackets in (6.2) vanishes identically for any δm, because it reduces to the *linear* homogeneous equation based on (2.6b). One may now verify that the contribution from asymptotically large p to (6.2) is in fact finite.

All terms of $O((\delta m)^4)$ or higher in Ω are finite, by a naive power-counting argument, since Ω has dimension M^4. [Potential logarithmic divergences in the $(\delta m)^4$ term are saved by the asymptotic decrease of $\Sigma_v(p) \sim (p^2)^{-\epsilon}$.] Thus we conclude that Ω, the difference of two quartically divergent objects, is finite.

However, a much stronger conclusion seems warranted in the case at hand: It appears that $\Omega \equiv 0$. This conclusion is based on the following theorem: If Ω_v depends on a *continuous* symmetry-breaking parameter δm which can be chosen arbitrarily within a certain neighborhood of $\delta m = 0$, then Ω_v is independent of δm. In this case, the variation of Ω_v with respect to δm vanishes identically, because the square brackets in (6.2) vanish identically for any δm (and so for all the other Dyson equations). A simpler proof invokes the Feynman-Hellwarth theorem: Let

$$E(\delta m) = \langle \psi | H | \psi \rangle \tag{6.3}$$

be the vacuum energy (i.e., expectation) value of H in the *normalized* vacuum state $|\psi\rangle$. Then we have

$$\frac{\partial E}{\partial \delta m} = \langle \psi | \frac{\partial H}{\partial \delta m} | \psi \rangle = 0 \tag{6.4}$$

because H is independent of δm.

Thus, $\Omega = 0$ if δm can be chosen from a continuum of values. Otherwise, Ω is finite. At the moment, it is an open question whether or not the full Dyson equations [e.g., (2.3)] determine that $\delta m/m$ takes on discrete values. It is our current belief that δm can be chosen from a continuum, based on studies of equations with nonlinearities similar to those of (2.3); thus $\Omega \equiv 0$. Needless to say, this is quite different from conventional Higgs theories.

One might question whether it makes physical sense to compare the vacuum energies of two relativistic field theories, since there seems to be no physical mechanism for causing transitions between the two vacuums. However, all the remarks of this section hold for the ground-state energy of a system of particles at finite density and temperature, and in this case it is physically possible for a phase transition to occur. Of course, the required densities and temperatures must be very high; the only possible relevant circumstances seem to be the first moment of the "big-bang" model of the universe. Kirzhnits and Linde[15] have discussed the possibility of a phase transition in conventional Higgs theories, at temperatures comparable to particle masses. At the moment, it seems unlikely that consideration of finite temperature and density can change our conclusion that $\Omega \equiv 0$, but it remains an interesting and open question.

VII. CONCLUSIONS

It seems to be an important advance to rid spontaneously broken gauge theories (SBGT) of Higgs scalars, for several reasons. The first is the obvious consideration that no such scalars have been observed, and in most SBGTs the scalar masses are chosen to ensure unobservability for many years to come. Perhaps more important, there are no arbitrary parameters in $\mathcal{L}$ which characterize the coupling of scalars to themselves and to fermions, which ultimately allows for the calculation of a larger number of symmetry-breaking effects than in more conventional Higgs theories.

Clearly, the model discussed here is too restrictive in a number of respects. We have dealt explicitly only with the Abelian case, largely to avoid the tedious complications of ghost scalars which accompany closed loops of vector mesons.[1] However, it is easy to generalize the treatment of the linear, homogeneous equation for Σ_v by enlarging the B meson and the fermion to multiplets of fields. Then δm becomes a matrix which can be expanded into irreducible group representations, for each one of which there is a separate ϵ, as determined by Eq. (2.8). It is interesting to observe that the contribution to each ϵ from the B multiplet may have either sign (it is negative for an Abelian case). For example, let the fermions and B mesons both be in an $I=1$ representation of O(3); then δm can be $I=1$ or $I=2$. The B contribution is positive for $\epsilon_{I=1}$, but negative for $\epsilon_{I=2}$. Likewise, if the fermions and B mesons are each in an SU(3) octet, then ϵ_{8d} and ϵ_{8f} have the same positive contribution, while ϵ_{27} is negative. In the absence of coupling to the singlet A meson, then, in the first case $I=2$ symmetry breaking is forbidden, and in the second case only octet symmetry breaking is allowed. This is in satisfying agreement with what appears to be happening in the real world.

All our considerations have been based on the assumption of specific asymptotic forms for the propagation and vertex functions; specifically, we assumed the meson propagators to go like k^{-2} at infinity. It is well known that this is true only if the Gell-Mann–Low eigenvalue condition is satisfied,[5] namely $\beta(\alpha_0)=0$, where β is the coefficient of $k^2 \ln k^2$ in $\Pi_B(k^2)$ for large k^2, and α_0 is the bare fine-structure constant; $\beta = O(\alpha_0)$ for small α_0. If $\beta \neq 0$, the vector propagators have the asymptotic behavior $k^{-2}[\sum_0 C_N(\beta \ln k^2)^N]$, where the C_N are uninteresting numerical constants. If this asymptotic form is used in the linearized version of Eq. (2.6) for Σ_v, the resulting equation for ϵ is (schematically) $\epsilon \sim (g^2 - g'^2)F(\beta/\epsilon, \beta'/\epsilon)$. In consequence, ϵ still scales with g^2 and g'^2 if these coupling constants are small, but the actual numerical value of ϵ depends on the C_N. Thus violation of the Gell-Mann–Low eigenvalue condition does not violate the spirit of the platform approximation of Sec. V.

It remains to work out a systematic perturbation theory, starting with the linearized platform approximation and going to higher orders in the symmetry-breaking parameter δm. Such a perturbation theory will be reminiscent of the insertion of scalar tadpoles on fermion and vector lines, as in the usual Higgs theories. However, there is a difference: Our "tadpole" (e.g., Σ_v) decreases at large momenta.

Our considerations of the vacuum self-energy in Sec. VI have been very brief. It is now clear, as we shall discuss elsewhere,[14] that the techniques alluded to there allow for a full statement of relativistic statistical mechanics, including vacuum fluctuations, which is free of infinities. It is possible that this may find application in certain astrophysical processes.

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[1]For a list of references through September 1972 see B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 249.

[2]See, for example, A. L. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill,

New York, 1971), Chap. 14.
[3]While this paper was in preparation we received a preprint of R. Jackiw and K. Johnson [Phys. Rev. D 8, 2386 (1973)] dealing with the same issues in the context of a model with an axial vector. A variant of this model is the same as the Abelian model discussed here, upon suitable transformation of the fields. Some time ago the possibility of a dynamical origin to spontaneous symmetry breakdown was discussed by F. Englert and R. Brout [Phys. Rev. Lett. 13, 321 (1964)].
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[7]It has been known for some time that a bound state which communicates with an elementary particle channel does not appear in the S matrix. This was argued by C. J. Goebel and B. Sakita [Phys. Rev. Lett. 11, 293 (1963)] for nonrelativistic bound states and by Y. S. Jin and S. W. MacDowell [Phys. Rev. 137, B688 (1965)] for bound-state poles which move with the coupling constant—this does not take place for the Goldstone boson, which is fixed at $q^2=0$. A general proof of this cancellation, based on the Dyson equations, was given by J. M. Cornwall and D. J. Levy [Phys. Rev. 178, 2356 (1968)].
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Coupled Anharmonic Oscillators. I. Equal-Mass Case

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This is the first in a series of papers on the large-order behavior of perturbation theory for coupled anharmonic oscillators. We exploit previously published dispersion techniques to convert the calculation of perturbation theory in large order into a barrier-penetration problem. We then introduce new semiclassical methods for describing tunneling through nonspherically symmetric, N-dimensional potentials. To illustrate our new methods, we calculate the large-order behavior of perturbation theory for a simple system of two equal-mass oscillators with quartic coupling. Our predictions are in complete agreement with computer calculations. We then extend our results to oscillators with x^{2N} coupling, N-oscillator systems, and some infinite-oscillator systems.

I. INTRODUCTION

In a recent paper Adler[1] argues that α, the physical charge on the electron, is an essential singularity of the Gell-Mann–Low function. Since the location of an essential singularity cannot be affected by the low-order terms in a perturbation expansion, an asymptotic study of perturbation theory for quantum electrodynamics in extremely large order seems indicated.

There has already been much work on the large-order behavior of perturbation series in quantum