$K^{\pm}\to\pi^{\pm}\pi^0\gamma$ Decays in a Current-Current Quark Model*

R. Rockmore
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey 08903

J. Smith
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 and Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790

T. F. Wong
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 and Department of Physics, Rutgers, The State University, New Brunswick, New Jersey 08903

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The decay rates for $K^{\pm}\to\pi^{\pm}\pi^0\gamma$ are calculated in a zero-parameter modified fermion-loop model first proposed by Rockmore and Wong. The weak Hamiltonian is phenomenologically constructed from one-baryon octet matrix elements. The predicted branching ratio $r = R(K^{\pm}\to\pi^{\pm}\pi^0\gamma;\ 55\le T_{\pi^{\pm}}\le 90\ \text{MeV})/R(K^{\pm}\to\text{all}) = 1.56\times10^{-5}$ is in excellent agreement with the recent experimental result of Abrams *et al.*

Recently two of us[1] have shown that when the baryon-loop model, first introduced by Steinberger[2] to explain the decay $\pi^0\to\gamma\gamma$, is suitably modified for weak interactions,[1] it unexpectedly provides a qualitative explanation for the decay $K_2^0\to\gamma\gamma$. In a subsequent paper,[3] the same authors calculated the decay rate for

$$K_2^0\to\pi^+\pi^-\gamma \tag{1}$$

and found that the same *zero*-parameter model gives a result which is just below the experimental upper limit.[4]

In a recent publication, Abrams *et al.*[5] reported the observation of a direct emission amplitude in the decays

$$K^+\to\pi^+\pi^0\gamma \tag{2}$$

and

$$K^-\to\pi^-\pi^0\gamma\,. \tag{3}$$

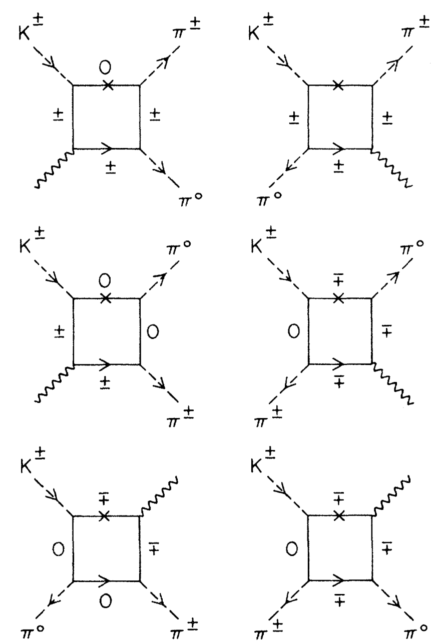
The experimental branching ratio is

$$\frac{R(K^{\pm}\to\pi^{\pm}\pi^0\gamma)}{R(K^{\pm}\to\text{all})}=(1.56\pm0.35)\times10^{-5}, \tag{4}$$

with $55\le T_{\pi^{\pm}}\le 90$ MeV. This number presents a direct challenge to our model.

In this note, we give the result of a calculation of the decay rates for reactions (2) and (3). The calculation is very similar to the one for the decay (1), and we refer to Ref. 3 for the details. As in Ref. 3, we describe the decays in terms of the two possible mechanisms graphically illustrated in Figs. 1 and 2. Their contributions to the decay amplitudes are denoted by $A^{(\pm)}$ and $A_\rho^{(\pm)}$, respectively, where

$$\begin{aligned}\epsilon[\epsilon(q,\lambda)p_K p_{\pi^+}p_{\pi^0}]&\\ \times[A^{(+)}+A_\rho^{(+)}](p_{\pi^+}{}^2, p_{\pi^0}{}^2, p_K{}^2, p_K\cdot p_{\pi^+}, p_K\cdot p_{\pi^0}, p_{\pi^+}\cdot p_{\pi^0})&\\ =(16m_K E_{\pi^+}E_{\pi^0}E_\gamma)^{1/2}&\\ \times\langle\gamma(q)\pi^+(p_{\pi^+})\pi^0(p_{\pi^0})\ \text{out}\,|\mathcal{H}_W(0)|K^+(p_K)\rangle\,.&\end{aligned} \tag{5}$$



+ (Three Sets of Similar Diagrams With The × on Other Sides)

FIG. 1. Baryon-loop graphs for emission of "uncorrelated" pions in $K^{\pm}\to\pi^{\pm}\pi^0\gamma$ decays.

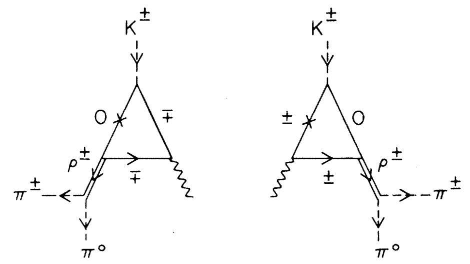


FIG. 2. Baryon-loop graphs for emission of "correlated" pions (from virtual ρ decay) in $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ decays.

The baryons traveling around the loop can be p, Σ^+, etc., with the appropriate charge and SU(3) index. A straightforward calculation gives

$$A^{(+)}=\frac{\sqrt{2}eg^3}{(4\pi)^2m^4}2[fF(3f^2-\tfrac{73}{3}d^2)+dD(13f^2+3d^2)] \tag{6}$$

and

$$A_\rho^{(+)}=\frac{\sqrt{2}egg_\rho\phi}{(4\pi)^2m^2}\,\frac{64}{9}\left[dD+\frac{\delta}{\phi}(fD-2dF)\right]\times\frac{1}{(p_{\pi^+}+p_{\pi^0})^2-m_\rho^{\,2}}. \tag{7}$$

The definitions of the various quantities can be found in Ref. 3.

We remark that Eqs. (6) and (7) are the result of complicated sums of many terms, and they *cannot* be obtained from Eqs. (6) and (8) of Ref. 3 by a simple isospin argument.[6] On the other hand, we do have

$$A^{(+)}=-A^{(-)},\qquad A_\rho^{(+)}=-A_\rho^{(-)}, \tag{8}$$

as can be seen from the following observation. Consider, for example, the diagrams in Fig. 3(a). They are identical except for the direction of the loop momenta, which gives rise to a different sign from the tensor structure. In the case of Fig. 3(b), however, the direction of the loop momenta does not matter, but the $\rho\pi\pi$ vertex changes sign.

Finally, the decay rate is given by

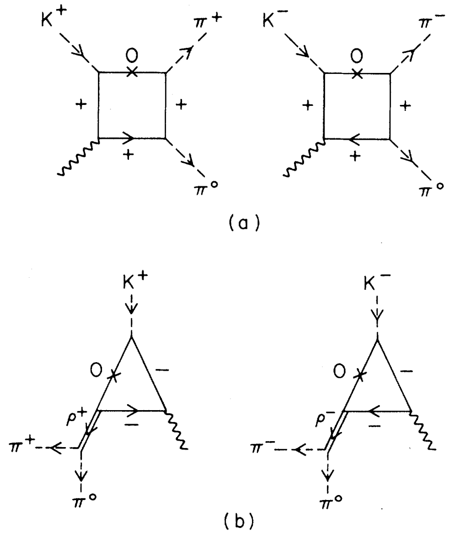


FIG. 3. Examples of diagrams in $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ which are equal to each other and opposite in sign.

$$R=\frac{1}{64\pi^3m_K}\int dE_{\pi^+}dE_{\pi^0}\times\theta(4(E_{\pi^+}{}^2-m_{\pi^+}{}^2)(E_{\pi^0}{}^2-m_{\pi^0}{}^2)-[m_K{}^2-2m_K(E_{\pi^+}+E_{\pi^0})+2E_{\pi^+}E_{\pi^0}+m_{\pi^+}{}^2+m_{\pi^0}{}^2]^2)\times\sum_{\text{pol}}\epsilon(\epsilon(q,\lambda)p_Kp_{\pi^+}p_{\pi^0})\epsilon(\epsilon(q,\lambda)p_Kp_\pi+p_{\pi^0})|A^{(+)}+A_\rho^{(+)}|^2, \tag{9}$$

with $55\leq T_{\pi^+}\equiv(E_{\pi^+}-m_{\pi^+})\leq 90$ MeV.

A two-dimensional numerical integration of Eq. (9) gives

$$R(K^{\pm}\to\pi^{\pm}\pi^{0}\gamma)=0.832\times10^{-12}\ \text{eV} \tag{10}$$

or

$$\left[\frac{R(K^{\pm}\to\pi^{\pm}\pi^{0}\gamma)}{R(K^{\pm}\to\text{all})}\right]_{\text{theo}}=1.56\times10^{-5}, \tag{11}$$

which is in excellent agreement with the experimental value in Eq. (4).

As a check on our program we also calculated the inner-bremsstrahlung contribution to the decays (2) and (3) in the same energy interval, finding the branching ratio 2.43×10^{-4}. This agrees with the number quoted in Ref. 5.

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[6]A similar situation occurred before in the calculation of $\gamma+\gamma\to\pi^0\pi^0\pi^0$ and $\gamma+\gamma\to\pi^+\pi^-\pi^0$. See, for example, T. F. Wong, Phys. Rev. Lett. 27, 1617 (1971).

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Baryon-Antibaryon Contributions to the $K_2^0 - K_1^0$ Mass Difference in a Current-Current Quark Model

R. Rockmore*

Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada

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The baryon-antibaryon contributions to the $K_2^0 - K_1^0$ mass difference are studied in an extended fermion-loop model as a further consistency test of a current-current quark model. It is found that contributions arising from the parity-conserving weak Hamiltonian phenomenologically constructed of one-baryon octet matrix elements are negligible compared to the crude estimate of the K_1^0 mass shift, $\Delta(m_{K_1}^0) = -(\hbar/2\tau_s)\cot\delta_{00}(m_K^2)$. Although the contribution to the K_2^0 mass shift arising from the parity-violating weak Hamiltonian via an S-wave effective meson-baryon-baryon interaction turns out to be comparable to $\Delta(m_{K_1}^0)$ and negative, the extended loop model is not incompatible with present theoretical understanding of the $K_2^0 - K_1^0$ mass difference.

I. INTRODUCTION

The fermion-loop model,[1] suitably modified[2] for strangeness-changing nonleptonic weak interactions, has lately proved successful in providing (1) a qualitative[2] explanation for $K_2^0\to\gamma\gamma$ decay, (2) a predicted branching ratio for the CP-conserving decay $K_2^0\to\pi^+\pi^-\gamma$, (Ref. 3) $r_0 = R(K_2^0\to\pi^+\pi^-\gamma)/R(K_2^0\to\text{all modes}) = 3.0\times10^{-4}$, consistent with the tree-graph estimate, $2.6\times10^{-4} < r_0 < 4\times10^{-4}$ of Moshe and Singer[4] and below the present[5] experimental upper limit ($r_0 < 4\times10^{-4}$), and (3) a predicted[6] branching ratio $r_\pm = R(K^\pm\to\pi^\pm\pi^0\gamma;\ 55\leq T_{\pi^\pm}\leq 90\ \text{MeV})/R(K^\pm\to\text{all modes}) = 1.56\times10^{-5}$, in excellent agreement with the recent experiment of Abrams *et al.*[7] As we noted in these earlier calculations,[2,3] the extended fermion-loop model furnishes an attractive alternative to the usual tree-graph description of these ($B=0$) processes[4,8] [we have in mind the most successful of such (current-current) models, that of Moshe and Singer,[4] which also seems to be the "simplest" (i.e., with a minimum of neutral currents)], since unlike the tree-graph model, one obtains results with *no adjustable parameters*. [Recall that the parameters of our model are fixed in Gronau's[9] remarkable fit to the experimental amplitudes for nonleptonic hyperon decay ($B=1$ processes).] In this paper, in continuation of our program of analysis of ($B=0$) strangeness-changing processes in terms of the extended baryon-loop model, we discuss the baryon-antibaryon contribution to the ($\Delta S=2$) $K_2^0 - K_1^0$ mass difference.

It has long been argued[10] from (a) the source of the mass difference: the weak interactions, (b) the difference in the respective lifetimes of K_2^0 ($\sim 10^{-7}$ sec) and K_1^0 ($\sim 10^{-10}$ sec), and (c) the predominant two-pion decay mode (principally in the $I=0$ state) of the K_1^0, that the mass difference may be primarily due to the weak mass shift of the K_1^0 arising from its coupling to the ($I=0$) two-pion state. This K_1^0 mass shift has been shown to have the form[11]

$$\Delta(m_{K_1^0}) = -\frac{\hbar}{2\tau_s}\cot\delta_{00}(s=m_K^2) + \text{correction due to left-hand contribution.} \quad (1)$$

While we do not propose to deal here with the problem posed by the correction term in Eq. (1), we want to point out that *if it is small* (indeed it vanishes in an effective-range theory of unitarized current algebra which fits the "up-up" $\pi\pi$ data), then the "main term,"

$$-(\hbar/2\tau_s)\cot\delta_{00}(m_K^2) \simeq -3.73\times10^{-6}\ \text{eV}, \quad \text{for } \delta_{00}(m_K^2)\simeq\tfrac{1}{4}\pi,$$

is rather close to the present experimental value,[12]