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Scaling Behavior of the Ratio of Longitudinal to Transverse Total Virtual-Photoabsorption Cross Sections*

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The Bjorken scaling of $R=\sigma_L/\sigma_T$ is studied. We show that the ratio ν^2R/M^2Q^2 [which scales in canonical spin-(1/2) theories] behaves as $1/x^2$ $(x=+Q^2/2\nu)$ for small x and as $(1-x)^{-\alpha}$ $(0<\alpha<2)$ for x near unity. We compare the scaling of νW_2 as extracted from the differential cross-section data using this form of R with that of νW_2 extracted using a form of R appropriate when some charged spin-0 constituents are present.

Inelastic electron scattering experiments performed at SLAC[1] have shown that the ratio, R, of longitudinal to transverse total virtual-photoabsorption cross sections is small. This is taken as evidence that the carriers of electric charge in the proton are primarily spin-$\frac{1}{2}$.[2] More precise determinations of R are expected in the near future.[3] In this paper we present the behavior expected of R in canonical field theories. Although we use the language (momentum space) of the parton model, our results, except where noted, may also be derived from light-cone considerations. R is related to structure functions with well-determined scaling, Regge and threshold behaviors which severely constrain its form. We analyze the presently available differential cross-section data with a form of R appropriate to an admixture of spin-0 and spin-$\frac{1}{2}$ constituents as well as with a form corresponding to purely spin $\frac{1}{2}$. The scaling of $\nu W_2(q^2,\nu)$ is examined for both choices.

R is defined by

$$R\equiv\frac{\sigma_L}{\sigma_T}=\frac{(1+\nu^2/M^2Q^2)W_2(q^2,\nu)-W_1(q^2,\nu)}{W_1(q^2,\nu)}\equiv\frac{W_L(q^2,\nu)}{W_1(q^2,\nu)}, \tag{1}$$

where $Q^2=-q^2>0$ is the four-momentum transfer from the electron and $\nu=P\cdot q$. W_1 and W_2 are the usual structure functions.

The current data on R are presented in Fig. 1. It was noted in Ref. 1 that the data are consistent with (a) $R=$ constant and with (b) $R=M^2Q^2/\nu^2$. Recently, however, Sakurai[4] has shown that a bound derived from vector-meson photoproduction is inconsistent with (b).

We shall show that canonical scaling theories predict neither (a) nor (b). Though (a) may be a reasonable approximation in models with some spin-zero constituents, (b) is not a good approximation for any combination of spin-0 and spin-$\frac{1}{2}$ constituents.

In the Bjorken limit $(\lim_{\text{Bj}})$ structure functions

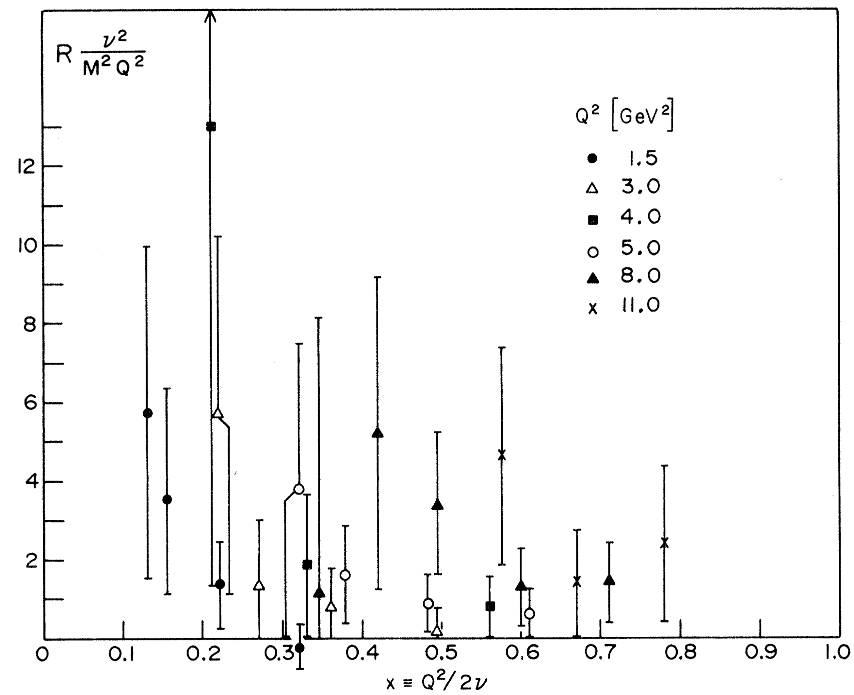

FIG. 1. $\nu^2 R/M^2Q^2$ is plotted as a function of $x=-q^2/2p\cdot q$. The data are from Ref. 1.

have the following scaling properties, if some spin-0 partons are present:

$$\lim_{\text{Bj}} W_L(q^2,\nu)=\frac{1}{2x}F_2^{(0)}(x)\,,\quad x\equiv\frac{-q^2}{2\nu}\,,$$
$$\lim_{\text{Bj}} W_1(q^2,\nu)=\frac{1}{2x}F_2^{(1/2)}(x)\,, \tag{2}$$

$$\lim_{\text{Bj}}\frac{\nu}{M^2}W_2(q^2,\nu)=F_2^{(0)}(x)+F_2^{(1/2)}(x)\,, \tag{3}$$

where the superscripts $(\frac{1}{2})$ and (0) denote the contribution to the structure function from spin-$\frac{1}{2}$ and spin-0 partons, respectively. Thus in this case

$$\lim_{\text{Bj}} R(q^2,\nu)=\frac{F_2^{(0)}(x)}{F_2^{(1/2)}(x)}\,. \tag{4}$$

With only spin-$\frac{1}{2}$ partons it is well known that R and W_L should vanish in the Bjorken limit.[2] Moreover νW_L should scale in such a model[5]:

$$\lim_{\text{Bj}}\frac{\nu W_L(q^2,\nu)}{M^2}=F_G(x)\,. \tag{5}$$

We obtain then

$$\lim_{\text{Bj}}\frac{\nu^2}{M^2Q^2}R(q^2,\nu)=\frac{F_G(x)}{F_2(x)}\,, \tag{6}$$

where we have used Eq. (3).

In the presence of some spin-zero constituents, Eq. (4) gives the correct generalization of the simple R = constant form. Since $F_2^{(0)}$ and $F_2^{(1/2)}$ should have the same Regge behavior for small x, $\lim_{x\to 0}R$ = constant in this case.

Equation (6) places several useful constraints on the behavior of $(\nu^2/M^2Q^2)R$ in spin-$\frac{1}{2}$ theories. According to the dictates of Regge theory, $W_L(q^2,\nu)\sim\nu^\alpha$ and $W_2(q^2,\nu)\sim\nu^{\alpha-2}$ for fixed q^2, $\nu\to\infty$. If Regge behavior persists in the Bjorken limit we expect therefore $F_G(x)\sim x^{-\alpha-1}$, $F_2(x)\sim x^{1-\alpha}$ as $x\to 0$. The latter is in reasonable agreement with present data. Thus for small x, Eq. (6) diverges:

$$\lim_{x\to 0}\left[\lim_{\text{Bj}}\frac{\nu^2}{M^2Q^2}R(q^2,\nu)\right]\sim\frac{1}{x^2}\,. \tag{7}$$

The data of Fig. 1 are as consistent with Eqs. (6) and (7) as with either R = constant or $R=M^2Q^2/\nu^2$. Because of the quadratic divergence in Eq. (7) there is no conflict with Sakurai's bound. The hypothesis that R itself scales [cf. Eq. (4)] is also consistent with the data. Clearly more accurate measurements are necessary.

Assuming that partons are purely spin-$\frac{1}{2}$ we fit the data of Fig. 1 with the approximate form

$$\frac{\nu^2}{M^2Q^2}R=\frac{1}{x^2} \tag{8}$$

$(R=0.4M^2/Q^2)$ consistent with the scaling and Regge behavior discussed above. A more elaborate parametrization (including, for example, a threshold dependence as discussed below) would be unjustified in light of the poor quality of the R data. In particular, a possible x-independent term is omitted from Eq. (8) since its inclusion would force R to be large in regions of low Q^2. $\nu W_2(q^2, \nu)$ may then be extracted from the much larger amount of data (for which separation of σ_L and σ_T was not experimentally possible) using Eq. (8) for R. One may then ask if the resulting values of $\nu W_2(q^2, \nu)$ are consistent with a universal scaling curve. To answer this question quantitatively we have fitted the resultant νW_2 to the form

$$\nu W_2(q^2, \nu) = a(1-\bar{x})^3 + b(1-\bar{x})^4 + c(1-\bar{x})^5$$

using both $\bar{x} = x$ and $\bar{x} = x' = Q^2/(2\nu + M^2)$. The fit was performed over the range $1.25 < \omega' < 10$ and $Q^2 > 1.0\ \text{GeV}^2$. The χ^2 deviation was then computed over this same range. This procedure was repeated for the standard choice $R = 0.18$ consistent with an admixture of spin-0 and spin-$\frac{1}{2}$ partons. The results are as follows. For $R = 0.4M^2/Q^2$ and $R = 0.18$ the fits (which include 170 data points) in ω' had χ^2 of 177 and 132, respectively, whereas the fits in ω had χ^2 of 587 and 387, respectively. Scaling is clearly best in ω'. In both cases the spin-0 admixture choice of $R = 0.18$ results in the best scaling but the distinction is not statistically significant for the variable ω'. More accurate measurements of R would be very valuable.[6] In the absence of conclusive data on R, very accurate measurements of the experimental cross section can test the compatibility of scaling with particular choices of R.

So far we have not discussed the expected threshold behavior of R. To do so requires a more detailed model than the study of scaling or Regge behavior. The most general framework in which this question may be examined is that of the covariant parton model of Landshoff, Polkinghorne, and Short,[7] to which we now turn. We present the analysis for spin-$\frac{1}{2}$ partons. We show that in such a model

$$\lim_{x\to 1}\frac{F_G(x)}{F_2(x)} \sim \left(\frac{1}{1-x}\right)^{\alpha}, \quad 0 \leq \alpha \leq 2. \tag{9}$$

This result does not depend upon the validity of the Drell-Yan-West relation.

The derivations will be brief. For notation see Ref. 8. For spin-$\frac{1}{2}$ partons the scaling functions F_G and F_2 are given by

$$F_2(x) = \frac{x}{x-1}\int ds'\,d^2k_\perp\,[V_1(k^2, s') + x V_2(k^2, s')], \tag{10}$$

$$F_G(x) = \frac{x}{x-1}\int ds'\,d^2k_\perp \times \left[V_1(k^2, s') + \left(\frac{s' + k_\perp{}^2}{M^2(x-1)} + 1\right)V_2(k^2, s')\right], \tag{11}$$

where s' is the center-of-mass energy squared for the parton-proton scattering amplitude, and $k_\perp{}^2$ is related to the virtual-parton mass squared, k^2

$$k^2 = x\left(\frac{s' + k_\perp{}^2}{x-1} + M^2\right) - k_\perp{}^2. \tag{12}$$

The integral is over the imaginary part of the parton-proton amplitude, $V(p, k)$, along its right-hand cut ($s' > 0$). $V(p, k)$ has the spin decomposition

$$V_{\alpha\beta}(p, k) = V_1(s', k^2)(p\cdot\gamma)_{\alpha\beta} + V_2(s', k^2)(k\cdot\gamma)_{\alpha\beta} + R_{\alpha\beta}, \tag{13}$$

where α and β are Dirac indices and $R_{\alpha\beta}$ does not contribute to spin-averaged electron scattering.

In Ref. 7 it was shown that power-law behavior of $F_2(x)$ near $x = 1$ follows if one assumes

$$\lim_{k^2\to\infty} V_i(k^2, s') = (k^2)^{-\gamma_i} f_i(s') \tag{14}$$

which obtains in simple models.[7,9] Scaling of νW_2 requires the integrals $\int ds'\,s'^{1-\gamma_i} f_i(s')$ to be finite while scaling of νW_L requires additionally that $\int ds'\,s'^{2-\gamma_2} f_2(s')$ be finite. Thus if $f_i(s') \sim s'^{\delta_i - 1}$ for large s' we require $\delta_1 < \gamma_1 - 1$ and $\delta_2 < \gamma_2 - 2$. To obtain Eq. (9) substitute Eq. (14) into Eqs. (10) and (11) and let x approach unity ($k^2 \to \infty$):

$$\lim_{x\to 1}\frac{F_G(x)}{F_2(x)} = \frac{A(1-x)^{\gamma_1} + B(1-x)^{\gamma_2 - 1}}{A(1-x)^{\gamma_1} + C(1-x)^{\gamma_2}}. \tag{15}$$

A, B, and C are integrals (which we assume do not vanish) over $f_i(s)$ which converge if $F_2(x)$ and $F_G(x)$ scale. If $A \neq -C$ the ratio diverges like $1/(1-x)^{\alpha}$ with $0 < \alpha \leq 1$, but if $\gamma_1 = \gamma_2$ and $A = -C$ $[f_1(s') = -f_2(s')]$, one obtains $\alpha = 2$. Replacing Eq. (14) by a sum of terms with different γ's allows α to assume any value between 0 and 2.

In the simple models of Ref. 9 the spin structure of $V_{\alpha\beta}(p, k)$ is governed by the Born graph for parton-proton scattering. A Born graph with scalar or pseudoscalar exchange yields

$$V_1(k^2, s') = -V_2(k^2, s') \quad \text{as} \quad k^2 \to \infty,$$

which implies $\alpha = 2$. A Born graph with vector exchange yields

$$V_1(k^2, s') = -\frac{k^2}{s'}V_2(k^2, s') \quad \text{as} \quad k^2 \to \infty,$$

and therefore $\alpha = 0$. A mixture of exchanged spin-0 and spin-1 yields any α between 0 and 2. Clearly the threshold behavior of R is highly model-dependent. Unfortunately the data are not sufficient to indicate the behavior of R near $x=1$.

If one persists and uses the spin structure of Born graphs in a parton-model calculation of the elastic form factors, asymptotic scaling of G_E/G_M obtains only in the case of vector exchange for which $\gamma_1 \leq \gamma_2 - 1$ and $\lim_{x\to 1} F_G(x)/F_2(x)$ constant.

Strict, local duality predicts that F_G/F_2 must be constant near threshold if G_E/G_M is to scale asymptotically.[10] However, one must be skeptical of local duality arguments based only upon elastic contributions since they are disjoint from the rest of the inelastic structure functions [they are proportional to $\delta(\omega - 1)$].

Landshoff and Polkinghorne have argued[11] that the structure function $\overline{F}_2(x)$ for e^+e^- annihilation is given by an equation identical to Eq. (10) with $F_2 \to \overline{F}_2$ and $V_i \to \overline{V}_i$. They claim that for large k^2, $\overline{V}_i(k^2, s')$ factors exactly like $V_i(k^2, s')$ with the same exponent γ_i and function f_i (up to a factor) as in Eq. (14). Their claim is based on a study of Feynman graphs in softened field theories which produce scaling. If we accept this, a strong bound on $\lim_{x\to\infty} \overline{F}_2(x)$ may be derived.

As $x \to \infty$ according to Eq. (12) k^2 again becomes infinite. The $x\to\infty$ limit is best extracted by redefining the variable $k_\perp{}^2$ [cf. Eq. (12)] by $k_\perp'^2 = x^2 k_\perp{}^2$ which leads to

$$\lim_{x\to\infty} \overline{F}_2(x) = x^{2-\gamma_1} \int ds' f_1(s') + x^{3-\gamma_2} \int ds' f_2(s') \quad (16)$$

provided δ_i defined above are negative. If they are positive[12] we obtain instead

$$\lim_{x\to\infty} \overline{F}_2(x) \sim x^\beta, \qquad \beta = \max(2+\delta_1 - \gamma_1, 3 + \delta_2 - \gamma_2). \quad (17)$$

The bounds on δ_i required by the scaling of $F_2(x)$ and $F_G(x)$ together with the observed threshold behavior of $F_2(x)$ [$\sim (1-x)^3$] imply $\lim_{x\to\infty} \overline{F}_2(x) \sim x^n (n<1)$, a more restrictive limit than that obtained in Ref. 11. In both cases the multiplicity in e^+e^- annihilation is finite.

The restrictions $\delta_1 < \gamma_1 - 1$, $\delta_2 < \gamma_2 - 2$ also bound Landshoff and Polkinghorne's prediction[13] for inclusive pion production at high $p_\perp$ in pp collisions. If $\alpha = 2$ or if $\delta_1 < \delta_2 + 1$ (for instance, $\alpha = 0$, but $\delta_1 = \delta_2 - 1$ if the spin structure is governed by the vector Born graph) the bound is more restrictive than that which they give.

To summarize we have found that in canonical theories with only spin-$\frac{1}{2}$ constituents $\nu^2 R/M^2 Q^2$ scales and has rather striking behavior near $x=0$ [Eq. (7)] and perhaps also near $x=1$ [Eq. (9)], the latter depending on an assumption regarding the off-mass-shell behavior of parton-proton amplitudes. Constraints on $\overline{F}_2(x)$ and inclusive pion production in parton models follow from Eq. (9) and similar assumptions. In models with some spin-zero constituents R scales and should be constant near $x=0$. Present data are inconclusive regarding both the form of R and the consistency of a particular choice of R with the scaling of $\nu W_2(Q^2, \nu)$.

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