Critical Comment on Resonance Saturation of Sum Rules

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Using a generalization of the sum rules obtained by equating a fixed-t and a backward dispersion relation, it is argued and illustrated for πN elastic scattering that saturation with a few resonances can easily lead to inconsistent results.

Dispersion relations which receive contributions from both the direct channel (s) and the crossed channel (t) provide a means of investigating the dymanics of the t-channel exchanges. In particular, such dispersion relations have often been used in combination with fixed-t dispersion relations to calculate coupling constants of t-channel resonances. Usually one equates a fixed-t dispersion relation to a backward dispersion relation at the threshold of the direct channel and then makes the assumption that it is possible to saturate the dispersion integrals in a narrow-width approximation with a few resonances.[1] Unfortunately, it was in general not possible to check the consistency of such a calculation.

Recently dispersion relations have been proposed[2] which may be considered to be a generalization of the well-known backward dispersion relations of elastic scattering. These dispersion relations are written along parametrized curves in the Mandelstam plane that allow the path of integration to be varied from the boundary of the physical region to paths within the physical region. If such dispersion relations are combined with fixed-t dispersion relations and the assumption of narrow-width resonance saturation is made one is led to the important observation that values calculated for the t-channel coupling constants depend on the path of integration. This of course means that the coupling constants calculated by such a method are not unique and consequently that such a narrow-width saturation cannot be considered as a reliable method to determine t-channel coupling constants.

To illustrate this point we consider elastic πN scattering. In the case of elastic scattering the family of curves which contain the backward direction and pass through the s-channel threshold point consists of hyperbolas described by one parameter, c, which determines the asymptotes $s=-c$, $u=-c$. For $c=0$, the dispersion relation is just the backward dispersion relation, whereas for $c>0$ the paths of integration are within the s-channel physical region.

The sum rule is obtained by equating such a dispersion relation and a fixed-t dispersion relation at threshold, i.e., at $t=0$, $s=s_0=(M+1)^2$, $M=$ nucleon mass, $m_\pi=1$. In the following we consider the invariant amplitude $B^{(-)}$ and equate

$$B^{(-)}(s_0, t=0, c)=G^2\left(\frac{1}{M^2-s_0}+\frac{1}{M^2-u_0}-\frac{1}{M^2+c}\right)+\frac{1}{\pi}\int_{s_0}^{\infty} ds'\,\mathrm{Im}\,B^{(-)}(s', z_s')\left(\frac{1}{s'-s_0}+\frac{1}{s'-u_0}-\frac{1}{s'+c}\right)+\frac{1}{\pi}\int_4^{\infty} dt'\,\mathrm{Im}\,B^{(-)}(t', z_t')/t' \tag{1}$$

to

$$B^{(-)}(s_0, t=0)=G^2\left(\frac{1}{M^2-s_0}+\frac{1}{M^2-u_0}\right)+\frac{1}{\pi}\int_{s_0}^{\infty} ds'\,\mathrm{Im}\,B^{(-)}(s', t=0)\left(\frac{1}{s'-s_0}+\frac{1}{s'-u_0}\right), \tag{2}$$

where $u_0=(M-1)^2$, $G^2/4\pi=$ pion-nucleon coupling constant $\simeq 14.6$ (Ref. 3), and the cosines of the scattering angles in the s and t channels are given, respectively, by

$$z_s'=\frac{c-s'}{c+s'},\qquad z_t'^2=1-\frac{4ct'}{(t'-4M^2)(t'-4)}. \tag{3}$$

Saturating in the narrow-width approximation the fixed-t dispersion relation, Eq. (2), with N and Δ and the hyperbola dispersion relation, Eq. (1), with N, Δ, and ρ, we obtain the sum rule

$$S_\rho = S_N(c) + S_\Delta(c), \tag{4}$$

where

$$S_\rho = \frac{G_{\rho\pi\pi} G^V_{\rho NN}}{4\pi m_\rho^2}\left(1 + 2\frac{G^T_{\rho NN}}{G^V_{\rho NN}}\right),$$
$$S_N = \frac{G^2}{4\pi}(M^2+c)^{-1} \tag{5}$$
$$S_\Delta = \gamma^{(1)}_\Delta (M_\Delta^2 + c)^{-1} + \gamma^{(2)}_\Delta (M_\Delta^2 + c)^{-2}$$

and

$$\gamma^{(1)}_\Delta = \frac{1}{9}\frac{G_{\pi N\Delta}^2}{4\pi} \times [(E_\Delta + M)^2 - 3(M_\Delta^2 - M^2 - 1 + q_\Delta^2)],$$
$$\gamma^{(2)}_\Delta = \frac{2}{3}\frac{G_{\pi N\Delta}^2}{4\pi} q_\Delta^2, \tag{6}$$
$$E_\Delta + M = \frac{(M_\Delta + M) - 1}{2M_\Delta},$$
$$4M_\Delta^2 q_\Delta^2 = (M_\Delta^2 - s_0)(M_\Delta^2 - u_0).$$

In the above expressions m_ρ and M_Δ are the ρ and $\Delta(1236)$ masses, respectively, $G^{V,T}_{\rho NN}$ denotes the vector (V) and tensor (T) ρNN coupling constants, $G_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling constant, and $G_{\pi N\Delta}^2/4\pi \simeq 0.26$ is the $\pi N\Delta$ coupling constant.[3,4] The numerical value for the meson contribution S_ρ differs from author to author[3]:

$$0.42 \lesssim S_\rho \lesssim 0.59. \tag{7}$$

Equations (4) and (5) clearly demonstrate that the c dependence is different for the s- and t-channel resonances. In particular, an s-channel resonance introduces pole terms in c of the form $(M_R^2 + c)^{-n}$ while a t-channel resonance of spin J introduces a power series in c of order $(J-1)/2$ due to the angular dependence $P_J'(z_t)$, e.g., the g meson ($J^P = 3^-$) gives a linear c dependence.

TABLE I. The dependence of S_N, S_Δ, and $S_N + S_\Delta$ on the parameter c.

c	$S_N(c)$	$S_\Delta(c)$	$S_N(c) + S_\Delta(c)$
0	0.32	0.04	0.36
5	0.30	0.03	0.33
10	0.27	0.03	0.30
15	0.24	0.03	0.27
20	0.22	0.03	0.25
25	0.21	0.03	0.23
30	0.20	0.03	0.22
35	0.18	0.02	0.21

Within the ρ, N, Δ saturation scheme, Eq. (4), it is clear that the sum rule cannot be used to determine the ρ couplings, assuming the N and Δ coupling constants to be given, since the right-hand side of the sum rule, Eq. (4), depends on c whereas the left-hand side is independent of c. For convenience, we give a table of S_N, S_Δ, and $S_\rho + S_\Delta$ for various values of c.[5]

As is seen from the table the right-hand side of the sum rule, Eq. (4), varies by more than 40% as c varies from 0 to 35. Thus, various determinations of the coupling constant using this sum rule can differ by 40%. Since there is no reason to prefer the $c=0$ (backward) sum rule over those for other values of c, the assumption of a narrow-width saturation with ρ, N, and Δ resonances is clearly not valid.

Although we have demonstrated our point that a simple narrow-width saturation scheme can easily lead to inconsistencies, we could hazard a few guesses as to the origin of the discrepancy. Clearly the correct way to calculate s-channel contributions is to use experimental phase-shift solutions for the low-energy region and reasonable parametrization, e.g., Regge-pole fits, for the high-energy region. For $c=0$, this has been done by Engels *et al.*[1] and one finds a correction to $S_N + S_\Delta$ of the order of 0.24, i.e., $S_\rho = 0.60$. This is in reasonable agreement with the values given in Eq. (7). One could also include higher-J t-channel resonance contributions which would introduce a c dependence on the left-hand side of the sum rule, but the fact that the $c=0$ sum rule with a proper treatment of s-channel contributions is reasonably satisfied by the ρ alone suggests that such contributions are not of major importance.

It should be noticed that we have written our sum rule at $t=0$. This is not necessary and one could consider various values of t. For t a free variable, one finds that the s-channel resonances introduce terms of the form t^n coming from their angular variation, whereas the t-channel resonances introduce poles in t, e.g., $(t - m_\rho^2)^{-1}$, etc. Since the dependence on t for the s- and t-channel resonances is different, calculations using such sum rules would result in an ambiguity similar to that considered above and would not lead to unique values of t-channel coupling constants.[6]

In conclusion, the use of parametrized dispersion relations has shown us that simple narrow-width resonance saturation of sum rules is in general not a reliable method to determine coupling constants.

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Flux-Independent Measurements of Deep-Inelastic Neutrino Processes*

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The scaling variable $v = xy = 2(E_\mu/M)\sin^2(\frac{1}{2}\theta_\mu)$ is useful for the description of deep-inelastic neutrino-nucleon scattering processes. This variable is determined solely by the momentum and angle of the outgoing lepton. The normalized scattering distribution in this variable is independent of incident lepton energy and flux, provided scale invariance is valid. The sensitivity to various hypothetical scale-breaking mechanisms is discussed.

Experiments which attempt to measure deep-inelastic neutrino-nucleon cross sections are plagued with the difficult problem of determining the neutrino flux. It is clearly desirable to find ways of extracting information from such experiments which is flux-independent. Experimental studies along these lines have already been carried out by Myatt and Perkins.[1] Recently, Cline and Paschos[2] have analyzed moments of the scaling variable $x = Q^2/2M\nu$, which yield information regarding current-algebra sum rules, provided scaling is correct. Paschos and Zakharov[3] have also put bounds on $\langle E_\mu/E_\nu\rangle$ and $\langle Q^2/2ME_\nu\rangle$, which depend only on the hypothesis of scale invariance.

Here we consider the case where the only accurately determined quantities in an experiment are the secondary muon momentum E' and its production angle θ. We find that some theoretical questions, in particular the validity of dimensional scaling, can be answered from this information alone. The key lies in the fact that the scale-invariant quantity