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Pion Emission in Proton-Antiproton Annihilation Processes

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Using the formalism of current algebra and PCAC (partially conserved axial-vector current) we calculate the amplitudes for emission of two neutral pions in $\bar{p}p \to K^+K^-$ and in $\bar{p}p \to K^+K^-K^+K^-$, and from these the ratios of cross sections $(\bar{p}p \to K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-)$ and $(\bar{p}p \to K^+K^-K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-K^+K^-)$ at various energies.

Recent work utilizing the methods of current algebra and of the partially conserved axial-vector current (PCAC) hypothesis has included applications to annihilation of an electron-positron pair[1] and of a proton-antiproton pair.[2] Specifically, in Ref. 2 this formalism was used by Uritam to deduce the differential rate for $\bar{p}p \to K^+K^-\pi^+\pi^-$, normalized to the $\bar{p}p \to K^+K^-$ rate, as well as the branching ratio $(\bar{p}p \to K^+K^-\pi^+\pi^-)/(\bar{p}p \to K^+K^-)$ for annihilation in flight at vanishing momentum.

In the present communication we extend and generalize the work of Ref. 2 in a number of ways. First, by considering the process $\bar{p}p \to K^+K^-\pi^0\pi^0$ where the emitted pions are neutral, we are dealing with a reaction where there is no contribution from "Low's soft-photon term."[3] For $\pi^+\pi^-$ emission that term is present,[2] and leads to an expression that depends on which pion is soft, a consequence of simultaneously taking both pions off the mass shell that was first noted in Weinberg's result for the K_{e4} form factor.[4] This effect then is absent for $\pi^0\pi^0$ emission.

Secondly we calculate the ratio of cross sections $(\bar{p}p \to K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-)$ for annihilation in flight at nonzero incident momenta. Finally we consider a different reaction, $\bar{p}p \to K^+K^-K^+K^-$, and calculate the analogous ratio $(\bar{p}p \to K^+K^-K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-K^+K^-)$ at various incident momenta.

As is customary we begin by considering the reaction

$$i \to f + \pi^\alpha(k_1) + \pi^\beta(k_2)$$

for multiparticle hadronic states i and f, where pions of momenta k_1 and k_2 and isospin α and β are emitted. After defining the quantity,

$$M_{\mu\nu}^{\alpha\beta} = \int d^4x d^4y\, e^{-ik_1x} e^{-ik_2y} \times \langle f|\, T(A_\mu^\alpha(x) A_\nu^\beta(y))\,|i\rangle\,, \tag{1}$$

where the operators A are axial-vector currents with the specified Lorentz and isospin indices, we contract it with the momenta k_1^μ and k_2^ν, use PCAC to replace axial-vector current divergences with pion field operators, evaluate the commutators using the SU(3) current algebra, and obtain in the "soft-pion limit" ($k_1^\mu \to 0$, $k_2^\nu \to 0$)

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = \frac{-c_\pi^{\,2}}{2\mu^2} \int d^4x d^4y\, e^{-ik_1x} e^{-ik_2y} (\mu^2 - \Box_x)(\mu^2 - \Box_y) \langle f|\, T(\phi_\pi^\alpha(x)\phi_\pi^\beta(y))\,|i\rangle + \epsilon_{\alpha\beta\gamma}\tfrac{1}{2}(k_1 - k_2)^\lambda \int d^4x\, e^{-i(k_1+k_2)x} \langle f|\, V_\lambda^\gamma(x)|i\rangle\,. \tag{2}$$

Here c_π is the PCAC constant defined by

$$\partial^\mu A_\mu^\alpha = \frac{c_\pi}{\sqrt{2}}\phi_\pi^\alpha\,, \tag{3}$$

where

$$c_\pi = \frac{\sqrt{2}\, M_N \mu^2 g_A}{g_r}\,. \tag{4}$$

M_N is the nucleon mass; μ, the pion mass; g_A, the renormalized axial-vector coupling constant ($g_A \sim 1.2$); and g_r, the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi \sim 14.6$).

Through the Lehmann-Symanzik-Zimmermann (LSZ) reduction formulas, Eq. (2) yields

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = \frac{-c_\pi^2}{2\mu^4} M_{2\pi}^{\alpha\beta} + \epsilon_{\alpha\beta\gamma}\frac{(k_1-k_2)^\lambda}{2} M_\lambda^\gamma , \quad (5)$$

which connects the invariant amplitudes for three "processes": $M_{\mu\nu}^{\alpha\beta}$, the emission of two axial-vector currents of isospin α and β; $M_{2\pi}^{\alpha\beta}$, the emission of two pions of isospin α and β; and M_λ^γ, the emission of an isovector photon.

If we consider emission of π^+ and π^- the isovector photon term contributes because $\epsilon_{\alpha\beta\gamma}$ is nonvanishing (this case was considered in Ref. 2, and M_λ^γ was evaluated through Low's soft-photon theorem[3]). For emission of two neutral pions $\epsilon_{\alpha\beta\gamma}$ vanishes and the isovector photon term is absent. Thus we have for the amplitude for $\pi^0\pi^0$ emission

$$M_{\pi^0\pi^0} = -\frac{2\mu^4}{c_\pi^2} k_1^\mu k_2^\nu M_{\mu\nu}^{33} . \quad (6)$$

Even in the soft-pion limit that we are considering ($k_1^\mu \to 0, k_2^\nu \to 0$) the right-hand side does not vanish since $M_{\mu\nu}^{33}$, which explicitly is given by

$$M_{\mu\nu}^{33} = \int d^4x d^4y\, e^{-ik_1x} e^{-ik_2y} \times \langle f |\, T(A^3(x)A^3(y)) \,| i \rangle , \quad (7)$$

has contributions of order k^{-2}.

Specializing to the case $i=\bar{p}p$, $f=K^+K^-$, the terms of order k^{-2} result from diagrams where the axial-vector currents are attached to the external $\bar{p}$ and p lines; there are six such diagrams.[5] In evaluating the diagrams we write the central interaction explicitly in terms of invariants as $\mathfrak{M} = A + B\gamma\cdot Q$, where $Q = q_1 - q_2$; q_1 and q_2 are the kaon momenta. As the proton and antiproton momenta approach zero, only the B term survives, and we have

$$M_{\pi^0\pi^0} = \frac{-2\mu^4 g_A^2}{c_\pi^2 k_1^0 k_2^0 (k_1^0 + k_2^0)} \bar{v}_s B F_B u_r , \quad (8)$$

where

$$\begin{aligned} F_B = {} & k_1^0(-\gamma\cdot Q\gamma\cdot k_1\gamma\cdot k_2 + \gamma\cdot k_1\gamma\cdot Q\gamma\cdot k_2 - \gamma\cdot k_2\gamma\cdot k_1\gamma\cdot Q + \gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1) \\ & + k_2^0(-\gamma\cdot Q\gamma\cdot k_2\gamma\cdot k_1 + \gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1 - \gamma\cdot k_1\gamma\cdot k_2\gamma\cdot Q + \gamma\cdot k_1\gamma\cdot Q\gamma\cdot k_2) \\ & + 2ik_1^0k_2^0(\gamma\cdot Q\gamma\cdot k_2 - \gamma\cdot k_2\gamma\cdot Q + \gamma\cdot Q\gamma\cdot k_1 - \gamma\cdot k_1\gamma\cdot Q) . \end{aligned} \quad (9)$$

From this we compute in a straightforward way the spin-averaged rate for $\bar{p}p \to K^+K^-\pi^0\pi^0$, differential in five kinematic variables, normalized to the $\bar{p}p \to K^+K^-$ total rate, and obtain

$$\begin{aligned} \frac{d^5\omega(\bar{p}p \to K^+K^-\pi^0\pi^0)}{\omega(\bar{p}p\to K^+K^-)} = {} & dm_{\pi\pi}^2 dm_{KK}^2 d(\cos\theta_\pi) d(\cos\theta_K) d\phi \frac{1}{m_{\pi\pi} m_{KK} M_{\bar{p}p}} (m_{\pi\pi}^2 - 4m_\pi^2)^{1/2} (m_{KK}^2 - 4m_K^2)^{1/2} \\ & \times [(M_{\bar{p}p}^2 + m_{\pi\pi}^2 - m_{KK}^2)^2 - 4\, m_{\pi\pi}^2 M_{\bar{p}p}^2]^{1/2} \frac{1}{64\pi^5 M_N^4} [k_1^0k_2^0(k_1^0+k_2^0)]^{-2} g_r^4 |\tilde{M}_{\pi^0\pi^0}|^2 , \end{aligned} \quad (10)$$

where

$$\begin{aligned} |\tilde{M}_{\pi^0\pi^0}|^2 = {} & (k_1^0)^2[\vec{k}_2^2\vec{k}_1^2\vec{Q}^2 - \vec{k}_2^2(\vec{k}_1\cdot\vec{Q})^2 - (\vec{k}_2\times\vec{k}_1\cdot\vec{Q})^2] + (k_2^0)^2[\vec{k}_1^2\vec{k}_2^2\vec{Q}^2 - \vec{k}_1^2(\vec{k}_2\cdot\vec{Q})^2 - (\vec{k}_1\times\vec{k}_2\cdot\vec{Q})^2] \\ & + 2k_1^0k_2^0[(\vec{k}_1\cdot\vec{k}_2)^2\vec{Q}^2 - (\vec{k}_2\cdot\vec{k}_1)(\vec{k}_1\cdot\vec{Q})(\vec{Q}\cdot\vec{k}_2)] . \end{aligned} \quad (11)$$

The spectrum is displayed in the five variables: $m_{\pi\pi}^2$, the invariant mass squared of the dipion system; m_{KK}^2 the invariant mass squared of the dikaon system; θ_π, the angle in the dikaon rest frame between kaon 1 and $-$(total momentum); and ϕ, the azimuthal angle between the dipion and dikaon decay planes. This set has attractive symmetry and lends itself to calculation since the variables are not restricted, except in a simple way, i.e., $2m_\pi + 2m_K \le m_{\pi\pi} + m_{KK} \le M_{\bar{p}p}$, $0<\theta_\pi<\pi$, $0<\theta_K<\pi$, $0<\phi<2\pi$. The formulas relating all variables in $|\tilde{M}|^2$ to this preferred set are given in the Appendix of Ref. 2.

Appealing to the basic tenet of all soft-pion calculations—that one can smoothly extrapolate the soft-pion results to nonzero momenta, retaining the same functional form for the amplitude—we have performed the (highly problematic) integration over all the kinematic variables to yield the ratio of cross sections, $R = (\bar{p}p \to K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-)$ at various energies[6]; the results are shown in Table I.

It is clear that in this case the pions are no longer at all soft and in fact for annihilation at $E_{\text{c.m.}} = 3.62$ GeV the pion momentum can reach 1.325 GeV/c. Our formalism also does not consider any

TABLE I. Ratio of calculated cross sections $R = (\bar{p}p \to K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-)$ at various energies.

$\bar{p}$ lab. momentum (GeV/c)	Center-of-mass energy (GeV)	R
0.0	1.87	0.09
0.3	1.90	0.10
1.5	2.25	0.61
3.0	2.77	2.96
4.5	3.22	7.82
6.0	3.62	15.15

resonances, such as the C meson [or $K_A(1240)$], produced in annihilation at rest into four particles.[7] Both considerations may seriously interfere with the direct application of our procedures to experimental results.

Experimental data on annihilation into $K\bar{K}(m\pi)$ final states are not abundant for any final charge state and energy desired. For a general comparison of our results with experiment we summarize in Table II cross sections for $\bar{p}p$ annihilation into various two kaon final states and into various $K\bar{K}\pi\pi$ final states.

We see that the ratio of experimental cross sections also exhibits the over-all behavior of the theoretical R, although its values are in general somewhat larger, as one sees by forming some ratios from Table II: $R_{\text{exp}}(1.2\text{ GeV}/c) \sim 2.5$, $R_{\text{exp}}(2.5\text{ GeV}/c) \sim 7$; $R_{\text{exp}}(3.7\text{ GeV}/c) \sim 15$.

Next we apply a similar procedure to pion emission in $\bar{p}p \to K^+K^-K^+K^-$. Thus in Eq. (2) $i = \bar{p}p$ and $f = K^+K^-K^+K^-$. Let q_1, q_2, q_3, and q_4 be the four-momenta of the kaons; p_1 and p_2 are the proton and antiproton momenta. Define

TABLE II. Measured cross sections for $\bar{p}p$ annihilation into various final states. (Complete references are given in Ref. 8.)

Final state	$\bar{p}$ lab momentum (GeV/c)	Cross section (μb)	Ref. 8
K^+K^-	0.5	120 ± 60	Bizzarri
K^+K^-	1.0	99 ± 5	Nicholson
K^+K^-	2.0	22 ± 3	Nicholson
K^+K^-	2.4	10 ± 1	Nicholson
$K_S^0K_S^0$	2.5	12 ± 5	Badier
K^+K^-	3.7	~ 2	Katz
$K^+K^-\pi^+\pi^-$	1.2	260 ± 40	Frodesen
$K_S^0K_S^0\pi^+\pi^-$	1.2	208 ± 20	Barlow
$K_S^0K_S^0\pi^+\pi^-$	2.5	81 ± 7	Badier
$K_S^0K_S^0\pi^+\pi^-$	3.7	31 ± 4	Baltay
$K_S^0K_S^0\pi^+\pi^-$	5.7	18 ± 3	Atherton

$$
\begin{aligned}
&P = p_1 + p_2, \quad K = p_1 - p_2, \\
&Q_1 = q_1 - q_2 + q_3 - q_4, \\
&Q_2 = q_1 + q_2 - q_3 - q_4, \\
&Q_3 = q_1 - q_2 - q_3 + q_4.
\end{aligned}
\tag{12}
$$

The amplitude for $\bar{p}p \to K^+K^-K^+K^-$ can be written as

$$M \sim \bar{v}(p_2)\mathfrak{M}u(p_1), \tag{13}$$

where the structure of $\mathfrak{M}$ in terms of invariants is

$$
\begin{aligned}
\mathfrak{M} = {} & A + B\gamma \cdot Q + B'\gamma \cdot Q_2 + B''\gamma \cdot Q_3 \\
& + C\sigma_{\mu\nu} Q_1^\mu Q_2^\nu + C'\sigma_{\mu\nu} Q_2^\mu Q_3^\nu + C''\sigma_{\mu\nu} Q_3^\mu Q_1^\nu .
\end{aligned}
\tag{14}
$$

This expression is used in finding the terms of order k^{-2} in $M_{\mu\nu}^{33}$ in the expression identical to Eq. (6) that gives the amplitude for $\pi^0\pi^0$ emission,

$$M_{\pi^0\pi^0} = -\frac{2\mu^4}{c_\pi^{\,2}} k_1^\mu k_2^\nu M_{\mu\nu}^{33}. \tag{15}$$

A useful simplification (that makes the computation tractable) occurs if we consider the production of kaons at threshold. Thus $Q_1 = Q_2 = Q_3 = 0$, and only the A term in Eq. (14) survives; since the reaction is energetically allowed only if the proton and antiproton are in relative motion the A term does not vanish as in the previous reaction. Using this form for the $\bar{p}p \to K^+K^-K^+K^-$ amplitude we can calculate the differential cross section for this process (exact for threshold kaon production, an approximation for the general case) and using Eq. (15) we have the amplitude, and hence the differential cross section, for $\bar{p}p \to K^+K^-K^+K^-\pi^0\pi^0$.[9]

There are not enough data at present to make a comparison of differential cross sections with experiment.[10] Since both cross sections are proportional to A^2, we can integrate over all kinematic variables (noting again that we are thereby including cases where the pions are far from "soft") and obtain the ratio of total cross sections, $R = (\bar{p}p \to K^+K^-K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-K^+K^-)$ at various energies; the results appear in Table III.

The small numerical value of R at low energy is a kinematic effect, since just above the

TABLE III. Ratio of calculated cross sections $R = (\bar{p}p \to K^+K^-K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-K^+K^-)$ at various energies.

$\bar{p}$ lab momentum (GeV/c)	Center-of-mass energy (GeV)	R
2.0	2.49	0.000 05
2.5	2.62	0.0018
3.0	2.77	0.012
5.0	3.36	1.97
10.0	4.54	10.25
15.0	5.47	41.9

$K^+K^-K^+K^-\pi^0\pi^0$ threshold phase space is severely limited (this effect was present, but less pronounced, for $\bar{p}p \to K^+K^-\pi^0\pi^0$). Although there is no data for this process at present, our computation of it has value in stretching the limits of a straightforward soft-pion technique and it yields, at the very least, a ratio of cross sections of the right order of magnitude that exhibits an energy dependence similar to that for the $(\bar{p}p \to K^+K^-\pi^0\pi^0)/(\bar{p}p \to K^+K^-)$ case.

For both pairs of reactions studied our procedure does not take into account production of resonant intermediate states,[7] or deal with any particular model of the annihilation process.[11] Rather we have confined ourselves to a relatively straightforward application of the soft-pion formalism. We conclude that such a procedure gives a reasonable picture of pion emission for the two annihilation processes considered, even when annihilation takes place at nonvanishing incident $\bar{p}$ momentum.

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[5]See Fig. 3 of Ref. 2. In the present case all six diagrams yield nonvanishing contributions.

[6]Preliminary results for annihilation at vanishing momentum were reported in P. Nuthakki and R. A. Uritam, Bull. Am. Phys. Soc. 17, 777 (1972).

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[9]The (relatively lengthy) expressions for these differential cross sections, displayed with a judicious choice of variables for the spectrum, are presented in P. Nuthakki and R. A. Uritam, Boston College Report No. PH-EP7301, 1973 (unpublished).

[10]We reiterate our point of view that experimental data and theory should be presented and compared in as fully differential form as possible. This is not possible at present for the annihilation processes considered. See R. A. Uritam, Ref. 2; see also L. Van Hove, Phys. Rep. 1C, 347 (1971); J. D. Bjorken, in *Particles and Fields–1971,* edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971), p. 110.

[11]Some references to these are cited in Ref. 2.