sults to him.
[15]M. Gourdin, lectures presented at the International School of Physics "Ettore Majorana," Erice, 1971 (unpublished); and Nucl. Phys. B29, 601 (1971). Note that in Feynman's notation, D_1, D_2, and D_3 correspond to $\mathcal{P}$, $\mathfrak{N}$, and λ quark-parton distribution functions.
[16]See Ref. 13.
[17]Recent experimental information was presented by D. H. Perkins and Ph. Heusse at the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972 (unpublished).
[18]The results for the simple model of Sec. IV A and model (B1) differ from the results of model (B2) presented in the tables and figures typically by 5% or less.

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Model Approach to the High-Energy Asymptotic Limit*

K. S. Soh
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 26 June 1973)

The approach to the high-energy asymptotic limit of the pp total cross section and elastic cross section is calculated using a unitary field-theoretic model, in which simplified versions of all eikonal, checkerboard graphs are included. The sign of approach to the asymptotic limit is negative.

The approach to the high-energy asymptotic limit of the pp total cross section and the elastic cross section has been an interesting problem.[1] In this note, a unitary field-theoretic model will be presented, in which it is shown that the approach is from below, in agreement with Blankenbecler's argument, wherein this property is generated by strong, absorptive, unitary corrections to inelastic amplitudes.

The model interaction Lagrangian coupling nucleon, neutral vector meson (NVM), and scalar pion fields is given by

$$\mathcal{L}' = ig\bar{\psi}\sum_\mu \gamma_\mu W_\mu \psi + \tfrac{1}{2}\lambda\Pi\sum_\mu W_\mu{}^2 . \tag{1}$$

A formal construction of the eikonal function for nucleon-nucleon scattering has been given elsewhere[2]; it is

$$e^{i\chi} = \exp\left(-\tfrac{1}{2}i\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\right)\exp\left[ig^2\int\mathfrak{F}_{\rm I}^\mu\,\bar{\Delta}_c(\Pi)\mathfrak{F}_{\rm II}^\mu\right]\bigg|_{\Pi=0}, \tag{2}$$

where

$$\bar{\Delta}_c(\Pi) = \Delta_c(1+\lambda\Pi\Delta_c)^{-1},$$

$$\mathfrak{F}^\mu_{\rm I,II}(\omega) = p^\mu_{1,2}\int_{-\infty}^{\infty}d\xi\,\delta(\omega - z_{1,2} + \xi p_{1,2}).$$

In the high-energy limit ($s\to\infty$, $t/s\to 0$) it may be shown that

$$\int \mathfrak{F}_{\rm I}^\mu\,\bar{\Delta}_c(\Pi)\mathfrak{F}_{\rm II}^\mu = -\int\frac{dz_1^{(+)}}{2}\,dz_2^{(-)}\bar{\Delta}_c\,(z_1, z_2|\lambda\Pi). \tag{3}$$

A simplified model, in which the emission of arbitrary numbers of pions in the manner of Fig. 1(a) is replaced by pion emission in the form of Fig. 1(b), will be fully solvable in the sense that all operations of (2) may be performed. In this model $\bar{\Delta}_c(\Pi)$ is greatly simplified, and it is a straightforward calculation to show that

$$\int\frac{dz_1^{(+)}}{2}\,dz_2^{(-)}\,\bar{\Delta}_c(z_1, z_2|\Pi) = \frac{K_0(mb)}{2\pi} - m^2\int d^2x\,\frac{K_0(m|\vec{b}-\vec{x}|)}{2\pi}\frac{K_0(m|\vec{x}|)}{2\pi} + m^2\int d^4x\,\frac{K_0(m|\vec{b}-\vec{x}|)}{2\pi}\,\frac{K_0(m|\vec{x}|)}{2\pi}\,\delta(z_1-x)^{(-)}\delta\left(\frac{z_2-x}{2}\right)^{(+)}\frac{1}{1-(\lambda/m^2)\Pi(z_2-x)}, \tag{4}$$

where $\vec{b} = \vec{z}_1 - \vec{z}_2$. A further simplification, used where appropriate under the integrals of (4), is obtained by the replacement

$$\frac{K_0(m|\vec{x}|)}{2\pi} \to \frac{\delta^2(\vec{x})}{m^2} \tag{5}$$

and gives

$$\int \frac{dz_1^{(+)}}{2}\, dz_2^{(-)}\, \overline{\Delta}_c(z_1, z_2|\Pi) = \frac{K_0(mb)}{2\pi}\, \frac{1}{1-(\lambda/m^2)\Pi(\overline{z}_2, z_1^{(-)}, z_2^{(+)})}. \tag{6}$$

With these approximations, one has, easily,

$$e^{i\chi} = \exp\left(-\tfrac{1}{2} i \int \frac{\delta}{\delta\Pi} D_c \frac{\delta}{\delta\Pi}\right) \exp\left[\frac{-ig^2}{2\pi} K_0(mb)\left(1-\frac{\lambda}{m^2}\Pi(\overline{z}_2, z_1^{(-)}, z_2^{(+)})\right)^{-1}\right]\Bigg|_{\Pi=0}$$

$$= \int_{-\infty}^{\infty} d\xi \int_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{da}{2\pi} \exp\left[ia\xi + \frac{ig^2}{a}\frac{K_0(mb)}{2\pi} - \tfrac{1}{2}\xi^2 \frac{\lambda^2}{m^4}[-iD_c(0)]\right]. \tag{7}$$

The divergent quantity $-iD_c(0)$ has been interpreted and identified physically elsewhere[3]; it is

$$-iD_c(0) = \frac{\pi\kappa^2}{m^2} \ln\frac{s}{s_0} = \frac{\pi\kappa^2}{m^2} Y,$$

where κ^2 is a pion transverse-momentum cutoff parameter, and the phase space associated with each virtual pion grows $\sim Y$.

The two-dimensional impact parameter representation of the small-angle c.m. scattering amplitude is then

$$T(s,t) = \frac{is}{2m^2}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}(1-e^{i\chi(s,b)})$$

$$= \frac{is}{2m^2}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\left\{1-\int_{-\infty}^{\infty} d\xi \exp\left[-\xi^2\left(\frac{\lambda^2\pi\kappa^2}{2m^5}\right)Y\right]\int_{-\infty}^{\infty}\frac{da}{2\pi}\exp\left[ia\xi + \frac{ig^2}{a}\frac{K_0(mb)}{2\pi}\right]\right\}, \tag{8}$$

with $s=-(p_1+p_2)^2$ and $t=-(p_1-p_1')^2=q^2$. If we replace ξ by ξ' and a by a', respectively, where

$$\xi = \xi' Y_c^{-1/2}, \quad a = a' Y_c^{1/2}, \quad \text{and} \quad Y_c = \frac{\lambda^2\pi\kappa^2}{2m^5} Y, \tag{9}$$

there follows

$$T(s,t) = \frac{is}{2m^2}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\left[1-\int_{-\infty}^{\infty} d\xi'\, e^{-\xi'^2}\int_{-\infty}^{\infty}\frac{da'}{2\pi}\exp\left(ia\xi' + \frac{i}{a'}\frac{g^2K_0(mb)}{2\pi\sqrt{Y_c}}\right)\right]. \tag{10}$$

Using the optical theorem $\sigma_{\text{tot}} = (4m^2/s)\,\text{Im}T(s,0)$ and expanding with respect to $g^2K_0(mb)/2\pi\sqrt{Y_c}$, one finds

$$\sigma_{\text{tot}} = 2\int d^2b\, \frac{g^2K_0(mb)}{2\pi\sqrt{Y_c}}\left(\sqrt{\pi} - \frac{g^2K_0(mb)\sqrt{2}}{2\pi\sqrt{Y_c}}\right). \tag{11}$$

Even though this model gives a vanishing total cross section as $Y_c \to \infty$, the significance of the calculation is that it provides an explicit demonstration of the general mechanism in which the approach to the asymptotic total cross section is from below.

To study multiparticle production processes, one can use the formula[4]

$$\Sigma z^n P_n = \int \frac{d^3P_1}{(2\pi)^3}\frac{d^3P_2}{(2\pi)^3}\exp\left(+iz\int\frac{\delta}{\delta\Pi_1}D_+\frac{\delta}{\delta\Pi_2}\right) \times T(\Pi_1)\,T^\dagger(\Pi_2)\Bigg|_{\Pi_1,\Pi_2\to 0}, \tag{12}$$

where z is the "fugacity" and P_n is the unnormalized probability to produce n pions. This may be calculated here in a straightforward way:

$$\sigma_{\text{tot}}(z) \equiv \sum_{n=0}^{\infty} z^n\sigma_n = \int d^2b[1-2\,\text{Re}\,e^{i\chi}+I(z)], \tag{13}$$

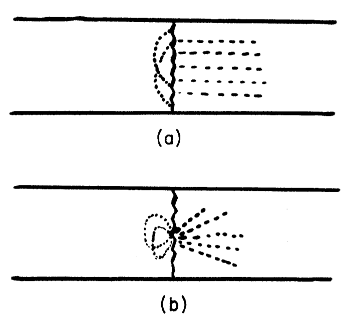

FIG. 1. (a) A typical fundamental graph of the exact theory. (b) The pictorial representation of (a) in the limit of large NVM mass between pion emissions.

where

$$I(z)=\int_{-\infty}^{\infty}\frac{da}{\sqrt{2\pi}}\exp\left[-\tfrac{1}{2}a^2-\frac{1}{a^4}\frac{g^2}{2\pi}\frac{K_0{}^2(mb)}{Y_c}(1-z^2)-\frac{i}{a}\frac{g^2K_0(mb)\sqrt{2}}{2\pi\sqrt{Y_c}}(1-z)\right]. \quad (14)$$

From (13) and (14), one sees that

$$\sigma_{\text{tot}}=2\int d^2b(1-\text{Re}\,e^{i\chi}), \quad (15)$$

$$\sigma_{\text{el}}=\tfrac{1}{2}\sigma_{\text{tot}}-\int d^2b\frac{1}{2\sqrt{\pi}}g^2K_0(mb)\frac{1}{\sqrt{Y_c}}, \quad (16)$$

showing that, asymptotically,

$$\tfrac{1}{2}\sigma_{\text{tot}}\sim\sigma_{\text{el}}\sim\sigma_{\text{inel}} .$$

Also, from (14) one finds at $z=1$ a singularity of $I(z)$ of type $e^{a(1-z)^{1/2}}$, and hence one cannot calculate the multiplicity $\langle n\rangle$ without inserting appropriate cutoffs to ensure energy conservation.[5]

In summary: In this simple model, it is seen that the asymptotic approach to $\sigma_{\text{tot}}(pp)$ is from below. In the absence of explicit energy conservation for inelastic processes, a singularity in the fugacity plane at $z=1$ appears in $\sigma_{\text{tot}}(z)$.

The author thanks Professor H. M. Fried for guidance and discussions throughout this work.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]The question concerns the sign of the two-Reggeon cut contribution to a cross section approaching its asymptotic limit from above or below. For arguments leading to a positive sign, see, for example, G. F. Chew, Phys. Rev. D 7, 934 (1973); H. D. I. Abarbanel, *ibid.* 6, 2788 (1972). For a negative sign, see, for example, R. Blankenbecler, SLAC Report No. SLAC-TN-72-13, 1972 (unpublished). See also A. R. White, CERN Report No. TH-1646, 1973 (unpublished); V. N. Gribov, Zh. Eksp. Teor. Fiz. 53, 654 (1967) [Sov. Phys.—JETP 26, 414 (1968)].

[2]R. Blankenbecler and H. M. Fried, Phys. Rev. D 8, 678 (1973). See also H. M. Fried, *Functional Methods and Models in Quantum Field Theory* (MIT Press, Cambridge, Mass., 1972), Chap. 10.

[3]H. M. Fried, Phys. Rev. D 6, 3562 (1972).

[4]Reference 2, Chap. 9.

[5]For example, see the method used by H. Cheng and T. T. Wu, Harvard Univ. report, 1973 (unpublished).

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Anomalous Magnetic Moment of the Electron

M. J. Levine*

Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 51213

Jon Wright†

Physics Department, University of Illinois, Urbana, Illinois 61801

(Received 8 June 1973)

We present detailed results of our previously published calculation of the sixth-order magnetic moment of the electron. The numerical accuracy has been improved, resulting in a value $a_e=\tfrac{1}{2}\alpha/\pi-0.32847(\alpha/\pi)^2+1.21(\alpha/\pi)^3$, which is in reasonable agreement with the latest experimental result.

I. INTRODUCTION

In a previous paper[1,2] we reported on a numerical calculation of the sixth-order anomalous magnetic moment of the electron. In this paper we present detailed results and a discussion of our method. At present there appears to be reasonable agreement between theoretical calculations and experiment. In addition to the actual calculation we present a new method of handling infrared singularities in Feynman graphs in numerical calculations.

In Sec. II we summarize past $g-2$ calculations and present our results for individual graphs. We also compare theory and experiment there. In Sec. III we briefly review the method of reducing the momentum-space integrals to parametric integrals. In Sec. IV we discuss the introduction of ultraviolet and infrared counterterms. We made no attempt to make use of Ward identities and we just evaluated each diagram in "cookbook" fashion. In Sec. V we discuss the actual numerical integration.

The results presented in Sec. II suggest that a