Quark-Parton Models with the Weinberg Neutral Current

Carl H. Albright*
Department of Physics, Northern Illinois University, DeKalb, Illinois 60115
(Received 2 July 1973)

The Weinberg neutral current is incorporated into several quark-parton models in order to investigate the neutral-current-induced neutrino and antineutrino cross sections. The results are rather insensitive to the particular model, provided a good fit is made to the charged-current data. Typically, the $\sigma^{\bar{\nu}}/\sigma^{\nu}$ ratio for the neutral-current reactions can be considerably larger than $\frac{1}{3}$ for $\sin^2\theta_W \gtrsim 0.25$. Minima of 0.20 and 0.38 occur in the neutral-current to charged-current cross-section ratios for neutrino and antineutrino reactions, respectively.

I. INTRODUCTION

The gauge theory of spontaneous symmetry breaking[1] of Weinberg and Salam is an appealing attempt to unite the weak and electromagnetic interactions into a coherent scheme in which the mutual renormalization of the interactions is effected. As orginally proposed,[1] this theory applied only to leptons and vector gauge bosons and required the existence of a weak neutral current coupling the leptons together.

Efforts to extend this theory to include the weak and electromagnetic interactions of hadrons confront the experimental fact that neutral currents of the strangeness-changing variety are strongly suppressed, if they exist at all.[2] This problem can be surmounted, as shown by Weinberg,[3] by ignoring the strangeness-changing weak interactions entirely or by using the four-quark model of Glashow, Iliopoulos, and Maiani,[4] which neatly finesses the difficulty. Many other schemes have since been put forward[5] which eliminate some or all of the neutral current couplings, generally with the introduction of heavy leptons.

With a plethora of theoretical models to choose from, the experimentalist is faced with the task of eliminating as many candidates as possible. In this paper, we attempt to cast the Weinberg neutral-current model into a framework where it can most easily be checked against the neutrino data.

To date only theoretical lower limits have been placed on the neutrino cross sections involving the Weinberg neutral current of hadrons. Pais and Treiman[6] and Paschos and Wolfenstein[7] have bounded the total inclusive cross section for $\nu_l + N \to \nu_l +$ anything, while Albright, Lee, Paschos, and Wolfenstein[8] have bounded the weak pion production cross sections for $\nu_l + N \to \nu_l + \pi + N$ and $\nu_l + N \to \nu_l + \pi +$ anything. The cross sections for the purely leptonic processes $\nu_l + e^- \to \nu_l + e^-$ and $\bar{\nu}_l + e^- \to \bar{\nu}_l + e^-$ can be calculated exactly and have been compared to the data by Chen and Lee[9] and 't Hooft.[9]

In order to arrive at firm predictions for the hadronic neutrino cross sections, one needs to invoke some theoretical model or calculational framework. The quark-parton model and the light-cone picture are two such fashionable schemes. Both of these techniques were used independently by Budny and Scharbach[10] and Riazuddin and Fayyazuddin[11] to obtain sum rules for the structure functions and bounds for the neutral-current cross sections. In this paper we reexamine the Weinberg scheme[3] in the quark-parton model by considering several specific models which lead to firm predictions for both neutrino and antineutrino cross sections.[12]

In the abovementioned work of Refs. 10 and 11, the authors used the four-quark model in the manner of Weinberg. In this version, the weak neutral current has the form[3]

$$J_\lambda^{(0)} = (V-A)_\lambda^3 - 2\sin^2\theta_W J_\lambda^{em} + \tfrac{1}{2} J_\lambda^c - \tfrac{1}{2} J_\lambda^s \quad , \tag{1.1a}$$

where θ_W represents the Weinberg mixing angle, the first and second terms refer to the neutral components of $V-A$ isospin current and electromagnetic current, respectively, and the third and fourth terms represent isoscalar contributions from the charmed and strange quark fields, respectively. In what follows, we shall neglect the latter two isoscalar contributions and write instead

$$J_\lambda^{(0)} = (V-A)_\lambda^3 - 2\sin^2\theta_W J_\lambda^{em} \; . \tag{1.1b}$$

The electromagnetic and Cabibbo currents can be expressed in the octet forms

$$J_\lambda^{em} = V_\lambda^{3+8/\sqrt{3}} \,, \tag{1.2}$$

$$J_\lambda^{(\pm)} = \cos\theta_C (V-A)_\lambda^{1\pm i2} + \sin\theta_C (V-A)_\lambda^{4\pm i5} \, . \tag{1.3}$$

This simplification for the weak neutral current allows us to use the three-quark parton model and the results of Gourdin,[13] who fitted this model to the charged-current neutrino data. The approximation introduced is not expected to be a serious one and should not distort our results to any great extent.[14]

In the next section we summarize briefly the quark-parton-model formalism. The general cross-section relations are given in Sec. III. In Sec. IV we specialize to several special models consistent with the electromagnetic and charged current data, and we present the numerical implications for the neutrino and antineutrino cross sections in Sec. V.

II. QUARK-PARTON-MODEL FORMALISM

The basic assumption in the parton model is that the electromagnetic and weak structure functions in the scaling region can be expressed as linear combinations of parton distribution functions. In the notation of Gourdin,[15] one can write the structure functions in terms of the parton distribution functions $D_j(x)$, $j=1,2,3,-1,-2,-3$, and the square of the electromagnetic and weak charges Q_j^2, I_j^2, and V_j^2 according to

$$2F_T^e(x)=\sum_j D_j(x)Q_j^2 \tag{2.1}$$

for the transverse electromagnetic function; for the helicity structure functions of the weak charged current one has

$$F_\pm^\nu(x)=\sum_j D_j(x)(1\mp\epsilon_j)(\cos^2\theta_C I_j^2+\sin^2\theta_C V_j^2) \equiv\cos^2\theta_C G_\pm^\nu+\sin^2\theta_C H_\pm^\nu \tag{2.2a}$$

and

$$F_\pm^{\bar\nu}(x)=\sum_j D_j(x)(1\mp\epsilon_j)(\cos^2\theta_C \bar I_j^2+\sin^2\theta_C \bar V_j^2), \tag{2.2b}$$

where

$$I_j^2=\langle j|I^-I^+|j\rangle,$$
$$\bar I_j^2=\langle j|I^+I^-|j\rangle,$$
$$V_j^2=\langle j|V^-V^+|j\rangle,$$
$$\bar V_j^2=\langle j|V^+V^-|j\rangle,$$

θ_C is the Cabibbo angle, ϵ_j is a signature factor which is $+1$ for partons and -1 for antipartons, and $x=q^2/2M\nu$ is the scaling variable.

The structure functions for the Weinberg neutral current of (1.1b) can be written in the three-quark model as

$$f_\pm^\nu(x)=f_\pm^{\bar\nu}=\sum_j D_j(x)[(1\mp\epsilon_j)(I_{3j}^2-2\sin^2\theta_W I_{3j}Q_j)+2\sin^4\theta_W Q_j^2] =g_\pm+h_\pm\sin^2\theta_W+i_\pm\sin^4\theta_W \tag{2.3}$$

in terms of the above charges with

$$I_{3j}^2=\langle j|I^3I^3|j\rangle,$$
$$2I_{3j}Q_j=\langle j|I^3Q+QI^3|j\rangle,$$

where I^3 is the third component of the $V-A$ isospin current generator.

The longitudinal structure functions $F_L^e(x)$, $F_0^\nu(x)$, and $f_0^\nu(x)$ receive contributions only from the gluons present in the nucleon. The structure functions used here are related to the more familiar W's in the scaling region by the relations

$$F_T^e=MW_1^e,$$
$$F_L^e=M[(1+\nu^2/q^2)W_2^e-W_1^e],$$
$$F_0^\nu=M[(1+\nu^2/q^2)W_2^\nu-W_1^\nu],$$
$$F_\pm^\nu=MW_1^\nu\pm\tfrac{1}{2}(\nu^2+q^2)^{1/2}W_3^\nu,$$

and likewise for f_0^ν, f_+^ν, and f_-^ν.

It is a simple matter to spell out the equations given in (2.1)–(2.3). While many of the relations have been tabulated previously,[15] they will be quite useful in Secs. III and IV, so we summarize them here for easy reference:

$$2F_T^e=\tfrac{4}{9}(D_1+D_{-1})+\tfrac{1}{9}(D_2+D_{-2}+D_3+D_{-3}), \tag{2.4a}$$
$$G_-^\nu=2D_2,\quad G_-^{\bar\nu}=2D_1, \tag{2.4b}$$
$$G_+^\nu=2D_{-1},\quad G_+^{\bar\nu}=2D_{-2}, \tag{2.4c}$$
$$H_-^\nu=2D_3,\quad H_-^{\bar\nu}=2D_1, \tag{2.4d}$$
$$H_+^\nu=2D_{-1},\quad H_+^{\bar\nu}=2D_{-3}, \tag{2.4e}$$
$$g_-^\nu=g_-^{\bar\nu}=\tfrac{1}{2}(D_1+D_2), \tag{2.5a}$$
$$g_+^\nu=g_+^{\bar\nu}=\tfrac{1}{2}(D_{-1}+D_{-2}), \tag{2.5b}$$
$$h_-^\nu=h_-^{\bar\nu}=-\tfrac{2}{3}(2D_1+D_2), \tag{2.5c}$$
$$h_+^\nu=h_+^{\bar\nu}=-\tfrac{2}{3}(2D_{-1}+D_{-2}), \tag{2.5d}$$
$$i_+^\nu=i_+^{\bar\nu}=i_-^\nu=i_-^{\bar\nu}=\tfrac{8}{9}(D_1+D_{-1})+\tfrac{2}{9}(D_2+D_{-2}+D_3+D_{-3}). \tag{2.5e}$$

Since the proton and neutron are related by interchanging the nonstrange quarks, one can relate the neutron and proton distribution functions by

$$D_{\pm1}^n=D_{\pm2}^p,\quad D_{\pm2}^n=D_{\pm1}^p,\quad D_{\pm3}^n=D_{\pm3}^p, \tag{2.6}$$

which in turn imply that

$$G_\pm^{\nu n}=G_\pm^{\bar\nu p},\quad G_\pm^{\bar\nu n}=G_\pm^{\nu p},$$
$$H_+^{\nu n}=G_+^{\bar\nu p},\quad H_+^{\bar\nu n}=H_+^{\bar\nu p},$$
$$H_-^{\nu n}=H_-^{\nu p},\quad H_-^{\bar\nu n}=G_-^{\nu p},$$
$$g_\pm^{\nu n}=g_\pm^{\bar\nu n}=g_\pm^{\nu p}=g_\pm^{\bar\nu p}. \tag{2.7}$$

It is a simple matter to derive sum rules from the above identities by computing zeroth and first moments of the parton distribution functions. For this purpose, one notes that the average number of quark partons of type j is just equal to the integral over the quark-parton distribution function $D_j(x)$:

$$\langle N_j \rangle = \int_0^1 dx\, D_j(x) . \tag{2.8}$$

The following restrictions hold for a proton target:

$$\begin{aligned} &\langle N_1 \rangle \geq 2, \\ &\langle N_2 \rangle \geq 1, \\ &\langle N_j \rangle \geq 0, \quad j = 3, -1, -2, -3 . \end{aligned} \tag{2.9}$$

The restrictions for a neutron target follow from (2.6).

Also of interest is the average fractional momentum d_j carried by the jth quark, which is given by

$$d_j = \int_0^1 dx\, x D_j(x) . \tag{2.10}$$

If all the momentum were carried by the quarks inside a nucleon, the sum $\sum_j d_j$ would add up to unity. This is not the case,[15] and it is conventional to introduce a parameter ϵ which represents the fractional longitudinal momentum carried by the gluons so that

$$0 \leq \epsilon \equiv 1 - \sum d_j < 1 . \tag{2.11}$$

The zeroth- and first-moment sum rules then follow with the help of (2.8)–(2.11). Since the results obtained are identical to those presented in Refs. 10 and 11, we refer the reader to those papers.

III. CROSS-SECTION RELATIONS

The differential cross sections for the electromagnetic and weak inclusive reactions are conventionally written as

$$\frac{d^2\sigma^{em}}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E} [2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)] \tag{3.1}$$

and

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dq^2 d\nu} = \frac{G^2}{2\pi} \frac{E'}{E} [2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2) \mp \frac{E+E'}{M} W_3 \sin^2(\theta/2)] \tag{3.2}$$

in terms of the incident lepton energy E, the final lepton energy E', the lepton scattering angle θ, and the invariants $q^2 = 4EE' \sin^2(\theta/2)$ and $\nu = E - E'$, all in the lab frame of reference.

In the scaling region, the above can be reexpressed in terms of the F's of Eqs. (2.1) and (2.2) and the variables $x = q^2/2M\nu$ and $y = E'/E$ by

$$\frac{d^2\sigma^{em}}{dx dy} = \frac{4\pi\alpha^2}{sx} \frac{1}{(1-y)^2} [(1+y^2) F_T^e(x) + 2y F_L^e(x)] , \tag{3.3}$$

$$\frac{d^2\sigma^{\nu}}{dx dy} = \frac{G^2}{2\pi} sx (y^2 F_+^{\nu} + F_-^{\nu} + 2y F_0^{\nu}) , \tag{3.4a}$$

and

$$\frac{d^2\sigma^{\bar{\nu}}}{dx dy} = \frac{G^2}{2\pi} sx (F_+^{\bar{\nu}} + y^2 F_-^{\bar{\nu}} + 2y F_0^{\bar{\nu}}) . \tag{3.4b}$$

The total neutrino cross sections follow from the above according to

$$\sigma^{\nu} = \frac{G^2}{2\pi} s \int_0^1 dx\, x (\tfrac{1}{3} F_+^{\nu} + F_-^{\nu} + F_0^{\nu}) , \tag{3.5a}$$

$$\sigma^{\bar{\nu}} = \frac{G^2}{2\pi} s \int_0^1 dx\, x (F_+^{\bar{\nu}} + \tfrac{1}{3} F_-^{\bar{\nu}} + F_0^{\bar{\nu}}) . \tag{3.5b}$$

They rise linearly with E in the scaling region since $s = 2ME$ at high energy. The neutral-current neutrino cross sections are obtained from Eqs. (3.5) by replacing the F's by f's.

We now express the cross sections on protons and neutrons in terms of the first moments of the parton distribution functions, i.e., in terms of the d_j's. For this purpose, we refer the cross sections to the proton distribution functions by means of Eqs. (2.5). Hence for the charged-current cross sections

$$\sigma_{ch}^{\nu p} = \sigma_0 [\cos^2\theta_C (\tfrac{2}{3} d_{-1} + 2d_2) + \sin^2\theta_C (\tfrac{2}{3} d_{-1} + 2d_3)] , \tag{3.6a}$$

$$\sigma_{ch}^{\nu n} = \sigma_0 [\cos^2\theta_C (\tfrac{2}{3} d_{-2} + 2d_1) + \sin^2\theta_C (\tfrac{2}{3} d_{-2} + 2d_3)] , \tag{3.6b}$$

where the definition $\sigma_0 = G^2 s/2\pi$ has been introduced. The antineutrino cross sections follow by replacing d_j by d_{-j}. For the neutral-current cross sections we find

$$\sigma_0^{\nu p} = \sigma_0 \{ \tfrac{1}{2}(d_1 + d_2) + \tfrac{1}{6}(d_{-1} + d_{-2}) - \tfrac{2}{3}(\tfrac{2}{3} d_{-1} + \tfrac{1}{3} d_{-2} + 2d_1 + d_2) \sin^2\theta_W + \tfrac{4}{27}[8(d_1 + d_{-1}) + 2(d_2 + d_{-2} + d_3 + d_{-3})] \sin^4\theta_W \} , \tag{3.7a}$$

$$\sigma_0^{\nu n} = \sigma_0 \{ \tfrac{1}{2}(d_1 + d_2) + \tfrac{1}{6}(d_{-1} + d_{-2}) - \tfrac{2}{3}(\tfrac{2}{3} d_{-2} + \tfrac{1}{3} d_{-1} + 2d_2 + d_1) \sin^2\theta_W + \tfrac{4}{27}[8(d_2 + d_{-2}) + 2(d_1 + d_{-1} + d_3 + d_{-3}) \sin^4\theta_W] \} . \tag{3.7b}$$

To obtain the antineutrino cross sections, the same prescription applies as above. We shall later find useful the averaged nucleon cross sections, which are defined by

$$\sigma^{\nu N} \equiv \tfrac{1}{2}(\sigma^{\nu p} + \sigma^{\nu n}) \tag{3.8}$$

and likewise for the antineutrino cross section.

The present experimental information involves the first moments of the electromagnetic structure functions[16] and the slopes and ratio of the charged-current cross sections.[17] We simply state the results below:

$$I^{ep}_{\rm expt} = 2\int_0^1 dx\, x F_T^{ep} = 0.18 \pm 0.018\,, \tag{3.9a}$$

$$I^{en}_{\rm expt} = 2\int_0^1 dx\, x F_T^{en} = 0.12 \pm 0.012\,, \tag{3.9b}$$

$$\sigma^{\nu N}_{\rm expt} = \sigma_0(0.450 \pm 0.090)\,, \tag{3.10a}$$

$$\sigma^{\bar\nu N}_{\rm expt} = \sigma_0(0.170 \pm 0.034)\,, \tag{3.10b}$$

$$R^{\rm ch}_{\rm expt} = \sigma^{\bar\nu N}_{\rm ch}/\sigma^{\nu N}_{\rm ch} = 0.377 \pm 0.023\,. \tag{3.10c}$$

In the next section we shall consider a few specific quark-parton models which are consistent with the above results.

IV. SPECIAL MODELS FITTING KNOWN DATA

In order to obtain numerical predictions for the neutral-current cross sections, we consider several quark-parton models which give reasonably good agreement with the charged-current cross sections. Gourdin has already phenomenologically fitted the data with his so-called equipartition model.[13] We look at several models here in order to measure the sensitivity of the predictions obtained for the neutral-current results.

A. A Simple Model

A characteristic feature of the neutrino data which must hold to a reasonable degree in any quark-parton model to be proposed is the value of the ratio $R^{\rm ch}_{\rm expt} \simeq \frac{1}{3}$ given in (3.10c). It follows from Eqs. (3.6) that the nonstrange-antiquark contributions d_{-1} and d_{-2} must be small. Here we set

$$d_{-1} = d_{-2} = 0 \tag{4.1a}$$

and choose

$$d_1 = d_2 = \tfrac{1}{3}(1-\epsilon)\,, \quad d_3 = d_{-3} = \tfrac{1}{6}(1-\epsilon)\,. \tag{4.1b}$$

This model is too simple in that the electroproduction integrals and neutrino cross sections must be equal for both proton and neutron targets, contrary to (3.9). We shall ignore this difficulty in this instance and look only at the results averaged over proton and neutron targets such as in (3.10).

From Eqs. (2.4c), (2.6), (2.10), (4.1), and (3.9)

$$\begin{aligned} I^{eN} &\equiv \tfrac{1}{2}(I^{ep} + I^{en}) \\ &= \tfrac{5}{18}(d_1 + d_2) + \tfrac{1}{9}(d_3 + d_{-3}) \\ &= \tfrac{2}{9}(1-\epsilon) \\ &= 0.15 \pm 0.015 \end{aligned} \tag{4.2}$$

or $\epsilon \simeq 0.32$. In other words, one third of the fractional longitudinal momentum is carried by the gluons. The implications for the charged-current neutrino cross sections follow from Eqs. (3.6):

$$\begin{aligned} \sigma^{\nu N}_{\rm ch} &= \sigma_0[\tfrac{1}{3}(d_{-1} + d_{-2}) + (d_1 + d_2)\cos^2\theta_C + 2d_3 \sin^2\theta_C] \\ &= \sigma_0(0.441)\,, \end{aligned} \tag{4.3a}$$

$$\sigma^{\bar\nu N}_{\rm ch} = \sigma_0(0.170)\,, \tag{4.3b}$$

$$R^{\rm ch} = 0.370\,. \tag{4.3c}$$

These results are all in very good agreement with the experimental values quoted in Eqs. (3.10).

The predictions for the neutral-current reactions are obtained from Eqs. (3.7), and we find

$$\begin{aligned} \sigma_0^{\nu N} &= \sigma_0\{\tfrac{1}{2}(d_1 + d_2) + \tfrac{1}{6}(d_{-1} + d_{-2}) - [d_1 + d_2 + \tfrac{1}{3}(d_{-1} + d_{-2})]\sin^2\theta_W + \tfrac{4}{27}[5(d_1 + d_2 + d_{-1} + d_{-2}) + 2(d_3 + d_{-3})]\sin^4\theta_W\} \\ &= \sigma_0[\tfrac{1}{3} - \tfrac{2}{3}\sin^2\theta_W + \tfrac{16}{27}\sin^4\theta_W](1-\epsilon)\,, \end{aligned} \tag{4.4a}$$

$$\sigma_0^{\bar\nu N} = \sigma_0[\tfrac{1}{9} - \tfrac{2}{9}\sin^2\theta_W + \tfrac{16}{27}\sin^4\theta_W](1-\epsilon)\,. \tag{4.4b}$$

The results depend critically on the Weinberg angle, and we shall defer numerical estimates to Sec. V.

B. Equipartition Model of Gourdin

A somewhat more attractive model is that of Gourdin[13] in which the partons are assumed to share the longitudinal momentum equally among themselves. In particular,

$$\begin{aligned} d_j &= \langle N_j/N\rangle\,, \\ d_1 - d_{-1} &= \langle 2/N\rangle\,, \\ d_2 - d_{-2} &= \langle 1/N\rangle\,, \\ d_3 - d_{-3} &= 0\,, \end{aligned} \tag{4.5}$$

where N is the total number of partons (quarks plus gluons) and the d's refer to the proton constituents as before. In this language, ϵ is the fractional number of partons which are gluons. Hence we can also write

$$d_j = \langle N_j/N_Q \rangle (1-\epsilon) \,, \tag{4.6}$$

where N_Q is the total number of quarks and antiquarks in the proton.

We consider two models in this framework:

$$\text{(B1)} \quad \langle N_1 \rangle = 2 \,, \quad \langle N_2 \rangle = 1 \,, \quad \langle N_3 \rangle = \langle N_{-3} \rangle = 1 \,, \quad \langle N_{-1} \rangle = \langle N_{-2} \rangle = 0 \,; \tag{4.7a}$$

$$d_1 = \tfrac{2}{5}(1-\epsilon) \,, \quad d_2 = \tfrac{1}{5}(1-\epsilon) \,, \quad d_3 = d_{-3} = \tfrac{1}{5}(1-\epsilon) \,, \quad d_{-1} = d_{-2} = 0 \,. \tag{4.7b}$$

$$\text{(B2)} \quad \langle N_1 \rangle = 2 \,, \quad \langle N_2 \rangle = 1 \,, \quad \langle N_3 \rangle = \langle N_{-3} \rangle = \tfrac{1}{2} \,, \quad \langle N_{-1} \rangle = \langle N_{-2} \rangle = 0 \,; \tag{4.8a}$$

$$d_1 = \tfrac{1}{2}(1-\epsilon) \,, \quad d_2 = \tfrac{1}{4}(1-\epsilon) \,, \quad d_3 = d_{-3} = \tfrac{1}{8}(1-\epsilon) \,, \quad d_{-1} = d_{-2} = 0 \,. \tag{4.8b}$$

In terms of model (B1), the gluon contribution is estimated to be $\epsilon \simeq 0.29$, so that the electroproduction integrals become

$$I^{ep}_{B1} = 0.174 \,, \quad I^{en}_{B1} = 0.126 \,. \tag{4.9}$$

The two integrals are different and well within the experimental values quoted in (3.9). The charged-current neutrino cross sections are given by

$$\sigma^{\nu p}_{ch,B1} = \sigma_0(0.284) \,, \quad \sigma^{\nu n}_{ch,B1} = \sigma_0(0.553) \,, \quad \sigma^{\bar\nu p}_{ch,B1} = \sigma_0(0.204) \,, \quad \sigma^{\bar\nu n}_{ch,B1} = \sigma_0(0.109) \,. \tag{4.10a}$$

Averaged over protons and neutrons,

$$\sigma^{\nu N}_{ch,B1} = \sigma_0(0.418) \,, \quad \sigma^{\bar\nu N}_{ch,B1} = \sigma_0(0.157) \,, \quad R^{ch}_{B1} = 0.375 \,. \tag{4.10b}$$

The averaged values of the cross sections and ratio are also in good agreement with (3.10), while $\sigma^{\nu n}$ is nearly twice as large as $\sigma^{\nu p}$. This has been observed already by Gourdin and appears to be consistent with the present experimental information.

The predictions for the neutral-current cross sections follow from Eqs. (3.7):

$$\begin{aligned}
\sigma^{\nu p}_{0,B1} &= \sigma_0\left[\tfrac{3}{10} - \tfrac{2}{3}\sin^2\theta_W + \tfrac{88}{135}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\nu n}_{0,B1} &= \sigma_0\left[\tfrac{3}{10} - \tfrac{8}{15}\sin^2\theta_W + \tfrac{64}{135}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\bar\nu p}_{0,B1} &= \sigma_0\left[\tfrac{1}{10} - \tfrac{2}{9}\sin^2\theta_W + \tfrac{88}{135}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\bar\nu n}_{0,B1} &= \sigma_0\left[\tfrac{1}{10} - \tfrac{8}{45}\sin^2\theta_W + \tfrac{64}{135}\sin^4\theta_W\right](1-\epsilon) \,.
\end{aligned} \tag{4.10c}$$

In terms of model (B2), the corresponding numbers are $\epsilon \simeq 0.365$ and

$$I^{ep}_{B2} = 0.177 \,, \quad I^{en}_{B2} = 0.124 \,; \tag{4.11}$$

$$\sigma^{\nu p}_{ch,B2} = \sigma_0(0.310) \,, \quad \sigma^{\nu n}_{ch,B2} = \sigma_0(0.610) \,, \quad \sigma^{\bar\nu p}_{ch,B2} = \sigma_0(0.220) \,, \quad \sigma^{\bar\nu n}_{ch,B2} = \sigma_0(0.114) \,; \tag{4.12a}$$

$$\sigma^{\nu N}_{ch,B2} = \sigma_0(0.460) \,, \quad \sigma^{\bar\nu N}_{ch,B2} = \sigma_0(0.167) \,, \quad R^{ch}_{B2} = 0.364 \,. \tag{4.12b}$$

The general remarks following Eqs. (4.10) also apply here. The neutral-current cross sections are given by

$$\begin{aligned}
\sigma^{\nu p}_{0,B2} &= \sigma_0\left[\tfrac{3}{8} - \tfrac{5}{6}\sin^2\theta_W + \tfrac{20}{27}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\nu n}_{0,B2} &= \sigma_0\left[\tfrac{3}{8} - \tfrac{2}{3}\sin^2\theta_W + \tfrac{14}{27}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\bar\nu p}_{0,B2} &= \sigma_0\left[\tfrac{1}{8} - \tfrac{5}{18}\sin^2\theta_W + \tfrac{20}{27}\sin^4\theta_W\right](1-\epsilon) \,, \\
\sigma^{\bar\nu n}_{0,B2} &= \sigma_0\left[\tfrac{1}{8} - \tfrac{2}{9}\sin^2\theta_W + \tfrac{14}{27}\sin^4\theta_W\right](1-\epsilon) \,.
\end{aligned} \tag{4.12c}$$

The results for the two models (B1) and (B2) differ little, since we confined our attention to the situation where $d_{-1} = d_{-2} = 0$ in order to satisfy the charged-current relations (3.10). We close this section by noting that the parameters for the second model are very close to those obtained by Gourdin for the phenomenological fit to the charged-current data[13]:

$$\begin{aligned}
d_1 &= 0.31 \pm 0.06 \,, \\
d_{-1} &= 0.01 \pm 0.06 \,, \\
d_2 &= 0.15 \pm 0.04 \,, \\
d_{-2} &= 0 \pm 0.04 \,, \\
d_3 &= d_{-3} = 0.087 \pm 0.023 \,, \\
\epsilon &= 0.355 \pm 0.185 \,.
\end{aligned} \tag{4.13}$$

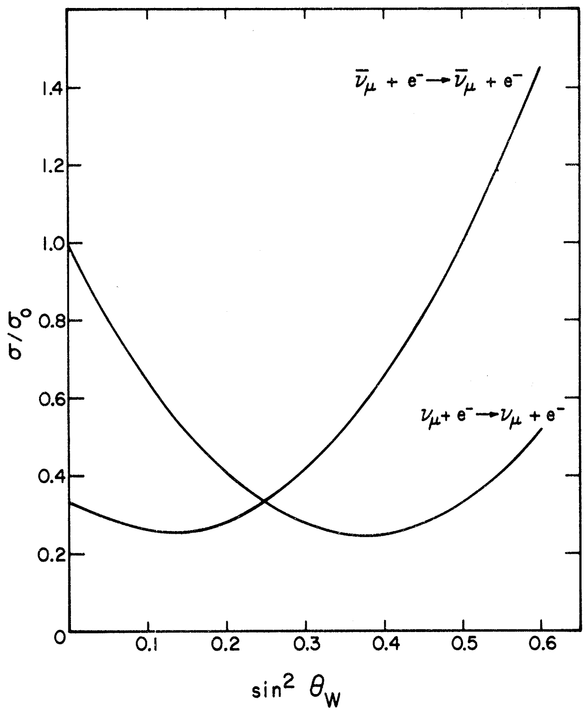

FIG. 1. Neutral-current cross sections for the leptonic reactions $\nu_\mu + e^- \to \nu_\mu + e^-$ and $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ in units of $\sigma_0 = G^2 s/2\pi$.

V. NUMERICAL ESTIMATES FOR THE NEUTRAL-CURRENT CROSS SECTIONS

While the Weinberg angle θ_W, like the Cabibbo angle, is not determined by theory except possibly at some deep level, it is of course restricted by experiment. The same Weinberg angle appears in the purely leptonic reactions

$$\nu_\mu + e^- \to \nu_\mu + e^- , \qquad (5.1a)$$

$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^- , \qquad (5.1b)$$

and it is here that the gauge theory has been most severely tested.

The matrix elements for the above reactions are given simply by

$$M(\nu_\mu + e^- \to \nu_\mu + e^-) = \frac{G}{\sqrt{2}}\, \bar{u}_e \gamma_\lambda [(\tfrac{1}{2} - 2\sin^2\theta_W) + \tfrac{1}{2}\gamma_5] \times u_e \bar{u}_\nu \gamma_\lambda (1+\gamma_5) u_\nu , \qquad (5.2a)$$

$$M(\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-) = \frac{G}{\sqrt{2}}\, \bar{u}_e \gamma_\lambda [(\tfrac{1}{2} - 2\sin^2\theta_W) + \tfrac{1}{2}\gamma_5] \times u_e \bar{v}_\nu \gamma_\lambda (1+\gamma_5) v_\nu , \qquad (5.2b)$$

from which the cross sections are computed to be

$$\sigma(\nu_\mu + e^- \to \nu_\mu + e^-) = \sigma_0 (1 - 4\sin^2\theta_W + \tfrac{16}{3}\sin^4\theta_W) , \qquad (5.3a)$$

$$\sigma(\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-) = \sigma_0 (\tfrac{1}{3} - \tfrac{4}{3}\sin^2\theta_W + \tfrac{16}{3}\sin^4\theta_W). \qquad (5.3b)$$

The angle dependences of these cross sections are given in Fig. 1. The present experimental information[17] from CERN restricts the Weinberg angle to lie in the range $\sin^2\theta_W \lesssim 0.60$.

The results for the neutral-current cross sections calculated in Sec. IV are so similar that we present results only for the (B2) equipartition model.[18] In Table I we give the neutrino and antineutrino cross sections on protons and neutrons, while the cross sections and ratios averaged over nucleons are presented in Table II. These numerical results are graphed in Figs. 2 and 3. For $\sin^2\theta_W = 0.33$, the cross-section ratio, $\sigma_0^{\nu N}/\sigma_{ch}^{\nu N}$, has the value 0.27, which is just slightly higher than the theoretical minimum of 0.23 obtained by Pais and Treiman[6] and Paschos and Wolfenstein.[7]

The most striking feature of our results is that the antineutrino-neutrino ratio for the neutral-current cross sections, $\sigma_0^{\bar{\nu}N}/\sigma_0^{\nu N}$, is noticeably different from the value of 0.377 measured for the charged-current ratio, $\sigma_{ch}^{\bar{\nu}N}/\sigma_{ch}^{\nu N}$, so long as $\sin^2\theta_W \gtrsim 0.25$. This fact plus the relatively large values for $\sigma_0^{\bar{\nu}N}/\sigma_{ch}^{\bar{\nu}N}$ should play a key role in helping the

TABLE I. Neutral-current neutrino and antineutrino cross sections on protons and neutrons scaled to $\sigma_0 = G^2 s/2\pi$.

$\sin^2\theta_W$	$\sigma_0^{\nu p}/\sigma_0$	$\sigma_0^{\nu n}/\sigma_0$	$\sigma_0^{\bar{\nu} p}/\sigma_0$	$\sigma_0^{\bar{\nu} n}/\sigma_0$
0	0.238	0.238	0.079	0.079
0.20	0.151	0.167	0.063	0.065
0.33	0.114	0.134	0.072	0.069
0.40	0.102	0.121	0.084	0.076
0.60	0.090	0.103	0.143	0.114
0.80	0.116	0.110	0.239	0.178
1.00	0.180	0.144	0.374	0.268

TABLE II. Neutral-current neutrino and antineutrino cross sections on nucleons in units of $\sigma_0 = G^2 s/2\pi$. Ratios of these cross sections relative to each other and to the charged-current cross sections are also given.

$\sin^2\theta_W$	$\sigma_0^{\nu N}/\sigma_0$	$\sigma_0^{\bar{\nu} N}/\sigma_0$	$\sigma_0^{\bar{\nu} N}/\sigma_0^{\nu N}$	$\sigma_0^{\nu N}/\sigma_{ch}^{\nu N}$	$\sigma_0^{\bar{\nu} N}/\sigma_{ch}^{\bar{\nu} N}$
0	0.238	0.079	0.33	0.52	0.47
0.20	0.159	0.064	0.40	0.35	0.38
0.33	0.124	0.071	0.57	0.27	0.43
0.40	0.112	0.080	0.71	0.24	0.48
0.60	0.097	0.128	1.32	0.21	0.77
0.80	0.113	0.208	1.84	0.25	1.24
1.00	0.162	0.321	1.98	0.35	1.92

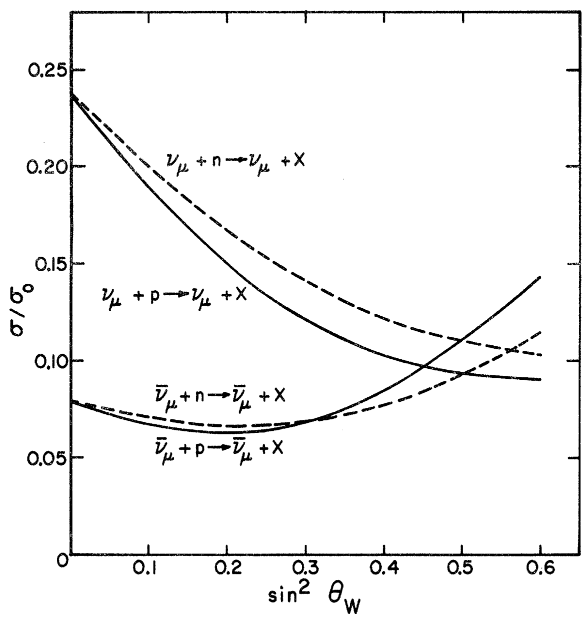

FIG. 2. Neutral-current cross sections scaled to $\sigma_0 = G^2 s/2\pi$ as functions of $\sin^2\theta_W$ for the model (B2).

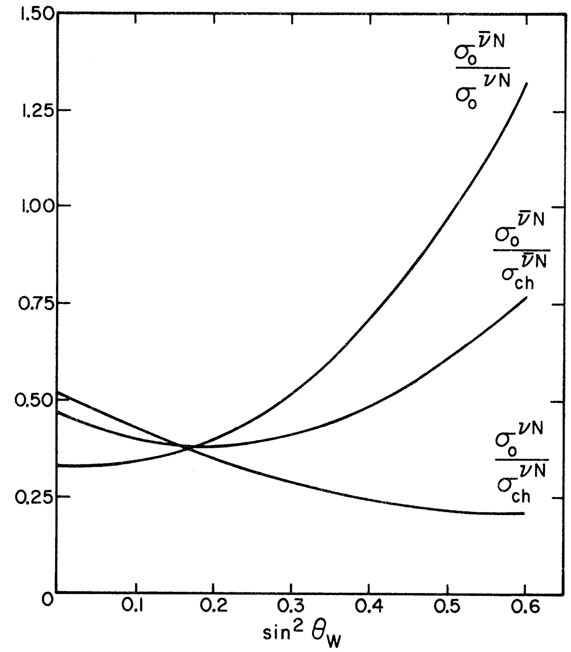

FIG. 3. Cross-section ratios as functions of $\sin^2\theta_W$ for the model (B2).

experimentalists to discriminate a true neutral-current reaction from background.

We conclude by remarking that our results derived from a three-quark ($\mathcal{P}$, $\mathfrak{N}$, and λ) version of the quark-parton model differ little from those recently obtained by Sehgal,[14] who used a four-quark version with different assumptions. In particular, our neutral-current cross-section predictions nearly coincide with Sehgal's for small Weinberg angles and depart toward 10–20% larger values as $\sin^2\theta_W$ approaches unity. It appears that the divergence arises due to different handling of the isoscalar contributions, which are more important at large θ_W. This observation is in keeping with the remarks made in the Introduction, where the simplifying assumption was proposed.

ACKNOWLEDGMENT

The author wishes to acknowledge the hospitality of the theory group at the National Accelerator Laboratory, where this work was initiated.

*On sabbatical leave during the 1973-74 academic year at the Theoretical Division, CERN, 1211 Geneva 23.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory*, edited by N. Svartholm (Almquist, Stockholm, 1968).

[2]See e.g., C. H. Albright and R. J. Oakes, Phys. Rev. D 2, 1883 (1970).

[3]S. Weinberg, Phys. Rev. D 5, 1412 (1972).

[4]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[5]We cite just a few: B. W. Lee, Phys. Rev. D 6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972); I. Bars, A. B. Halpern, and M. Yoshimura, Phys. Rev. Lett. 29, 969 (1972).

[6]A. Pais and S. B. Treiman, Phys. Rev. D 6, 2700 (1972).

[7]E. A. Paschos and L. Wolfenstein, Phys. Rev. D 7, 91 (1973).

[8]C. H. Albright, B. W. Lee, E. A. Paschos, and L. Wolfenstein, Phys. Rev. D 7, 2220 (1973).

[9]G. 't Hooft, Phys. Lett. 37B, 195 (1971); H. H. Chen and B. W. Lee, Phys. Rev. D 5, 1874 (1972).

[10]R. Budny, Phys. Lett. 39B, 533 (1972); R. Budny and P. N. Scharbach, Phys. Rev. D 6, 3651 (1972).

[11]Riazuddin and Fayyazuddin, Phys. Rev. D 6, 2032 (1972).

[12]This paper is an expansion of a note circulated earlier as NAL Report No. NAL-PUB-73/23-THY (unpublished).

[13]M. Gourdin, Nucl. Phys. B53, 509 (1973).

[14]While this paper was in preparation, the author received a copy of a report by L. M. Sehgal in which predictions of the Weinberg model for neutral currents in inclusive neutrino reactions were also investigated, but from the four-quark-model point of view. A comparison of the results is made in Sec. V. The author wishes to thank Dr. Sehgal for communicating his re-

sults to him.

[15]M. Gourdin, lectures presented at the International School of Physics "Ettore Majorana," Erice, 1971 (unpublished); and Nucl. Phys. B29, 601 (1971). Note that in Feynman's notation, D_1, D_2, and D_3 correspond to $\mathcal{P}$, $\mathcal{N}$, and λ quark-parton distribution functions.

[16]See Ref. 13.

[17]Recent experimental information was presented by D. H. Perkins and Ph. Heusse at the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972 (unpublished).

[18]The results for the simple model of Sec. IV A and model (B1) differ from the results of model (B2) presented in the tables and figures typically by 5% or less.

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Model Approach to the High-Energy Asymptotic Limit*

K. S. Soh

Department of Physics, Brown University, Providence, Rhode Island 02912

(Received 26 June 1973)

The approach to the high-energy asymptotic limit of the pp total cross section and elastic cross section is calculated using a unitary field-theoretic model, in which simplified versions of all eikonal, checkerboard graphs are included. The sign of approach to the asymptotic limit is negative.

The approach to the high-energy asymptotic limit of the pp total cross section and the elastic cross section has been an interesting problem.[1] In this note, a unitary field-theoretic model will be presented, in which it is shown that the approach is from below, in agreement with Blankenbecler's argument, wherein this property is generated by strong, absorptive, unitary corrections to inelastic amplitudes.

The model interaction Lagrangian coupling nucleon, neutral vector meson (NVM), and scalar pion fields is given by

$$\mathcal{L}' = ig\bar{\psi}\sum_\mu \gamma_\mu W_\mu \psi + \tfrac{1}{2}\lambda\Pi\sum_\mu W_\mu{}^2 . \tag{1}$$

A formal construction of the eikonal function for nucleon-nucleon scattering has been given elsewhere[2]; it is

$$e^{i\chi} = \exp\left(-\tfrac{1}{2}i\int \frac{\delta}{\delta\Pi} D_c \frac{\delta}{\delta\Pi}\right)\exp\left[ig^2\int \mathcal{F}_{\rm I}^\mu \,\overline{\Delta}_c(\Pi)\mathcal{F}_{\rm II}^\mu\right]\Bigg|_{\Pi=0} , \tag{2}$$

where

$$\overline{\Delta}_c(\Pi) = \Delta_c(1+\lambda\Pi\Delta_c)^{-1} ,$$

$$\mathcal{F}^\mu_{\rm I,II}(\omega) = p^\mu_{1,2}\int_{-\infty}^{\infty} d\xi\, \delta(\omega - z_{1,2} + \xi p_{1,2}) .$$

In the high-energy limit ($s\to\infty$, $t/s\to 0$) it may be shown that

$$\int \mathcal{F}_{\rm I}^\mu\, \overline{\Delta}_c(\Pi)\mathcal{F}^\mu_{\rm II} = -\int \frac{dz_1^{(+)}}{2} dz_2^{(-)}\overline{\Delta}_c(z_1, z_2|\lambda\Pi) . \tag{3}$$

A simplified model, in which the emission of arbitrary numbers of pions in the manner of Fig. 1(a) is replaced by pion emission in the form of Fig. 1(b), will be fully solvable in the sense that all operations of (2) may be performed. In this model $\overline{\Delta}_c(\Pi)$ is greatly simplified, and it is a straightforward calculation to show that

$$\int \frac{dz_1^{(+)}}{2} dz_2^{(-)}\overline{\Delta}_c(z_1, z_2|\Pi) = \frac{K_0(mb)}{2\pi} - m^2\int d^2x\, \frac{K_0(m|\vec{b}-\vec{x}|)}{2\pi}\frac{K_0(m|\vec{x}|)}{2\pi} + m^2\int d^4x \frac{K_0(m|\vec{b}-\vec{x}|)}{2\pi}\frac{K_0(m|\vec{x}|)}{2\pi}\,\delta(z_1-x)^{(-)}\delta\left(\frac{z_2-x}{2}\right)^{(+)}\frac{1}{1-(\lambda/m^2)\Pi(z_2-x)} , \tag{4}$$

where $\vec{b}=\vec{z}_1-\vec{z}_2$. A further simplification, used where appropriate under the integrals of (4), is obtained by the replacement

$$\frac{K_0(m|\vec{x}|)}{2\pi} \to \frac{\delta^2(\vec{x})}{m^2} \tag{5}$$