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PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

$|\Delta I| = \frac{1}{2}$ Rule From the Symmetric Quark Model

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(Received 9 March 1973)

Two topics are treated in this paper: the explanation of the $|\Delta I| = \frac{1}{2}$ rule based on the symmetric quark model and the tests for this explanation in Ω^- nonleptonic decays. From the "color-quark," the three-triplet, and the paraquark models, the $|\Delta I| = \frac{1}{2}$ rule follows for the octet-hyperon, the Ω^-, and the kaon weak nonleptonic decays as a consequence of current algebra, pion PCAC (partially conserved axial-vector current), and dispersion relations. Gronau's successful numerical results for the octet-hyperon decay amplitudes also follow in these alternatives to the Bose-quark model. However, though the origin of both explanations is the Fierz reshuffling property of the $V\pm A$ interactions, the explanations of the $|\Delta I| = \frac{1}{2}$ rule are quite distinct, e.g., in the Bose-quark model this rule is *exact* whereas in the other versions it is only *approximate*, being violated by continuum contributions to the absorptive parts. Because $\langle 0|\mathcal{H}_w|K\rangle$ and $\langle \pi|\mathcal{H}_w|K\rangle$ vanish in the symmetric quark model, the usual current-algebra soft-pion argument for $|\Delta I| = \frac{1}{2}$ rule and the K^*-pole-dominance assumption (as a Feynman diagram) for $K_1^0 \to 2\pi$ are *not* convincing. On the other hand, the ordinary Fermi-quark model supplemented with octet dominance can be excluded, as it predicts $D/F = 3$ in the SU(3) limit for the matrix element of the parity-conserving Hamiltonian for two baryons in the nucleon octet ($D/F \simeq -0.85$ from P-wave fits). The ΛK^- decay mode of the Ω^- should be *predominantly* P wave (parity-conserving), whereas the $\Xi\pi$ mode should have the P wave strongly *suppressed* and comparable to the D wave (parity-violating). This implies $\Gamma(\Omega^- \to \Xi\pi)/\Gamma(\Omega^- \to \Lambda K^-) \ll 1$. The estimated total Ω^- decay rate is consistent with the present experimental number.

I. INTRODUCTION

One of the important questions in the phenomenological theory of weak interactions is whether the current-current form of the effective Hamiltonian can successfully explain the observed $|\Delta S| = 1$ nonleptonic weak decays. Unfortunately, even the most striking qualitative aspect of these decays, namely, the well-established $|\Delta I| = \frac{1}{2}$ rule, does not immediately follow from the current-current theory because the product of an $I = \frac{1}{2}$ current with an $I = 1$ current can belong to an $I = \frac{1}{2}$ or $\frac{3}{2}$ representation of the isospin group. Recently, using the Fierz reshuffling property of the $V\pm A$ interaction, several authors[1] have shown that in the version of the *symmetric quark model*[2] in which the quarks are bosons (Bose-quark model), the nonleptonic weak Hamiltonian is a member of an SU(3) octet and so the $|\Delta I| = \frac{1}{2}$ rule immediately follows. Furthermore, Gronau[3] has shown that the Bose-quark model leads to the following interesting relations:

$$\langle \Sigma^0|\mathcal{H}_w|\Xi^0\rangle = 0, \tag{1}$$

$$\langle B|\mathcal{H}_w|B_{10}\rangle = 0, \tag{2}$$

$$|\langle B^*|\mathcal{H}_w|B_g\rangle| \ll |\langle B_g|\mathcal{H}_w|B_g\rangle|, \tag{3}$$

and

$$|\langle M^*|\mathcal{H}_w|M_g\rangle| \ll |\langle M_g|\mathcal{H}_w|M_g\rangle|, \tag{4}$$

where $\mathcal{H}_w$ is the parity-conserving (pc) or parity-violating (pv) piece of the nonleptonic weak Hamiltonian density. B_{10} is a $3q$ baryon belonging to the $\frac{3}{2}^+$ s-wave decuplet while B is any $3q$ baryon. B_g is a ground-state s-wave $3q$ baryon (56, $L^P = 0^+$) in the quark model and B^* is any baryon whose three quarks are in an orbitally or radially excited state. Similarly, M_g is a ground-state s-

wave $q\bar{q}$ meson ($\underline{36}$, $L^P = 0^-$), whereas M^* is a spatially excited $\overline{q\bar{q}}$ meson. The derivation of these equations does not prescribe the range of momentum transfer and in this paper we will assume they are valid at least when the squared four-momentum transfer vanishes. Using Eqs. (1) through (4), Gronau[3] has demonstrated that one can quantitatively explain the nonleptonic weak decays of all the hyperons in the nucleon octet in the framework of current algebra and PCAC (partially conserved axial-vector current), with SU(3) conserved at the vertices but broken in the hadronic masses.[4] The two-parameter fit[3] of the eight independent S- and P-wave amplitudes is in very good agreement with experiment. It should be noted that in order to obtain this good fit, contributions of the K^* pole diagram to the parity-violating decay amplitudes had to be included.

Although these results are impressive, the Bose-quark model itself suffers from some serious difficulties. First of all, from a fundamental point of view it is disturbing because in local field theory the association of Bose statistics with half-integer-spin quarks leads to a conflict with the microcausality condition for physical currents constructed from these quark fields. Hence, there probably exists a conflict with basic analytic consequences, such as dispersion relations involving these currents. Second, if the constituents of hadrons are bosons, then the fermion character of the baryons becomes a deep mystery. It is even more troubling that in the Bose-quark model, the physical currents formed out of the quark fields have the wrong charge-conjugation properties. These are serious difficulties even if the quarks are fictitious mathematical objects.

It is more natural to resolve the statistics problem of the ordinary Fermi-quark model by introducing a second degree of freedom for each of the quarks, as is done in the "color-quark model"[5] and in the three-triplet quark model in either the SUB (see Ref. 6) or Han-Nambu[7] versions. It is interesting to ask whether the $|\Delta I| = \frac{1}{2}$ rule can be derived in these models where the quarks obey Fermi statistics. It was partly answered by the work of Pati and Woo.[8,9] In the three-triplet model, they took the simplest choice[10] for the weak current and demonstrated that although $\mathcal{H}_w$ does not entirely belong to an SU(3) octet, only the octet part of $\mathcal{H}_w$ will contribute to the matrix element $\langle B_1|\mathcal{H}_w|B_2\rangle$, where B_1 and B_2 are $3q$ baryon states and at least one[11] is an SU(3)$''$ singlet. An equivalent result holds in the "color-quark" model (see Appendix A), as it is quite similar to the three-triplet model—the principal difference is that in it a different choice is made for the electromagnetic charges for the nine fundamental quarks. Similarly in the paraquark model[12] (see Appendix B), only the octet part of $\mathcal{H}_w$ contributes between two $3q$-baryon states even though the para-Fermi fields of order three do not Fierz reshuffle. The octet dominance of this matrix element when supplemented with current algebra and PCAC, Pati and Woo argued, will lead[13] to the $|\Delta I| = \frac{1}{2}$ rule for the nonleptonic decays of the hyperons in the nucleon octet.

However, unlike in the Bose-quark model, it is *not* obvious that these versions of the symmetric quark model imply octet dominance in the nonleptonic decays of the K meson and the Ω^- hyperon. In fact, even if one accepts the extrapolation of one or more pions to zero four-momentum in K meson decays, the usual current-algebra explanation[8,9] is *not* convincing, because in the symmetric quark model the matrix elements $\langle 0|\mathcal{H}_w|K\rangle$ and $\langle \pi|\mathcal{H}_w|K\rangle$ vanish.[13] [The vanishing of the latter is easily found (see Sec. III), as a side result of dispersion calculations for the Ω^- decays.] Second, although Eqs. (1) through (4) of Gronau also hold in the color-quark, the three-triplet, and the paraquark models (see Appendixes), the vanishing of these matrix elements is not consistent with the assumption of Gronau that the $K_1^0 \to 2\pi$ amplitude is dominated by the K^* pole in the sense of a Feynman diagram.

These matters are resolved in Sec. IV: We first show that the $|\Delta I| = \frac{1}{2}$ rule and Gronau's successful numerical fit[3] of the S- and P-wave amplitudes of the octet hyperon nonleptonic decays can be reproduced by a dispersion-theoretic approach valid in all versions of the symmetric quark model. Throughout this paper we apply current algebra and PCAC working in a dispersion-relation framework based on Regge asymptotic behavior and resonance saturation of absorptive parts according to Eqs. (1) through (4). Second, we then argue for K^* dominance in the appropriate once-subtracted dispersion relations for the invariant amplitudes involved in the process $K_1^0 + S^{\rm pv} \to 2\pi$ and calculate the physical $K_1^0 \to 2\pi$ decay amplitude by means of current algebra. The numerical value of the important matrix element $\langle \pi|\mathcal{H}_w^{\rm pv}|K^*\rangle$ is evaluated by comparing the theoretical expression for the $K_1^0 \to 2\pi$ amplitude with the experimental decay rate. Next, exploiting analyticity in the form of finite-energy sum rules, we argue that the octet property of the matrix element $\langle B_1|\mathcal{H}_w|B_2\rangle$, valid in all versions of the symmetric quark model, *leads to* the octet dominance of matrix elements of $\mathcal{H}_w$ between two mesons. The approximate $|\Delta I| = \frac{1}{2}$ rule in the K and Ω^- decays then directly follows.

While the color-quark, three-triplet, and paraquark models are as successful as the Bose-quark model in describing the nonleptonic weak decays, the ordinary singlet triplet Fermi-quark model[14] is *not*. In it, the Fierz reshuffling property of the $V \pm A$ interaction leads to a weak Hamiltonian which is symmetric in the SU(3) indices under interchange of two Fermi-quark fields. Consequently, this model must be supplemented with an additional assumption of octet dominance, but then in the SU(3) limit, it follows[15] that $D/F = 3$ for the matrix element of $\mathcal{H}_w^{pc}$ between two baryons in the nucleon octet, instead of $D/F = -1$ as obtained[3] from Eq. (1) in the symmetric quark model. Experimentally, a good fit[3] of the P-wave amplitudes requires $D/F = -0.85$. Therefore, the Fermi-quark model supplemented with octet dominance can be ruled out.

Nevertheless, the truth of the proposed explanation of the $|\Delta I| = \frac{1}{2}$ rule based on the symmetric quark model is not *established* by these results and this consistency. One needs further predictions (preferably strong qualitative ones) and more tests of Eqs. (1) through (4). Therefore the second goal of this paper is to apply these equations to the nonleptonic decays of the Ω^- hyperon. This is of interest principally for three reasons. First, the tests in Ω^- are *stronger* than in the case of the decays of the octet hyperons. These predictions depend only on Eqs. (1) through (4), current algebra, PCAC for pions, dispersion relations, Regge asymptotic behavior, and resonance saturation of their absorption parts. Second, experimentally, in about two years there should be a thousand or so Ω^- events, enough to test these predictions. Third, there seems to be some confusion in the literature[16] about the application of current algebra and PCAC to the Ω^- decays. We would like to dispel this confusion by making a more careful analysis.

The format of the paper regarding the Ω^- test is as follows: In Sec. II we consider the consequences of current algebra and Eq. (2) for the Ω^- decay amplitudes. We obtain the soft-meson conditions and formulate the problem in such a way that dispersion relations can be easily applied. In Sec. III, we argue, on the basis of Regge asymptotics, for unsubtracted dispersion relations for the invariant amplitudes that are involved and calculate the dispersion integrals. We make use of PCAC for pions and the results derived in Sec. II to obtain information on the physical parity-conserving and -violating Ω^- decay amplitudes, namely A and B. In Sec. V, we estimate the Ω^- nonleptonic decay rates. The predicted total nonleptonic decay rate is consistent with the present experimental value. The strong prediction is that the branching ratio

$$\Gamma(\Omega^- \to \Xi\pi)/\Gamma(\Omega^- \to \Lambda K^-) \ll 1\,.$$

The reader who is mainly interested in the first topic of this paper—the explanation of the $|\Delta I| = \frac{1}{2}$ rule based on the symmetric quark model—need only read Sec. IV and Sec. VI, which summarizes the predictions and discusses the different origins of the breaking of the $|\Delta I| = \frac{1}{2}$ rule in alternative versions of the symmetric quark model.

II. EQUATION (2) AND THE SOFT-MESON CONDITIONS

There are three nonleptonic weak decay modes for the $\frac{3}{2}^+$ Ω^- hyperon:

$$\Omega^- \to \Xi^- + \pi^0\,,$$
$$\to \Xi^0 + \pi^-\,,$$

and

$$\to \Lambda + K^-\,.$$

In general the amplitude for any of the above three processes can be written

$$\mathfrak{M}_{ij} \equiv (2\pi)^{9/2}(2q_0)^{1/2}\left(\frac{p_0 p_0'}{M_\Omega M_j}\right)^{1/2} \times \langle B_j(p')\pi_i(q)|\mathcal{H}_w(0)|\Omega^-(p)\rangle$$
$$\equiv \bar{u}_j(p')q^\mu(A+\gamma_5 B)u_\mu(p)\,, \tag{5a}$$

where

$$p = p' + q \tag{5b}$$

and A and B are, respectively, the parity-conserving (P-wave) and parity-violating (D-wave) amplitudes. Next, we define

$$T_{ij}(q,p,p') \equiv i(2\pi)^3\left(\frac{p_0 p_0'}{M_\Omega M_j}\right)^{1/2} \frac{(-q^2+m_i^2)}{f_i m_i^2}\sqrt{2} \times \int d^4x\, e^{iq\cdot x} \times \langle B_j(p')|T(\partial^\mu A_\mu^i(x)\mathcal{H}_w(0))|\Omega^-(p)\rangle\,, \tag{6}$$

where m_i is the mass of the pseudoscalar meson in the final state of the Ω^- decay. By the LSZ (Lehmann-Symanzik-Zimmermann) reduction technique, we have

$$T_{ij}(q^2 = m_i^2, p, p')|_{p=p'+q} = \mathfrak{M}_{ij}\,. \tag{7}$$

In general, T_{ij} defined by Eq. (6) can be considered to be the amplitude for the reaction

$$\Omega^-(p) + S(h) \to B_j(p') + P_i'(q)\,,$$

where the four-vector h is defined by

$$p + h = p' + q\,, \tag{8a}$$

$$h^2 = 0\,. \tag{8b}$$

$S(h)$ is a spurion carrying the four-momentum h

and it is represented by the local field $\mathcal{H}_w(x)$. $P_i'(q)$ is an off-mass-shell pseudoscalar meson represented by the interpolating field $\sqrt{2}\,\partial^\mu A_\mu^i(x)/f_i m_i^2$. The Lorentz-invariant functions devoid of kinematical singularities and zeros are defined by

$$T_{ij} = q^\mu \bar{u}(p')[(F_1 + \gamma_5 F_2) + \tfrac{1}{2}(\not{q} + \not{h})(G_1 + \gamma_5 G_2)]u_\mu(p) + h^\mu u(p')[(F_1' + \gamma_5 F_2') + \tfrac{1}{2}(\not{q} + \not{h})(G_1' + \gamma_5 G_2')]u_\mu(p) . \tag{9}$$

The F_i's, G_i's, F_i''s, and G_i''s ($i = 1, 2$) are functions of s, t, q^2, and h^2 where

$$s = (p + h)^2 = (p' + q)^2 , \tag{10a}$$

$$u = (p - q)^2 = (p' - h)^2 , \tag{10b}$$

$$t = (p - p')^2 = (q - h)^2 , \tag{10c}$$

with

$$s + t + u = M_\Omega^2 + M_j^2 + q^2 + h^2 . \tag{11}$$

When $h \to 0$ and $q^2 \to m_i^2$, we have the physical amplitude for the decay

$$\Omega^-(p) \to B_j(p') + P_i(q) .$$

At this limit

$$s \to p^2 = M_\Omega^2 , \quad u \to M_j^2 , \quad t \to q^2 , \quad q^2 \to m_i^2 , \tag{12}$$

and

$$h^2 \to 0 ,$$

so by taking the $h \to 0$ limit of Eq. (9) we obtain

$$A = F_1(u = M_j^2,\ t = m_i^2,\ q^2 = m_i^2,\ h^2 = 0) + \tfrac{1}{2}(M_\Omega - M_j) \times G_1(u = M_j^2,\ t = m_i^2,\ q^2 = m_i^2,\ h^2 = 0) , \tag{13}$$

$$B = F_2(u = M_j^2,\ t = m_i^2,\ q^2 = m_i^2,\ h^2 = 0) - \tfrac{1}{2}(M_\Omega + M_j) \times G_2(u = M_j^2,\ t = m_i^2,\ q^2 = m_i^2,\ h^2 = 0) . \tag{14}$$

In order to consider the $q \to 0$ limit of Eq. (6), we rewrite it in the usual way, namely,

$$T_{ij}(q, p, p') = (2\pi)^3 \left(\frac{p_0 p_0'}{M_\Omega M_j}\right)^{1/2} \sqrt{2}\, \frac{(-q^2 + m_i^2)}{f_i m_i^2} \left[q^\mu \int d^4x\, e^{iq\cdot x} \langle B_j(p') | T(A_\mu^i(x)\mathcal{H}_w(0)) | \Omega^-(p)\rangle - i \int d^4x\, e^{iq\cdot x} \langle B_j(p') | \delta(x^0)[A_0^i(x), \mathcal{H}_w(0)] | \Omega^-(p)\rangle \right] . \tag{15}$$

Because of Eq. (2), the term multiplying q^μ on the right-hand side of Eq. (15) has no singularity as $q \to 0$, so we obtain

$$T_{ij}(q = 0, p, p') = -i(2\pi)^3 \left(\frac{p_0 p_0'}{M_\Omega M_j}\right)^{1/2} \frac{\sqrt{2}}{f_i} \langle B_j(p') | [F_5^i(0), \mathcal{H}_w(0)] | \Omega^-(p)\rangle . \tag{16}$$

But in the current-current theory,

$$[F_5^i(0), \mathcal{H}_w(0)] = [F^i(0), \mathcal{H}_w(0)] , \quad i = 1, 2, 3 \tag{17}$$

and thus for pions the right-hand side of Eq. (16) vanishes *because of Eq. (2)*. For kaons, the right-hand side will vanish in the SU(3) limit where $\langle B' | F^K | \Omega^-\rangle = 0$ for B' not in the $\frac{3}{2}^+$ s-wave $\underline{10}$. Then Eqs. (16), (9), and (10) lead to

$$F_1'(u = M_\Omega^2,\ t = 0,\ q^2 = 0,\ h^2 = 0) - \tfrac{1}{2}(M_\Omega - M_j) G_1'(u = M_\Omega^2,\ t = 0,\ q^2 = 0,\ h^2 = 0) = 0 , \tag{18}$$

$$F_2'(u = M_\Omega^2,\ t = 0,\ q^2 = 0,\ h^2 = 0) + \tfrac{1}{2}(M_\Omega + M_j) G_2'(u = M_\Omega^2,\ t = 0,\ q^2 = 0,\ h^2 = 0) = 0 . \tag{19}$$

It should be noted that Eqs. (18) and (19) hold exactly for pions, whereas for kaons they are valid only in the SU(3) limit. This need not concern us in this paper, however, since we will not use Eqs. (18) and (19) to determine the amplitudes for the $\Omega^- \to \Lambda K^-$ decay. In general for either pions or kaons, Eqs. (18) and (19) do not impose any restrictions[17] on A and B given by Eqs. (13) and (14) since we are dealing with two different sets of functions in the two sets of equations. But within the framework of a model for calculating the invariant amplitudes, Eqs. (18) and (19) may help to determine A and B. In this paper we assume a model in which dispersion relations and Regge asymptotic behavior for the invariant amplitudes are valid. We also assume the validity of the quark-model relations, Eqs. (1) through (4).

III. DISPERSION RELATIONS AND THE CALCULATION OF THE PHYSICAL AMPLITUDES A AND B

For the application of dispersion relations we have to consider the $\Xi\pi$ and ΛK decay modes of Ω^- separately.

A. $\Omega^- \to \Xi\pi$ Decay

To be specific let us consider the $\Omega^- \to \Xi^- + \pi^0$. The amplitude for the process $\Omega^- \to \Xi^0 + \pi^-$ will be automatically determined since the model predicts the $|\Delta I| = \frac{1}{2}$ rule (see Sec. IV). First of all let us try to calculate the parity-conserving amplitude A. For this consider the reaction

$$\Omega^-(p) + S^{pc}(h) \to \Xi^-(p') + \pi^0(q),$$

where $S^{pc}(h)$ is the parity-conserving spurion carrying the four-momentum h. In the u channel of this reaction, no known resonances exist since the quantum numbers of this channel are $S=-3$ and $I=1$. Therefore, according to Regge asymptotic behavior, the fixed-u dispersion relations are unsubtracted and moreover the dispersion integrals must be highly convergent. Dispersion relations are written for F_1' and G_1' dispersing in the variable t, keeping $u = M_\Omega{}^2$ and $q^2 = m_\pi{}^2$. Thus,

$$F_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t=0) = \frac{1}{\pi}\int_{-\infty}^{\infty} dt'\, \frac{\mathrm{Abs}_{t'} F_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t')}{(t' + i\epsilon)} \tag{20}$$

and

$$G_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t=0) = \frac{1}{\pi}\int_{-\infty}^{\infty} dt'\, \frac{\mathrm{Abs}_{t'} G_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t')}{(t' + i\epsilon)}. \tag{21}$$

The absorptive parts in t' of the invariant amplitudes, for fixed u, can be determined from the relation

$$\mathrm{Abs}_{t'} T^{pc}_{\pi^0\Xi^-} = \frac{1}{2}\left(\frac{p_0}{M_\Omega}\right)^{1/2}(2q_0)^{1/2}(2\pi)^3\Big[\sum_n (2\pi)^4\delta^4(p'+p_n-p)\langle\pi^0(q)|\mathcal{H}_w^{pc}(0)|n\rangle\langle n|j_{\Xi^-}(0)|\Omega^-(p)\rangle$$
$$- \sum_n (2\pi)^4\delta^4(p'+q-p_n)\langle\pi^0(q)|j_{\Xi^-}(0)|n\rangle\langle n|\mathcal{H}_w^{pc}(0)|\Omega^-(p)\rangle\Big]. \tag{22}$$

We now assume[18] that single-particle and resonance intermediate states alone will saturate the summation over the complete set of states in the absorptive part in Eq. (22). Then, because of Eq. (2) the second term on the right-hand side of Eq. (22) does not contribute. In the first term $\overline{K}^0$ is the lowest mass state in the sum over intermediate states. The contribution of the higher-mass intermediate states to the dispersion integrals in Eqs. (20) and (21) must be highly suppressed, first because of Eq. (4) and second, because inside the dispersion integrals they appear with a higher denominator. Therefore in the symmetric quark model it is reasonable to make the assumption that $\overline{K}^0$ saturates the dispersion integrals. Thus, using Eqs. (9), (20), and (22), we are led to the results

$$F_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t=0) \simeq \frac{f_{\pi^0\overline{K}^0}(0)G_{\overline{K}^0\Xi^-\Omega^-}}{m_K{}^2} \tag{23}$$

and

$$G_1'(u = M_\Omega{}^2,\ q^2 = m_\pi{}^2,\ t=0) \simeq 0, \tag{24}$$

where

$$(2\pi)^3(4k_0q_0)^{1/2}\langle\pi^0(q)|\mathcal{H}_w^{pc}(0)|\overline{K}^0(k)\rangle = f_{\pi^0\overline{K}^0}((k-q)^2) \tag{25}$$

and

$$\overline{u}(p')\langle\overline{K}^0(k)|j_{\Xi^-}(0)|\Omega^-(p)\rangle(2\pi)^3\left(\frac{2k_0p_0}{M_\Omega}\right)^{1/2}\Bigg|_{(p-k)^2\to M_\Xi{}^2} = G_{\overline{K}^0\Xi^-\Omega^-}\overline{u}(p')k^\mu u_\mu(p). \tag{26}$$

Because of the PCAC assumption of smoothness, either in the "strong" or "weak"[19,20] version, Eqs. (23) and (24) will hold even if F_1' and G_1' are evaluated at $q^2=0$ rather than at $q^2 = m_\pi{}^2$ with u and t at the same values. Then, using Eq. (18), we obtain

$$\frac{f_{\pi^0\overline{K}^0}(0)G_{\overline{K}^0\Xi^-\Omega^-}}{m_K{}^2} \simeq 0. \tag{27}$$

Equation (27) has only one possible solution, namely,

$$f_{\pi^0\overline{K}^0}(0) \simeq 0. \tag{28a}$$

By considering the $\Xi^0\pi^-$ decay mode, we can also derive

$$f_{\pi^-K^-}(0) \simeq 0\,. \tag{28b}$$

By writing unsubtracted dispersion relations for F_1 and G_1 in the variable t, keeping u at M_Ξ^2 and q^2 at m_π^2, and then using Eq. (13), we obtain in a similar manner

$$A \simeq \frac{f_{\pi^0\bar{K}^0}(0)G_{\bar{K}^0\Xi^-\Omega^-}}{(m_K^2 - m_\pi^2)} \simeq 0 \tag{29}$$

because of Eq. (28). Therefore in our model the parity-conserving (P-wave) amplitude of the decay $\Omega^- \to \Xi\pi$ is essentially zero, or in any case small.[21]

Next let us consider the parity-violating amplitude B. For this, consider the reaction

$$\Omega^-(p) + S^{\rm pv}(h) \to \Xi^-(p') + \pi^0(q)\,.$$

As in the case of the parity-conserving amplitude, we can consider fixed-u unsubtracted dispersion relations in the variable t for the invariant functions of this reaction. The role of $\bar{K}^0$ is now taken by the vector meson $\bar{K}^{*0}$. Using arguments mentioned before, we obtain

$$F_2'(u = M_\Omega^2,\ q^2 = m_\pi^2,\ t = 0) \simeq \frac{1}{2}\frac{g_{\pi^0\bar{K}^{*0}}(0)}{m_{K^*}^2}[G^{(1)} - \tfrac{1}{2}G^{(3)}(M_\Xi^2 - M_\Omega^2 - m_{K^*}^2 + 2m_\pi^2)], \tag{30}$$

$$G_2'(u = M_\Omega^2,\ q^2 = m_\pi^2,\ t = 0) \simeq -\frac{g_{\pi^0\bar{K}^{*0}}(0)G^{(3)}}{m_{K^*}^2}, \tag{31}$$

where

$$(2\pi)^3(4k_0q_0)^{1/2}\langle\pi^0(q)|\mathcal{H}_w^{\rm pv}(0)|\bar{K}^{*0}(k,\lambda)\rangle = g_{\pi^0\bar{K}^{*0}}[(k-q)^2]\epsilon_{\bar{K}^{*0}}(k,\lambda)\cdot q \tag{32}$$

and

$$\begin{aligned}(2\pi)^3\left(\frac{2k_0p_0}{M_\Omega}\right)^{1/2}\bar{u}_\Xi(p')\langle\bar{K}^{*0}(k,\lambda)|j_{\Xi^-}(0)|\Omega^-(p)\rangle &= -[G^{(1)}\bar{u}_\Xi(p')\gamma_5u_\mu(p)\epsilon_{K^*}^\mu(k,\lambda)\\ &\quad + G^{(2)}\bar{u}_\Xi(p')\gamma_\nu\gamma_5u_\mu(p)k^\mu\epsilon_{K^*}^\nu(k,\lambda)\\ &\quad + G^{(3)}\bar{u}_\Xi(p')\gamma_5p_\nu u_\mu(p)k^\mu\epsilon_{K^*}^\nu(k,\lambda)]\,.\end{aligned} \tag{33}$$

As before, using the PCAC assumption in its "strong" or "weak" version,[19,20] we argue that Eqs. (30) and (31) will be valid even when F_2' and G_2' are evaluated at $q^2 = 0$ rather than at $q^2 = m_\pi^2$, with u and t at the same values. With this in mind, we substitute Eqs. (30) and (31) into Eq. (19) (the soft-pion constraint on F_2' and G_2'), and obtain

$$G^{(1)} - (M_\Omega + M_\Xi)G^{(2)} + \tfrac{1}{2}(M_\Omega^2 + m_{K^*}^2 - M_\Xi^2 - 2m_\pi^2)G^{(3)} \simeq 0\,. \tag{34}$$

Next, by writing unsubtracted dispersion relations for F_2 and G_2 in the variable t, keeping u at M_Ξ^2 and q^2 at m_π^2, and using Eqs. (14) and (34), we have, in the approximation of saturation of the dispersion integral by K^* which is justified by Eq. (4),

$$B \simeq -\tfrac{1}{2}g_{\pi^0\bar{K}^{*0}}(0)G^{(3)}\frac{[m_{K^*}^2 - 2m_\pi^2]}{[m_{K^*}^2 - m_\pi^2]}\,. \tag{35}$$

B. $\Omega^- \to \Lambda K^-$ Decay

First let us calculate the parity-conserving amplitude A of this decay defined by Eq. (5a). For this, consider the reaction

$$\Omega^-(p) + S^{\rm pc}(h) \to \Lambda(p') + K^-(q)\,.$$

As we will see, even though K meson is involved, *we do not have to invoke the smoothness assumption of kaon PCAC* to obtain the results we seek.

The t channel of the above reaction is exotic since there is no known meson with $S = -2$. Therefore, according to Regge hypothesis, fixed-t dispersion relations must be highly convergent and in particular we can write unsubtracted dispersion relations for the invariant amplitudes F_i's, F_i''s, G_i's, and G_i''s in the variable u for fixed t. The absorptive parts in u of these invariant functions can be calculated from the expression

$$\begin{aligned}{\rm Abs}_uT^{\rm pc}_{K^-\Lambda} = \tfrac{1}{2}(2\pi)^3\left(\frac{p_0p_0'}{M_\Omega M_\Lambda}\right)^{1/2}\frac{(-q^2 + m_K^2)}{f_Km_K^2}\Big[&\sum_n(2\pi)^4\delta^4(q + p_n - p)\langle\Lambda(p')|\mathcal{H}_w^{\rm pc}(0)|n\rangle\langle n|\partial^\mu A_\mu^{K^-}(0)|\Omega^-(p)\rangle\\ &- \sum_n(2\pi)^4\delta^4(q + p' - p_n)\langle\Lambda(p')|\partial^\mu A_\mu^{K^-}(0)|n\rangle\langle n|\mathcal{H}_w^{\rm pc}(0)|\Omega^-(p)\rangle\Big]\,.\end{aligned} \tag{36}$$

Again, as in Eq. (22), we assume[18] that the single-particle and resonance-intermediate states alone will saturate the summation over the complete set of states in Eq. (36). Then, because of Eq. (2), the second term (the contributions from s-channel-resonance intermediate states) will be zero. In the first term the Ξ^0 intermediate state will dominate over all the others because of Eq. (3).

In passing, we note that if we use this Ξ^0 dominance (justified in the quark models) in the unsubtracted dispersion relations written *for fixed* $q^2=0$ and fixed $t=0$, we would obtain

$$F_1'(u=M_\Omega{}^2,\ q^2=0,\ t=0)\simeq 0\,, \tag{37a}$$

$$G_1'(u=M_\Omega{}^2,\ q^2=0,\ t=0)\simeq 0\,. \tag{37b}$$

So the soft-kaon constraint given by current algebra and the quark models, namely Eq. (18), is trivially satisfied. Thus, *no conditions* on the coupling constants are imposed. In deriving Eq. (37) it is gratifying to note that one *does not have to make use of kaon PCAC* since here we write the unsubtracted dispersion relations for fixed $q^2=0$ and can argue that in general (that is, even for $q^2\neq 0$) the contribution from Ξ^0 to the absorptive part of $T^{\mathrm{pc}}_{K^-\Lambda}$ does not contain a term multiplied by h^μ.

By considering the dispersion relations for F_1 and G_1 for fixed $t=m_K{}^2$ and $q^2=m_K{}^2$, we obtain the physical decay amplitudes

$$A\simeq\frac{a_{\Lambda\Xi^0}(0)G^{D}_{\Xi^0K^+\Omega^-}}{(M_\Xi-M_\Lambda)}\,, \tag{38}$$

where

$$(2\pi)^3\left(\frac{p_0p_0'}{M_\Xi M_\Lambda}\right)^{1/2}\langle\Lambda(p')|\mathcal{H}_w^{\mathrm{pc}}(0)|\Xi^0(p)\rangle = a_{\Lambda\Xi^0}((p-p')^2)\bar u_\Lambda(p')u_\Xi(p) \tag{39}$$

and

$$(2\pi)^3\left(\frac{p_0p_0'}{M_\Omega M_\Xi}\right)^{1/2}\frac{(-q^2+m_K{}^2)}{f_Km_K{}^2}\times\langle\Xi^0(p')|\partial^\mu A_\mu^{K^-}(0)|\Omega^-(p)\rangle\Big|_{q^2\to m_K{}^2} = G^{D}_{\Xi^0K^+\Omega^-}\bar u_\Xi(p')q^\mu u_\mu(p)\,. \tag{40}$$

By considering the reaction

$$\Omega^-(p)+S^{\mathrm{pv}}(h)\to\Lambda(p')+K^-(q)$$

and using the same arguments used to obtain Eq. (38), we can now derive for the parity-violating amplitude B [given by Eq. (14)] that

$$B\simeq\frac{b_{\Lambda\Xi^0}(0)G^{D}_{\Xi^0K^+\Omega^-}}{(M_\Xi+M_\Lambda)}\,, \tag{41}$$

where

$$(2\pi)^3\left(\frac{p_0p_0'}{M_\Xi M_\Lambda}\right)^{1/2}\langle\Lambda(p')|\mathcal{H}_w^{\mathrm{pv}}(0)|\Xi^0(p)\rangle = b_{\Lambda\Xi^0}((p-p')^2)\bar u_\Lambda(p')\gamma_5u_\Xi(p)\,. \tag{42}$$

Also,

$$F_2'(u=M_\Omega{}^2,q^2=0,t=0)=G_2'(u=M_\Omega{}^2,q^2=0,t=0)=0$$

since the absorptive part of $T^{\mathrm{pv}}_{K^-\Lambda}$ does not contain a term multiplied by h^μ. From Eqs. (38) and (41),

$$\frac{B}{A}\simeq\frac{b_{\Lambda\Xi^0}(0)}{a_{\Lambda\Xi^0}(0)}\frac{(M_\Xi-M_\Lambda)}{(M_\Xi+M_\Lambda)}\,. \tag{43}$$

It should be noticed that $b_{\Lambda\Xi^0}(0)$ vanishes in the limit of exact SU(3) symmetry whereas $a_{\Lambda\Xi^0}(0)$ does not. Therefore from Eq. (43), we may safely conclude

$$\frac{B}{A}\ll 1\,. \tag{44}$$

In other words, the ΛK^- decay mode of Ω^- is predominantly[22] P wave with little admixture of the D-wave amplitude.

IV. $|\Delta I|=\frac{1}{2}$ RULE FROM THE SYMMETRIC QUARK MODEL

A. The Vanishing of $f_{\pi K}(0)$

There is a very important consequence for $K_1^0\to 2\pi$ decay of our analysis of the Ω^- nonleptonic decays: From Eqs. (25) and (28) we have

$$f_{\pi K}(0)\equiv(2\pi)^3(4k_0q_0)^{1/2}\langle\pi(q)|\mathcal{H}_w^{\mathrm{pc}}(0)|K(k)\rangle\Big|_{(q-k)^2=0}\simeq 0\,. \tag{45}$$

The principal ingredients in the derivation of Eq. (45) were the dominance [following from the symmetric-quark-model relation, Eq. (4)] of the K intermediate states in the unsubtracted dispersion relations and the current-algebra soft-pion constraint given by Eq. (18). It should be noted that there is a direct derivation of $f_{\pi K}(0)=0$ from the symmetric quark model. Since $\mathcal{H}_w^{\mathrm{pv}}$ has the quark structure $(\bar qq)(\bar qq)$ whereas the K meson is a $\bar qq$ state, it is obvious that

$$\langle 0|\mathcal{H}_w^{\mathrm{pv}}(0)|K\rangle=0\,, \tag{46}$$

so if a once-subtracted dispersion relation in the momentum-transfer variable is assumed for the vertex $\langle\pi(q)|\mathcal{H}_w^{\mathrm{pc}}(0)|K(k)\rangle$ with the subtraction-point chosen at the soft-pion limit [given by Eq. (46)], and if the dispersion integral is saturated by resonances alone, one also obtains Eq. (45). Both the subtraction constant and the dispersion integral vanish as a consequence of Eqs. (1) through (4).

Equation (45) is inconsistent with Gronau's assumption[3] that the $K_1^0 \to 2\pi$ amplitude is dominated by the K^*-pole diagram. This is easily seen as follows: In the notation of Sakurai,[23] the $K_1^0 \to 2\pi$ decay amplitude in the approximation of K^*-pole dominance (in the sense of a Feynman diagram) is

$$T(K_1^0(p_K) + S^{pv}(h) \to \pi^+(p_+) + \pi^-(p_-); h=0) \simeq \frac{\lambda f_V}{m_{K^*}^2}(m_K^2 - m_\pi^2). \quad (47)$$

However, in the soft-pion limit,

$$T(K_1^0(p_K) + S^{pv}(h) \to \pi^+(p_+) + \pi^-(p_-); p_+=0) = \frac{1}{f_\pi}\langle \pi^-(p_-)|[F^{\pi^+}(0), \mathcal{H}_w^{pc}(0)]|K_1^0(p_K)\rangle\Big|_{(p_K-p_-)^2=0} \simeq \frac{\lambda f_V}{4m_{K^*}^2}(m_K^2 + m_\pi^2). \quad (48)$$

Only the π^0 and K^- intermediate states contribute to the commutator in Eq. (48) and thus by Eq. (45), $\lambda f_V \simeq 0$. Therefore, by Eq. (47), the $K_1^0 \to 2\pi$ decay rate vanishes.

B. Reproduction of Gronau's Fit of the $\underline{8}$ Hyperon-Decay Amplitudes in the Dispersion Approach

We will now demonstrate that Gronau's successful fit of the $\underline{8}$ hyperon-decay amplitudes can be reproduced in a dispersion calculation, in spite of the technical difficulty mentioned above. This dispersion approach is also needed to derive the $|\Delta I| = \frac{1}{2}$ rule for these decays in the color-quark, the three-triplet, and the paraquark models. Following a method suggested by Okubo,[24-26] we consider the amplitude defined by

$$\begin{aligned} T_{ijk}(q,p,p') &\equiv i(2\pi)^3\left(\frac{p_0 p_0'}{M_j M_k}\right)^{1/2}\sqrt{2}\,\frac{(-q^2+m_\pi^2)}{f_\pi m_\pi^2} \\ &\quad\times\int d^4x\, e^{iq\cdot x} \\ &\quad\times\langle B_k(p')|\,T(\partial^\mu A_\mu^i(x)\mathcal{H}_w(0))\,|B_j(p)\rangle \\ &\equiv \bar{u}_k(p')[(F_1+\gamma_5F_2) \\ &\quad+\tfrac{1}{2}(\not{k}+\not{q})(G_1+\gamma_5G_2)]u_j(p) \\ &\equiv \bar{u}_k(p')[(H_1+\gamma_5H_2) \\ &\quad+[\not{k},\not{q}](J_1+\gamma_5J_2)]u_j(p). \end{aligned} \quad (49)$$

Equation (49) can be considered to be the amplitude for the process

$$B_j(p) + S_w(h) \to B_k(p) + \pi_i'(q),$$

where $S_w(h)$ is a massless spurion of four-momentum h and π_i' is an off-mass-shell pion defined by the interpolating field $(\sqrt{2}/f_\pi m_\pi^2)\partial^\mu A_\mu^i(x)$. The Lorentz invariants F_i, G_i, H_i, and J_i $(i=1,2)$ in Eq. (49) can be considered to be the functions of the variables s, t, and q^2. At the limit $h\to 0$ and $q^2 \to m_\pi^2$, Eq. (49) gives the parity-violating and parity-conserving amplitudes A_{ji} and B_{ji} of the physical decay

$$B_j(p) \to B_k(p') + \pi_i(q).$$

It is clear,[24] that

$$A_{ji} = H_1(s=M_j^2,\ t=m_\pi^2,\ q^2=m_\pi^2), \quad (50)$$

$$B_{ji} = H_2(s=M_j^2,\ t=m_\pi^2,\ q^2=m_\pi^2). \quad (51)$$

The method[24] consists of writing once-subtracted dispersion relations for the H_i's $(i=1,2)$, dispersing in s (with fixed $t=q^2=m_\pi^2$), and choosing the subtraction point s_0 to have the value of the soft-pion limit, that is,

$$H_i(s=M_j^2,\ t=m_\pi^2,\ q^2=m_\pi^2) = H_i(s_0=M_k^2,\ t=m_\pi^2,\ q^2=m_\pi^2) + \frac{(M_j^2-M_k^2)}{\pi}\int_{-\infty}^{\infty} ds'\,\frac{\mathrm{Abs}_{s'}H_i(s', t=m_\pi^2, q^2=m_\pi^2)}{(s'-M_k^2)(s'-M_j^2-i\epsilon)}. \quad (52)$$

Next, the PCAC smoothness assumption can be used to identify the subtraction constant with the value of the amplitude at the soft-pion limit, namely, $H_i(s_0=M_k^2,\ t=0,\ q^2=0)$ given by the familiar equal-time commutator term (ETC).[24] The absorptive parts of H_1 and H_2 can be calculated from the expression

$$\begin{aligned} \mathrm{Abs}_s T_{ijk}^{pv(pc)} = \frac{(2\pi)^3}{2}\left(\frac{p_0p_0'}{M_jM_k}\right)^{1/2}\Big[&\sum_n (2\pi)^4\delta^4(q+p'-p_n)\langle B_k(p')|j_{\pi_i}(0)|n\rangle\langle n|\mathcal{H}_w^{pv(pc)}(0)|B_j(p)\rangle \\ &-\sum_n (2\pi)^4\delta^4(q+p_n-p)\langle B_k(p')|\mathcal{H}_w^{pv(pc)}(0)|n\rangle\langle n|j_{\pi_i}(0)|B_j(p)\rangle\Big]. \end{aligned} \quad (53)$$

The sums of the subtraction constant and of the contribution from the nucleon-octet intermediate states to the dispersion integral give[24] expressions for the hyperon-decay amplitudes identical in form with those (see Gronau) where the baryon Born terms are included in addition to the ETC terms in the current algebraic treatment. However, in the dispersion approach, there is the pleasant difference that the intermediate baryons are on the mass shell as they should be, strictly speaking, to make use of Eqs. (1) through (4). In the approximation of resonance saturation,[18] because of Eq. (3), one might expect the contribution of the higher-mass resonances to $A_{ji}(H_1)$ and $B_{ji}(H_2)$ to be negligible. However, Gronau's work[3] has demonstrated that for H_1, the addition of a K^*-pole term to the equal-time commutator term [baryon-Born terms for A_{ji} are assumed to be negligible since they vanish in the SU(3) limit] is necessary to obtain an over-all good fit simultaneously of the P-wave and the S-wave decay amplitudes. In the dispersion theory this K^*-pole term must come from the resonance contributions to the dispersion integral in the s variable [see Eq. (52)]. In spite of Eq. (3), such contributions to H_1 may be important for the following reasons: (i) The dispersion integral is not as highly convergent as in the case of the Ω^- amplitudes where the crossed channel was exotic, or as in the case of H_2 where K and K_A Regge poles are exchanged in the crossed channel. (ii) The K and K_A Regge contributions to H_2 must be suppressed since the corresponding Regge residues should be small because of the symmetric-quark-model relations, Eqs. (4) and (45). (iii) Finally, due to the presence of very closely spaced baryon resonances (unlike the case of meson resonances) the sum of such resonance contributions to H_1 may be significant even though each contribution separately may be small. Several authors[27] have shown that this higher-energy contribution to H_1 gives a term with the structure of the K^*-pole term, provided Regge asymptotic behavior (for H_1, the leading Regge pole exchanged in the crossed channel is that of K^*) is assumed for the amplitude in question. Furthermore, in the approximation of resonance saturation[18] of the dispersion integral, such a K^*-polelike term must satisfy the octet property and the $|\Delta I| = \frac{1}{2}$ rule since the matrix elements of the form $\langle B_1|\mathcal{H}_w|B_2\rangle$ have these properties in the symmetric quark model (see Appendixes A and B). With the SU(3) approximation for the vertices involved in the Regge residues, there are two unknown parameters coming from this term, instead of only one as in Gronau's treatment.[3] The numerical fit of the eight independent hyperon-decay amplitudes will be even better. Nevertheless, the values of the fitted parameters should be approximately equal to those obtained by Gronau[3] in his "best" fit as it is already in good agreement with experiment.

C. The Dispersion Calculation of the $K_1^0 \to 2\pi$ Amplitude

Here we calculate the amplitude for the $K_1^0 \to 2\pi$ decay, using once-subtracted dispersion relations, current algebra, and Eqs. (3) and (45). Such calculations using subtracted dispersion relations and current algebra have already been done by other authors[25,28] in the past, but not in the framework of the symmetric-quark-model relations, Eqs. (1) through (4) and (45). As we will see, Eqs. (3) and (45) lead to important differences in the final result. We present this calculation here for two reasons: (a) The final expression we get for $K_1^0 \to 2\pi$ amplitude will enable us to understand the $|\Delta I| = \frac{1}{2}$ rule in all nonleptonic decays of the K meson, and (b) it will help us to evaluate the constant $g_{\pi^0 K^{*0}}$ appearing in Eq. (35), which will lead to an estimate of the $\Omega^- \to \Xi\pi$ decay rate.

As in Secs. II and III we consider

$$T \equiv i(2\pi)^3(4p_K^0 p_-^0)^{1/2}\frac{(-p_+{}^2 + m_\pi{}^2)}{f_\pi m_\pi{}^2} \times \int d^4x\, e^{ip_+\cdot x} \times \langle \pi^-(p_-)|T(\partial_\mu A^\mu_{\pi^+}(x)\mathcal{H}_w^{\text{pv}}(0))|K_1^0(p_K)\rangle \tag{54}$$

to be the amplitude of the reaction

$$K_1^0(p_K) + S^{\text{pv}}(h) \to \pi^{+\prime}(p_+) + \pi^-(p_-),$$

where $\pi^{+\prime}(p_+)$ in general is an off-mass-shell pion with $\partial^\mu A_\mu^{\pi^+}(x)/f_\pi m_\pi{}^2$ as its interpolating field. $S^{\text{pv}}(h)$ is a massless spurion of four-momentum h defined by energy-momentum conservation

$$p_K + h = p_+ + p_- . \tag{55}$$

T defined by Eq. (54) is a function of any two of the Mandelstam variables s, t, and u and q^2, where

$$\begin{aligned} s &= (p_K + h)^2 \\ &= (p_+ + p_-)^2, \\ t &= (p_K - p_+)^2 \\ &= (p_- - h)^2, \\ u &= (p_K - p_-)^2 \\ &= (p_+ - h)^2. \end{aligned} \tag{56}$$

At the limit $h \to 0$ and $p_+{}^2 \to m_\pi{}^2$ we have the physical decay amplitude of $K_1^0(p_K)$. That is,

$$\begin{aligned} \mathfrak{M} &\equiv (2\pi)^{9/2}(8p_K^0 p_+^0 p_-^0)^{1/2} \\ &\quad\times \langle \pi^+(p_+)\pi^-(p_-)|\mathcal{H}_w^{\text{pv}}(0)|K_1^0(p_K)\rangle \\ &= T(s = m_K{}^2,\ t = m_\pi{}^2,\ u = m_\pi{}^2,\ p_+{}^2 = m_\pi{}^2). \end{aligned} \tag{57}$$

Next we write a fixed-u ($u = m_\pi{}^2$) once-subtracted dispersion relation, dispersing in the variable t and choosing the subtraction point to be the value of t in the soft-pion limit, (i.e., $t_0 = m_K{}^2$). Thus we have

$$\begin{aligned}\mathfrak{M} &= T(t=m_\pi^2,\ u=m_\pi^2,\ p_+^2=m_\pi^2)\\ &= T(t_0=m_K^2,\ u=m_\pi^2,\ p_+^2=m_\pi^2)+\frac{(m_K^2-m_\pi^2)}{\pi}\\ &\quad\times\int_{-\infty}^{\infty}dt'\,\frac{\mathrm{Abs}_{t'}T(t',\ u=m_\pi^2,\ p_+^2=m_\pi^2)}{(t'-m_K^2)(t'-m_\pi^2-i\epsilon)}.\end{aligned}\tag{58}$$

Note that for fixed $u=m_\pi^2$, the dispersion relation in the t variable must have at least one subtraction in order to have a convergent dispersion integral since K^* is the leading Regge pole exchanged in the u channel. The subtraction constant in Eq. (58) vanishes since by using the PCAC smoothness assumption,[29] we can identify it with the value of T at the soft-pion limit, namely, $T(t_0=m_K^2,\ u=0,\ p_+^2=0)$, and the latter is zero because of Eqs. (45) and (48). Moreover, if we assume resonance saturation, by Eq. (4) the dispersion integral is approximately saturated by the contribution of the K^* intermediate state alone. Thus Eq. (58) leads to

$$\mathfrak{M}\simeq-\frac{1}{2}\,\frac{g_{\pi^-K^{*-}}(0)G_{K^{*-}K_1^0\pi^+}(m_K^2-m_\pi^2)}{m_{K^*}^2},\tag{59}$$

where $g_{\pi K^*}$ and $G_{K^*K_1^0\pi^+}$ are defined by

$$\begin{aligned}(2\pi)^3(4p_0p_-^0)^{1/2}\langle\pi^-(p_-)|\mathcal{H}_w^{\mathrm{pv}}(0)|K^{*-}(p,\lambda)\rangle\\ =g_{\pi^-K^{*-}}(h^2)\epsilon_{K^*}(p,\lambda)\cdot p_-\end{aligned}\tag{60}$$

and

$$\begin{aligned}(2\pi)^3(4p_0p_K^0)^{1/2}\langle K^{*-}(p,\lambda)|j_{\pi^+}(0)|K_1^0(p_K)\rangle\\ =-G_{K^{*-}K_1^0\pi^+}\epsilon_{K^*}\cdot(p_++p_K).\end{aligned}\tag{61}$$

It is amusing to note that if the $K_1^0\to2\pi$ amplitude were dominated by the K^* pole in the sense of a Feynman diagram, in place of Eq. (59) we would have obtained

$$\mathfrak{M}\simeq-2\,\frac{g_{\pi^-K^{*-}}(0)G_{K^{*-}K_1^0\pi^+}(m_K^2-m_\pi^2)}{m_{K^*}^2},$$

which is four times the value given by Eq. (59).

Using Eq. (59) and the experimental widths[30]

$$\Gamma(K_1^0\to\pi^+\pi^-)\simeq0.799\times10^{10}\ \mathrm{sec}^{-1},$$

$$\Gamma(K^*\to K\pi)\simeq50.1\ \mathrm{MeV},$$

we obtain

$$|g_{\pi^-K^{*-}}|\simeq6.7\times10^{-7}\ \mathrm{GeV}.\tag{62}$$

D. The $|\Delta I|=\frac{1}{2}$ Rule for the K and Ω^- Decays

Unlike in the Bose-quark model, in the color-quark, the three-triplet, and the paraquark models the nonleptonic weak Hamiltonian $\mathcal{H}_w$ does not entirely belong to an SU(3) octet and so the $|\Delta I|=\frac{1}{2}$ rule of the nonleptonic weak decays does not immediately follow. In particular, in these models we still have to prove the rule for the K and Ω^- decays. From Eqs. (32), (35), and (59) it is clear that the octet property and the $|\Delta I|=\frac{1}{2}$ rule for $\Omega^-\to\Xi\pi$ and $K\to2\pi$ decays will follow if the matrix element $\langle\pi|\mathcal{H}_w^{\mathrm{pv}}|K^*\rangle$, i.e., $g_{\pi K^*}$, is octet-dominated. We will prove the octet dominance of this matrix element by means of finite energy sum rules applied to the amplitudes involved in the hypothetical reaction

$$B_j(p)+S^{\mathrm{pv}}(h)\to B_k(p')+\pi_i(q),$$

and by means of the symmetric-quark-model result that $\langle B_1|\mathcal{H}_w^{\mathrm{pv}}|B_2\rangle$ is octet-dominated.

The invariant amplitudes F_1 and G_1 [see Eq. (49)] of the above reaction can be taken to be functions of the two variables $\nu=(s-u)/(M_j+M_k)$ and t, assuming the pion is on the mass shell. Let $F_1^{(-)}$ be the part of F_1 which is odd under s-u crossing. Since the leading odd-signature Regge pole exchanged in the t channel is that of K^*, one can write[18] for $|\nu|>N$ (N being sufficiently large)

$$F_1^{(-)}(\nu,t)=\beta_{K^*}(t)\,\frac{[1-e^{-i\pi\alpha_{K^*}(t)}]}{\sin(\pi\alpha_{K^*}(t))}\,\nu^{\alpha_{K^*}(t)}.\tag{63}$$

The lowest-moment finite-energy sum rule written for $F_1^{(-)}$ is[18]

$$\int_0^N d\nu\,\mathrm{Abs}_\nu F_1^{(-)}(\nu,t)=\beta_{K^*}(t)\,\frac{N^{\alpha_{K^*}(t)}}{\alpha_{K^*}(t)+1}.\tag{64}$$

As is well known,[18] only analyticity and Regge asymptotic behavior [Eq. (63)] go into the derivation of Eq. (64), and for sufficiently large N one expects it to be very accurate. By Eq. (53), $\mathrm{Abs}_\nu F_1^{(-)}$, in the approximation of resonance saturation, involves only matrix elements of the type $\langle B_R|\mathcal{H}_w^{\mathrm{pv}}|B_j\rangle$ and $\langle B_k|\mathcal{H}_w^{\mathrm{pv}}|B_R\rangle$. In the color-quark, the three-triplet, and the paraquark models, only the octet part of $\mathcal{H}_w^{\mathrm{pv}}$ contributes to these matrix elements and so the left-hand side of Eq. (64) must be octet-dominated. This result in turn implies the octet dominance of the Regge residue $\beta_{K^*}(t)$. Since within irrelevant factors

$$\beta_{K^*}(t=m_{K^*}^2)\sim g_{\pi K^*}G_{jkK^*},\tag{65}$$

we have the result we sought, namely, the octet dominance of the matrix-element $\langle\pi|\mathcal{H}_w^{\mathrm{pv}}|K^*\rangle$. In fact, by considering Eq. (64) at appropriate values of t it easily follows that the matrix element of $\mathcal{H}_w^{\mathrm{pv}}$ between the pion and any Regge recurrence (with spin-parity 3^-, 5^-, 7^-, etc.) of $K^*(1^-,890)$ is also octet-dominated. Similarly, by the use of finite energy sum rules written for $F_1^{(+)}$, the part of F_1 which is even under s-u crossing, we also derive the octet dominance for the matrix element

$\langle\pi|\mathcal{H}_w^{pv}|K_N\rangle$, where K_N is either $K_N(2^+, 1420)$ or any of its Regge recurrence with spin-parity 0^+, 4^+, 6^+, etc. By analogous considerations applied to the process

$$B_j(p)+S^{pc}(h)\to B_k(p')+\pi_i(q)\,,$$

we can furthermore argue for the octet dominance of the matrix elements $\langle\pi|\mathcal{H}_w^{pc}|K\rangle$ and $\langle\pi|\mathcal{H}_w^{pc}|K_A\rangle$, where K and K_A could be $K(0^-, 495)$ and $K_A(1^+, 1240)$, respectively, or any of their Regge recurrences.

Two further points are worth noticing. First, the $|\Delta I|=\frac{1}{2}$ rule for $K\to 2\pi$ and $\Omega^-\to\Xi\pi$ decays will hold even if we do not neglect the higher resonance contributions to the dispersion relations, so long as these higher resonances are Regge recurrences of K^*, K_A, or K meson. Second, if the $|\Delta I|=\frac{1}{2}$ rule holds for $K\to 2\pi$ decays, it can be shown to be valid[31] for $K\to 3\pi$ decays by making use of two facts, namely (1) the amplitude for the latter is proportional to the former in the limit of any one soft pion, and (2) experimentally, the amplitude for $K\to 3\pi$ is linear in the energies of each of the pions to a good approximation.

V. NONLEPTONIC Ω^- DECAY RATES

By means of the general results of the preceding sections plus the additional assumptions of vector-meson dominance for the electromagnetic form factors and of SU(3) symmetry for the vertices, the Ω^- nonleptonic decay rates can be estimated.

A. $\Omega^-\to\Xi\pi$ Decay Rate

The decay rate for the process $\Omega^-(\frac{3}{2}^+)\to B(\frac{1}{2}^+)+\pi(0^-)$ in terms of the amplitudes A and B defined by Eq. (5a) is

$$\Gamma(\Omega^-\to B\pi)=\frac{|p_B|^3}{24\pi M_\Omega^2}\left[(M_\Omega+M_B)^2-m_\pi^2\right]\times\left[|A|^2+|B|^2\frac{(M_\Omega-M_B)^2-m_\pi^2}{(M_\Omega+M_B)^2-m_\pi^2}\right]. \tag{66}$$

So, from Eqs. (29) and (35) it is clear that the constants $g_{\pi^0 k^{*0}}$ and $G^{(3)}$ determine the $\Omega^-\to\Xi\pi$ decay rates. From Eq. (62) and the fact that the matrix element $\langle\pi|\mathcal{H}_w^{pv}|K^*\rangle$ is octet-dominated,

$$|g_{\pi^0\bar{K}^{*0}}|=|(\tfrac{1}{2})^{1/2}g_{\pi^-K^{*-}}|\simeq 4.8\times10^{-7}\,\text{GeV}\,. \tag{67}$$

To estimate the constant $G^{(3)}$ of Eq. (35), we use vector-meson dominance to relate it to the π^0 photoproduction reaction

$$\gamma+p\to p+\pi^0$$

in the region of $N^*(1236)$ resonance. The decay process

$$N^{*+}(p)\to p(p')+\gamma(q)$$

can be described by the Gourdin-Salin invariants[32,33] C_3, C_4, and C_5 defined by

$$(2\pi)^3\left(\frac{p_0p_0'}{M_pM_{n^*}}\right)^{1/2}\langle p(p')|J_\mu^{em}(0)|N^{*+}(p)\rangle = e\bar{u}_p(p')\gamma_5[C_3(q^2)(q\cdot\gamma g_{\mu\nu}-\gamma_\mu q_\nu)+C_4(q^2)(q\cdot p g_{\mu\nu}-p_\mu q_\nu)+C_5(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)]u_{N^*}^\nu(p)\,. \tag{68}$$

In terms of these invariants, the magnetic-dipole and electric-quadrupole contributions to this decay are[32,33]

$$\Gamma_{M_1}=\frac{\alpha}{96}\frac{(M_{N^*}^2-M_p^2)^3}{M_{N^*}^5}\times|(3M_{N^*}+M_p)C_3(0)+M_{N^*}(M_{N^*}-M_p)C_4(0)|^2\,, \tag{69}$$

$$\Gamma_{E_2}=\frac{\alpha}{32}\frac{(M_{N^*}^2-M_p^2)^3}{M_{N^*}^5}\times|M_{N^*}-M_p)C_3(0)+M_{N^*}(M_{N^*}-M_p)C_4(0)|^2\,, \tag{70}$$

where α is the fine-structure constant. However, $SU(6)_W$ symmetry implies that the $E2$ transition is forbidden[34] and the empirical estimates[35] also indicate that the $E2$ contribution to N^* excitation in π^0 photoproduction is probably not more than 0.1% at the resonance energy. Hence from Eq. (70),

$$C_3(0)\simeq -M_{N^*}C_4(0)\,. \tag{71}$$

At resonance, assuming a pure $M1$ transition, the total cross section for π^0 photoproduction is given by

$$\sigma_{\text{Res}}=(\gamma p\to p\pi^0)=\frac{2}{3}\left(\frac{4\pi}{q^{*2}}\right)\frac{\Gamma_{M1}}{\Gamma_T}\,, \tag{72}$$

where Γ_T denotes the total $N^*(1236)$ width and q^*, the c.m. photon momentum. We take the values used by Dalitz and Sutherland[36] in their analysis of $M1$ photoexcitation of the $N^*(1236)$,

$$\sigma_{\text{Res}}(\gamma p\to p\pi^0)=260\pm6\,\mu\text{b}\,, \tag{73}$$

$$\Gamma_T=0.119\text{ GeV}\,, \tag{74}$$

and find

$$C_4(0)\approx-1.77\text{ GeV}^{-2}\,. \tag{75}$$

Next we use vector-meson dominance[32,33] to connect $C_4(0)$ to $G^{(3)}$. In the spirit of the rest of the paper we follow the dispersion theory approach[37] to the vector-meson dominance. To implement the dispersion approach, we consider an alternative redundant choice for the form factors

$$(2\pi)^3\left(\frac{p_0 p_0'}{M_p M_{N^*}}\right)^{1/2} \langle p(p')\,|\,J_\mu^{\text{em}}(0)\,|\,N^{*+}(p)\rangle = e\bar{u}_p(p')\gamma_5[B_1(q^2)g_{\mu\nu} + B_2(q^2)\gamma_\mu q_\nu + B_3(q^2)p_\mu q_\nu + B_3(q^2)p_\mu q_\nu]u_{N^*}^\nu(p) \tag{76}$$

and write unsubtracted dispersion relations for the invariant B_i's, dispersing in q^2. Saturating the dispersion integral by the contributions of the intermediate ρ meson (ω and φ do not contribute since this is an isovector transition), we obtain

$$B_1(0) = (M_{N^*} + M_p)C_3(0) + \tfrac{1}{2}(M_{N^*}{}^2 - M_p{}^2)\; C_4(0) \simeq \left(\frac{2}{3}\right)^{1/2}\frac{G^{(1)}}{f_\rho}, \tag{77}$$

$$B_2(0) = -C_3(0) \simeq -\left(\frac{2}{3}\right)^{1/2}\frac{G^{(2)}}{f_\rho}, \tag{78}$$

$$B_3(0) = -C_4(0) \simeq \left(\frac{2}{3}\right)^{1/2}\frac{G^{(3)}}{f_\rho}, \tag{79}$$

where f_ρ is defined by

$$(2\pi)^{3/2}(2k_0)^{1/2}\langle 0\,|\,J_\nu^{\text{em}}(0)\,|\,\rho^0(k,\lambda)\rangle = e\frac{m_\rho{}^2}{f_\rho}\epsilon_\mu(k,\lambda), \tag{80}$$

and the $(2/3)^{1/2}$ comes from the SU(3) relation

$$G^{(i)}_{\bar{\rho}^0\bar{p}N^{*+}} = \left(\frac{2}{3}\right)^{1/2} G^{(i)}_{\bar{K}^{*0}\Xi^-\Omega^-}, \quad i = 1, 2, 3 \tag{81}$$

with $G^{(i)}_{\bar{K}^*\Xi\Omega}$ ($i = 1, 2, 3$) defined by Eq. (33). By Eqs. (75) and (79) we obtain, for $f_\rho{}^2/4\pi = 2.0 \pm 0.1$,[38]

$$|G^{(3)}| \simeq 10.8\ \text{GeV}^{-2}. \tag{82}$$

Substituting Eqs. (67) and (82) into Eq. (35) we obtain

$$|B| \simeq 25\times 10^{-7}\ \text{GeV}^{-1}. \tag{83}$$

Now, using Eqs. (29), (83), and (66), we have the decay rates

$$\begin{aligned}\Gamma(\Omega^- \to \Xi^-\pi^0) &\simeq 1.2\times 10^8\ \text{sec}^{-1},\\ \Gamma(\Omega^- \to \Xi^0\pi^-) &\simeq 2.4\times 10^8\ \text{sec}^{-1}.\end{aligned} \tag{84}$$

Besides the SU(3) assumption for the vertices and the assumption of vector-meson dominance, there is another possible source of error in the estimate of Eq. (54). We have assumed that the parity-conserving amplitude A is strictly zero for the $\Omega^- \to \Xi\pi$ mode. Strictly speaking, the quark models through Eq. (3) only predict that $A/B \ll 1$. Therefore, because $\Gamma(\Omega^- \to \Xi\pi) \propto [\,|P|^2 + |D|^2]$, where

$$P = A, \qquad D = B\left[\frac{(M_\Omega - M_\Xi)^2 - m_\pi{}^2}{(M_\Omega + M_\Xi)^2 + m_\pi{}^2}\right]^{1/2}, \tag{85}$$

A and B will contribute almost equally to the decay rates even if $A/B \simeq 1/10$. Hence, the estimates given by Eqs. (84) should be expected to be valid to no better than a factor of two.

It is encouraging to notice that Eqs. (71), (77), (78), and (79) are consistent with Eq. (34) within approximately 15%. Because of Eq. (71), Eqs. (77) through (79) predict a relation among $G^{(1)}$, $G^{(2)}$, and $G^{(3)}$. This relation will be equivalent to Eq. (34), provided

$$(M_{N^*} + M_p)M_{N^*} - \tfrac{1}{2}(M_{N^*}{}^2 - M_p{}^2) = (M_\Omega + M_\Xi)M_{N^*} - \tfrac{1}{2}(M_\Omega{}^2 + m_{k^*}{}^2 - M_\Xi{}^2 - 2m_\pi{}^2). \tag{86}$$

The left-hand side of Eq. (86) is about 2.4 GeV^2, whereas the right-hand side is about 2.8, only a 15% discrepancy. In view of the SU(3) approximation for the vertices used in deriving Eqs. (77), (78), and (79), this result is quite remarkable. We emphasize that this is a nontrivial consistency check of the model we are using.

B. $\Omega^- \to \Lambda K^-$ Decay Rate

The parity-conserving amplitude A for $\Omega^- \to \Lambda K^-$, given by Eq. (38), is simpler to estimate numerically. Since we have shown in Sec. IVB that Gronau's fit[3] of the octet hyperon-decay amplitudes can be essentially reproduced in the framework of dispersion relations and the symmetric quark model, we accept the "best fit" value for $a_{\Lambda\Xi^0}$, namely,

$$a_{\Lambda\Xi^0} \simeq 10.5\times 10^{-8}. \tag{87}$$

While the coupling constant $G^D_{\Xi^0 K^+\Omega^-}$ is not directly measurable, it can of course be related to the experimental width $(N^{*++} \to p\pi^+) \simeq 120$ MeV by SU(3). However, in the application of SU(3) symmetry to the couplings of the $\frac{3}{2}^+$ hyperon decuplet to the nucleon and pseudoscalar meson octets, Rotelli and Scadron[39] have found that some account of SU(3) breaking at the vertices is necessary in order to obtain good agreement between the predicted decay rates and experiment. They have shown that a simple, successful procedure is to define SU(3) symmetric, dimensionless coupling constants $g_{DB\pi}$ by

$$(2\pi)^{9/2}\left(\frac{2q_0p_0p_0'}{M_DM_B}\right)^{1/2}\langle \pi B(p')\mathcal{H}_I(0)\,|\,D(p)\rangle = \frac{g_{DB\pi}}{2(M_BM_D)^{1/2}}\, q^{\mu}\bar{u}_B(p')u_{\mu}^{D}(p)\,. \quad (88)$$

We choose this form for the reduced vertex for kaon couplings and so applying SU(3) symmetry, obtain

$$|\,G^{D}_{\Xi^0K^+\Omega^-}| = \left|\frac{g_{\Xi^0K^+\Omega^-}}{(M_\Omega M_\Xi)^{1/2}}\right| \simeq 11\,. \quad (89)$$

Substituting Eqs. (87) and (89) into Eq. (38), we find

$$A \simeq 6\times 10^{-6}\ \mathrm{GeV}^{-1} \quad (90)$$

and a decay rate

$$\Gamma(\Omega^- \to \Lambda K^-) \simeq 1.5\times 10^{10}\ \mathrm{sec}^{-1}\,. \quad (91)$$

From Eqs. (84) and (91) we find the total nonleptonic decay rate to be

$$\Gamma^{\mathrm{th}}_{\Omega^-} \simeq 1.5\times 10^{10}\ \mathrm{sec}^{-1}\,, \quad (92)$$

to be compared with the experimental value, namely,

$$\Gamma^{\mathrm{exp}}_{\Omega^-} = (1\pm 0.3)\times 10^{10}\ \mathrm{sec}^{-1}\,. \quad (93)$$

It must be stressed that the decay rates are very *sensitive* to the errors in the amplitudes, e.g., a reduction by a factor of two in the latter will reduce the former by a factor of four.

VI. SUMMARY OF PREDICTIONS AND CONCLUDING REMARKS

In spite of the tentative nature of the estimates of the Ω^- decay rates presented in Sec. V, there are strong qualitative predictions which are insensitive to the errors in these estimates and which can be put to an unambiguous experimental test in the near future. If we accept the usage of dispersion relations,[40] Regge asymptotic behavior, and the resonance saturation of the absorptive part, the results summarized below pose a test of Eqs. (1) through (4) based on the symmetric quark model. Since the proposed explanation of the $|\Delta I| = \frac{1}{2}$ rule is likewise based on the symmetric quark model, these results also stand as a test of the truth of this expalantion:

(i) From Eq. (43) we find that the parity-violating amplitude B of the ΛK^- mode is highly suppressed compared to the parity-conserving amplitude A. This prediction implies that the D-wave effects should be almost entirely negligible in this decay mode.

(ii) The prediction of Eq. (29), that the parity-conserving amplitude A vanishes for $\Omega^- \to \Xi\pi$ decay, should not be taken literally. Strictly speaking, through Eqs. (2) and (4) the symmetric quark model only predicts that the A amplitude is suppressed compared to B, that is to say, $A/B \ll 1$. Because of the inherent angular momentum suppression of the D-wave amplitude, there could still exist a P-wave amplitude comparable to that of the D wave. So here the qualitative consequence of the quark model Eqs. (1) through (4) is that the D-wave amplitude is at least equal in strength, whereas ordinarily without the suppression of A we would expect the natural dominance of the P wave.

(iii) From the results of (i) and (ii) discussed above, we have a very strong qualitative prediction, easily testable against experiment when more Ω^- decay events are available. Because of the suppression of the parity-conserving amplitudes in $\Omega^- \to \Xi\pi$ decays, the $\Omega^- \to \Lambda K^-$ decay mode is much more probable than the $\Omega^- \to \Xi\pi$ mode, i.e., $\Gamma(\Omega^- \to \Xi\pi)/\Gamma(\Omega^- \to \Lambda K^-) \ll 1$. Preliminary estimates in Sec. V indicate that the ratio of the decay rates may be as small as $(\frac{1}{10} - \frac{1}{30})$. We find this prediction to be quite striking since it is distinct from that of any other model discussed in the literature.[41]

Two features of the results are already encouraging. First, the predicted total nonleptonic decay rate of the Ω^- is consistent with experiment. Second, Eq. (34) relating the strong-interaction form factors, which was derived using Eqs. (2) and (4), implies a mass formula Eq. (86) that is satisfied empirically to 15%.

In closing, we make brief remark on the contrast between the origins of breaking the $|\Delta I| = \frac{1}{2}$ rule in the Bose-quark model and in the other versions of the symmetric quark model. In Sec. IV D we found that in the color-quark, the three-triplet, and the paraquark model, the octet dominance and the $|\Delta I| = \frac{1}{2}$ rule of all the observed nonleptonic weak decays can be obtained by making use of analyticity, Regge asymptotic behavior, and the resonance saturation of the absorptive parts. Besides resolving the serious spin and statistics difficulties associated with the Bose-quark model, the explanation of the $|\Delta I| = \frac{1}{2}$ rule using one of these other models has a further advantage. In the Bose-quark model[1,3] the nonleptonic weak Hamiltonian is a member of an SU(3) octet and the $|\Delta I| = \frac{1}{2}$ rules are *exact*. So in this model the observed breaking—e.g., 5–10% in the amplitudes for $K \to 2\pi, 3\pi$ decays[42]—of the $|\Delta I| = \frac{1}{2}$ rule must come either from electromagnetism or from an intermediate W particle of large mass ($M_W \simeq 37$ GeV). Both these effects could be of order α and hence, too small. On the other hand, in the color-quark, the three-triplet, and the paraquark models, there is the gratifying difference (see Sec. IV) that the $|\Delta I| = \frac{1}{2}$ rule is only *approximate*—it will be broken by continuum contributions to the dispersion integrals.

Note added in proof. We used the old-fashioned pre-gauge-theory point of view in this paper. In gauge theories all the above arguments still apply in lowest order to ordinary weak processes (those not involving real W mesons) when the momentum transfers involved are less than M_W. This is the only region which has been studied in any detail experimentally.

We would like to point out an important assumption made in the paper, namely, that the current and constituent quarks could be taken to be the same objects as far as taking the matrix element of $\mathcal{H}_w$ (constructed out of the current quarks) between two baryons (made of three constituent quarks). This may look like an unreasonable assumption, since the distinction between current and constituent quarks seems to be necessary, especially if we want to relate partons to quarks. We would like to justify this assumption by proposing the point of view that as far as matrix elements of currents involving small invariant momentum transfer are concerned, the current and constituents quarks can be taken to be the same objects. On the other hand, from considerations of deep-inelastic scattering of electrons by protons one should conclude that in matrix elements of currents involving high momentum transfer the distinction between current and constituent quarks is required. Besides preserving the results of this paper, this attitude, we think, is a very reasonable one since all of the so-called "bad" quantitative results of the naive quark model in low-energy phenomena can be avoided without losing the identification between current and constituent quarks in low-energy matrix elements. The quark-model results, $G_A/G_V = \frac{5}{3}$, $\Gamma(\Delta^{++} \to p\pi^+) \simeq 200$ MeV, $\Gamma(B' \to B\pi) \simeq 0$ for B' and B belonging to two different $SU(6)_W$ multiplets, etc. are obtained *only if* we assume $SU(6)_W$ wave functions for the constituent quarks of the baryons. $SU(6)_W$ is, of course, far from being a perfect symmetry of strong interactions. So by modifying the wave function of the three-quark system from the pure SU(6) type (or by properly taking into account the relativistic correction) one has the freedom to improve all of the above results. It is not necessary to set the current and constituent quarks different for the calculation of these matrix elements. It should be emphasized that SU(6)-type wave functions are not necessary for the derivation of any of the quark-model results presented in this paper. We would also like to point out that in a hypothetical world, where $SU(6)_W$ is a perfect symmetry, there is no need to distinguish between current and constituent quarks as has been shown by S. P. de Alwis and J. Stern.[42a]

ACKNOWLEDGMENTS

Part of this work was done while the authors were visiting the Center for Particle Theory of the University of Texas at Austin. They wish to thank Professor E. C. G. Sudarshan and Professor A. M. Gleeson for the gracious hospitality extended to them. They also wish to thank Dr. C. Y. Chang of the University of Maryland for information concerning experiments on the Ω^- nonleptonic decays, and Dr. S. Meshkov and Dr. J. C. Pati for useful comments.

APPENDIX A: THE COLOR-QUARK AND THREE-TRIPLET QUARK MODELS

The purpose of this appendix is to show, for the current-current weak Hamiltonian density $\mathcal{H}_w$ constructed out of the "color-quark" or three-triplet quark fields that (a) only the octet part of $\mathcal{H}_w^{\text{pv(pc)}}$ contributes to the matrix elements of the type $\langle B_1|\mathcal{H}_w^{\text{pv(pc)}}|B_2\rangle$, where B_1 and B_2 are any two $3q$ baryon states of which at least one is an $SU(3)_{\text{color}}$ or $SU(3)''$ singlet state, and that (b) Eqs. (1) through (4) of the text hold.

For the sake of definiteness we will prove these assertions in the *SUB* version[6] of the three-triplet model which has the approximate symmetry group $SU(3)\times SU(3)''$. By a mere relabeling they can also be proven in the Han-Nambu[7] version, where the basic group is $SU(3)'\times SU(3)''$, as well as in the color-quark model, where it is $SU(3)\times SU(3)_{\text{color}}$. The hadronic weak current in the *SUB* model can be written as[8]

$$J_\mu = \sum_{\alpha,\,\beta=S,\,U,\,B} C_{\alpha\beta}\bar{\psi}_{(\alpha,\,i)}\gamma_\mu(1+\gamma_5) \times[(\lambda_1+i\lambda_2)_{ij}\cos\theta+(\lambda_4+i\lambda_5)_{ij}\sin\theta]\psi_{(\beta,\,j)}\,. \tag{A1}$$

The usual SU(3) group (with the familiar I_3 and Y generators) acts on the Latin indices (i, j, etc.) in Eq. (A1), whereas a second SU(3) group, called $SU(3)''$, acts on the Greek indices (α, β, etc.). The coefficients $C_{\alpha\beta}$ determine the $SU(3)''$ structure of the current. If the nonleptonic weak Hamiltonian density is taken as

$$\mathcal{H}_w = (\tfrac{1}{2})^{1/2}G\{J_\mu, J^{\mu\dagger}\}\,, \tag{A2}$$

the $|\Delta S| = 1$ part of it contains, in general, the SU(3) octet and 27-plet parts. We will now show that the 27-plet part will not contribute to the matrix element $\langle B_1|\mathcal{H}_w|B_2\rangle$.

Consider the $|\Delta S|=1$ part of $\mathcal{H}_w$. It can be written as a linear combination of terms of the form

$$T^{(\beta,j)(\delta,l)}_{(\alpha,i)(\gamma,k)}\equiv\{\bar\psi_{(\alpha,i)}(x)\gamma_\mu(1+\gamma_5)\psi_{(\beta,j)}(x)\}\times\{\bar\psi_{(\gamma,k)}(x)\gamma^\mu(1+\gamma_5)\psi_{(\delta,l)}(x)\},\quad (A3)$$

each of which by Fierz reshuffling[43] is *symmetric* under the interchange $(\alpha,i)\leftrightarrow(\gamma,k)$ or $(\beta,j)\leftrightarrow(\delta,l)$. To be definite, let us assume that B_2 is the SU(3)″ singlet state. When the matrix element of $\mathcal{H}_w$ is taken between B_1 and B_2, a nonvanishing contribution occurs only when $\psi_{(\beta,j)}$ and $\psi_{(\delta,l)}$ act on the quarks in the same baryon. The same statement will hold for $\bar\psi_{(\alpha,i)}$ and $\bar\psi_{(\gamma,k)}$. Let the quark fields $\psi_{(\beta,j)}$ and $\psi_{(\delta,l)}$ lead to the destruction of the two quarks in B_2. Only that part of (A3) which is antisymmetric under the interchange $(\beta\leftrightarrow\delta)$ will contribute to this process, since B_2 is an SU(3)″ singlet baryon whose quark wave function is completely antisymmetric in the SU(3)″ indices of any two constituent quarks. Because of the symmetry of (A3) under the interchange $(\beta,j)\leftrightarrow(\delta,l)$, it then follows that only the part of (A3) which is antisymmetric in the SU(3) indices $(j\leftrightarrow l)$ will contribute. Since the 27-plet part of $\mathcal{H}_w$ must be symmetric under the interchange $(j\leftrightarrow l)$, it is now obvious that only the octet part of $\mathcal{H}_w$ will contribute to the matrix element $\langle B_1|\mathcal{H}_w|B_2\rangle$. Furthermore, since[43]

$$T'^{(\beta,j)(\delta,l)}_{(\alpha,i)(\gamma,k)}\equiv\{\bar\psi_{(\alpha,i)}(x)\gamma_\mu(1-\gamma_5)\psi_{(\beta,j)}(x)\}\times\{\bar\psi_{(\gamma,k)}(x)\gamma^\mu(1-\gamma_5)\psi_{(\delta,l)}(x)\}\quad (A4)$$

is also symmetric under the interchange $(\alpha,i)\leftrightarrow(\gamma,k)$ or $(\beta,j)\leftrightarrow(\delta,l)$, the matrix elements $\langle B_1|\mathcal{H}_w^{\rm pv}|B_2\rangle$ and $\langle B_1|\mathcal{H}_w^{\rm pc}|B_2\rangle$ separately have the octet properties.

Equations (1) and (2) of the text can now be proven. $\langle\Xi^\circ|\mathcal{H}_w^{\rm pv(pc)}|\Sigma^\circ\rangle$ will vanish since the quark wave function of Σ° is symmetric in the SU(3) indices u and d, while only the part of $\mathcal{H}_w^{\rm pc(pv)}$ antisymmetric in the u and d indices will contribute to this matrix element. The matrix element $\langle B|\mathcal{H}_w^{\rm pc(pv)}|B_{10}\rangle$, where B is any $3q$ state, will be zero since the states in the $\underline{10}$ are totally symmetric in the quark SU(3) indices.

If we treat only orbitally and radially excited $3q$ and $q\bar q$ states B^* and M^*, Eqs. (3) and (4) will also follow since $\mathcal{H}_w^{\rm pc(pv)}$ describes the interaction of four quarks at the same space-time point and any two quarks in B^* and M^* are much less likely to be found at the same space-time point than their counterparts in ground states $B_g(56,0^+)$ and $M_g(36,0^-)$. However, for $q\bar q$-pair excited or SU(3)″ excited states Eqs. (3) and (4) can be violated. But this need not concern us because at present there are no established resonances of these types and even if they exist at higher energies they should not contribute significantly to low-energy processes in a dispersion-theoretic approach.

APPENDIX B: THE PARAQUARK MODEL

In this appendix we show the following results for the nonleptonic weak Hamiltonian in the paraquark model[12]: (a) The parafermion fields of order three entering into the nonleptonic weak Hamiltonian density do not undergo a Fierz reshuffling, unlike boson or fermion fields. (b) Only the octet part of the nonleptonic $\mathcal{H}_w^{\rm pc(pv)}$ contributes to the matrix element between two three-quark baryon states, namely, $\langle B_1|\mathcal{H}_w^{\rm pc(pv)}|B_2\rangle$. (c) Equations (1) through (4) of the text hold also in the paraquark model.

The hadronic weak current in the paraquark model can be chosen as

$$J_\mu(x)=\cos\theta[\bar\psi_u(x),\gamma_\mu(1+\gamma_5)\psi_d(x)]_- +\sin\theta[\bar\psi_u(x),\gamma_\mu(1+\gamma_5)\psi_s(x)],\quad (B1)$$

where the ψ's are parafermion fields of order three. These local currents (in the sense of spacelike commutativity) and their commutation relations are the usual ones, e.g., one has all the results of current algebra. The nonleptonic weak Hamiltonian density is

$$\mathcal{H}_w(x)=(\tfrac{1}{2})^{1/2}G\{J_\mu(x),J^{\mu\dagger}(x)]_+-\langle 0|\,\text{same}\,|0\rangle\},\quad (B2)$$

where the vacuum expectation value has been subtracted off to remove the usual divergences. This procedure is used instead of normal ordering since the commutation relations are trilinear.

The $\Delta S=1$ part of Eq. (B2) can be written as

$$\mathcal{H}_w^{(\Delta S=1)}=(\tfrac{1}{2})^{1/2}G\cos\theta\sin\theta\{[[\bar\psi_u(x),\gamma_\mu(1+\gamma_5)\psi_d(x)]_-,[\bar\psi_s(x),\gamma^\mu(1+\gamma_5)\psi_u(x)]_-]_+-\langle 0|\,\text{same}\,|0\rangle\},\quad (B3)$$

where each $\psi(x)$ has the usual expansion:

$$\psi_l(x)=\sum_L[a_{l,L}U_{l,L}(x)+b^\dagger_{l,L}V_{l,L}(x)],\quad (B4)$$

where the index L (over which the sum is taken) denotes the momentum and spin quantum numbers and l is the SU(3) quantum number of the quark, namely $l=u$, d, or s. The U's and V's are defined by

$$U_{l,L}(x) = \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{E_p}\right)^{1/2} u_l(p,s)e^{-ip\cdot x}\,, \qquad V_{l,L}(x) = \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{E_p}\right)^{1/2} v_l(p,s)e^{ip\cdot x}\,. \tag{B5}$$

In Eq. (B4) $a_{l,L}$ is the annihilation operator for the quark, whereas $b_{l,L}^\dagger$ is the creation operator for the antiquark. The part of Eq. (B3) which will contribute to the matrix element between two $3q$ baryon states can be written in the form

$$\mathcal{H}_{lm}^{kn} = (\tfrac{1}{2})^{1/2} G\cos\theta\sin\theta \sum_{L,K,M,N}\{[[a_{l,L}^\dagger \overline{U}_{l,L}(x), \gamma_\mu(1+\gamma_5)a_{k,K}U_{k,K}(x)]_-, [a_{m,M}^\dagger \overline{U}_{m,M}(x), \gamma^\mu(1+\gamma_5)a_{n,N}U_{n,N}(x)]_-]_+ - \langle 0|\text{same}|0\rangle\}\,. \tag{B6}$$

There are also other pieces in Eq. (B3) which will involve the creation and annihilation operators of antiquarks. If the quark fields undergo Fierz reshuffling in (B3), it is clear that each of these pieces must undergo Fierz reshuffling separately. That is, a necessary condition for Fierz reshuffling in Eq. (B3) is that $\mathcal{H}_{lm}^{kn}$ given by Eq. (B6) is either symmetric or antisymmetric in the upper and lower indices. We will now prove that neither is true in the case of para-Fermi fields of order three.

It is easily checked that the interchange of $(\overline{U}_{l,L} \leftrightarrow \overline{U}_{m,M})$ or $(U_{k,K} \leftrightarrow U_{n,N})$ in Eq. (B6) introduces only a minus sign. So the symmetry or antisymmetry of $\mathcal{H}_{m}^{kn}$ will depend, respectively, on the antisymmetry or symmetry of the object,

$$T_{\alpha\beta}^{\gamma\delta} = \overline{T}_{\alpha\beta}^{\gamma\delta} - \langle 0|\overline{T}_{\alpha\beta}^{\gamma\delta}|0\rangle\,, \tag{B7}$$

where

$$\overline{T}_{\alpha\beta}^{\gamma\delta} = [[a_\alpha^\dagger, a_\gamma]_-, [a_\beta^\dagger, a_\delta]_-]_+ \tag{B8}$$

and the Greek symbols α, β, γ, and δ denote both the SU(3) index and the momentum and spin quantum numbers, so $\alpha = (l, L), \ldots$ The question now is: Is $T_{\alpha\beta}^{\gamma\delta}$ either (i) purely antisymmetric in α and β, (ii) purely symmetric, or (iii) neither? We proceed to eliminate (i) and (ii), so (iii) is true.

If (i) were true, the following object:

$$B \equiv \sum_{\alpha,\beta=1}^{\alpha,\beta=N} \langle 0|[a_\alpha, a_\beta]_+ T_{\alpha\beta}^{\gamma\delta}[a_\gamma^\dagger, a_\delta^\dagger]_+|0\rangle \tag{B9}$$

should be obviously zero. Using the trilinear commutation relations for parafermions,[12]

$$[[a_\mu^\dagger, a_\nu]_-, a_\rho]_- = -2\delta_{\mu\rho}a_\nu\,, \tag{B10}$$

$$[[a_\mu, a_\nu]_-, a_\rho]_- = 0\,, \tag{B11}$$

$$[[a_\mu, a_\nu]_-, a_\rho^\dagger]_- = 2\delta_{\nu\rho}a_\mu - 2\delta_{\mu\rho}a_\nu\,, \tag{B12}$$

and the equations

$$a_\mu|0\rangle = 0 \ \ \forall\mu\,, \tag{B13}$$

$$a_\mu a_\nu^\dagger|0\rangle = p\delta_{\mu\nu}|0\rangle \ \ \forall\mu,\nu\,, \tag{B14}$$

where p is the order of the parafermions ($p=1$ for ordinary fermions and $p=3$ for parafermions of order three), we can calculate B given by Eq. (B9). If we take for simplicity $N=1$, and all the indices α, β, γ, and δ the same and equal to 1, we find, using Eqs. (B10)–(B14),

$$B = -16p(8p-16)(p-1)\,, \tag{B15}$$

which does *not* vanish for $p=3$. (As expected, it does vanish for $p=1$ so that $\mathcal{H}_{lm}^{kn}$ of Eq. (B6) is indeed symmetric in l and m for ordinary fermions.)

Next consider

$$C \equiv \sum_{\alpha,\beta=1}^{\alpha,\beta=N} \langle 0|[a_\alpha, a_\beta]_- T_{\alpha\beta}^{\gamma\delta}[a_\gamma^\dagger, a_\delta^\dagger]_-|0\rangle\,, \tag{B16}$$

where $\gamma \neq \delta$ and fixed. C should be zero if $T_{\alpha\beta}^{\gamma\delta}$ is symmetric in α and β, i.e., if case (ii) holds. On the other hand, using Eqs. (B10)–(B14) we find

$$C = -32p[p - pN + N^2 - 2N + 1]\,. \tag{B17}$$

Since N is arbitrary, this only vanishes for $p=0$.

Hence only case (iii) can be true, $T_{\alpha\beta}^{\gamma\delta}$ is neither symmetric nor antisymmetric in α and β. This implies that $\mathcal{H}_{lm}^{kn}$ defined by Eq. (B6) is neither symmetric nor antisymmetric in l and m and so it is not a pure SU(3) octet operator. In spite of this we will now show that only the octet part of $\mathcal{H}_{lm}^{kn}$ will contribute to the matrix element $\langle B_1|\mathcal{H}_w|B_2\rangle$, where B_1 and B_2 are $3q$ baryon states, and that Eqs. (1) through (4) of the text still hold.

The crucial point is to notice that the baryon states[12] in the paraquark model are linear combinations of states

$$|\alpha,\beta,\gamma\rangle = [[a_\alpha^\dagger, a_\beta^\dagger]_+, a_\gamma^\dagger]_+|0\rangle\,, \tag{B18}$$

each of which is completely symmetric in the quantum numbers α, β, and γ. Therefore in the matrix element of $\mathcal{H}_{lm}^{kn}$ between two single baryon states, only the symmetric part of $T_{\alpha\beta}^{\gamma\delta}$ (symmetric under $\alpha \leftrightarrow \beta$ and $\gamma \leftrightarrow \delta$) can contribute. Therefore, *as far as taking the matrix element between two single baryon states is concerned,* $\mathcal{H}_{lm}^{kn}$ of Eq. (B6) can be rewritten as [the interchange of $\overline{U}_\alpha$ and $\overline{U}_\beta$ introduces an additional negative sign in Eq. (B6)]

$$\mathcal{H}^{kn}_{lm} = -(\tfrac{1}{2})^{1/2} G\cos\theta\sin\theta \sum_{L,K,M,N} \{[[a^{\dagger}_{m,M}\overline{U}_{m,M}(x), \gamma_\mu(1+\gamma_5)a_{k,K}U_{k,K}(x)]_-, [a^{\dagger}_{l,L}\overline{U}_{l,L}(x), \gamma^\mu(1+\gamma_5)a_{n,N}U_{n,N}(x)]_-]_+ - \langle 0|\text{same}|0\rangle\}$$

$$= -(\tfrac{1}{2})^{1/2} G\cos\theta\sin\theta \sum_{L,K,M,N} \{[[a^{\dagger}_{m,L}\overline{U}_{m,L}(x), \gamma_\mu(1+\gamma_5)a_{k,K}U_{k,K}(x)]_-, [a^{\dagger}_{l,M}\overline{U}_{l,M}(x), \gamma^\mu(1+\gamma_5)a_{n,N}U_{n,N}(x)]_-]_+ - \langle 0|\text{same}|0\rangle\}. \tag{B19}$$

The last line of Eq. (B19), on comparison with Eq. (B6), is seen to be $-\mathcal{H}^{kn}_{ml}$. Therefore, *between two 3q baryon states,*

$$\mathcal{H}^{kn}_{lm} = -\mathcal{H}^{kn}_{ml}. \tag{B20}$$

Equation (B20) of course implies that only the octet part of the nonleptonic $\mathcal{H}_w$ contributes to any matrix element of the form $\langle B_1|\mathcal{H}_w|B_2\rangle$.[44] By the arguments given at the end of Appendix A, it is now obvious that Eqs. (1) through (4) of the text will also hold for the paraquark model.

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[2]The term "symmetric quark model", as used in the literature and here, refers to a model in which the over-all wave function of three quarks in the baryon ground state is symmetric in the visible quantum number. This includes the Bose-quark model, the color-quark and three-triplet models, and the paraquark model. See O. W. Greenberg, invited talk at Washington APS meeting (April, 1967), University of Maryland Report No. 680 (unpublished); O. W. Greenberg and M. Resnikoff, Phys. Rev. 163, 1844 (1967).

[3]M. Gronau, Phys. Rev. Lett. 28, 188 (1972); Phys. Rev. D 5, 118; 5, 1877(E) (1972); while there are four parameters varied in obtaining the "best fit", the "best fit" values of the parameters D/F and d/f are about equal to the quark model values -1 and $+1.5$, respectively, the fitted values being $F = 4.7\times10^{-5}$ MeV, $D/F = -0.85$, $d/f = 1.8$, and $\delta/\varphi = -0.5$. The d (δ) and f (φ) parameters are, respectively, the symmetric and antisymmetric ***PBB*** (***VBB***) couplings. Matrix elements of the octet, parity-conserving weak Hamiltonian between two s-wave 8 baryons can be parametrized as

$$(2\pi)^3\left(\frac{4p_0p_0'}{M_iM_j}\right)^{1/2}\langle B_j(p)|\mathcal{H}_w^{pc}(0)|B_i(p')\rangle = 2\sqrt{2}\,\bar{u}_j(d_{i6j}D + if_{i6j}F)u_i$$

in the framework of SU(3) symmetry.

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[11]For proof of this statement, see Appendix A.

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[13]See Sec. IV.

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[15]In the Fermi-quark model, the Fierz-reshuffling property of the Fermi-quark fields in the $V\pm A$ interaction leads to

$$\langle\Lambda^0|\mathcal{H}_w^{pc}|\Xi^0\rangle = 0,$$

because Λ^0 is antisymmetric under interchange of the u and d quarks while $\mathcal{H}_w^{pc}$ is symmetric. Then SU(3) parametrization of the octet part of this matrix element (see Ref. 3) implies $D/F = 3$.

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[17]This simple fact is *not* well appreciated in the literature. See Ref. 16.

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[20]Assuming Regge asymptotic behavior, F_1' and G_1' (as well as F_2' and G_2') can be shown to obey unsubtracted dispersion relations in the variable s or u for fixed q^2 and t. Also because of Eq. (3), the dispersion relations are saturated by low-mass resonance intermediate states. So "weak PCAC" is applicable to these invariant functions.

[21]See Sec. V for the observable consequences of this smallness of the P-wave amplitude in $\Omega^- \to \Xi\pi$.

[22]There is also a natural suppression factor for the D-wave amplitude coming from the relation connecting it to the parity-violating amplitude B. See Eq. (85) in Sec. V.

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[44]For another argument to show the same result, see Kingsley, Ref. 10. This result seems to contradict footnote 17 of Ref. 8.