PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Muon Polarization in the Neutrino-Induced Inclusive Reaction—A Test of $V-A$ Weak Interaction at High Energy*

Ilmun Ju
Stanford Linear Accelerator Center, Standord University, Stanford, California 94305
and Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan 48104†
(Received 2 July 1973)

A possible modification of the $V-A$ weak interaction at high energy is studied through the introduction of an intermediate vector boson with derivative couplings or of a phenomenological momentum-dependent leptonic vector current. It will be shown that a small contamination of a certain derivative coupling both changes the sign of the longitudinal polarization of the muon in the neutrino-nucleon inelastic scattering in the scaling region and lets the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ increase as the neutrino energy gets larger. A different momentum-dependent vector current leaves the sign of the polarization and the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ unchanged.

I. INTRODUCTION

The conventional $V-A$ theory of the weak interaction[1] has been able to describe low-energy leptonic interactions remarkably well, in spite of the fact that higher-order contributions in perturbation theory badly diverge. But this seemingly satisfactory consistancy of the $V-A$ theory with experiment cannot be regarded as proof of the validity of the theory; rather it implies that present data are not sufficient to fix uniquely all the general nonderivative coupling constants. In fact Jarskog[2] claims that up to 30% scalar and tensor couplings would not be contradictory to the experimental muon decay data. Moreover, even at the phenomenological level, there is no fundamental reason why the current-current theory should prevail at higher momentum transfers. The idea of a mediating particle for the weak interaction analogous to the photon in quantum electrodynamics or the Yukawa-type pion field in the strong interactions has long been attractive, because this nonlocal theory reduces to the local current-current interaction at low energies. Although the intermediate-vector-boson theory is no less renormalizable, theoretical consequences were examined extensively in the literature. But this line of effort has always been embarrassed by the fact that intermediate vector bosons are not observed experimentally. This is also the case for the renormalizable theories of spontaneous symmetry breakdown, which employ unobserved gauge bosons and/or heavy leptons.

On the other hand, it would be very interesting to know what kind of theory, even with a phenomenological Hamiltonian, would explain the experimental data far above the unitary limit.[3] Recently a model[4] of weak interactions at high energy by Appelquist, Bjorken, and Chanowitz extending the conventional current-curıent theory to incorporate possible higher-order effects has been proposed. In this paper we investigate some theoretical consequences of a derivatively coupled intermediate vector boson (IVB), or, equivalently, a phenomenological momentum-dependent vector current, in the Bjorken-Johnson-Low scaling limit. Previously, IVB theorists excluded the possibility of a derivative coupling on the grounds that: (a) in the observed leptonic weak interactions, momentum transfer q is so small as to be neglected, and (b) introduction of a derivative coupling provides a momentum q at each vertex in higher orders, making already divergent Feynman diagrams more so. Nevertheless high-energy accelerators available at present or in the future may be able to detect the momentum dependence of the leptonic weak vector current directly, if it exists at all. Also the argument of divergent graphs may be bypassed in the meanwhile because we are dealing with the phenomenological Lagrangian anyway. In fact, it turns out that a certain derivative coupling is no worse than the conventional $V-A$ intermediate-vector-boson theory of weak interactions as we will see later. In Sec. II we present a model Lagrangian and vector current. This model is used to calculate the polarization of the muon in the deep-inelastic neutrino-nucleon scattering in Sec. III.

II. PHENOMENOLOGICAL LAGRANGIAN AND VECTOR CURRENT

Consider a general coupling of lepton and charged vector boson fields,[5]

$$\begin{aligned}\mathcal{L} = & g_W^S\bar{\psi}_l(x)(a_1-b_1\gamma_5)\psi_\nu(x)\partial_\lambda\overline{W}_\lambda(x)\\ & -ig_W^V\bar{\psi}_l(x)\gamma_\lambda(1-\gamma_5)\psi_\nu(x)\overline{W}_\lambda(x)\\ & +\tfrac{1}{2}ig_W^T\bar{\psi}_l(x)\sigma_{\lambda\mu}(a_2-b_2\gamma_5)\psi_\nu(x)[\partial_\lambda\overline{W}_\mu(x)-\partial_\mu\overline{W}_\lambda(x)]\\ & +g_W^V J_\lambda(x)\overline{W}_\lambda(x),\end{aligned} \qquad (1)$$

where $J_\lambda(x)$ is the hadronic current. We can write this in another form, viz.,

$$\mathcal{L}=g_W^V[j_\lambda(x)+J_\lambda(x)]\overline{W}_\lambda(x)+\text{H.c.}\,, \tag{2}$$

where

$$\begin{aligned} j_\lambda(x)=\overline{\psi}_l(x)\Big[i\frac{\eta}{m}(a_1-b_1\gamma_5)q_\lambda - i\gamma_\lambda(1-\gamma_5) \\ - i\frac{\xi}{m}\sigma_{\lambda\mu}q_\mu(a_2-b_2\gamma_5)\Big]\psi_\nu(x)\,, \\ \overline{\psi}_l(x)q_\lambda\psi_\nu(x)\equiv -i\frac{\partial}{\partial x_\lambda}[\overline{\psi}_l(x)\psi_\nu(x)]\,, \end{aligned} \tag{3}$$

and

$$\frac{\eta}{m}=\frac{g_W^S}{g_W^V}\,,\quad \frac{\xi}{m}=\frac{g_W^T}{g_W^V}\,, \tag{4}$$

where m is the muon mass. We introduce the muon mass in (3) to make ξ and η dimensionless. In general, a_i and b_i are invariant functions of the momentum transfer q. We may regard (3) as a natural modification of the conventional $V-A$ leptonic weak current. Notice that these derivative-coupling terms when applied to hadronic physics with strongly interacting intermediate fields yield Weinberg's form factors[6] with a specific choice of a_i and b_i. Derivative couplings similar to (1) in principle have been used by Igarashi *et al.* in their strong-meson-dominance model[7] to get the $V-\alpha A$ theory of β decay even though they used the $V-A$ leptonic current. Also it is well known that the decay $\pi^-\to\mu^-\overline{\nu}_\mu$ can be described phenomenologically by the derivative coupling of the pion directly to the lepton current,[8] i.e.,

$$\mathcal{L}=g_\pi[\overline{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu+\overline{e}\gamma_\lambda(1-\gamma_5)\nu_l]\,\partial_\lambda\Phi_{\pi^-}+\text{H.c.} \tag{5}$$

Therefore, it may be interesting to see the consequences of a current-current theory with $j_\lambda(x)$ defined as in (3). This would not change the well-established low-energy predictions of the $V-A$ theory for sufficiently small ξ and η. Then this result can readily be translated to that of the intermediate-vector-boson theory, Lagrangian (1).

Cheng and Tung[9] have discussed tests of the $V-A$ theory in neutrino-scattering processes assuming the most general form of local (nonderivative $V-A$ and $S-T$) interaction in the helicity formalism. Here we are interested in the nonlocal deviation from the $V-A$ theory, which does not originate from the second-order radiative corrections. As for the scalar and tensor hadronic currents, their commutators, hence those hadronic structure functions, are relatively unknown to us simply because only those of the vector and axial-vector hadronic currents have been the focus of attention up to now. Therefore we will restrict ourselves to the vector and axial-vector hadronic currents.

III. POLARIZATION OF THE MUON IN $\nu N\to\mu X$

The phenomenological current (3) discussed in Sec. II shall be used to compute the polarization of the muon in the neutrino-induced inclusive reaction. To simplify the calculation we invoke the following assumptions:

(1) Only the left-handed neutrino takes part in the leptonic current, i.e., $a_1=b_1$, $a_2=b_2$. Choose $a_1=a_2=1$ without loss of generality.

(2) Time-reversal invariance, i.e., ξ and η are real. Then if we put

$$\frac{(g_W^V)^2}{m_W{}^2}=\frac{G}{\sqrt{2}}\,,$$

we have[10]

$$\frac{d^2\sigma}{dQ^2d\nu}=\frac{G^2}{4\pi M^2E^2}\mathfrak{M}_{\mu\lambda}W_{\mu\lambda}\,,$$

where

$$\mathfrak{M}_{\mu\lambda}=\overline{\sum_{\nu\,\text{spin}}}mm_\nu\langle\mu(k')|j_\mu(0)|\nu(k)\rangle\langle\nu(k)|j_\lambda^\dagger(0)|\mu(k')\rangle \tag{6}$$

and

$$\begin{aligned} W_{\mu\lambda}&=\int\frac{d^4x}{4\pi}e^{iq\cdot x}\langle N(p)|[J_\mu(x),J_\nu^\dagger(0)]|X(p')\rangle \\ &=-\left(g_{\mu\nu}-\frac{q_\mu q_\lambda}{q^2}\right)W_1 \\ &\quad+\frac{1}{M^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\lambda-\frac{\nu}{q^2}q_\lambda\right)W_2 \\ &\quad-i\frac{\epsilon_{\mu\lambda\alpha\beta}p^\alpha q^\beta}{2M^2}W_3+\frac{q_\mu q_\lambda}{M^2}W_4+\frac{(p_\mu q_\lambda+p_\lambda q_\mu)}{2M^2}W_5\,. \end{aligned} \tag{7}$$

The average over the proton spin is taken in $W_{\mu\lambda}$ and $q=k-k'=p'-p$, $\nu=q\cdot p=M(E-E')$, $q^2=-Q^2<0$, where E and E' are the laboratory energies of the neutrino and the muon, respectively. For the sake of illustration, we use the quark model version of the structure functions, which exhibits a scaling behavior. A more general case can be worked out, leading to similar conclusions. Thus we assume that[11,12]:

$$\begin{aligned} &\lim_{\text{Bj}}W_1(q^2,\nu)=F_1(x)\,, \\ &\lim_{\text{Bj}}\left(\frac{\nu}{M^2}\right)W_{2,3}(q^2,\nu)=F_{2,3}(x)\,, \\ &\lim_{\text{Bj}}\left(\frac{\nu}{M^2}\right)^2W_{4,5}(q^2,\nu)=F_{4,5}(x)\,, \end{aligned} \tag{8}$$

where $x=-(q^2/2\nu)$. On the other hand, from (3) to (6), it follows that[10]

$$\mathfrak{M}_{\mu\nu}=\mathfrak{M}_{\mu\nu}^{(0)}+\mathfrak{M}_{\mu\nu}^{(s)}\,,$$

where

$$\mathfrak{M}_{\mu\nu}^{(0)}=(k'_\mu k_\nu+k'_\mu k_\nu-\delta_{\mu\nu}k\cdot k')(1+\xi)^2\mp i\epsilon_{\mu\nu\sigma\rho}k'^\sigma k^\rho(1+\xi)^2$$
$$+\frac{1}{m^2}(k\cdot k')q_\mu q_\nu\eta^2\mp\frac{1}{m^2}(k\cdot k')[q_\mu(k+k')_\nu+q_\nu(k+k')_\mu]\eta\xi+\frac{1}{m^2}(k\cdot k')(k+k')_\mu(k+k')_\nu\xi^2$$
$$\pm(k_\nu q_\mu+k_\mu q_\nu)(\eta+\eta\xi)-[k_\mu(k+k')_\nu+k_\nu(k+k')_\mu](\xi+\xi^2)$$

and

$$\mp\mathfrak{M}_{\mu\nu}^{(s)}=-m[s_\mu k_\nu+s_\nu k_\mu-\delta_{\mu\nu}s\cdot k](1+\xi)^2\pm im\epsilon_{\mu\nu\sigma\rho}s^\sigma k^\rho(1+\xi)^2$$
$$+\frac{\eta^2}{m}(s\cdot k)q_\mu q_\nu+\frac{i}{m}\epsilon_{\sigma\rho\lambda\nu}k'^\sigma s^\rho k^\lambda[-q_\mu(\eta+\eta\xi)\pm(k+k')_\mu(\xi+\xi^2)]$$
$$-\frac{i}{m}\epsilon_{\sigma\rho\lambda\mu}k'^\sigma s^\rho k^\lambda[-q_\nu(\eta+\eta\xi)\pm(k+k')_\nu(\xi+\xi^2)]$$
$$\pm\frac{1}{m}[(s\cdot k)k'_\nu q_\mu-(k\cdot k')s_\nu q_\mu](\eta+\eta\xi)\pm\frac{1}{m}[(s\cdot k)k'_\mu q_\nu-(k\cdot k')s_\mu q_\nu](\eta+\eta\xi)$$
$$\mp\frac{1}{m}(s\cdot k)[q_\mu(k+k')_\nu+q_\nu(k+k')_\mu]\eta\xi+\frac{1}{m}(s\cdot k)(k+k')_\mu(k+k')_\nu\xi^2$$
$$+\frac{1}{m}[(k\cdot k')s_\mu(k+k')_\nu-k'_\mu(k+k')_\nu(s\cdot k)](\xi+\xi^2)+\frac{1}{m}[(k\cdot k')s_\nu(k+k')_\mu-k'_\nu(k+k')_\mu(s\cdot k)](\xi+\xi^2)\,, \tag{9}$$

where the upper and lower signs refer to the neutrino and antineutrino, respectively. Furthermore, from now on, the following expressions will be used:

$$E=-\frac{q^2}{2Mxy}\,,\quad E'=-\frac{(1-y)}{2Mxy}q^2\,,$$
$$s\cdot k\cong\frac{1}{2m}\left(-q^2-m^2\frac{1+y}{1-y}-m^2\frac{2M^2x^2y^2}{(1-y)^2q^2}\right), \tag{10}$$
$$s\cdot p\cong-\frac{(1-y)}{2mxy}q^2+m\frac{M^2xy}{(1-y)q^2}\,,$$

where $y=\nu/ME=1-E'/E$ and s is the longitudinal polarization vector of the muon. Now averaging over the muon spin, we get with the Callan-Gross relation

$$F_1=F_2/2x\,,$$
$$\frac{d^2\sigma}{dQ^2d\nu}=\frac{G^2x^2y^2}{\pi q^4}\overline{\sum\sum}\mathfrak{M}_{\mu\nu}W_{\mu\nu}$$
$$=\frac{G^2x}{\pi}\left\{(1-y)\frac{\xi^2}{m^2}F_2(x)+\frac{1}{2Q^2}\left[F_2(x)[y^2+2(1-y)+2y^2\xi-y(2-y)\xi^2]\right.\right.$$
$$\left.\left.\mp xF_3(x)\left(\frac{y(2-y)}{2}\right)(1+\xi)^2+\frac{M^2\gamma\eta^2y^2}{m^2}F_2(x)\pm\frac{2M^2x^2y(2-y)}{m^2}F_5\eta\xi\right]\right\}. \tag{11}$$

Thus we see that

$$\lim_{\mathrm{Bj}}\frac{d^2\sigma}{dQ^2d\nu}=(1-y)\frac{G^2\xi^2}{\pi m^2}xF_2(x)+O\left(\frac{1}{Q^2}\right), \tag{12}$$

while the $V-A$ theory yields the term of the order of $1/Q^2$.

Next we turn to the computation of the muon polarization. We define the polarization P and the longitudinal muon polarization vector S such that

$$P=\frac{d\sigma(\leftarrow)-d\sigma(\rightarrow)}{d\sigma(\leftarrow)+d\sigma(\rightarrow)}\,,\quad s=\left(\frac{E'}{m}\hat{k}',\frac{|k'|}{m}\right). \tag{13}$$

Then a straightforward calculation yields the longitudinal polarization P^L:

$$P^L = \pm \frac{\frac{q^4}{2m^2}\xi^2\left[-F_1+\frac{F_2}{2x}\frac{(2-y)^2}{y^2}\right]+q^2\left[F_1a_1+\frac{F_2}{2x}a_2\mp F_3a_3+\left(F_1-\frac{F_2}{2x}\right)a_4-\frac{F_2M^2\gamma\eta^2}{2xm^2}\mp\frac{F_5 2M^2x^2}{2xm^2}\frac{(2-y)}{y}\eta\xi\right]}{\frac{q^4}{2m^2}\xi^2\left[-F_1+\frac{F_2}{2x}\frac{(2-y)^2}{y^2}\right]-q^2\left[F_1b_1+\frac{F_2}{2x}b_2\mp F_3b_3+\left(F_1-\frac{F_2}{2x}\right)b_4+\frac{F_2M^2\gamma\eta^2}{2xm^2}\pm\frac{F_5 2M^2x^2}{2xm^2}\frac{(2-y)}{y}\eta\xi\right]}, \tag{14}$$

where

$$\begin{aligned}
&a_1 = 1+\xi+\xi^2, \quad b_1 = 1+2\xi,\\
&a_2 = \frac{2(1-y)}{y^2}-\frac{(4-y)}{y}\xi+\frac{2}{y}\xi^2, \quad b_2=\frac{2(1-y)}{y^2}-\frac{(2-y)}{y}\xi^2,\\
&a_3 = \frac{1}{2}\left[\frac{(2-y)}{y}+2\frac{(4-y)}{y}\xi+\frac{(6-y)}{y}\xi^2\right], \quad b_3=\frac{(2-y)}{2y}(1+\xi)^2,\\
&a_4 = -\frac{(1+y)}{1-y}\xi^2, \quad b_4 = \mp\eta\mp\eta\xi+\xi+\tfrac{1}{2}(\eta^2+\xi^2),\\
&\gamma = \frac{4x^3F_4-2x^2F_5}{F_2}.
\end{aligned} \tag{15}$$

Furthermore, if we adopt the spin-$\frac{1}{2}$ parton model, we have the Callan-Gross relation $F_1 = F_2/2x$. Then,

$$P^L = \pm\frac{\frac{(1-y)}{y^2}\xi^2-\frac{m^2}{2Q^2}\left[(1+\xi)^2+\frac{2(1-y)}{y^2}-\frac{4}{y}\xi+\frac{2}{y}\xi^2\mp\frac{xF_3}{F_2}\left(\frac{(2-y)}{y}(1+\xi)^2+\frac{4}{y}\xi+\frac{4}{y}\xi^2\right)-\frac{M^2\gamma\eta^2}{m^2}\mp\frac{2M^2x^2}{m^2}\frac{F_5}{F_2}\frac{(2-y)}{y}\eta\xi\right]}{\frac{(1-y)}{y^2}\xi^2+\frac{m^2}{2Q^2}\left[(1+\xi)^2+\frac{2(1-y)}{y^2}-\frac{2}{y}\xi^2\mp\frac{xF_3}{F_3}\frac{2-y}{y}(1+\xi)^2+\frac{M^2\gamma\eta^2}{m^2}\pm\frac{2M^2x^2}{m^2}\frac{F_5}{F_2}\frac{(2-y)}{y}\eta\xi\right]}. \tag{16}$$

We note that in the vector-gluon model[12] the parameter γ in Eq. (15) is a constant given by $\gamma=(m_\rho/M)^2$, where m_ρ is the bare proton quark mass. The constants $\xi/m<0.1\ \text{GeV}^{-1}$, $\eta/m<0.1\ \text{GeV}^{-1}$, i.e., $\xi<10^{-2}$, $\eta<10^{-2}$ seem to be reasonable from the low-energy experiments which agree with the $V-A$ theory. Moreover, as is well known, the positivity condition gives $0\leqslant -xF_3\leqslant F_2$. Therefore for practical purposes we may write, if $y\neq 1$,

$$P^L=\mp 1\pm\frac{m^2xy^2\left[2F_1\mp F_3-(1-y)\left(F_1-\frac{F_2}{2x}\right)\right]}{Q^2(1-y)\left[xy^2F_1+(1-y)F_2\mp xy\left(\frac{2-y}{2}\right)F_3\right]}, \quad \text{for pure } V-A, \tag{17a}$$

$$P^L=\pm 1\mp\frac{m^2}{Q^2\xi^2}\left[\frac{y^2}{1-y}+2\mp\frac{xF_3}{F_2}\frac{y(2-y)}{(1-y)}\right], \quad \text{for } \eta,\ \xi, \text{ and } V-A, \tag{17b}$$

$$P^L=\mp\frac{1+\frac{2(1-y)}{y^2}\mp\frac{xF_3}{F_2}\left(\frac{2-y}{y}\right)-\frac{M^2\gamma\eta^2}{m^2}}{1+\frac{2(1-y)}{y^2}\mp\frac{xF_3}{F_2}\left(\frac{2-y}{y}\right)+\frac{M^2\gamma\eta^2}{m^2}}, \quad \text{for } \xi=0, \text{ and } V-A. \tag{17c}$$

These results indicate that P^L approaches 1 instead of -1 for large Q^2 and $0<y<1$ if $\xi\neq 0$. We illustrate P^L for $y=\frac{1}{2}$ in Fig. 1. But P^L will not change sign if $\xi=0$ and $(M^2\gamma\eta^2/m^2)\ll(2/y^2)$, as long as the assumption $F_2=-xF_3$ is used. This corresponds to $\gamma\ll 8\times 10^2$, which means that $m_\rho\ll 28M$ in the vector-gluon model, when $\eta=10^{-2}$ and $y=\frac{1}{2}$. An estimate[12] is that $\gamma=1.6\times 10^{-2}$, i.e., $m_\rho=120$ MeV, which leads to $P^L=-1$.

An important feature in the estimate described above is that the momentum transfer at which the change of the muon polarization occurs is very sensitive to the value of ξ and η. Because complete right-handed polarization needs extremely large Q^2, it would be interesting to see the value of Q^2 at which P goes to zero before it changes sign. In Fig. 1, we see that, for $\xi/m=0.1\ \text{GeV}^{-1}$, P^L vanishes at $Q^2=200\ \text{GeV}^2$, while for $\xi/m=0.05\ \text{GeV}^{-1}$ that occurs at $Q^2=800\ \text{GeV}^2$. The perpendicular polarization would not given any insight about the ξ or η term, because it is proportional to $m\sin\theta$, which is very small at high energy.

If we do the above calculation in the intermediate-vector-boson theory, i.e., Lagrangian (1), we can easily obtain the result by simply replacing ξ, η, and G in Eqs. (11)–(17) by

$$\eta \to \frac{m^2}{m_W{}^2} + \eta\left(\frac{q^2}{m_W{}^2} - 1\right), \quad \xi \to \xi\,,$$
$$G^2 \to \frac{G^2}{(1 + 2MxyE/m_W{}^2)^2}\,. \tag{18}$$

Therefore we conclude that unless $Q^2/m_W{}^2 \gg 1$, the previous results would not be modified. Notice that as far as the ξ term and polarization are concerned we cannot distinguish between the intermediate-vector-boson theory, or a momentum-dependent weak current. In fact the merit of the polarization measurement rather than that of the cross section in view of testing the $V-A$ theory as formulated here, is that the change of the polarization is not as sensitive to the validity of the Callan-Gross relation or the breakdown of scaling as the cross section is. Nevertheless, one might expect that the general IVB theory given by the Lagrangian (1) would be appropriate to describe high-energy phenomena rather than the current-current theory with $j_\lambda(x)$ in (3). In that case, if we assume $F_2 = 2xF_1 = -xF_3$ and $F_4 = F_5 = 0$, the violation of scaling in the expression $d^2\sigma/dxdy$ may be parametrized by

$$F_2^\nu(x) \approx F_2^\nu(x)\left[2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]\Big/\left(1 + \frac{Q^2}{m_W{}^2}\right)^2 \tag{19a}$$

and

$$F_2^{\bar\nu}(x) \approx F_2^{\bar\nu}(x)\left[2(1-y)^2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]\Big/\left(1 + \frac{Q^2}{m_W{}^2}\right)^2. \tag{19b}$$

As far as power counting in Feynman diagrams is concerned, the ξ term is not worse than the $V-A$ theory with an IVB, because $\gamma_\mu q_\mu q_\alpha \approx m q_\alpha$, and $\sigma_{\mu\nu} q_\nu q_\mu q_\alpha = 0$. We summarize all the results in Table I. Obviously the realization of the limits in Table I depends upon the value of $y = 1 - E'/E$. A larger y requires higher Q^2.

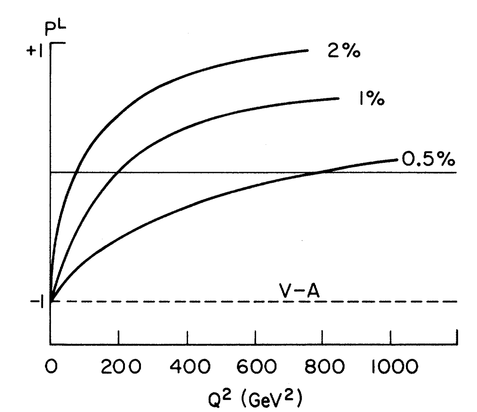

FIG. 1. Longitudinal polarization of the muon in $\nu N \to \mu X$ with the assumptions $F_4 = F_5 = 0$, $-xF_3 = F_2$, $F_1 = F_2/2x$, only for illustration. Percentage on the curve refers to that of mg_W^T with respect to the vector coupling constant g_W^V. We put $y = \frac{1}{2}$, which is the average value of y calculated in Ref. 13.

In the recent analysis of momentum-independent S-T-P weak currents, Cheng and Tung[9] state that "to the extent that the lepton mass can be neglected, a purely left-handed outgoing lepton indicates V interaction, a purely right-handed one indicates $S-T$ interaction, and the coexistence of both indicates a mixture of the two." Accepting the premise that no matter what the additional term is, it must be small compared with the $V-A$ term, Cheng and Tung predict $P^L = -1 \pm \alpha$ when there exists a small contamination of $S-T$ interaction. Here α is a small positive constant. However, our result is that, as can be seen in Table I, a purely right-handed outgoing muon at high energy indicates the existence of the ξ term whether the interaction is via current-current theory or IVB theory. It is interesting to note that even though the η term contains momentum transfer q which makes us anticipate that it might dominate over the $V-A$ term at large momentum transfer, it turns out that all the higher momentum-dependent parts involving η cancel out so that it does not dominate over the $V-A$ term. But a small contamination of the η term would mean $P^L = -1 \pm \alpha$ as in Cheng's and Tung's case with the momentum-independent $S-T$ interaction. Therefore if a future experiment indicates that $P^L = -1 \pm \alpha$, a very elaborate analysis would be necessary to decide whether that is due to Cheng's and Tung's momentum-independent $S-T$ interaction or our momentum-dependent η-type coupling. However, we notice that the result described above concerning the η term is a consequence of the assumption (8). If we assume a different scaling law, such as

TABLE I. Longitudinal polarization of the muon in the $\nu N \to \mu X$ at the scaling limit. MDV refers to the momentum-dependent weak vector current. The values of P^L given are the limit of P^L as $Q^2 \to \infty$. These values seem to be realized, because of the smallness of ξ and η, at much larger Q^2 than the one at which structure functions begin to scale in the Bjorken-Johnson–Low limit.

	$V-A$	$P=\mp 1$	IVB	$V-A$	$P=\mp 1$
MDV	$\xi, V-A$	± 1		$\xi, V-A$	± 1
	$\xi, \eta, V-A$	± 1		$\xi, \eta, V-A$	± 1
	$\eta, V-A$	depend upon y		$\eta, V-A$	± 1 if $\frac{Q^2}{m_W{}^2} \gg 1$
					∓ 1 if $\frac{Q^2}{m_W{}^2} \ll 1$

$$\lim_{\text{Bj}}\left(\frac{\nu}{M^2}\right)W_{4,5}=F_{4,5}(x)\ ,$$

the role of the η term would be similar to that of the ξ term.

Another interesting prediction of the momentum-dependent vector current (ξ type) is that the ratio $\sigma(\bar\nu)/\sigma(\nu)$ increases as the neutrino energy gets larger, i.e.,

$$R\equiv\frac{\sigma(\bar\nu)}{\sigma(\nu)}$$

$$=\frac{\dfrac{ME\xi^2}{6m^2}+\left(\frac{4}{3}-\frac{2}{3}\dfrac{|\langle xF_3\rangle|}{\langle F_2\rangle}+\dfrac{M^2\gamma\eta^2}{2m^2}-\dfrac{2M^2\eta\xi\langle x^2F_5\rangle}{3m^2\langle F_2\rangle}\right)}{\dfrac{ME\xi^2}{6m^2}+\left(\frac{4}{3}+\frac{2}{3}\dfrac{|\langle xF_3\rangle|}{\langle F_2\rangle}+\dfrac{M^2\gamma\eta^2}{2m^2}+\dfrac{2M^2\eta\xi\langle x^2F_5\rangle}{3m^2\langle F_2\rangle}\right)}$$

and

$$\lim_{\text{Bj}}R=1\ , \tag{20}$$

where

$$\langle x^nF_m\rangle\equiv\frac{\displaystyle\int_0^1dx\int_0^1dy\,x^nF_m\frac{d^2\sigma}{dxdy}}{\displaystyle\int_0^1dx\int_0^1dy\frac{d^2\sigma}{dxdy}}\ .$$

The above conclusion will not be modified as long as the ratios F_3/F_2, F_4/F_2, and F_5/F_2 scale, as in the IVB theory. The CERN result[13,14] $R=0.377\pm0.023$ may indicate the deviation from $R=\frac{1}{3}$, although it is obviously too early to draw any definite conclusion. However, various models predict different values of R. The spin-$\frac{1}{2}$ parton model yields $R=\frac{1}{3}$, while other constituents would give $\frac{1}{3}<R<3$. In general, from the positive semidefinite property of $W_{\mu\nu}$ and the scaling behavior of structure functions given in (8), it follows that $F_2\geqslant 2xF_1\geqslant -xF_3$, which leads to $R\geqslant\frac{1}{3}$. However, the above ratio is constant unless scaling breaks down to the extent $F_2(x)=xF_3(x)G(E)=xF_3(x)G(Q^2/2Mxy)$, where $G(E)$ is some function of E. Therefore in order to detect the ξ or η term, we have to look at the energy dependence of the ratio R.

One might expect that the ξ term (or η term) could be obtained as a result of the radiative corrections in the framework of the $V-A$ theory as in the eeA vertex correction in quantum electrodynamics. However, that this is not the case can be seen by the following argument. First, the radiative corrections[15] to the $\mu\nu W$ coupling (which do not have vertex corrections in the second order) fail to induce the ξ-type effective coupling. Similarly in the Weinberg theory,[16] the $\mu\nu W$ vertex correction[17] due to a neutral vector boson Z_μ rules out this possibility because of the $1\pm\gamma_5$ structure of the interactions. Thus we cannot as of now tell where the extra derivative coupling may come from. Nevertheless, the other higher-order corrections might well lead to the same conclusions about the polarization of the muon deduced from our formalism.[4] On the phenomenological level, however, we conclude that the polarization measurement of the muon in the inclusive reaction $\nu N\to\mu X$ at high energy will provide a good criterion for the existence or nonexistence of a phenomenological momentum-dependent weak vector current or of the intermediate-vector-boson theory with Lagrangian (1).

ACKNOWLEDGMENT

I would like to thank Professor Byron Roe and Professor Yukio Tomozawa for suggesting this problem and for many useful discussions. Also thanks are due to Douglas Fairobent and Robert Pearson for reading the manuscript and to SLAC for their hospitality extended to me during the first half of 1973.

---

*Work supported in part by the U. S. Atomic Energy Commission.

†Permanent address.

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); E. C. G. Sudarshan and R. Marshak, *ibid.* 109, 1860 (1958).

[2] Cecilia Jarskog, Nucl. Phys. 75, 659 (1966); C. Jarskog, Nuovo Cimento Lett. 4, 377 (1970).

[3] A. O. Barut and T. Nagylaki, Nuovo Cimento Lett. 4, 169 (1970); Amar Maheshwari and Hironari Miyazawa, Prog. Theor. Phys. 44, 772 (1970); J. D. Bjorken and J. Kogut, Phys. Rev. D 8, 1341 (1973).

[4] T. Appelquist, James D. Bjorken, and Michael Chanowitz, Phys. Rev. D 7, 2225 (1973).

[5] R. E. Marshak, Riazuddin and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

[6] S. Weinberg, Phys. Rev. 112, 1375 (1958).

[7] Masataka Igarashi, Seitaro Nakamura, and Shigeo Sato, Prog. Theor. Phys. 46, 874 (1971); Masataka Igarashi, *ibid.* 48, 1237 (1972).

[8] D. Lurié, *Particles and Fields* (Wiley-Interscience, New York, 1968), p. 246.

[9] T. P. Cheng and Wu-Ki Tung, Phys. Rev. D 3, 733 (1971).

[10] When we use the Pauli-Dirac convention for four-vectors, we should change $W_1=-W_1$, $(A\cdot B)\to-(A\cdot B)$. However, the first terms of $\mathfrak{M}^{(0)}_{\mu\nu}$ and $\mathfrak{M}^{(s)}_{\mu\nu}$, in the expressions for $W_{\mu\nu}$ and $\mathfrak{M}_{\mu\nu}$, remain as given in the text.

[11]D. J. Broadhurst, and J. F. Gunion, and R. L. Jaffe, Phys. Rev. D 8, 566 (1973).
[12]R. L. Jaffe and C. H. Llewellyn Smith, Phys. Rev. D 7, 2506 (1973); David J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969).
[13]E. A. Paschos and V. I. Zakharov, Phys. Rev. D 8, 215 (1973).
[14]Preliminary results of Gargamelle Collaboration, presented by Ph. Heusse at the *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973); D. H. Perkins, *ibid.* Vol. 4, p. 189.
[15]David Bailin, Phys. Rev. 135, B166 (1964).
[16]Steven Weinberg, Phys. Rev. Lett. 27, 1688 (1971); 19, 1264 (1967).
[17]D. A. Ross, Nucl. Phys. B51, 116 (1973).

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Improved Weizsäcker-Williams Method and Its Application to Lepton and W-Boson Pair Production*

Kwang Je Kim and Yung-Su Tsai
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 30 April 1973)

A Weizsäcker-Williams method is derived which handles the elastic and inelastic target form factors properly. The method is applied to calculate energy-angle distributions of photoproduced lepton pairs: electrons, muons, and heavy leptons, using target form factors appropriate to each case. The agreement with the exact result is found to be excellent. Simple formulas for pair production of spin-0 and spin-1 particles are also given.

I. INTRODUCTION

Some of the cross sections which involve one-photon exchange can be quite complicated. The best example is the calculation of the W pair production, $\gamma + Z \to W^+ + W^- +$anything, which involves threefold integration of roughly 3000 terms. With the advancement in the modern computer technique, even such a complicated calculation can be handled easily. However, it is often desirable to have a simple expression which shows all the gross features of the problem, such as the dependence of the cross section on the incident energy, outgoing energy, angle, mass, magnetic moment, radius of the target, etc. The way in which one can do this quickly was originally suggested in 1924 by Fermi,[1] who noted the similarity between the electromagnetic fields of a rapidly moving charged particle and a pulse of radiation. Basing their work on this observation, Weizsäcker[2] and Williams[2] showed independently in 1934 that an incident particle with charge Ze, mass M, and energy $E=\gamma M$ would produce the same effect as a beam of photons with a spectrum $\rho(\omega)$ given by[3]

$$\rho(\omega)=\frac{Z^2\alpha}{\pi\omega}\{2xK_0(x)K_1(x)-x^2[K_1^2(x)-K_0^2(x)]\}$$
$$\underset{x\ll 1}{\sim}\frac{2Z^2\alpha}{\pi\omega}\left[\ln\left(\frac{1.123\gamma}{\omega b_{\min}}\right)-\frac{1}{2}\right], \tag{1.1}$$

where ω is the photon energy, $x=\omega b_{\min}/\gamma$, $b_{\min}$ is the minimum impact parameter, and K_0 and K_1 are the usual Bessel functions. The second expression is valid when $x\ll 1$, which is the usual case when γ is large.

The above formula, which is known as the pseudophoton flux of the classical Weizsäcker-Williams (W.W.) method, has enjoyed wide applications in processes involving one-photon exchange in the past because of its conceptual and mathematical simplicity. However, it can sometimes lead to a numerical value which deviates considerably from the correct one, mainly because it does not properly take into account the effect due to the rapid variations of the form factors. As an example, let us consider the pseudophoton flux of a nucleus with a form factor $G_e^2=Z^2/(1+t/d)^2$ to be used for the pair production of particles of mass m [see Appendix C, Eqs. (C6) and C9), and also Eq. (3.23)]:

$$\rho(\omega)=\frac{2Z^2\alpha}{\pi\omega}\left\{\left[1+2\left(\frac{\omega}{\gamma}\right)^2\frac{1}{d}\right]\ln\left[\frac{1+d(\gamma/\omega)^2}{1+d/t_{\text{up}}}\right]-\left[1+\frac{(\omega/\gamma)^2}{t_{\text{up}}}\right]\frac{1+2(t_{\text{up}}/d)}{1+(t_{\text{up}}/d)}\right\}, \tag{1.2}$$

where $t_{\text{up}}=m^2(1+l)^2$. In the limit $d\to\infty$, i.e., the case of a point particle, we recover (1.1) if we identify

$$b_{\min}=1.123/(t_{\text{up}})^{1/2}. \tag{1.3}$$