sentations $(\frac{3}{2},\frac{1}{2})$. The quantity $\mathrm{Re}\beta_0\beta_-^*$ depends upon details of the vacuum expectation values of the Higgs particles and upon their couplings to the muonic leptons. Without perversely constraining these parameters, the quantity $\mathrm{Re}\beta_0\beta_-^*$ is of order unity.

[24]See especially Ref. 14, Sec. IV. As explained in Ref. 14, Sec. I, the notion of superweakness as used here is in conformance with its initial introduction by L. Wolfenstein [Phys. Rev. Lett. 13, 562 (1964)].

[25]See especially Ref. 14, Sec. VII.

[26]For the definitions of σ and ρ see Ref. 14, Eqs. (3.16) and (3.17).

[27]Ref. 14, Eq. (3.19).

[28]Ref. 14, Eq. (7.24).

[29]Ref. 14, Sec. VII. Other contributions to the *real* mass difference stem from the so-called box graphs, for example (see Ref. 19).

[30]For D_{muon}, the set of graphs is as in Fig. 6, with obvious changes in the particle labels.

[31]Ref. 14, Eqs. (4.13), (4.14), (7.23).

[32]At this point we make use of the fact that the $(\frac{1}{2},\frac{1}{2})^0$ scalar multiplets may be chosen to be real in the sense defined in Ref. 14, Sec. II; see also Ref. 14, Sec. VII. The reader will also verify that the presence of singly-charged scalar fields in $(\frac{1}{2},\frac{1}{2})^1$ does not modify the m_e proportionality of the dipole moments.

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Difference Between e^+ and e^- Deep-Inelastic Scattering*

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(Received 28 June 1973)

We show that the existence of pointlike constituents within the nucleon makes it plausible that scale-breaking effects due to higher-order electromagnetic corrections in deep-inelastic scattering will be of order $\alpha\ln^2(-q^2/\mu^2)$ in the region $s_{ep}\gg 1$ (GeV)^2 and large electron scattering angles. Additionally we are led to conclude that, when the final electron energy is finite in the laboratory frame, the difference of electron and positron deep-inelastic scattering is of order $\alpha\ln^2(-q^2/\mu^2)$ rather than of order α. We discuss the possible measurement of this difference and what we may learn from such a measurement.

I. INTRODUCTION

Whatever the deeper reasons behind this fact, deep-inelastic lepton-nucleon scattering at present day accelerators[1] behaves as if the nucleon contained pointlike constituents.[2,3] We shall try to use this feature of the data to make some very brief comments on electromagnetic corrections to e-e and deep-inelastic scattering and on the possible difference between deep-inelastic electron and positron scattering. The difference between $e^++p\to e^++X$ and $e^-+p\to e^-+X$ was previously studied by Kingsley[4] using a "softened" parton model[5] as a method of effecting the cutoff necessary to bring Bjorken scaling into a field-theoretic picture. He came to the conclusion that the ratio r between the deep-inelastic electron and positron cross sections[6] σ^- and σ^+ takes the form

$$r=1+O(\alpha), \tag{1.1}$$

up to possible factors involving logarithms of q^2. While we are not so ambitious in the sense of studying a definite model, the method we use seems to us to be of compelling simplicity. We come to the conclusion, based on the pointlike constituent idea, that when $2m\nu\to s_{ep}\gg$ rest masses squared of the problem (which corresponds to large electron scattering angles and finite values of the final electron energy E'),

$$r=1+O(\alpha\ln^2(-q^2/\mu^2)), \tag{1.2}$$

where μ is a scale mass to be discussed (Sec. II), which is plausibly of hadronic size. This is therefore a scale-breaking effect. On the other hand, we expect a form like Eq. (1.1) in small momentum-transfer experiments,[7,8] or when $-q^2/s_{ep}\to 0$.

The factor $\ln^2(-q^2/\mu^2)$ enhances r and thereby gives us an extra chance to test our ideas on quantum electrodynamics and deep-inelastic scattering. According to our reasoning, the over-all cross sections behave as

$$\sigma^\pm\sim\frac{\alpha^2}{q^4}[1\pm O(\alpha\ln^2(-q^2/\mu^2))]\,. \tag{1.3}$$

At presently available energies the logarithmic factor is ~5, which does not allow us to differentiate $r-1$ from the over-all experimental error of ~5–10%. We argue below (Sec. III), however, that we can reduce this experimental error in an experiment designed to measure r.

While it is of course desirable to test quantum electrodynamics, an expression like Eq. (1.3) also spells some trouble for the interpretation of future deep-inelastic experiments at larger values of $-q^2$, in that it introduces a built-in scale-breaking effect, whose removal will necessitate rather specific dynamical assumptions. When $-q^2=O(20\text{ GeV}^2)$, Eq. (1.3) gives an $O(10\%)$ correction to the term of primary interest.

Moreover, some of the $\alpha \ln^2 q^2$ enhancements we refer to exist in all limits of large q^2 in both electron and positron deep-inelastic scattering (i.e., they do not contribute to r). These enhancements are due to vertex corrections to the interaction of the pointlike constituents with the exchanged photon. While again their removal would be model-dependent, their discovery would constitute further evidence for pointlike constituents. Such an experimental discovery would, however, be very difficult.

In Sec. II we review the source of the difference between e^- and e^+ deep-inelastic scattering and discuss the simple reasoning which leads us to Eq. (1.2) and other conclusions. Section III contains a discussion on experimental measurement of r and what observation of nonzero $(r-1)$ can tell us. In an Appendix we show that the behavior (1.2) is a natural prediction of the softened-field parton model.[5]

II. DEEP-INELASTIC SCATTERING AND DIFFERENCE OF σ^- AND σ^+

Figure 1(a) shows the $O(\alpha)$ and Figs. 1(b)–1(f) show some of the $O(\alpha^2)$ contributions to the process $l+p \to l'+X$, where l and l' are the momenta

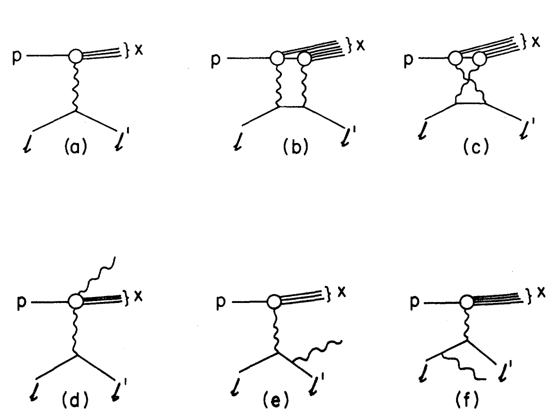

FIG. 1. $O(\alpha)$ and some relevant $O(\alpha^2)$ contributions to deep-inelastic scattering $e^{\pm}+p \to e^{\pm}+X$.

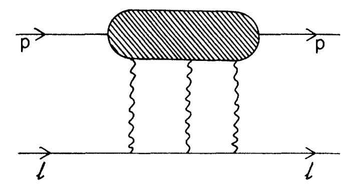

FIG. 2. Forward lepton-nucleon scattering with three-photon exchange. We find interference terms contributing to the difference of electron and positron deep-inelastic scattering by taking the discontinuity of this graph for fixed values of momentum transfers.

of the initial and final leptons. Because electrons and positrons couple to photons with opposite signs, we get a nonzero difference from the interference of the two-photon–exchange graphs (b) and (c) with the ordinary Born term (a) and from the interference of graph (d) and (e) and (f).[9] Put another way, the departure of r from 1 can be expressed as the connected part of the discontinuity of the three-photon–exchange contribution to forward elastic lepton-nucleon scattering (for certain values of internal photon momenta) as in Fig. 2.

From Fig. 2 it is clear that to $O(\alpha^3)$ the lepton-nucleon cross section requires more than the usual two inelastic form factors $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$. In other words, the Rosenbluth form for the deep-inelastic cross section, corresponding to single-photon exchange, no longer holds. The cross section may of course continue to scale up to necessary kinematic factors, e.g.,

$$\frac{d\sigma}{dq^2dx} \sim \frac{1}{(q^2)^2} F\left(\frac{q^2}{s_{ep}}, \frac{2m\nu}{q^2}\right) \tag{2.1}$$

(or some other scaling law) for large s_{ep}, q^2, and ν. We shall argue that this hope is not realized in parton models.

We now consider a particular contribution to $r-1$, namely, the interference of Fig. 1(a) with Figs. 1(b) and 1(c). Figure 1(a) represents the usual deep-inelastic amplitude. We therefore concentrate on the two-photon–exchange term. To estimate this amplitude, which we call $M^{(2)}(q)$, we recall the experimental result that summing inelastic states in a deep-inelastic process is equivalent to the elastic scattering of pointlike constituents. We therefore approximate Figs. 1(b) and 1(c) by Fig. 3, in which the nucleon is treated as a fixed-mass pointlike object[10] with no form factors at its vertices.

There are two relevant kinematic regimes which we shall consider in turn. The first regime is the limit $q^2/s_{ep} \to 0$. Such graphs in this regime are by now familiar from discussions of the eikonal[11] in elastic fermion-fermion scattering at momentum transfer q and large s_{ep}. The sum of the two graphs in Fig. 3 takes the form

$$M^{(2)}(q)=\frac{i s_{ep}}{2m^2}\int d^2b\, e^{-i\vec{q}\cdot\vec{b}}\,\frac{1}{2!}\times\left[-i4\pi\alpha\int\frac{d^2k}{(2\pi)^2}\,\frac{1}{\vec{k}^2+\mu_\gamma{}^2}\,e^{i\vec{k}\cdot\vec{b}}\right]^2, \tag{2.2}$$

where we have dropped factors associated with spinors. In this equation m is a fermion mass and $\mu_\gamma{}^2$ is the photon "mass." (The photon mass is, as usual, interpreted as the experimental resolution.) The factor s appears both here and in the expression for the Born term: It is a standard kinematic factor. By introducing a Feynman parameter x it is straightforward to reduce (2.2) to the form

$$M^{(2)}(q)=\frac{i s_{ep}}{m^2}\,\alpha^2\pi\frac{i}{q^2}\int_0^1\frac{dx}{\mu_\gamma{}^2/q^2+x(1-x)}. \tag{2.3}$$

By assuming $q^2 \gg \mu_\gamma{}^2$, we may approximate this result by

$$M^{(2)}(q)\approx\frac{s_{ep}}{m^2}\,2\alpha^2\pi\left(\ln\frac{q^2}{\mu_\gamma{}^2}\right)\frac{1}{q^2}, \tag{2.4}$$

accurate to $O(\ln(q^2/\mu_\gamma{}^2))$. When we note that the expression for Fig. 1(a), $M^{(1)}(q)$, takes the form

$$M^{(1)}(q)\sim i s_{ep}\frac{\alpha}{q^2}, \tag{2.5}$$

we see that although the two-photon—exchange terms contribute in a logarithmically enhanced way, they are 90° out of phase with the single-exchange term and therefore do not interfere.

The second regime which we consider is $q^2=O(s_{ep})\gg$ hadronic masses squared. While this limit is not so familiar, it has become of increasing interest[12] recently in hadronic scattering, particularly in inclusive scattering at the CERN intersecting storage rings. Simple intuitive techniques (analogous to the eikonal) have not yet been developed for this limit. However, in 1953 Redhead[13] worked out the full differential cross section for e^+e^+ and e^+e^- scattering to $O(\alpha^3)$. It is a simple matter to adapt his result to our situation; namely, we are interested in electron-parton scattering, and, in particular, with the interference between single-photon and double-photon exchanges as above.

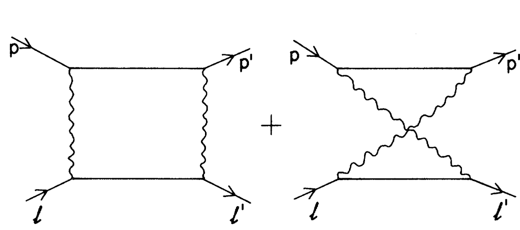

FIG. 3. Two-photon—exchange graphs for the elastic scattering of two pointlike fermions.

The electron-parton energy is $s=xs_{ep}$, with momentum transfer $t=-q^2$. We give the result (whose numerical coefficients have been independently checked) directly in terms of the differential cross section for this particular interference term, retaining only the leading behavior. We find

$$\left.\frac{d\sigma^\pm}{d\Omega^*}\right|_{\rm int}=\pm\frac{\alpha^3}{\pi}\frac{1}{\chi^2 s}\,[2uw(4-2\chi+\chi^2)+2vw(-4+6\chi-3\chi^2)-(v^2+u^2+2w^2)\chi(2-\chi)]. \tag{2.6}$$

In this expression,

$$\chi=\sin^2(\tfrac{1}{2}\theta^*)=-t/s,\quad u=\tfrac{1}{2}\ln(s/\mu^2),\quad w=\tfrac{1}{2}\ln(-t/\mu^2),\quad v=\tfrac{1}{2}\ln[(s+t)/\mu^2]. \tag{2.7}$$

We see that each of the coefficients in Eq. (2.6) represents a potential $\ln^2$ term. We can distinguish two interesting limits:

(i) $\theta^*\to 180°$. Then $\chi\to 1$ and $v\to$ const, while $u=w=\frac{1}{2}\ln(q^2/\mu^2)$. In this case

$$\left.\frac{d\sigma^\pm}{d\Omega^*}\right|_{\rm int}=\pm\frac{3\alpha^3}{4\pi s}\ln^2(q^2/\mu^2). \tag{2.8}$$

In terms of the electron laboratory scattering angle θ and its initial and final energy E and E', this limit corresponds to

$$2m\nu=s_{ep},\quad \sin^2(\tfrac{1}{2}\theta)=\tfrac{1}{2}x\left(\frac{m}{E'}\right),\quad E'\ \text{finite}. \tag{2.9}$$

(ii) $\theta^*\not\to 180°$. Then w, v, and u are all given to leading order by $\frac{1}{2}\ln(q^2/\mu^2)$, and computation of Eq. (2.6) shows that the $\ln^2$ terms cancel for arbitrary values of χ.

We see that only in the limit (i) does the $\ln^2$ enhancement remain.

Literal application of Redhead's result to electron-parton scattering would reveal the appropriate scale mass μ to be the reduced parton-electron mass; we then adopt the point of view that this scale mass is typically hadronic, i.e., $\mu^2=O(m_\rho{}^2)$. Since the experimental resolution of the relevant experiments is also hadronic, it does not seem necessary or desirable to be more precise on this point.

In order to check that the double logarithm of Eq. (2.8) is not canceled by other contributions [i.e., coming from Figs. 1(d), 1(e), and 1(f)] to the discontinuity of the three-photon—exchange ampli-

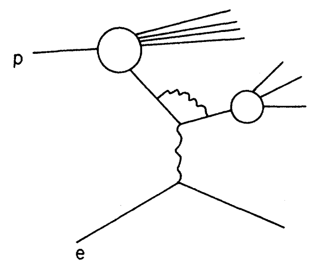

FIG. 4. Vertex corrections of $O(\alpha)$ to the single-photon–exchange deep-inelastic amplitude.

tude in this limit, we need only to work out the other $O(\alpha^3)$ pieces in Redhead's work, omitting those terms which correspond to form factor corrections on the fermions. We find that the $\ln^2$ behavior survives. Finally, we can check the full $O(\alpha^3)$ contribution to the cross section, and find that the $\ln^2$ behavior is not canceled.

One such $\ln^2 q^2$ contribution, which does not contribute to e^+-e^- interference, comes from the parton-vertex renormalization graph shown in Fig. 4. The off-shell form factor of a pointlike fermion is known to have a lowest-order electromagnetic correction of $\alpha \ln^2(q^2/\mu^2)$ from previous studies of the form factor in massive quantum electrodynamics[14]; this is the source of part of this behavior in Redhead's work and hence in deep-inelastic scattering in parton models.

These results confirm Eq. (1.3) and indicate a breakdown in the scaling behavior of Eq. (2.1).

III. FURTHER DISCUSSION

Our results emphasize the interest and importance of doing both e^+p and e^-p experiments. A nonzero difference has never been observed between any electron and positron process.[7] This is because in exclusive channels, where this search has been concentrated, there may be experimental difficulty in fixing the channel and because in such channels the difference is only $O(\alpha)$. We would argue that in deep-inelastic scattering at large $s = O(q^2)$ we have a better chance to observe any difference. This is not only because of the squared logarithmic enhancement of the difference, but also because of the relative ease of the experiment, given our present ability to measure deep-inelastic scattering. Indeed one of the experimental difficulties of the usual deep-inelastic experiment is *not* present in a measurement of r. This difficulty lies in the extraction of radiative corrections from the data.[1] A major radiative correction is precisely that corresponding to Figs. 1(e) and 1(f). But in a measurement of the difference between electron and positron deep-inelastic scattering, we certainly do not want to remove this correction, thereby relieving ourselves of a great deal of the experimental difficulty and uncertainty.

Let us return to the diagrams of Fig. 1 and ask ourselves which of them may be predicted using quantum electrodynamics and our present knowledge of deep-inelastic scattering. Figures 1(e) and 1(f) of course depend only on quantum electrodynamics. Figure 1(d) can be directly computed using the parton model.[2] It is simply a crossed version of the reaction $\gamma + p \to X + l^+ l^-$; in Fig. 1(d) the lepton pair has spacelike mass $-|q^2|$ rather than timelike mass $|q^2|$ as in the pair-production calculation. (This reaction cannot, however, be studied using standard light-cone techniques.[15])

The two-photon–exchange diagrams are the ones which remain problematical, i.e., which cannot be calculated using known models appropriate for deep-inelastic scattering. To see why this is so, let us ask how the large momentum transfer q is carried in terms of the momenta carried by the individual photons, q_1 and q_2. Since $q^2 = (q_1 + q_2)^2$, we see that q^2 is large (compared to scale masses) when either ${q_1}^2$ or ${q_2}^2$ is large, or when both ${q_1}^2$ and ${q_2}^2$ are large. Direct calculation[16] shows that the case where ${q_1}^2$ and ${q_2}^2 = O(\frac{1}{4}q^2)$ is indeed significant. In other words, the interaction does not prefer to proceed only via a "hard" exchange and then a "soft" exchange. Even if the case of one hard and one soft exchange were dominant, we would still have the problem of the soft exchange going to a different parton, which is not calculable from the data. Thus we cannot directly extract the two-photon–exchange contribution using known deep-inelastic data plus a calculable correction.

Even within the context of the single-photon–exchange terms, vertex corrections such as those in Fig. 4, contribute $\ln^2$ factors. If such terms could be observed, they would provide additional strong evidence for pointlike structure. Unfortunately the experimental difficulties of observing this q^2 dependence in e^-(or e^+)p scattering, rather than the departure of r from one, are formidable.

Thus we conclude that direct measurement of $\sigma_-/\sigma_+ - 1$, in addition to providing us with a way to test our old ideas on quantum electrodynamics and the parton model, gives us a way to measure the new features afforded by the two-photon–exchange graphs. Moreover, this measurement may be within reach of present experimental capability, although the interesting region is not the previously studied region of deep-inelastic scattering. If, for illustrative purposes, we set $-q^2 = s = 10\ \text{GeV}^2$, then $\alpha \ln^2(-q^2/\mu^2) = O(5\%)$.

On the negative side, the fact that we cannot compute the two-photon contribution in a model-independent way means that the interpretation of future deep-inelastic scattering experiments at larger values of $-q^2$ becomes more difficult, both because of the breakdown of the Rosenbluth form and because of the $\ln^2$ scale-breaking. For example, when $-q^2 = 10^3$ GeV^2, the correction term of $O(\alpha^3)$ becomes 50% of the $O(\alpha^2)$ term. One would then have to worry about the difficult theoretical problem of $O(\alpha^4 \ln^4 q^2)$ corrections.

ACKNOWLEDGMENT

One of us (P. M. F.) would like to thank the Brookhaven National Laboratory for their hospitality while this work was initiated and Professor C. N. Yang for his kind hospitality at Stony Brook. He would also like to thank S. Brodsky for a helpful communication. R. L. K. would like to acknowledge a grant from the Science Research Council.

APPENDIX

Here we show how these results are obtained in the specific parton model of Landshoff, Polkinghorne, and Short.[5] We have already seen[4] that in this model dominant contributions to the two-photon—exchange terms come from the diagrams of Figs. 5 and 6, where the lower bubbles represent the connected part of a hadronic interaction, softened according to the usual ideas of the model.[5] In the diagram of Fig. 6, it is simple to calculate the asymptotic behavior of the terms involving the k_1 integration, using scaling techniques[17] in the Feynman parameterization of the loop integration. Alternatively the results can be inserted from the analysis of Redhead.[13]. Then we obtain, including only terms from the two-photon—exchange diagram of Fig. 6, with the single-photon—exchange term and spin-$\frac{1}{2}$ partons, in the large electron-scattering-angle region,

$$\frac{d^2\sigma}{dE'\,d\Omega} = \frac{2\alpha^2}{|q^2|}\,\frac{E'}{ME}\left[\sum_i Q_i{}^2 F_1^i(\omega) - \frac{3\alpha}{2\pi}\ln^2|q^2| \sum_i Q_i{}^3 F_1^i(\omega)\right], \tag{A1}$$

where Q_i is the charge on the ith parton, and $Q_i{}^2 F_1^i(\omega)$ is the contribution of the ith parton to the structure function $F_1(\omega)$. The sign is that appropriate for electron-proton scattering. The sum rules of Brodsky, Gunion, and Jaffe[18] will then apply to the coefficient of this $\alpha \ln^2|q^2|$ term.

We can also examine limits in which $2m\nu/s_{ep} \xrightarrow[\nu\to\infty]{} \beta$ for $0<\beta<1$. $\alpha \ln^2 q^2$ behavior is then found by these methods for each of the two diagrams represented in Fig. 6, but the coefficient of this term cancels between the two diagrams. The transition from this result to the limit with $\beta = 1$ in which (A1) is valid is not smooth because of the existence of $\ln(1-\beta)$ terms which are dropped in evaluating the $\beta<1$ result. The transition to $\beta=0$, in which q^2, $2m\nu \ll s_{ep}$ is also nontrivial, but the cancellation is still found, as is well known from earlier work.[11]

The more complicated diagram of Fig. 5 is represented by Eq. (B15) of Ref. 4. In this equation the k_1 integration is represented in terms of Sudakov variables x_1, y_1', and K_1. It turns out that y_1' integration divides x_1 space into various regions in which the y_1' contour can be wrapped around different cuts of the hadronic amplitude. The points $x_1 = -1/\omega$ and $x_1 = 0$ are end points of such integrations, and logarithmic divergences occur in the x_1 integration at these points, arising from the singularities in the photon-propagator and electron-propagator terms, respectively. However, no double logarithms occur because of the softening in the other terms in the k_1-integration loop, and so the connected diagram of Fig. 5

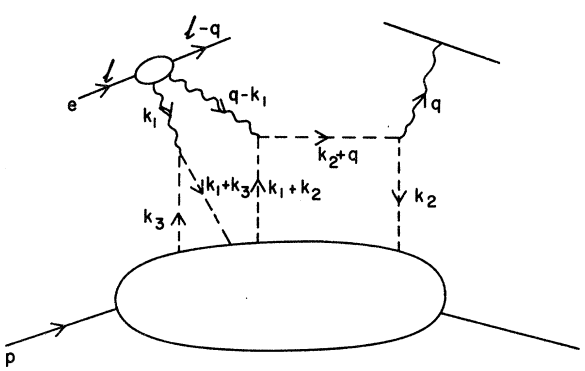

FIG. 5. The dominant two-photon—exchange interference term.

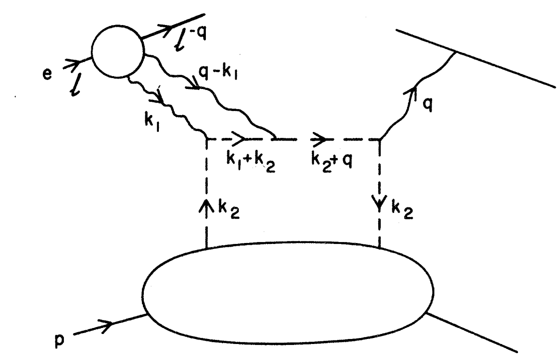

FIG. 6. The disconnected part of the previous diagram, which contains an extra logarithmic enhancement in certain limits.

contributes only terms of order $\alpha \ln|q^2|$ to (1.2), and is dominated by the $\alpha \ln^2|q^2|$ behavior of Fig. 6.

Redhead's work also shows that terms of order $\alpha \ln^2|q^2|$ will occur in this model from the electromagnetic renormalization of both the electron and the parton vertex as illustrated in Fig. 4. Clearly these terms will occur with the same sign in both the electron and positron scattering cross sections, and the photon-parton vertex renormalization part will involve model-dependent terms such as $\sum_i Q_i{}^4 F_1^i(\omega)$.

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*Work supported in part by the Center for Advanced Studies, University of Virginia, and in part by National Science Foundation Grant No. GP-32998X.

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[6]The inelastic process $e^\pm + p \to e^\pm + X$ for fixed ep energy s_{ep} is characterized by the two invariants q^2 and $\nu = m^{-1} p \cdot q$, where q_μ is the momentum transfer to the lepton and p_μ is the momentum of the target nucleon. The deep-inelastic process requires q^2, $\nu \to \infty$ but their ratio $x = -q^2/2m\nu$, which must lie between 0 and 1, to be finite. The cross sections σ^- and σ^+ may be any differential cross section in q^2 and ν.

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[9]We may also get $O(\alpha^2)$ terms from electromagnetic corrections internal to the hadronic blobs; these corrections we expect to be the same for electrons or positrons and hence we ignore them, save for comments on vertex corrections.

[10]We have used just one pointlike object for the example. We could also use many such constituents, leading to two types of graphs: those in which the two-photon interaction is on the same constituent (this is the type we are studying), and those on which the photons interact on different constituents. In the Appendix we show that in the softened-field parton model (Ref. 5) the latter type of graph contributes only $O(\alpha \ln q^2)$ to r and hence is nonleading. The contribution of this latter type of graph is not likely to affect the conclusions we draw from the former type in other models.

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