[10]J. H. Klems, R. H. Hildebrand, and R. Stiening, Phys. Rev. Lett. 25, 473 (1970); Phys. Rev. D 4, 66 (1971).
[11]D. Ljung and D. Cline, Phys. Rev. D 8, 1307 (1973).
[12]R. J. Abrams, A. S. Carroll, T. F. Kycia, K. K. Li, D. N. Michael, P. M. Mockett, and R. Rubinstein, Phys. Rev. D (to be published).
[13]Nothing is known experimentally about this decay mode. As $K_L \to 2\pi$ violates *CP* invariance, the unitarity bound for $K_L \to \pi^0 2\gamma$ is so small that it is not of interest at present.
[14]The latest published paper is by M. Moshe and P. Singer, Phys. Rev. D 6, 1379 (1972). Most of the previous models have been ruled out by the experiments referred to in Refs. 10, 11, and 12.
[15]L. M. Sehgal, Phys. Rev. D 6, 367 (1972).
[16]Analogous calculations for $K \to 2\mu$ decay have been given by L. M. Sehgal [Nuovo Cimento 45, 785 (1966)], and by B. R. Martin, E. de Rafael, and J. Smith, [Phys. Rev. D 2, 179 (1970); 272(E) (1971)]. We have done the calculation in a longhand fashion which allows us to check gauge invariance at every stage.
[17]R. J. Abrams, A. S. Carroll, T. F. Kycia, K. K. Li, J. Menes, D. N. Michael, P. M. Mockett, and R. Rubinstein, Phys. Rev. Lett. 29, 1118 (1972).

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# *CP* Violation and Electric Dipole Moments in Gauge Theories of Weak Interactions*

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We present some general considerations on computations of electric dipole moments ($D$) of leptons and of hadrons in gauge theories which incorporate *CP* violation. Technical aspects of the isolation of the *CP*-violating parts of the corresponding graphs are described. We emphasize the distinction (already familiar from $g$–2 constraints) between "mixed chirality" and "pure chirality" couplings. In the mixed-chirality case, $D$'s can potentially appear to third order in the semiweak-coupling constant; in the pure-chirality case $D$ is at least of fifth order. Estimates are given for prototypes of two classes of gauge models: (1) *CP* violation is implemented via the introduction of a small parameter. For the example considered, a model due to Mohapatra, $D_{\text{nucleon}}$ and $D_{\text{lepton}}$ are both of fifth order. (2) The violation is "maximal," as exemplified in the O(4) and O(4)×U(1) models. Here $D_{\text{muon}}$ appears to third order in the (mixed-chirality) O(4) case. All other $D$'s in the "maximal" examples are of fifth order. For the "maximal" model our estimates are below but may not be very far from the present experimental bounds. For the small-parameter model they are quite considerably smaller.

## I. INTRODUCTION AND RESULTS

The experimental discovery[1] of *CP* violation in $K_L$ decay implies the violation of $T$ invariance if one accepts the *CPT* theorem. In addition, the $K$ complex provides direct experimental tests of $T$ invariance.[2-4] In fact, it is not necessary to assume *CPT* invariance at the outset in the experimental analysis of these decays since they may serve to test independently $T$ invariance and *CPT* invariance.[5] Such tests have been performed[5] with the result that $T$ violation has been established with 10 standard deviations, while there is no evidence (here or anywhere else) for *CPT* violation.[6]

As is well known, experimental attempts to observe $T$-violating effects outside the $K$ complex have all turned out negative so far. This is particularly true in regard to searches for electric dipole moments ($D$) of nucleons and leptons. We recall that a nonvanishing $D$ can only occur if both $T$ and $P$ violation are present, and that the possibility of the existence of $D$'s for elementary particles and nuclei was first raised by Purcell and Ramsey[7] well before the discovery even of $P$ violation. For spin-$\frac{1}{2}$ particles, the presence of a $D$ implies the existence of an *effective* interaction of the form

$$iF_D(q^2)\bar{\psi}(p_1)\gamma_5\sigma_{\mu\nu}\psi(p_2)F_{\mu\nu}\ , \qquad (1.1)$$

where $F_{\mu\nu}$ is the electromagnetic field and $q^2 = (p_1-p_2)^2$. $D$ is defined by $D=F_D(0)$. Note that the possibility that $F_D(0)=0$ (though perhaps not natural) cannot be discarded out of hand.[8] That is to say, even in the presence of $P$ and $T$ violation the *static* quantity $F_D(0)$ could be zero.

It is the purpose of this paper to discuss the question of electric dipole moments for fermions in the context of the general strategy embodied in the gauge theories of weak and electromagnetic interactions.[9] In such theories currents appear which are associated with the generators of a compact Lie group. Like all relativistic local Lagrangian field theories, such theories are inherently *CPT*-invariant. They are $P$-violating by construction, since the particle states with left helicity are taken to enter inequivalently to those with right helicity into the representations of whatever group is chosen. Likewise, gauge theories can be $T$-noninvariant by construction. We shall indicate presently several ways in which this can be done, but first we would like to make the following general comments:

(1) It is inherent in all gauge theories that the electromagnetic current has a minimal structure. Thus the precepts of the gauge strategy, including the insistence on strict renormalizability, rule out from the start any attempt to include $CP$ violation via the structure of the electromagnetic coupling.

In particular, because renormalizability is at the core of this type of theory, it is inadmissible to introduce into the basic Lagrangian a term of the type $iF_{\mu\nu}(x)\bar{\psi}(x)\gamma_5\sigma_{\mu\nu}\psi(x)$, just as it is inadmissible to introduce the well-known Pauli term. This statement has a consequence which is as obvious as its implications are nontrivial: If a gauge theory allows a nonzero $D$ to appear at all, then $D$ must be "calculable." Indeed, since the renormalizability of the theory forbids the introduction of a counterterm of the electric dipole type, $D$ must be expressible in terms of the renormalized parameters such as masses and coupling constants which enter the theory.

(2) The local gauge principle associated with the compact Lie-group structure for any given model rules out of consideration certain other interactions as well which might induce a fermion $D$. An example is provided[10] by a coupling of the type $i\epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}(x)W^*_\lambda(x)W_\sigma(x)$, where $W_\sigma(x)$ is some (non-self-adjoint) vector-boson field.

(3) Since $D$ cannot be present as a first-order vertex, the next question is whether a $D$ can arise to third order, that is, in the one-loop approximation. This is possible for the following reason. Suppose we wish to compute to this order the $D$ of a fermion called $f_1$. Consider all the self-energy graphs of the type $f_1 \to f_2 + B \to f_1$, where $f_2$ is an intermediate fermion state and $B$ is some vector boson. If $f_1 \to f_2 + B$ goes via a coupling $\gamma_\mu(g_V + g_A\gamma_5)$ then $f_2 + B \to f_1$ goes via $\gamma_\mu(g_V^* + g_A^*\gamma_5)$. The electromagnetic vertex corrections corresponding to this graph are obtained by attaching an external photon line in all possible ways. A potential $D$ contribution is proportional to $e\,\mathrm{Im}\,g_A^* g_V$. To the order considered, the effects of various $(f_2, B)$ sets are additive. It follows that $f_1$ can only have a $D$ to third order if among the particle pairs $(f_2, B)$ there are some for which the corresponding pair $(g_A, g_V)$ of coupling constants are not relatively real. We shall encounter one instance in Sec. II and one in Sec. III where this situation obtains. In all other cases the leading contribution to the $D$'s will be at most of fifth order. (We need also to consider $f_1 \to f_2 + B \to f_1$, where $B$ is a Higgs scalar field. In no case encountered here do third-order contributions to $D$ arise from such couplings.)

One particular and important class of couplings in which the relative reality of $g_A$ and $g_V$ obtains are those which are of "pure chirality" (i.e., where the vertex structure is given by $\gamma_\mu P_L$, $P_L$ being the left chirality projection operator). Thus if only pure-chirality couplings are involved, $D$ is patently at least of fifth order.

Still holding off on any details about particular gauge models, let us then try next to get an idea of orders of magnitude for those instances where the leading effects are of third order. *If we were to assume* that there are no further cancellation mechanisms or characteristic small factors, then the following power-counting argument can be given. Let the loop be generated by a vector meson with mass $M$. Let us work in the gauge[11] where the corresponding propagators are $\delta_{\mu\nu}(k^2+M^2)^{-1}$ ("'t Hooft gauge"). Let the coupling constants $g$ in the theory be of general order $e$. Then by power counting and by dimensional reasoning, the general order of magnitude of the contribution to $D$ of a one-loop graph, barring cancellations and other small factors, would be $D \sim eGm/\pi^2$ (where the typical number of $\pi$ factors has also been recorded and where $m$ is a typical fermion mass in the problem). The additional one-loop contributions involving scalar fields in the 't Hooft gauge are at most of the same order of magnitude as the vector contributions.

It should be noted that the order of magnitude for $D$ just stated is not at all that small, as witness the current experimental limits on electric dipole moments given in Table I. Thus for $m \simeq 1$ GeV the order for $D$ would be $\sim 10^{-20}$ $e$ cm. Such orders of magnitude for $m$ are quite fair. In the case of the $D$ operator for nucleons, one should think of $m$ as a quark mass. For leptons, $m$ is typically a heavy lepton mass unless the model has no heavy leptons. It is obvious that electric dipole moments of this magnitude can be in considerable trouble with experiment. Even an additional factor-of-$\alpha$ suppression, which would be the naive guess if the leading order were fifth, would not necessarily be sufficient for $D_{\text{neutron}}$ and $D_{\text{electron}}$. We now hasten to add that *additional* suppression factors do appear in the gauge models to be discussed next. The only exception is $D_{\text{muon}}$, which in one model is actually $\sim 10^{-20}$ $e$ cm.

After these orienting remarks on the $D$ problem in gauge theories, we must now turn to the question how $CP$-violation is actually implemented in theories of this kind. Broadly speaking, this can

TABLE I. Experimental limits on electric dipole moments.

| Particle | Upper limit on electric dipole moment ($e$ cm) | Experiment |
|---|---|---|
| neutron | $1\times10^{-23}$ | Dress *et al.* (1973) [a] |
| proton | $2\times10^{-20}$ | Harrison *et al.* (1969) [b] |
| electron | $3\times10^{-24}$ | Weisskopf *et al.* (1968) [c]; Player and Sandars (1970) [c] |
| muon | $2\times10^{-17}$ | Charpak *et al.* (1961) [d] |
| | $4\times10^{-18}$ | From muon $g-2$ [e] |

[a] W. B. Dress, P. D. Miller, and N. F. Ramsey, Phys. Rev. D 7, 3147 (1973).
[b] G. E. Harrison, P. G. H. Sandars, and S. J. Wright, Phys. Rev. Lett. 22, 1263 (1969).
[c] M. C. Weisskopf, J. P. Carrico, H. Gould, E. Lipworth, and T. S. Stein, Phys. Rev. Lett. 21, 1645 (1968); M. A. Player and P. G. H. Sandars, J. Phys. B 3 1620 (1970).
[d] G. Charpak, F. J. M. Farley, R. L. Garwin, T. Muller, J. C. Sens, and A. Zichichi, Nuovo Cimento 22, 1043 (1961).
[e] If the muon has an electric dipole moment $D_\mu = f e\hbar/mc$ in addition to its anomalous magnetic moment $a=\frac{1}{2}(g-2)$, its precession rate in a muon storage ring is increased by the factor $1+2(f/a)^2$ [V. Bargmann, L. Michel, and V. Telegdi, Phys. Rev. Lett. 2, 435 (1959)]. Using $\Delta a = 6\times10^{-7}=2\sigma$ [J. Bailey *et al.*, Phys. Lett. 28B, 287 (1968)], $f \leq (a\Delta a/2)^{1/2} \lesssim 2\times10^{-5}$, which corresponds to the value quoted in the table. We would like to thank Warren Wilson and Dr. Alvaro de Rújula for pointing out to us this indirect way of obtaining an upper limit for $D_\mu$. Improved direct measurements of $D_\mu$ using the method of Ref. d are desirable, if only to establish a firmer basis for the measurement of $g-2$.

be done in two distinct ways.

(a) All $CP$-violating effects are proportional to a *small parameter* introduced into the theory for the very purpose of incorporating such effects. An example of this category is the scheme described recently by Mohapatra.[12] If we denote the small parameter by $\phi$, then all $D$'s are evidently proportional to $\phi$. A brief discussion of this scheme with particular emphasis on the $D$'s will be given in Sec. II.

(b) Examples have been given recently[13,14] of $CP$-violating gauge theories in which no small $\phi$ parameters are ever introduced, but in which the observable $CP$-violating effects are nevertheless quite tiny. This comes about not because of multiparticle cancellation effects (an often-used suppression mechanism in gauge models) but simply through the application of the quantum-mechanical description of $T$ violation. In this class of theories one is motivated to introduce a maximal $CP$ violation in the leptonic sector in order to account properly for the relative rates of $\mu$ decay and $\Delta S=0$ and $\Delta S=1$ semileptonic decays.

In spite of the quite different approaches employed in theories of type (a) as compared to (b), there is one feature common to both. Namely, one quite general way to implement CP violation is tied to the occurrence, within the *same* multiplet, of at least two fermions which have the same electric charge. In the model mentioned under (a) the relevant fermion pair is the $\mathfrak{N}$ and the $\lambda$ quark (each with right-handed chirality, as it happens). In the models of Refs. 13 and 14 the pairs are $\nu_e$ and a neutral heavy lepton $x^0$, and $\nu_\mu$ and another heavy lepton $y^0$. (Here, more specifically, one deals with left-handed pairs.) Quite generally, if a model is such that its representation content allows for the occurrence of particle pairs of the same electric charge, it is possible to give a $CP$-violating version of such a model. (We do not know whether or not this is the only way to implement $CP$ violation.)

Table II summarizes the results we have obtained for the two kinds of models mentioned above.[15] The details of the derivation are found in Sec. II and III. Let us right away explain some of the notations in this table and also indicate the qualitative reasons for the origin of further suppression factors which are manifest in Table II.

*(a) "Small parameter" CP violation.* Both $D_l$ and $D_{\rm neutron}$ are proportional to $\phi$, as already stated, and also to $\sin\theta$, where $\theta$ is the Cabibbo angle. $m_Q$ denotes a typical average quark mass. $D_l$ is proportional to $m_l$ since there are no heavy leptons in this model.

*(b) "Maximal" CP violation.* In these models neutral heavy leptons appear, one of the electron type and one of the muon type.[16] $m(x^0)$ denotes the typical mass of these neutral objects. There is also one charged heavy lepton of the electron type and one of the muon type. In the $D_{\rm neutron}$ column the quantity $d_0$ denotes the mass difference of two neutral charmed quarks $q^0$ and $r^0$ which are typical for these as for other 8-quark models.[17]

TABLE II. Electric dipole moments in various models.

| Model | $D_{\text{electron}}/e\,\text{cm}$ | $D_{\text{muon}}/e\,\text{cm}$ | $D_{\text{neutron}}/e\,\text{cm}$ |
|---|---|---|---|
| $CP$ violation in hadronic currents proportional to small parameter $\phi \approx 10^{-4}$ | $\frac{G\alpha}{\pi^3}\frac{m_Q{}^2}{M^2}m_l\,\phi\,\sin\theta$ | | $\frac{G\alpha}{\pi^3}m_Q\phi\,\sin\theta$ |
| SU(2)×U(1)—4 quarks [a] | $\sim\frac{m_Q{}^2}{M^2}10^{-30}$ | $\sim\frac{m_Q{}^2}{M^2}10^{-28}$ | $\sim\left(\frac{m_Q}{\text{GeV}}\right)10^{-27}$ |
| "Maximal" $CP$ violation in leptonic currents | | | |
| O(4)—8 quarks [b] | $\frac{G\alpha}{\pi^3}\delta_0{}^2m(x^0)$ | $\frac{G}{2\pi^2}m(y^0)$ | $\frac{G\alpha}{\pi^3}\delta_0{}^2d^0$ |
| $\delta_0=m(y^0)/M\lesssim 10^{-1}$ | $\lesssim\left(\frac{m(x^0)}{\text{GeV}}\right)10^{-24}$ | $\sim\left(\frac{m(y^0)}{\text{GeV}}\right)10^{-20}$ | $\lesssim\left(\frac{d^0}{\text{GeV}}\right)10^{-24}$ |
| O(4)×U(1)—8 quarks [b] | $\frac{G\alpha}{\pi^3}\delta_0{}^2m_l$ | | $\frac{G\alpha}{\pi^3}\delta_0{}^2d^0$ |
| $\delta_0<1$ | $\sim\delta_0{}^2 10^{-25}$ | $\sim\delta_0{}^2 10^{-23}$ | $\sim\delta_0{}^2\left(\frac{d^0}{\text{GeV}}\right)10^{-22}$ |

[a] Here $m_Q$ is a quark mass.
[b] The quantity $d^0$ is the difference in mass of the two neutral charmed quarks $q^0$ and $r^0$.

The factor $\delta_0{}^2=[m(y^0)/M]^2$ stems from a cancellation that would be exact for $m(y^0)=m_\nu(=0)$. As was pointed out in Ref. 14, Sec. III, for $m(y^0)=0$ one can absorb all phases in a redefinition of states in these "maximal" models, so that all $CP$ violation vanishes in this limit. The calculation in the Appendix shows explicitly how the factor $\delta_0{}^2$ arises.

There is another qualitative way in which one may divide gauge models with $CP$ violation into categories, namely as to whether the $CP$ violation enters via the hadron sector, the lepton sector, or the vector- or scalar-boson sector. (Some of these may be equivalent. Needless to say, one can also invent models in which the $CP$ violation enters in several sectors.) In the Mohapatra model,[12] which we consider in Sec. II, the $CP$ violation enters in the hadron sector. In the O(4) models discussed in Sec. III, $CP$ violation enters in the lepton sector. As is apparent from Table II, the magnitudes of the electric dipole moments depend on the structure of the model rather than just on the sector in which $CP$ violation resides.

We note an important difference between O(4) and O(4)×U(1) in regard to the electron $D$. In the former case there is a proportionality to $m_L$, in the latter to $m_e$, the electron mass. As will be spelled out in more detail in Sec. III and the Appendix, this quantitatively quite important distinction (at least three orders of magnitude) stems from the different "chiral mixing" properties of the two schemes. In O(4)×U(1) the charge-raising and -lowering currents (and one of the neutral currents) are made up purely of left-handed fermions, whereas for O(4) both left- and right-handed particles contribute to these very same currents. It is a much more general trait that gauge theories with little chiral mixing tend to be less constrained than those with more mixing. We shall also exemplify this in Sec. III by a brief discussion of a constraint imposed by $g-2$: $\delta_0<0.1$ in the O(4) model.

We conclude the Introduction with the following general comments.

(1) The results displayed in Table II are what we believe to be reasonable orders of magnitude, in spite of the following two circumstances. (a) Although in $D_{\text{neutron}}$ and $D_l$ there are no calculational ambiguities arising from the weak interactions, they are nevertheless not "computable" (that is, explicitly expressible in terms of measurable parameters of the theory) since, as we shall show, strong-interaction complications arise in varying degrees in each case. (In particular, we assume that $D_{\text{nucleon}}$ is of the order of $D_{\text{quark}}$.) (b) The actual detailed computation of two- and three-loop graphs is in any case an undertaking of considerable magnitude and importance, and one which we have not attempted.

(2) At this point we remark parenthetically that in the context of a renormalizable gauge theory there appears no reason for a form factor $F_D(q^2)$ to vanish as $q^2\to 0$, since, in such theories,

interactions which involve one or more derivatives[8] are generally inadmissible.

(3) From the point of view of the "maximal" $CP$-violation models of the sort considered here, the experimental search for electric dipole moments has now reached the interesting stage. On the other hand, if the "small parameter" approach is the correct line, present experimental techniques are entirely inadequate to see either neutron or lepton electric dipole moments.

It is regrettable that we cannot offer the experimentalists any precise numerical prediction to verify. We can only hope that the present considerations may convince them that, to say the very least, every improvement of the measurements will provide ever more significant and severe constraints on theoretical developments.

## II. AN EXAMPLE OF "SMALL PARAMETER" *CP* VIOLATION

The model of Ref. 12 employs the gauge group SU(2)×U(1) with the well-known set of vector mesons[13] $W^{\pm}$, $Z$, and the photon. The charge operator is $Q=t_3+\frac{1}{2}Y$; the $W^{\pm}$ are coupled to $t^{\pm}$, the raising and lowering operators of SU(2). $Z$ is coupled to $(g^2+g'^2)^{-1/2}(g^2t_3+g'^2\frac{1}{2}Y)$, where $Y$ is the U(1) generator. Lepton assignments are as in Ref. 18.

The model is of the four-quark variety, involving $\mathcal{P}$, $\mathfrak{N}$, $\lambda$ and a charmed quark $\mathcal{P}'$ with the same electric charge as $\mathcal{P}$. The variant which allows for the introduction of $CP$ violation conssists in a novel way of treating the right-handed quark states. The quark representations are three doublets and two singlets, namely,

$$Q_1=\begin{pmatrix}\mathcal{P}\\ \mathfrak{N}_C\end{pmatrix}_L,\quad Q_2=\begin{pmatrix}\mathcal{P}'\\ \lambda_C\end{pmatrix}_L,$$

$$Q_3=\begin{pmatrix}\mathcal{P}'\\ \mathfrak{N}\cos\phi+i\lambda\sin\phi\end{pmatrix}_R, \tag{2.1}$$

$$Q_4=\mathcal{P}_R,\quad Q_5=i\mathfrak{N}_R\sin\phi+\lambda_R\cos\phi,$$

with $\mathfrak{N}_C=\mathfrak{N}\cos\theta+\lambda\sin\theta$, $\lambda_C=-\mathfrak{N}\sin\theta+\lambda\cos\theta$. $CP$ violation is injected via the factor $i$ in the $Q_3$ doublet. A systematic treatment of the Higgs problem[12] shows that the Higgs set is more involved here as compared with CP-conserving versions of SU(2)×U(1).

As was noted in Ref. 12, in order to get the correct order for the $K_L\to 2\pi$ rate *and* to suppress unwanted processes like $K^+\to\pi^+\nu\bar{\nu}$, $\phi$ has to lie in the neighborhood of $10^{-4}$. (It is an interesting feature of this model that approximately the same value of $\phi$ satisfies both constraints.) Thus we deal with an expansion in $\phi$ along with other small parameters of the theory. From this it is easily recognized that to leading order the off-diagonal mass matrix elements in the $K$ system are purely real, so that this is a purely "on shell" theory of $CP$ violation insofar as the $K$ system is concerned (and without constraint on the $\Delta I=\frac{1}{2}$ vs $\frac{3}{2}$ parts in $K_L\to 2\pi$), in contradistinction to the models of Refs. 13 and 14 which are superweak.

We now turn to the estimation of the electric dipole moments. For $CP$ violation to be present in a one-loop graph, we must have $\mathrm{Im}\, g_A^* g_V\neq 0$. In the Mohapatra model the $CP$ violation enters through the right-handed $i\phi(\bar{\lambda}\mathfrak{N}Z)$ and $i\phi(\bar{\lambda}\mathcal{P}'W^-)$ couplings. Since there are no left-handed strangeness-changing *neutral* couplings in this model,[12] there is no $g_A$-$g_V$ interference in the $\mathfrak{N}\to\lambda+Z\to\mathfrak{N}$ self-energy graph, so no $D$ is generated by attaching a photon to this graph. (Note that in this model the quarks are not necessarily integrally charged.) The graph obtained by attaching a photon to $\lambda\to\mathcal{P}'+W^-\to\lambda$ does generate an electric dipole moment for the $\lambda$ quark of order $eGm_Q\phi$ $\sim(m_Q/\mathrm{GeV})10^{-23}\,e\,\mathrm{cm}$. This graph, embedded in the two-loop graph Fig. 1(a), gives a contribution to $D_{\mathcal{P}\text{ quark}}$,

$$D\sim e\frac{G\alpha}{\pi^3}\,m_Q\,\phi\sin^2\theta\approx(m_Q/\mathrm{GeV})\,10^{-28}\,e\,\mathrm{cm}.$$

Consider next the self-energy graph obtained by inserting a $\lambda$-$\mathcal{P}'$ loop in a $W$ propagator, Fig. 1(b). This graph generates a $D$ only if the photon is attached to the loop, since the $CP$-violating term is proportional to $\gamma_5$. The result then is propor-

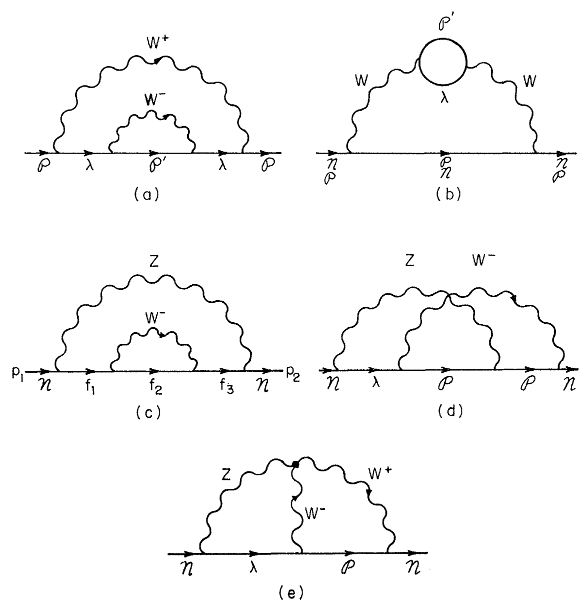


FIG. 1. Various topologies of self-energy graphs which contribute, upon attachment of a photon, to $D_{\text{nucleon}}$ in the "small parameter" model.

tional to $i\phi m_\lambda m_{\mathcal{P}'} \epsilon_{\alpha\beta\gamma\delta} q^\delta$ from the loop, and we estimate

$$D_{\text{quark}} \sim \frac{eG\alpha}{\pi^3} \left(\frac{m_Q}{M}\right)^2 m_Q \phi .$$

Here $m_Q$ represents a typical quark mass, which is assumed to be small[19] compared to a typical vector-boson mass $M$.

Figure 1(c) makes a contribution to $D$ of the same general order of magnitude. The intermediate fermions are $f_1 = \mathcal{N}$ or $\lambda$, $f_2 = \mathcal{P}$ or $\mathcal{P}'$, $f_3 = \mathcal{N}$ or $\lambda$. No $D$ contributions result if $f_1 = f_3 = \mathcal{N}$ or $f_1 = f_3 = \lambda$. For $f_2 = \mathcal{P}$ the contribution to $D$ is $\approx \phi$, for $f_2 = \mathcal{P}'$ it is $\approx \phi^2$. Thus we neglect $f_2 = \mathcal{P}'$. The remaining two cases, $f_1 = \mathcal{N}$, $f_3 = \lambda$ and $f_1 = \lambda$, $f_3 = \mathcal{N}$, have the opposite sign for the imaginary part of the coupling-constant product. The two cases cancel exactly for $m_{\mathcal{N}} = m_\lambda$. The $(\mathcal{N}, \lambda, Z)$ vertex gives a factor $\phi$, the $(\lambda, \mathcal{P}, W)$ vertex a factor $\sin\theta$. It is then readily seen that the contribution to $D$ is

$$\sim e\frac{G\alpha}{\pi^3}\left(\frac{m_Q{}^2}{M^2}\right)(m_{\mathcal{N}} - m_\lambda)\,\phi \sin\theta . \tag{2.2}$$

There is an analogous graph obtained from Fig. 1(c) by the substitution $Z \to W^-$, $W^- \to W^+$. The contributing $f$ combinations in this case are $f_1 = \mathcal{P}$, $f_3 = \mathcal{P}'$ or $f_1 = \mathcal{P}'$, $f_3 = \mathcal{P}$, while $f_2 = \lambda$. One obtains again a structure as in Eq. (2.2) but with $m_{\mathcal{N}} - m_\lambda$ replaced by $m_{\mathcal{P}} - m_{\mathcal{P}'}$.

Next let us consider Fig. 1(d) and also the "paired" graph obtained by running all lines backwards. Both graphs are proportional to $\phi \sin\theta$. The graphs cancel each other if $m_{\mathcal{P}} = m_\lambda$ *and* $M(W) = M(Z)$. This latter relation is unattainable in the SU(2)×U(1) model; thus we should treat $[M(W) - M(Z)]/(\text{average } M)$ as of order unity. The mass cancellation limit therefore does not induce any factor of importance and the order of these graphs is in fact the dominant one:

$$D \sim e\frac{G\alpha}{\pi^3} m_Q \phi \sin\theta , \tag{2.3}$$

recorded in Table II. There is another set of graphs symbolized in Fig. 1(e), involving a triple-vector-boson vertex. This set also gives a contribution of the order displayed in Eq. (2.3).

As an example of a graph which is pertinent to the $D$ of an electron (or muon) we show the graph of Fig. 2. One will again recognize the occurrence of a factor $\phi \sin\theta$. Furthermore, a factor $m_Q{}^2/M^2$ occurs because the $CP$ violation is generated in the hadronic loop to which the photon is attached. Moreover it is evident from the structure of the graph of Fig. 2 that $D_l$ must be proportional to $m_l$. In this way we arrive at the estimate given in Table II.

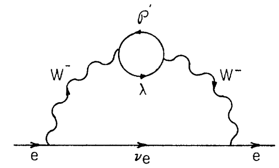


FIG. 2. A leading graph contributing to $D_{\text{electron}}$ in the "small parameter" model.

## III. MODELS WITH "MAXIMAL" *CP* VIOLATION

The models of Refs. 13 and 14 incorporate the Cabibbo angle dynamically. There are two pairs of charged vector bosons, $W_1^\pm$ and $W_2^\pm$, and the hadron multiplets are so arranged that the corresponding currents are respectively strangeness-conserving $(J^{(1)} \sim [\bar{\mathcal{P}}\mathcal{N} + \cdots])$ and strangeness-changing $(J^{(2)} \sim [\bar{\mathcal{P}}\lambda + \cdots])$. The spontaneous symmetry breaking generates the respective masses $M_1, M_2$ of $W_1$, $W_2$ and therefore generates the Cabibbo angle since

$$\frac{M_1{}^2}{M_2{}^2} = \tan\theta ,$$

in order that the usual semileptonic phenomenology is reproduced. (See Table III for more details on the vector bosons.)

We will discuss models built both on the gauge group O(4) and on the gauge group O(4)×U(1). In both cases the *left*-handed fermions are taken to lie in quartets $(\frac{1}{2}, \frac{1}{2})$, with charge structure $(+, 0, 0, -)$. The charge operator is taken to be

$$Q = t_3 + \rho_3, \quad \text{O(4)}$$

$$Q = t_3 + \rho_3 + Y, \quad \text{O(4)}\times\text{U(1)}$$

where

$$\vec{t} \times \vec{t} = i\vec{t}\,, \quad \vec{\rho} \times \vec{\rho} = i\vec{\rho}\,,$$

and $\vec{t}, \vec{\rho}$, and $Y$ all commute. The essential difference between the O(4) and O(4)×U(1) models has to do with the treatment of the *right*-handed fermions. In O(4) the charged fermions cannot be taken to be singlets. If we introduce four leptons of each variety ($e$ and $\mu$)— namely $(x^+, x^0, \nu_e, e)$ and $(y^+, y^0, \nu_\mu, \mu)$, with $x^+$ and $x^0$ heavy electron-type leptons and $y^+$ and $y^0$ heavy muon-type leptons—then we clearly cannot take the right-handed leptons to lie in quartets, since there are only three right-handed leptons of each type (assuming that $\nu_e$ and $\nu_\mu$ are Weyl particles). The solution is to put them in triplets, $(x^+, x^0, e)_R$ in (1, 0) and $(y^+, y^0, \mu)_R$ in (0, 1). An alternative solution is to introduce a fifth lepton of each variety, and place the massive right-handed leptons in quartets; this is the approach followed by Cheng.[16] In either of these O(4) treatments,

TABLE III. Models with "maximal" CP violation.[a]

| | O(4) | O(4)×U(1) |
|---|---|---|
| Vector bosons | $W_1^\pm, W_2^\pm, Z, \gamma$ | $W_1^\pm, W_2^\pm, Z, V, \gamma$ |
| Masses | $M_1=\frac{37}{(\cos\theta)^{1/2}}=37$, $M_2=\frac{37}{(\sin\theta)^{1/2}}=75$ | $M_1=\frac{37}{(\cos\theta)^{1/2}\sin\gamma}$, $M_2=\frac{37}{(\sin\theta)^{1/2}\sin\gamma}$ |
| | $M_1^2+M_2^2=M_0^2\xi$, $0<\xi<1$ | $M_1^2+M_2^2=M_0^2+M_V^2\cos^2\gamma$ |
| Higgs scalars | Two $(\frac{1}{2},\frac{1}{2})$; one pair $(\frac{1}{2},\frac{3}{2})$, $(\frac{3}{2},\frac{1}{2})$ | Two $(\frac{1}{2},\frac{1}{2})^0$; one $(\frac{1}{2},\frac{1}{2})^1$ |
| Fermion multiplets | $f^{lL}, f^{hL}$ in $(\frac{1}{2},\frac{1}{2})$ | $f^{lL}, f^{hL}$ in $(\frac{1}{2},\frac{1}{2})^0$ |
| | $f^{lR}, f^{hR}$ in $(1,0)$, $(0,1)$ | $f^{lR}, f^{hR}$ in $(0,0)^Q$ |
| Charge of quarks | Integral | Not necessarily integral |
| Chirality content of charged currents and of $Z$ current | Mixed chirality | Purely left-handed |
| Constraint from $(g-2)_\mu$ | $\delta_0=\frac{m(y^0)}{M}\lesssim 10^{-1}$ | No constraint |

[a] Notation: $f^{l,h}$ means leptonic, hadronic fermions; $L$, $R$ denote left-, right-handed chirality; $\theta=\theta_{\text{Cabibbo}}\simeq 12°$. In O(4)×U(1) a mixing angle $\gamma$ appears.

the charged currents are of "mixed chirality," having both left- and right-handed parts. (See Table IV.) In the O(4)×U(1) model, by contrast, the right-handed fermions may all be taken to be singlets with "leptonic hypercharge" $Y$ equal to their electromagnetic charge $Q$. Consequently, the charged currents are entirely of left-handed chirality.

As has previously been noted[20] in connection with the O(3) model of Georgi and Glashow,[21] strong constraints on heavy-lepton masses sometimes follow from the requirement that weak contributions $(a)_{\text{wk}}$ to the anomalous magnetic moments of muon and electron be sufficiently small that they do not destroy the excellent agreement between quantum electrodynamics and experiment. The relevant Feynman graph[22] is that of Fig. 3, and the potentially large contribution to $(a_\mu)_{\text{wk}}$ or $(a_e)_{\text{wk}}$ comes from the term proportional to $m(y^0)$ or $m(x^0)$. This term is present in the O(4) model but absent in the O(4)×U(1) model, where it is killed by the left-handed chiral projection operators $P_L$ associated with the charged currents: $\gamma_\alpha P_L(\not k+m)\gamma_\beta P_L=\gamma_\alpha \not k\gamma_\beta P_L$. The resulting constraints in the O(4) model are found by the methods of Ref. 20 to be

$$(a_e)_{\text{wk}}=-\frac{\alpha}{\pi\sqrt{2}}\,\frac{m(x^0)m(e)}{(37\text{ GeV})^2}\,(1-\sin 2\theta)^{1/2}\gtrsim -5\times10^{-9},$$

$$-3\times10^{-7}\lesssim(a_\mu)_{\text{wk}}=\frac{\alpha}{\pi\sqrt{2}}\,\frac{m(y^0)m(\mu)}{(37\text{ GeV})^2}\,[\text{Im}\beta_0^*\beta_-\cos\theta-\text{Re}\beta_0^*\beta_-\sin\theta]\lesssim 9\times10^{-7},$$

or[23]

$$m(x^0)\lesssim 11\text{ GeV},$$

$$m(y^0)\lesssim 8\text{ GeV}.$$

Accordingly, in the O(4) model the parameter $\delta^0=m(y^0)/M$, which enters many of our estimates below, satisfies

$$\delta_0\lesssim 10^{-1}.$$

We repeat that in the O(4)×U(1) model there is no such constraint, because the charged currents are "chirality pure."

Having chosen to place the left-handed leptons and quarks in quartet representations $(\frac{1}{2},\frac{1}{2})$, we are forced[13,14] to incorporate $CP$ violation "maximally." In order to see this, let us compare neutron, $\Lambda$, and muon decay. The relevant currents are

$$J^{(1)L}\propto i[\overline{\mathcal{P}}\mathfrak{N}+\alpha\overline{\nu}_e e+\beta\overline{\nu}_\mu\mu+\cdots]_L,$$
$$J^{(2)L}\propto i[\overline{\mathcal{P}}\Lambda+\gamma\overline{\nu}_e e+\delta\overline{\nu}_\mu\mu+\cdots]_L, \quad (3.1)$$

where $\alpha,\beta,\gamma,\delta$ are complex phases. We can without loss of generality take $\alpha=\beta=1$. In view of Eq. (3.1), the decay amplitudes are evidently in the ratio

$$a_{\text{neutron}}:a_\Lambda:a_{\text{muon}}=\cos\theta:\sin\theta:(\cos\theta+\gamma*\delta\sin\theta).$$

The requirement that

TABLE IV. Currents in O(4) and O(4)×U(1) models.[a]

O(4) leptonic currents

$$J^{(1)lL}=\frac{-ie}{\sqrt{2}}[\bar{x}^+(x^0+\nu_e)+(\bar{\nu}_e+\bar{x}^0)e+\bar{y}^+(\nu_\mu+iy^0)+(\bar{\nu}_\mu-i\bar{y}^0)\mu]_L$$

$$J^{(2)lL}=\frac{-ie}{\sqrt{2}}[\bar{x}^+(x^0-\nu_e)+(\bar{\nu}_e-\bar{x}^0)e+\bar{y}^+(y^0+i\nu_\mu)+(i\bar{\nu}_\mu-\bar{y}^0)\mu]_L$$

$$J^{(0)lL}=-\tfrac{1}{2}ie[\bar{\nu}_e\nu_e-\bar{x}^0x^0-\bar{\nu}_\mu y^0-\bar{y}^0\nu_\mu]_L$$

$$J^{(1,2)lR}=-ie[\bar{x}^0e+\bar{x}^+x^0\pm\beta_0^*\beta_-\bar{y}^0\mu\mp\beta_+^*\beta_0\bar{y}^+y^0]_R$$

$$J^{(0)lR}=-ie[\bar{x}^+x^+-\bar{e}e-\bar{y}^+y^++\bar{\mu}\mu]_R$$

O(4)×U(1) leptonic currents

$$J^{(1)l}=\frac{-ie}{\sqrt{2}\,\sin\gamma}[\bar{x}^+(x^0+\nu_e)+(\bar{x}^0+\bar{\nu}_e)e+\bar{y}^+(y^0+\nu_\mu)+(\bar{y}^0+\bar{\nu}_\mu)\mu]_L$$

$$J^{(2)l}=\frac{-ie}{\sqrt{2}\,\sin\gamma}[\epsilon^*\bar{x}^+(x^0-\nu_e)-\epsilon(\bar{x}^0-\bar{\nu}_e)e+\epsilon\bar{y}^+(y^0-\nu_\mu)-\epsilon^*(\bar{y}^0-\bar{\nu}_\mu)\mu]_L$$

$$J^{(0)}=\frac{ie}{\sqrt{2}\,\sin\gamma}[\bar{x}^0x^0+\bar{y}^0y^0-\bar{\nu}_e\nu_e-\bar{\nu}_\mu\nu_\mu-i(\bar{\nu}_ex^0-\bar{x}^0\nu_e-\bar{\nu}_\mu y^0-\bar{y}^0\nu_\mu)]_L$$

$$J^{(V)}=-ie[(\bar{x}^+x^++\bar{y}^+y^+-\bar{e}e-\bar{\mu}\mu)_L\cot\gamma-(\bar{x}^+x^++\bar{y}^+y^+-\bar{e}e-\bar{\mu}\mu)_R\tan\gamma]$$

Note: $J^{(1)R}=J^{(2)R}=J^{(0)R}=0,\ \epsilon\equiv e^{i\pi/4}$

O(4) hadronic currents

$$J^{(1)hL}=\frac{-ie}{\sqrt{2}}[\bar{\mathcal{P}}(\mathcal{N}+A_1^0)+(\bar{\mathcal{N}}+\bar{A}_1^0)q^-+\bar{q}^+(\mathcal{N}-A_1^0)+(\bar{\mathcal{N}}-\bar{A}_1^0)r^-]_L$$

$$J^{(2)hL}=\frac{-ie}{\sqrt{2}}[\bar{\mathcal{P}}(\lambda+A_2^0)-(\bar{\lambda}+\bar{A}_2^0)q^--\bar{q}^+(\lambda-A_2^0)+(\bar{\lambda}-\bar{A}_2^0)r^-]_L$$

$$J^{(0)hL}=\tfrac{1}{2}ie[(\bar{\mathcal{N}}+\bar{\lambda})q^0-(\bar{\mathcal{N}}-\bar{\lambda})r^0+\text{H.c.}]_L$$

$J^{(1)hR}$, $J^{(2)hR}$ analogous to $J^{lR}$

$$J^{(0)hR}=-ie[\bar{\mathcal{P}}\mathcal{P}-\bar{r}^-r^--\bar{q}^+q^++\bar{q}^-q^-]_R$$

O(4)×U(1) hadronic currents

$$J^{(i)hL}=\frac{1}{\sin\gamma}[J^{(i)hL}]_{O(4)},\quad J^{(i)hR}=0,\quad i=0,1,2$$

$$J^{(V)}=-ie(\Sigma_L\cot\gamma-\Sigma_R\tan\gamma),\quad \Sigma\equiv(\bar{\mathcal{P}}\mathcal{P}+\bar{q}^+q^+-\bar{q}^-q^--\bar{r}^-r^-)$$

[a] Notation: $[\bar{a}b]_{L,R}=\bar{a}\gamma_\mu P_{L,R}b$, where $P_L$ and $P_R$ are the left and right chirality projection operators. [$P_L=\frac{1}{2}(1+\gamma_5)$ in the metric used in Ref. 14 and $P_L=\frac{1}{2}(1-\gamma_5)$ in the metric used in the textbook of Bjorken and Drell.] The combinations $A_1^0=(q^0+r^0)/\sqrt{2}$ and $A_2^0=(q^0-r^0)/\sqrt{2}$ are not orthogonal since the quarks $q^0$ and $r^0$ are chosen not to be mass-degenerate. The $\Delta I=\frac{1}{2}$ rule for $|\Delta S|=1$, $\Delta Q=0$ processes arises in these models via $J^{(0)hL}$, as explained in Refs. 13 and 14. The quantities $\beta_{+,0,-}$ are phase factors.

$$|a_{\text{neutron}}|^2=|a_\Lambda|^2=|a_{\text{muon}}|^2=\cos^2\theta:\sin^2\theta:1$$

then *forces $\gamma$ and $\delta$ to be relatively imaginary:*

$$\gamma^*\delta=\pm i\ . \tag{3.2}$$

Other physical considerations[14] (see below) suggest the choices

$$\gamma=1,\ \delta=i,\quad O(4)$$

$$\gamma=\delta^*=e^{i\pi/4},\quad O(4)\times U(1).$$

Having thus introduced *CP* violation into the lepton sector of the model, we are next invited to inquire regarding the magnitude of the *CP* violation thereby induced. By general arguments it follows that all semileptonic *CP* violation is superweak,[24] even for $\delta_0$ as large as ~1. Due to strong-interaction complications, the extent to which nonleptonic decays are entirely superweak

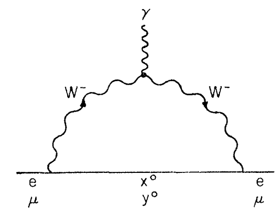


FIG. 3. Graph giving the largest weak contributions to the anomalous magnetic moment of the electron or muon and the leading contribution to $D_{\text{muon}}$ in the O(4) model.

is much harder to assess.[25] The crux of the matter is to find an estimate for $\Delta S=2$ imaginary mass mixing in the $K_1$-$K_2$ system (and to compare it with on-shell nonleptonic $\Delta S=1$ transition effects). The mechanism that generates $\text{Im}\Delta m_K$ is the imaginary part $\sigma$ of the $W_1$-$W_2$ mixing parameter, which is entirely due to lepton loop effects.[26] In regard to $\sigma$, the cases O(4) and O(4)×U(1) present distinct aspects. (1) In O(4), $\sigma$ is uncalculable, since before renormalization[27] $\sigma\approx(\alpha/\pi)\ \delta_0^2M^2\ln\Lambda^2$; it therefore seems reasonable to take $\sigma\approx(\alpha/\pi)\delta_0^2M^2$. (2) It was shown[28] that a finite expression for $\sigma$ in terms of the lepton and vector-boson masses can be obtained in O(4)×U(1), which is indeed $\approx(\alpha/\pi)\delta_0^2M^2$.

The ratio of real to imaginary $K$-mass mixing can be roughly estimated[29] by comparing Figs. 4(a) and 4(b), which yields

$$\frac{\text{Im}\Delta m_K}{\text{Re}\Delta m_K}=2\times10^{-3}\simeq\frac{\alpha}{\pi}\delta_0^2f_S\ , \tag{3.3}$$

where $f_S$ is a factor in which resides our ignorance of the strong interactions. In particular[29] $f_S$ is proportional to a *ratio* of neutral- and charged-quark mass differences $d^0/d^+$. For the O(4) model, the $g-2$ constraint (3.2) requires that $f_S\gtrsim100$, which certainly is possible although not expected *a priori*. The fact that $\delta_0$ is unconstrained by $g-2$ in the O(4)×U(1) model allows a larger $\delta_0$ and therefore a smaller $f_S$, which is perhaps more plausible.

In any event, it is clear that a common set of mass ratios about which we know little so far

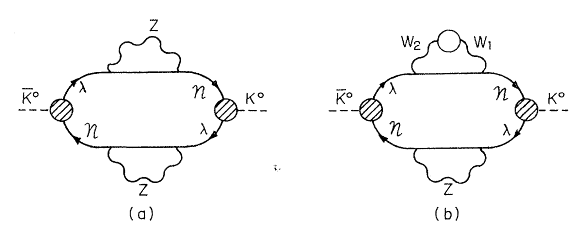


FIG. 4. (a) A graph contributing to $\text{Re}\Delta m_K$. (b) Leading contribution to $\text{Im}\Delta m_K$.

enters in three distinct places: In the ratio given in Eq. (3.3), in the $g-2$ constraints, and in the electric dipole moments, as we shall see next. The survival of the scheme therefore is quite dependent on the consistency with which all three effects can be treated.

Now then to the dipole moments. The leading graphs contributing to $D_{\text{neutron}}$ and to $D_{\text{electron}}$ in the O(4) model[30] all have the structure of Fig. 5. The resulting estimates for the electric dipole moments are given in Table II.

The muon electric dipole moment in the O(4) model is much larger, since it receives contributions of order $e^3$. The largest contribution comes from Fig. 3, which is also the source of the dominant weak contribution to the muon anomalous magnetic dipole moment (unless the Higgs scalar mass is very light[20]). The contribution of this graph to the electric dipole moment arises from a cross term $\operatorname{Im} g_A^* g_V$. It can be calculated in the same manner as the anomalous magnetic moment,[20] and equals

$$D_{\text{muon}} = e\frac{G}{2\pi^2}\, m(y^0)[\operatorname{Re}\beta_0^*\beta_-\cos\theta + \operatorname{Im}\beta_0^*\beta_-\sin\theta],$$

which leads to the estimate given in Table II.

The marked disparity between $D_{\text{electron}}$ and $D_{\text{muon}}$ for the O(4) case finds its origin in the asymmetric choice for the phases $\gamma$ and $\delta$ in this model. This asymmetry was so chosen[14] as to suppress the neutral current process $\nu_\mu + e \to \nu_\mu + e$ (see the expression for $J^{(0)l}$ in Table IV). We now see that, in O(4), $\gamma = 1$, $\delta = i$ corresponds to a $(\nu_\mu, e)$-scattering amplitude[14] $O(G\alpha)$ and a large dipole moment for the *muon*. We note that $\gamma = i$, $\delta = 1$ would have yielded a $(\nu_\mu, e)$-scattering amplitude $O(G)$ and dipole moment $\approx [m(x^0)/\text{GeV}] \times 10^{-20}\, e$ cm for the *electron,* a disastrously large value. Thus we have here a striking example of the connection, already stressed in Ref. 14, between the *CP*-violation effects and the neutral current structure. For the group O(4) $\times$U(1), this connection is quite different. Let us see next what happens to the $D$'s in this case.

The leading graphs contributing to $D_{\text{neutron}}$ in

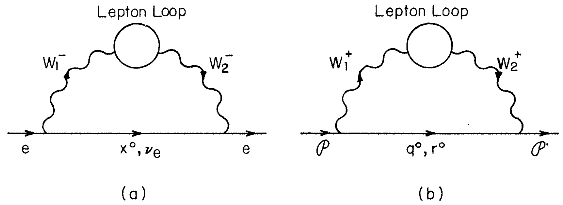


FIG. 5. Graphs contributing to $D_{\text{electron}}$ and $D_{\text{neutron}}$ in the O(4) and O(4)$\times$U(1) models, after attachment of an external photon.

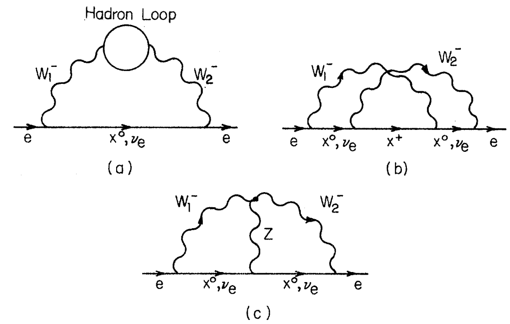


FIG. 6. Additional classes of graphs contributing to $D_{\text{electron}}$ in the O(4)$\times$U(1) model.

the O(4)$\times$U(1) model are again those of Fig. 5(b) where $\sigma$ now receives contributions[28] from electron-type as well as muon-type leptons. In addition to Fig. 5(a), several further sets of graphs contribute to $D_l$ in the O(4)$\times$U(1) model, as displayed in Fig. 6 for $l$ = electron. We draw attention to the fact that Fig. 6(a) contains a closed hadron loop which contributes only to the *real* $W_1$-$W_2$ mixing[26] parameter $\rho$. It is via this particular loop that hadronic effects enter to leading (fifth) order into the $D_l$. This $\rho$ parameter, finite in any event [for both O(4) and O(4)$\times$U(1)] has also a leptonic part; and the relative influence of hadronic to leptonic contributions is to a considerable extent (though certainly not fully) determined by the ratio of the hadronic to the leptonic contributions to $\rho$. Both contributions have been evaluated[31] (neglecting strong interactions) and the ratio is $\approx [m^2(q^0) - m^2(r^0)]/m^2(x^0)$. Our estimates as recorded in Table II are independent of the precise value of this ratio, as long as it is not much *larger* than 1. We emphasize, however, that in contradistinction to the magnetic moment, even in the electric dipole moment of *leptons* the hadronic contributions can enter in the same order in $\alpha$ as the leading leptonic contributions even though the *CP* violation lies entirely in the leptonic sector.

An important feature of the O(4)$\times$U(1) model is that all of these contributions to $D_l$ are proportional to $m_l$. This is in contrast to O(4) where instead of $m_l$ we meet the characteristic mass of *heavy* leptons. As in the discussion of weak contributions to the anomalous magnetic moment, this is because of the chiral projection operators associated with the charged currents in the O(4) $\times$U(1) model. The argument is given in detail in the Appendix.

*Note added:* After the completion of this work, we learned of work by T.D. Lee in which *CP* violation is associated with the Higgs scalar sector. Specifically, the *CP* violation arises through a relative phase between two identical multiplets of Higgs scalars. The dipole moment of a fermion $f$ is given in this model by $D_f \approx eGm_f^3F(m_H)$, where $F(m_H)\sim m_H^{-2}$ is a function of the (arbitrary) Higgs particle masses. The cube of the fermion mass arises as follows: Two powers come from the coupling of the Higgs particle to $f$, and a third from the spin structure of the graphs. Lee assumes $m_H \approx 10$ GeV, for which he finds $D_{\text{neutron}} \lesssim 10^{-23}$ $e$ cm, $D_\mu \approx 10^{-25}$ $e$ cm, and $D_e \approx 10^{-32}$ $e$ cm. We have appreciated the opportunity to learn of Professor Lee's work in advance of publication.

## APPENDIX

The purpose of this appendix is to amplify the general discussions of electric dipole moment contributions in Secs. II and III by a more explicit consideration of examples. We will restrict our considerations to the electric dipole moment of the electron in the O(4)×U(1) model, for definiteness.

Let us begin by considering Fig. 7, which is one of the graphs obtained by attaching a photon to Fig. 6(b). From Table IV one can verify that the $W_2$ couplings are proportional to the complex phase $\epsilon = e^{i\pi/4}$, while the $W_1$ couplings are real. Thus Fig. 7 and the graph obtained from it by interchanging $W_1$ and $W_2$ have potentially CP-violating imaginary parts. (Note that neither the combinations $W_1 W_1$ nor $W_2 W_2$ nor any uncrossed ladder graphs have such an imaginary part.) The contribution of these graphs to the electron's electromagnetic form factor is

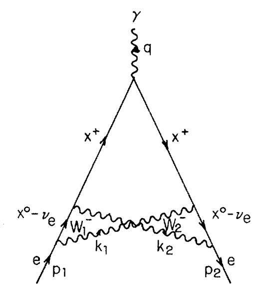


FIG. 7. Example of a graph contributing to $D_{\text{electron}}$ in the O(4)×U(1) model.

$$\frac{1}{4}e^4\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_2)\gamma_\beta P_L(\not{p}_2-\not{k}_2)\gamma_\alpha P_L[\not{p}_2-\not{k}_1-\not{k}_2+m(x^+)]\gamma_\mu[\not{p}_1-\not{k}_1-\not{k}_2+m(x^+)]\gamma_\beta P_L(\not{p}_1-\not{k}_1)\gamma_\alpha P_L u(p_1)$$

$$\times\left[\frac{1}{(p_2-k_2)^2-m(x^0)^2}-\frac{1}{(p_2-k_2)^2}\right]\left[\frac{1}{(p_1-k_1)^2-m(x^0)^2}-\frac{1}{(p_1-k_1)^2}\right]$$

$$\times\left(\epsilon^{*2}\frac{1}{k_1^2-M_1^2}\,\frac{1}{k_2^2-M_2^2}+\epsilon^2\frac{1}{k_1^2-M_2^2}\,\frac{1}{k_2^2-M_1^2}\right)\;.$$

We can now make the following comments: (1) The *CP*-violating part of the last bracket is proportional to $(\epsilon^{*2}-\epsilon^2)(M_1^2-M_2^2)$, but since $M_1\neq M_2$ in this model this results in no suppression factor. (2) The relative minus sign between the $x^0$ and $\nu_e$ propagators results from the relative minus sign in their couplings to $W_2$, and forces the appearance of an explicit factor of $\delta_0^2 = m(x^0)^2/M^2$ in our estimate of $D_l$. (3) Because of the chirality projection operators $P_L$, only terms with an odd number of $\gamma$ matrices survive. The only way to obtain an electric dipole moment term $(i\sigma_{\mu\nu}\gamma_5)$ is therefore to use the Dirac equation $(\not{p}_1-m_e)u(p_1)=\bar{u}(p_2)(\not{p}_2-m_e)=0$, which brings in a factor $m_e$.

All contributions to the electron's electric and magnetic dipole moments are in fact proportional to $m_e$ in the O(4)×U(1) model. In order to see this, let us consider the graph of Fig. 8, which represents any graph that can contribute to a dipole moment. The particles which couple to the electron diagonally in this model are the photon, the massive neutral vector boson $V$, and the neutral member $\phi^0$ of the $(\frac12,\frac12)^1$ Higgs scalar multiplet. The neutral vectors have coupling constants proportional to $e$, and the $\phi^0$ coupling is $\approx em_e/M$. Let us then first consider the case where the electron line flows through the graph remaining always an electron, i.e., with no charged bosons attached to it. It is evident that if $m_e=0$, only terms with odd numbers of $\gamma$ matrices appear.

The charged particles that couple to the elec-

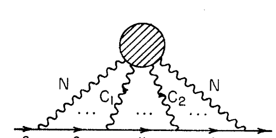


FIG. 8. General graph contributing to $D_{\text{electron}}$ in the O(4)×U(1) model. The lines labeled $C_1$ and $C_2$ are respectively the first and last charged bosons encountered by the electron line. The lines labeled $N$ are neutral bosons.

tron in the $O(4)\times U(1)$ model are $W_1, W_2$, and a doubly charged Higgs particle $\phi^{++}$ from the $(\frac{1}{2},\frac{1}{2})^1$ multiplet. [We can for present purposes work in the unitary formalism ($\xi=0$ gauge[11]) in which the fictitious charged $(\frac{1}{2},\frac{1}{2})^0$ scalars associated with $W_1$ and $W_2$ do not appear[32]]. The coupling of the $W_1$ and $W_2$ to the electron is purely left-handed, and that of the $\phi^{++}$ is

$$\frac{ie}{M}\,\bar{x}^+[m(x^+)P_L+m_eP_R]\,e\phi^{++},$$

so that the part proportional to the right-handed chiral projection operator $P_R$ vanishes for $m_e=0$. Now we must consider the cases $C_{1,2}=W$ or $\phi^{++}$.

For $C_1=W$, $C_2=W$, the part of the lepton line between $C_1$ and $C_2$ contributes the factor $(\gamma P_L)\cdots(\gamma P_L)$. Between the two $P_L$'s there must be an even number of $\gamma$ matrices, so this is, all together, an odd number of $\gamma$ matrices. For $m_e=0$, any neutral vectors attached before $C_1$ or after $C_2$ will contribute an even number of $\gamma$ matrices, so that the final number is always odd. Thus both electric and magnetic dipole moment contributions coming from such graphs must be proportional to $m_e$.

The other cases are similar. For $C_1=\phi^{++}$, $C_2=W$, the part between $C_1$ and $C_2$ gives the factor $(\gamma P_L)\cdots(P_L)$, and the argument is exactly as before. For $C_1=W$ and $C_2=\phi^{++}$, the factor is $(P_R)\cdots(\gamma P_L)$, which only survives if there are an odd number of $\gamma$ matrices between $P_R$ and $P_L$. Finally, for $C_1=\phi^{++}$, $C_2=\phi^{++}$, the factor is $(P_R)\cdots(P_L)$, and the argument is again the same. We remind the reader that this analysis follows immediately from the fact that the *charged* currents in the $O(4)\times U(1)$ model are "chirality-pure," which in turn follows essentially from the fact that the right-handed fermions are singlets. The original Weinberg model[18] is likewise chirality-pure to this same extent; the Georgi-Glashow[21] and O(4) models are not.

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).

[2] G. Charpak and M. Gourdin, CERN Report No. 67-18, 1967 (unpublished).

[3] R. C. Casella, Phys. Rev. Lett. 21, 1128 (1969); 22, 554 (1969).

[4] P. Kabir, Nature (London) 220, 1310 (1968).

[5] K. R. Schubert *et al.*, Phys. Lett. 31B, 662 (1970).

[6] For a recent review see K. Winter, in *Proceedings of the Amsterdam International Conference on Elementary Particles*, edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1972), p. 333.

[7] E. M. Purcell and N. F. Ramsey, Phys. Rev. 78, 807 (1950). For a list of references to models for dipole moments see P. D. Miller, W. B. Dress, J. K. Baird, and N. F. Ramsey, Phys. Rev. Lett. 19, 381 (1967). See also P. G. H. Sandars, *ibid.* 19, 1396 (1967), for a further comment on the proton moment.

[8] For an example see A. Pais and S. B. Treiman, Phys. Lett. 29B, 308 (1969), footnote **.

[9] For a recent survey of these theories see B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1972), Vol. 4, p. 249.

[10] F. Salzman and G. Salzman, Nuovo Cimento 41A, 443 (1966). The *CP*-violation mechanism proposed by Okubo, Marshak, and others [see R. Marshak, Y. Yang, and J. Rao, Phys. Rev. D 3, 1640 (1971)] is cutoff-dependent, at least in its present form. Therefore it cannot be incorporated in the gauge strategy without further ado.

[11] G. 't Hooft, Nucl. Phys. B35, 167 (1971); K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).

[12] R. N. Mohapatra, Phys. Rev. D 6, 2023 (1972).

[13] A. Pais, Phys. Rev. Lett. 29, 1712 (1972); 30, 114(E) (1973).

[14] A. Pais, Phys. Rev. D 8, 625 (1973).

[15] We believe that the number $D_{\rm neutron}\sim 10^{-23}\,e$ cm quoted in Ref. 12 is not correct.

[16] T. P. Cheng [Phys. Rev. D 8, 496 (1973)] discusses a variant of the O(4) model in which there are two heavy neutral leptons of each type. Here $D_l$ is of fifth order for both muon and electron.

[17] The suppression of $\Delta S\neq 0$, $\Delta Q=0$ processes is accomplished in this model (as in the Mohapatra model) by use of the mechanism of S. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)]. The quark charge structure is the same as in B. W. Lee, J. R. Primack, and S. B. Treiman, *ibid.* 7, 510 (1973).

[18] We follow the notation of S. Weinberg [Phys. Rev. Lett. 27, 1688 (1971); Phys. Rev. D 5, 1412 (1972)]. The devices for the elimination of anomalies, noted in Ref. 12, do not qualitatively alter the conclusions of this section.

[19] See the discussion of $K_L\to\bar{\mu}\mu$ and the comment on four-quark models in the paper by Lee, Primack, and Treiman, Ref. 17, Sec. IV.

[20] J. R. Primack and H. R. Quinn, Phys. Rev. D 6, 3171 (1972); Fujikawa *et al.*, Ref. 11.

[21] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972).

[22] We assume that the masses $m_H$ of the Higgs scalars are large enough that their contributions here and elsewhere can be neglected. In Ref. 20 it is shown that this will be true if $m_H{}^2 \gg m_\mu m(y^0)$.

[23] In order to lift an unwanted degeneracy in the mass spectrum of the muonic leptons in the O(4) model, it is necessary to introduce Higgs particles of repre-

sentations $(\frac{3}{2},\frac{1}{2})$. The quantity $\mathrm{Re}\beta_0\beta_-^*$ depends upon details of the vacuum expectation values of the Higgs particles and upon their couplings to the muonic leptons. Without perversely constraining these parameters, the quantity $\mathrm{Re}\beta_0\beta_-^*$ is of order unity.

[24]See especially Ref. 14, Sec. IV. As explained in Ref. 14, Sec. I, the notion of superweakness as used here is in conformance with its initial introduction by L. Wolfenstein [Phys. Rev. Lett. 13, 562 (1964)].

[25]See especially Ref. 14, Sec. VII.

[26]For the definitions of $\sigma$ and $\rho$ see Ref. 14, Eqs. (3.16) and (3.17).

[27]Ref. 14, Eq. (3.19).

[28]Ref. 14, Eq. (7.24).

[29]Ref. 14, Sec. VII. Other contributions to the *real* mass difference stem from the so-called box graphs, for example (see Ref. 19).

[30]For $D_{\mathrm{muon}}$, the set of graphs is as in Fig. 6, with obvious changes in the particle labels.

[31]Ref. 14, Eqs. (4.13), (4.14), (7.23).

[32]At this point we make use of the fact that the $(\frac{1}{2},\frac{1}{2})^0$ scalar multiplets may be chosen to be real in the sense defined in Ref. 14, Sec. II; see also Ref. 14, Sec. VII. The reader will also verify that the presence of singly-charged scalar fields in $(\frac{1}{2},\frac{1}{2})^1$ does not modify the $m_e$ proportionality of the dipole moments.

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# Difference Between $e^+$ and $e^-$ Deep-Inelastic Scattering*

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We show that the existence of pointlike constituents within the nucleon makes it plausible that scale-breaking effects due to higher-order electromagnetic corrections in deep-inelastic scattering will be of order $\alpha\ln^2(-q^2/\mu^2)$ in the region $s_{ep}\gg 1\ (\mathrm{GeV})^2$ and large electron scattering angles. Additionally we are led to conclude that, when the final electron energy is finite in the laboratory frame, the difference of electron and positron deep-inelastic scattering is of order $\alpha\ln^2(-q^2/\mu^2)$ rather than of order $\alpha$. We discuss the possible measurement of this difference and what we may learn from such a measurement.

## I. INTRODUCTION

Whatever the deeper reasons behind this fact, deep-inelastic lepton-nucleon scattering at present day accelerators[1] behaves as if the nucleon contained pointlike constituents.[2,3] We shall try to use this feature of the data to make some very brief comments on electromagnetic corrections to $e$-$e$ and deep-inelastic scattering and on the possible difference between deep-inelastic electron and positron scattering. The difference between $e^++p\to e^++X$ and $e^-+p\to e^-+X$ was previously studied by Kingsley[4] using a "softened" parton model[5] as a method of effecting the cutoff necessary to bring Bjorken scaling into a field-theoretic picture. He came to the conclusion that the ratio $r$ between the deep-inelastic electron and positron cross sections[6] $\sigma^-$ and $\sigma^+$ takes the form

$$r=1+O(\alpha), \tag{1.1}$$

up to possible factors involving logarithms of $q^2$. While we are not so ambitious in the sense of studying a definite model, the method we use seems to us to be of compelling simplicity. We come to the conclusion, based on the pointlike constituent idea, that when $2m\nu\to s_{ep}\gg$ rest masses squared of the problem (which corresponds to large electron scattering angles and finite values of the final electron energy $E'$),

$$r=1+O(\alpha\ln^2(-q^2/\mu^2)), \tag{1.2}$$

where $\mu$ is a scale mass to be discussed (Sec. II), which is plausibly of hadronic size. This is therefore a scale-breaking effect. On the other hand, we expect a form like Eq. (1.1) in small momentum-transfer experiments,[7,8] or when $-q^2/s_{ep}\to 0$.

The factor $\ln^2(-q^2/\mu^2)$ enhances $r$ and thereby gives us an extra chance to test our ideas on quantum electrodynamics and deep-inelastic scattering. According to our reasoning, the over-all cross sections behave as

$$\sigma^\pm\sim\frac{\alpha^2}{q^4}[1\pm O(\alpha\ln^2(-q^2/\mu^2))]. \tag{1.3}$$

At presently available energies the logarithmic factor is ~5, which does not allow us to differen-