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Absorptive Pomeranchukon and Quantum-Number Exchange*

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Quantum-number-exchange scattering is studied in the presence of a black-disk Pomeranchukon whose radius grows like the log of the energy. We find that there is a range of momentum transfers for which the Regge pole carrying the quantum numbers is completely absorbed and leaves the physical sheet of the angular momentum plane. The high-energy behavior of the scattering amplitude is controlled not by the Regge pole, but by a branch cut in the angular momentum plane.

I. INTRODUCTION

If total cross sections rise with energy, as suggested by CERN Intersecting Storage Rings (ISR) data,[1,2] the Regge description of processes involving quantum-number exchange may have some very peculiar features. These features arise if the Pomeranchukon is absorptive and saturates the Froissart bound at very high energies. For such a Pomeranchukon the s-channel partial-wave amplitude is

$$a_P(s,b) = \tfrac{1}{2}\theta(r_0 y - b)\,, \tag{1.1}$$

where the s channel is the high-energy channel, $y = \ln(s/s_0)$, and the partial-wave series has been replaced by the familiar integral over the impact parameter b. The corresponding invariant amplitude and t-channel partial-wave amplitude are

$$M(s,t) = 4s\int_0^\infty db\, bJ_0(b\sqrt{-t})\,a_P(s,b) = \frac{2ir_0 ys}{\sqrt{-t}}J_1(r_0 y\sqrt{-t})\,, \tag{1.2}$$

$$f(t,l) = \frac{2s_0 r_0^{\,2}}{[(l-1)^2 - r_0^{\,2}t]^{3/2}}\,. \tag{1.3}$$

This absorptive Pomeranchukon is consistent with the meagre evidence currently available. On the experimental side, the work of Yodh *et al.*[3] suggests that total cross sections rise like y^2 through cosmic-ray energies, and on the theoretical side, an absorptive Pomeranchukon is suggested by the Regge-eikonal model, and by work on the asymptotic behavior of cross sections in electrodynamics.[4]

An aspect of the Regge model for quantum-number exchange is that an appropriate Regge pole is exchanged, accompanied by Regge cuts involving the exchange of the pole and the Pomeranchuk singularity. The cuts are important because they lie to the right of the pole for $t < 0$; it is through the cuts that the character of the Pomeranchukon influences quantum-number exchange. The simplest recipe for calculating these cuts is the absorption model,[5] and we shall follow this recipe here. The absorption model has the virtues that it presents a physically attractive picture in impact-parameter space, and that the cuts it generates have the trajectories and threshold behavior stipulated by Reggeon unitarity.[6] It has also been criticized, and is, undoubtedly, an approximation.[7]

However, the conclusions we present are based on a simple physical argument in impact-parameter space, so it is possible their validity is more general than that of the absorption model.

The absorption model states that the s-channel partial-wave amplitude for quantum-number exchange is the product of a Regge pole amplitude and the Pomeranchukon S matrix, $S_P(s,b)=1+2ia_P(s,b)$. The S matrix for the Pomeranchukon enters here because we must include the possibility of Regge-pole exchange without any diffractive interaction between the incoming or outgoing particles. We take the Regge pole to lie on the linear trajectory $\alpha(t)=\alpha+\alpha' t$, with constant residue γ. We then find that the full amplitude is

$$a(s,b)=a_R(s,b)S_P(s,b)=\frac{\gamma(s/s_0)^{\alpha-1}}{8\alpha' y}e^{-b^2/4\alpha' y}\theta(b-r_0 y)\,, \tag{1.4}$$

where

$$a_R(s,b)=\frac{\gamma(s/s_0)^{\alpha-1}}{8\alpha' y}e^{-b^2/4\alpha' y}$$

is the amplitude for the exchange of a single Regge pole, and $\theta(b-r_0 y)=S_P(s,b)$.

The absorptive character of the Pomeranchukon is evident in that the scattering amplitude $a(s,b)$ is zero inside the black disk of the Pomeranchukon. The effects we explore in this paper arise from the fact that the radius of this disk expands linearly with y, that is, from the saturation of the Froissart bound. Since the Regge-pole Gaussian has a width that increases only as $y^{1/2}$, the Regge pole is absorbed away at high energy, and no longer contributes to the high-energy behavior of the quantum-number-exchange amplitude. We shall see in Sec. II that, in Regge language, this happens by having the Regge pole move to an unphysical sheet of the angular momentum plane for $t<(r_0/2\alpha')^2$. It is plausible that similar phenomena occur whenever the Pomeranchukon is completely absorptive and has a radius that grows faster than $y^{1/2}$.

When r_0 is sufficiently large we will have $m^2<(r_0/2\alpha')^2$, where m is the mass of some physical particle lying on the Regge trajectory. This raises the question whether particles disappear when their Regge pole is on an unphysical sheet. We shall see in Sec. II that such particles do not disappear provided two technical deficiencies of Eq. (1.4) are corrected. The t dependence of the Regge pole signature factor must be included, since this factor contains the particle poles. Likewise, the signature factor for the Pomeranchukon must be included; this amounts to giving $a_P(s,t)$ the real part required by s-channel dispersion relations.

Signature factors can be included in our calculation by expressing the s-channel partial-wave amplitude in terms of the t-channel partial-wave amplitude. For even signature we have

$$a(s,b)=\frac{1}{8s}\int_{-\infty}^{0}dt\,J_0(b\sqrt{-t})\int_{c-i\infty}^{c+i\infty}\frac{dl}{2\pi i}f(t,l)\left(\frac{s}{s_0}\right)^{l}\times(i-\cot\tfrac{1}{2}\pi l)\,. \tag{1.5}$$

Define the double partial-wave amplitude $a(l,b)$:

$$a(l,b)=\frac{1}{8s_0}\int_{-\infty}^{0}dt\,J_0(b\sqrt{-t})f(t,l)\,. \tag{1.6}$$

According to the absorption model, the partial-wave amplitude corresponding to the exchange of two t-channel singularities is $a_{12}(s,b)=2ia_1(s,b)\times a_2(s,b)$. Using the double partial-wave amplitude defined in Eq. (1.6), this leads to the following equations:

$$a_{12}(s,b)=2i\iint_{c-i\infty}^{c+i\infty}\frac{dl_1dl_2}{(2\pi i)^2}\left(\frac{s}{s_0}\right)^{l_1+l_2-2}(i-\cot\tfrac{1}{2}\pi l_1)(i-\cot\tfrac{1}{2}\pi l_2)a_1(l_1,b)a_2(l_2,b)\,, \tag{1.7}$$

$$M_{12}(s,t)=8is_0\iint_{c-i\infty}^{c+i\infty}\frac{dl_1dl_2}{(2\pi i)^2}\left(\frac{s}{s_0}\right)^{l_1+l_2-1}(i-\cot\tfrac{1}{2}\pi l_1)(i-\cot\tfrac{1}{2}\pi l_2)\int_0^\infty db\,bJ_0(b\sqrt{-t})a_1(l_1,b)a_2(l_2,b)\,, \tag{1.8}$$

$$f_{12}(t,l)=8s_0\iint_{c-i\infty}^{c+i\infty}\frac{dl_1dl_2}{(2\pi i)^2(l+1-l_1-l_2)}\frac{\cos\frac{1}{2}\pi(l_1+l_2)}{\sin\frac{1}{2}\pi l_1\sin\frac{1}{2}\pi l_2}\int_0^\infty db\,bJ_0(b\sqrt{-t})a_1(l_1,b)a_2(l_2,b)\,. \tag{1.9}$$

Equation (1.9) exhibits a Regge cut in the l plane due to a pinch involving the denominator $(l+1-l_1-l_2)^{-1}$. In calculating its discontinuity we set $l_1+l_2=l+1$ elsewhere in the integral, and this produces a factor $\cos\frac{1}{2}\pi(l+1)$ which vanishes at right signature (even) integers. This factor is a consequence of our proper treatment of signature, and is another reason Eq. (1.4) must be corrected. In the multiperipheral model, where signature is not properly handled, moving cuts in the t plane persist at right-signature integers, and the sign of the two-Pomeranchukon cut is reversed. Equation (1.9) forms the starting point of our calculation of the quantum-number-exchange amplitude in Sec. II.

II. QUANTUM-NUMBER EXCHANGE WITH ABSORPTION

In this section we discuss in detail the effect of an absorptive Pomeranchukon on Regge-pole exchange. Our starting point is the double partial-wave amplitude defined in Eq. (1.6). From Eqs. (1.3) and (1.4) we see that

$$a_P(l,b)=\frac{1}{2}\frac{e^{-(l-1)b/r_0}}{l-1} \tag{2.1}$$

and

$$a_R(l,b)=\frac{\gamma}{4\alpha'}K_0\left(b\left(\frac{l-\alpha}{\alpha'}\right)^{1/2}\right), \tag{2.2}$$

where K_0 is the modified Bessel function of order zero.

For simplicity let us begin by ignoring the signature factor of the Regge pole and approximating that of the Pomeranchukon by i. One then recovers Eqs. (1.1) and (1.4). In this approximation the t-channel partial-wave amplitude becomes

$$\bar{f}(t,l)=f_R(t,l)+\bar{f}_{RP}(t,l), \tag{2.3}$$

with

$$f_R(t,l)=\frac{\gamma s_0}{l-\alpha(t)} \tag{2.4}$$

and from Eq. (1.9),

$$\bar{f}_{RP}(t,l)=-\frac{\gamma s_0}{\alpha'}\int\int_{c-i\infty}^{c+i\infty}\frac{dl_1dl_2}{(2\pi i)^2}\frac{1}{l+1-l_1-l_2}\int_0^\infty bdb\,J_0(b\sqrt{-t})K_0\left(b\left(\frac{l_2-\alpha}{\alpha'}\right)^{1/2}\right)\frac{e^{-(l_1-1)b/r_0}}{l_1-1}. \tag{2.5}$$

The bars on $\bar{f}(t,l)$ and $\bar{f}_{RP}(t,l)$ are to remind us of the approximations we have made with the signature factors.

The contour of the l_1 integration in Eq. (2.5) runs to the left of the pole at $l_1=l+1-l_2$ and to the right of the pole at $l_1=1$, so closing the l_1 contour in the right half plane gives

$$\bar{f}_{RP}(t,l)=-\frac{\gamma s_0}{\alpha'}\int_{c-i\infty}^{c+i\infty}\frac{dl_2}{2\pi i}\int_0^\infty bdb\,J_0(b\sqrt{-t})\,K_0\left(b\left(\frac{l_2-\alpha}{\alpha'}\right)^{1/2}\right)\frac{e^{-(l-l_2)b/r_0}}{l-l_2}. \tag{2.6}$$

The contour of the l_2 integration runs to the left of the pole at $l_2=l$ and to the right of the branch point at $l_2=\alpha$. We now wrap the l_2 contour around the branch cut which runs along the real axis from $-\infty$ to α. Using the fact that $K_0(iz)-K_0(-iz)=-i\pi J_0(z)$ and writing $x=-(l_2-\alpha)$ gives

$$\bar{f}_{RP}(t,l)=-\frac{\gamma s_0}{2\alpha'}\int_0^\infty bdb\,J_0(b\sqrt{-t})\int_0^\infty dx\frac{e^{-(l-\alpha+x)b/r_0}}{l-\alpha+x}J_0\left(b\left(\frac{x}{\alpha'}\right)^{1/2}\right). \tag{2.7}$$

The remaining integrals can now be evaluated:

$$\begin{aligned}\bar{f}_{RP}(t,l)&=-\frac{\gamma s_0}{2\alpha'}\int_{1/r_0}^\infty dv\int_0^\infty b^2db\,J_0(b\sqrt{-t})\int_0^\infty dx\,e^{-(l-\alpha+x)bv}J_0\left(b\left(\frac{x}{\alpha'}\right)^{1/2}\right)\\&=-\frac{\gamma s_0}{2\alpha'}\int_{1/r_0}^\infty dv\int_0^\infty bdb\,J_0(b\sqrt{-t})e^{-(l-\alpha)bv}e^{-b/4\alpha'v}\\&=-\frac{\gamma s_0}{2\alpha'}\int_{1/r_0}^\infty\frac{dv}{v}\frac{[(l-\alpha)v+1/4\alpha'v]}{\{[(l-\alpha)v+1/4\alpha'v]^2-t\}^{3/2}}\\&=-\frac{\frac{1}{2}\gamma s_0}{l-\alpha(t)}\left\{1-\left(l-\alpha-\frac{r_0^2}{4\alpha'}\right)\left[\left(l-\alpha+\frac{r_0^2}{4\alpha'}\right)^2-r_0^2t\right]^{-1/2}\right\}.\end{aligned} \tag{2.8}$$

So, the full partial-wave amplitude is

$$\bar{f}(l,t)=\frac{\frac{1}{2}\gamma s_0}{l-\alpha(t)}\left\{1+\left(l-\alpha-\frac{r_0^2}{4\alpha'}\right)\left[\left(l-\alpha+\frac{r_0^2}{4\alpha'}\right)^2-r_0^2t\right]^{-1/2}\right\}. \tag{2.9}$$

The square root in Eq. (2.9) is positive when l is large and positive. Clearly $\bar{f}(l,t)$ has square-root branch points at $l=\alpha_c^\pm=\alpha-r_0^2/4\alpha'\pm r_0\sqrt{t}$, and a pole at $l=\alpha(t)$. The position of the branch cut is shown in Fig. 1. For $|t|>(r_0/2\alpha')^2$ the residue of the pole is γs_0 as expected, but for $|t|<(r_0/2\alpha')^2$ the residue vanishes so there is no pole. What happens is that the pole collides with the branch point at $l=\alpha_c^+$ when $t=(r_0/2\alpha')^2$. If t is decreased below this value, the pole moves through the cut onto the unphysical sheet. As t is decreased below $-(r_0/2\alpha')^2$ the pole reenters the physical sheet through the cut at the point $l=\alpha-r_0^2/4\alpha'$.

In this model for physical values of t the high-energy behavior of the charge-exchange amplitude $M(s,t)$ is always controlled by the branch cut rather than by the pole. This fact can be read off directly from Eqs. (1.2) and (1.4) since the maximum value of $a(s,b)$ for fixed s is[8]

$$a(s, r_0 y) = \frac{\gamma}{8\alpha' y}\left(\frac{s}{s_0}\right)^{\alpha - r_0^2/4\alpha' - 1}.$$

Notice that at $t=0$ the two branch points coalesce into a simple pole at $l=\alpha - r_0^2/4\alpha'$ with residue γs_0.

It is important to understand what happens to physical particles on the Regge trajectory when it leaves the physical sheet. In order to study this problem it is necessary to take into account the signature factor of the Regge trajectory since the particle pole is contained in it. We again write the t-channel partial-wave amplitude in the form

$$f(t, l) = f_R(t, l) + f_{RP}(t, l), \tag{2.10}$$

with $f_R(t, l)$ and $f_{RP}(t, l)$ given by Eqs. (2.4) and (1.9). For our present purposes we only need the Pomeranchuk partial-wave amplitude in the vicinity of $l_1 = 1$, so we can replace the factor of $\sin\frac{1}{2}\pi l_1$ in Eq. (1.9) by its value at $l_1 = 1$. The l_1 integration can then be done as before, and we have

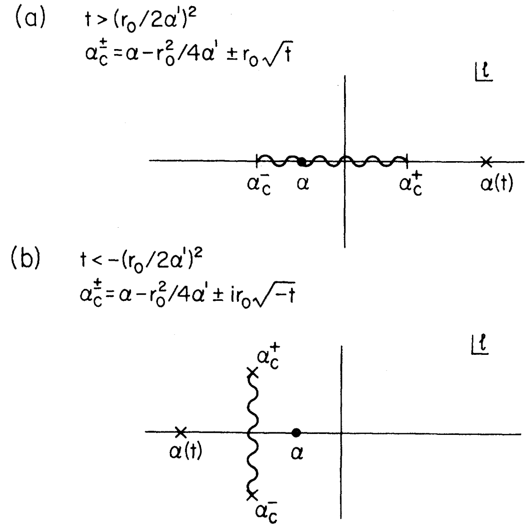

FIG. 1. The l-plane singularities of the partial-wave amplitude $\bar{f}(t, l)$ for (a) $t > (r_0/2\alpha')^2$ and (b) $t < -(r_0/2\alpha')^2$.

$$f_{RP}(t, l) = \frac{\gamma s_0}{\alpha'}\cos\tfrac{1}{2}\pi(l+1)\int_{c-i\infty}^{c+i\infty}\frac{dl_2}{2\pi i}\frac{1}{\sin\frac{1}{2}\pi l_2}\int_0^\infty b\,db\,J_0(b\sqrt{-t})\,K_0\left(b\left(\frac{l_2-\alpha}{\alpha'}\right)^{1/2}\right)\frac{e^{-(l-l_2)b/r_0}}{l-l_2}. \tag{2.11}$$

Using the identity

$$\frac{1}{l-l_2}\frac{1}{\sin\frac{1}{2}\pi l_2} = \frac{1}{l-l_2}\frac{1}{\sin\frac{1}{2}\pi l} + \frac{2}{\pi}\sum_{n=-\infty}^{\infty}\frac{(-)^n}{(l-2n)(l_2-2n)}, \tag{2.12}$$

we can rewrite Eq. (2.11) in the form

$$f_{RP}(t, l) = \bar{f}_{RP}(t, l) - \frac{2}{\pi}\sin\tfrac{1}{2}\pi l\sum_{n=-\infty}^{\infty}\frac{(-)^n}{l-2n}I_n(t, l), \tag{2.13}$$

where $\bar{f}_{RP}(t, l)$ is again given by Eq. (2.8) and

$$I_n(t, l) = \frac{\gamma s_0}{\alpha'}\int_{c-i\infty}^{c+i\infty}\frac{dl_2}{2\pi i}\frac{1}{l_2-2n}\times\int_0^\infty b\,db\,J_0(b\sqrt{-t})\,K_0\left(b\left(\frac{l_2-\alpha}{\alpha'}\right)^{1/2}\right)\times e^{-(l-l_2)b/r_0}. \tag{2.14}$$

Clearly the behavior of the Regge trajectory is unchanged by the inclusion of the signature factor. It only remains to study the analyticity properties of $I_n(t, l)$. For $n \geq 0$ the l_2 contour runs to the left of the poles at $l_2 = 2n$ and to the right of the branch point at $l_2 = \alpha$. For simplicity we take $\alpha < 0$. The l_2 contour can again be wrapped around the cut, and we obtain

$$I_n(t, l) = -\frac{\gamma s_0}{2\alpha'}\int_0^\infty b\,db\,J_0(b\sqrt{-t})\int_0^\infty dx\,\frac{e^{-(l-\alpha+x)b/r_0}}{x-\alpha+2n}\times J_0\left(b\left(\frac{x}{\alpha'}\right)^{1/2}\right) \tag{2.15}$$

for $n \geq 0$. It is convenient to take the Mellin transform of Eq. (2.15):

$$\begin{aligned}I_n(y, t) &= \int_{c-i\infty}^{c+i\infty}\frac{dl}{2\pi i}\left(\frac{s}{s_0}\right)^l I_n(t, l)\\
&= -\frac{\gamma s_0}{2\alpha'}r_0^2 y J_0(r_0 y\sqrt{-t})\times\int_0^\infty dx\,\frac{e^{-(x-\alpha)y}}{x-\alpha+2n}J_0\left(r_0 y\left(\frac{x}{\alpha'}\right)^{1/2}\right)\\
&= -\frac{\gamma s_0}{2\alpha'}r_0^2 y J_0(r_0 y\sqrt{-t})\,e^{2ny}\times\int_y^\infty dv\int_0^\infty dx\,e^{-(x-\alpha+2n)v}\times J_0\left(r_0 y\left(\frac{x}{\alpha'}\right)^{1/2}\right)\\
&= -\frac{\gamma s_0}{2\alpha'}r_0^2 y J_0(r_0 y\sqrt{-t})\,e^{2ny}\times\int_y^\infty\frac{dv}{v}\exp\left\{-\left[(2n-\alpha)v+\frac{r_0^2 y^2}{4\alpha' v}\right]\right\}.\end{aligned} \tag{2.16}$$

Notice that the quantity $[(2n-\alpha)v + r_0^2 y^2/4\alpha' v]$ has a minimum at $v_n = r_0 y/2\alpha' m_n$, where m_n is the mass of the nth particle on the Regge trajectory, $m_n^2 = (2n-\alpha)/\alpha'$. So, making use of the fact that

$$\int_0^\infty\frac{dv}{v}\exp\left\{-\left[(2n-\alpha)v+\frac{r_0^2 y^2}{4\alpha' v}\right]\right\} = 2K_0(r_0 y m_n), \tag{2.17}$$

we see that for large values of y

$$I_n(y,t) \simeq -\frac{\gamma s_0}{\alpha'} r_0^2 y J_0(r_0 y\sqrt{-t}) e^{2ny} K_0(r_0 y m_n) \times \theta\left(\frac{r_0}{2\alpha'} - m_n\right) - \frac{\gamma s_0}{2\alpha'} r_0^2 J_0(r_0 y\sqrt{-t}) e^{-(r_0^2/4\alpha' - \alpha) y} \times \left(\alpha' m_n^2 - \frac{r_0^2}{4\alpha'}\right)^{-1}. \qquad (2.18)$$

If $r_0/2\alpha' > m_n$, then the first term in Eq. (2.18) dominates. For $t \neq 0$ one finds to leading order in y that

$$I_n(y,t) \simeq -\frac{\gamma s_0}{2\alpha'(m_n\sqrt{-t})^{1/2}} e^{2ny} \times \left[e^{r_0 y(i\sqrt{-t} - m_n)} e^{-i\pi/4} + e^{r_0 y(-i\sqrt{-t} - m_n)} e^{i\pi/4}\right], \qquad (2.19)$$

which corresponds to a pair of poles in the t-channel partial-wave amplitude at $l = \alpha_n^{\pm}(t) = r_0(\pm\sqrt{t} - m_n) + 2n$. From Eq. (2.13) we see that the only physical particle on these trajectories is the one at $t = m_n^2$. The residue of the pole in $f(t, l)$ at $l = \alpha_n^+$ is

$$R_n = \frac{\gamma s_0 r_0}{2\alpha'(m_n\sqrt{t})^{1/2}} \frac{\sin\frac{1}{2}\pi(r_0\sqrt{t} - r_0 m_n)}{\frac{1}{2}\pi(r_0\sqrt{t} - r_0 m_n)}, \qquad (2.20)$$

so the physical partial-wave amplitude $f(t, 2n)$ has the required particle pole at $\sqrt{t} = m_n$, with the correct residue $-\gamma s_0/2\alpha' m_n$. Notice, however, that the nonleading powers of $1/y$ in the asymptotic expansion of $J_0(r_0 y\sqrt{-t})$ and $K_0(r_0 y m_n)$ give rise to branch points which are also located at $\alpha_n^{\pm}(t)$. Thus, except when l is an even integer, these new Regge poles are not isolated, but instead form the singular tips of branch cuts.

The second term in Eq. (2.18) corresponds to a pair of square-root branch points in the l-plane located at $l = \alpha_c^{\pm} = \alpha - r_0^2/4\alpha' \pm r_0\sqrt{t}$. As we have seen, these branch points are also present in $\bar{f}_{RP}(t, l)$.

Finally, for negative values of n we return to Eq. (2.14). The Regge residue function must have zeros at even negative integers, so there really is no pole at $l_2 = 2n$. We therefore write

$$I_n(t,l) \sim \frac{\gamma' s_0}{\alpha'} \int_{c-i\infty}^{c+i\infty} \frac{dl_2}{2\pi i} \int_0^\infty b\,db\,J_0(b\sqrt{-t}) \times K_0\left(b\left(\frac{l_2 - \alpha}{\alpha'}\right)^{1/2}\right) \times e^{-(l - l_2) b/r_0} = -\frac{\gamma s_0 r_0^2}{2\alpha'}\left[\left(l - \alpha + \frac{r_0^2}{4\alpha'}\right)^2 - r_0^2 t\right]^{-1/2}, \qquad (2.21)$$

so we again have the familiar branch points at $l = \alpha_c^{\pm}$.

III. QUANTUM-NUMBER EXCHANGE AT ISR ENERGIES: IS THE REGGE POLE VISIBLE?

We now estimate the effect of an absorptive Pomeranchukon on quantum-number-exchange amplitudes at NAL-ISR energies. Our motivation is to see how much an amplitude differs from simple Regge-pole exchange. However, even to make a qualitative statement we must modify the Pomeranchukon S matrix to fit the NAL-ISR data on the energy dependence of the proton-proton total cross section. Since cross sections are unequal at these energies, and only starting to rise, both the opacity and radius of the Pomeranchukon deviate from Eq. (1.1).

Perhaps it is correct to approximate the Pomeranchukon by a factorizing Gaussian, as is customary at lower energies. However, we are interested primarily in studying the extreme absorptive limit to see if the original Regge pole is still visible, and not in making a realistic prediction of what will be observed. We shall make the drastic assumption that the Pomeranchukon is totally absorptive, but change its radius:

$$a_P(s,b) = \tfrac{1}{2} i\,\theta(r_0 y + r_1 - b) + \frac{1}{2\pi}\ln\coth\left(\frac{1}{2r_0}|r_0 y + r_1 - b|\right). \qquad (3.1)$$

The real part can be computed by s-channel dispersion relations, or from the signature factor; it damps out rapidly away from the edge of the Pomeranchukon disk. We choose the parameters so that a_P fits the proton-proton total cross section at $s = 550$ GeV^2 and 2790 GeV^2, where $\sigma = 39.1$ and 43.2 mb, respectively.[1,2] We choose $s_0 = 1$ GeV^2, and find $r_0 = 0.125$ GeV^{-1}, and $r_1 = 3.16$ GeV^{-1}.

In treating the Regge-pole amplitude, we assume $\alpha(0)$ is not near a right-signature integer, and $\alpha' = 1$ GeV^{-2}. With this choice the t dependence of the signature factor is much less important than the t dependence of $(s/s_0)^{\alpha(t)}$, and the Regge-pole amplitude can be approximated by a Gaussian, as in Eq. (1.4). According to the absorption model, the quantum-number-exchange amplitude is now

$$M(s,t) = \gamma s^{\alpha + \alpha' t} - \frac{\gamma s^\alpha}{2\alpha' y}\int_0^R db\, bJ_0(b\sqrt{-t}) e^{-b^2/4\alpha' y} + \frac{i\gamma s^\alpha}{2\pi\alpha' y}\int_0^\infty db\, bJ_0(b\sqrt{-t}) e^{-b^2/4\alpha' y} \times \ln\coth\left(\frac{1}{2r_0}|r - R|\right), \qquad (3.2)$$

where $R = r_0 y + r_1$.

The angular distributions predicted by Eq. (3.2) at $s = 550$ GeV^2 and 2790 GeV^2 are shown in Figs.

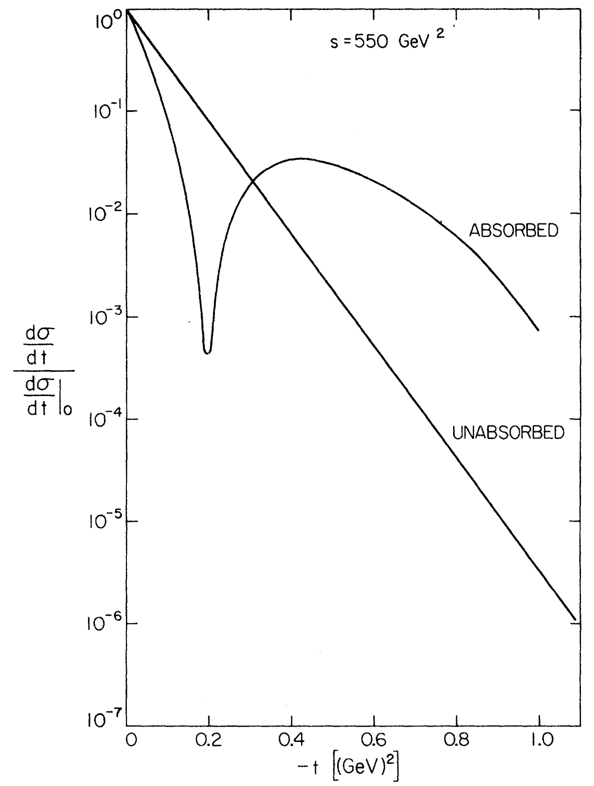

FIG. 2. The angular distribution predicted by Eq. (3.2) at $s=550\ \text{GeV}^2$, with $s_0=1\ \text{GeV}^2$, $\alpha'=1$, $r_0=0.125\ \text{GeV}^{-1}$, and $r_1=3.16\ \text{GeV}^{-1}$.

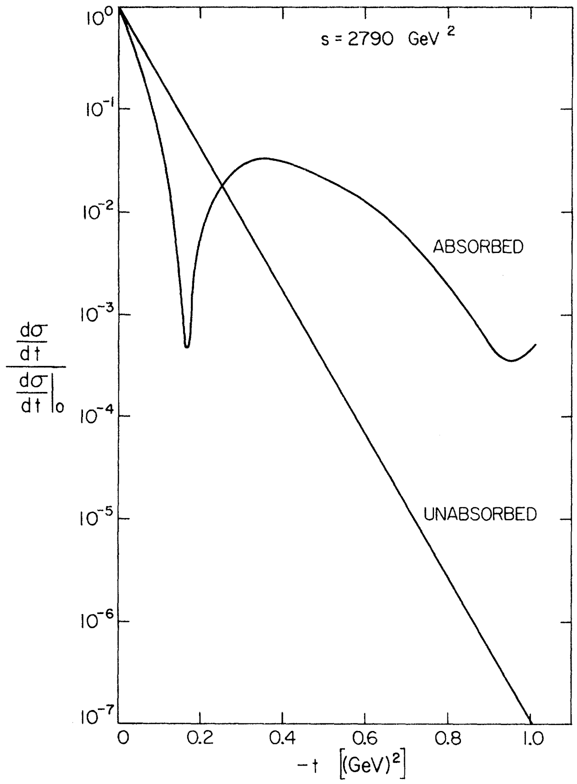

FIG. 3. The angular distribution predicted by Eq. (3.2) at $s=2790\ \text{GeV}^2$, with $s_0=1\ \text{GeV}^2$, $\alpha'=1$, $r_0=0.125\ \text{GeV}^{-1}$, and $r_1=3.16\ \text{GeV}^{-1}$.

2 and 3. The dips near $t=-0.2\ \text{GeV}^2$ are due to a destructive interference between the first two terms of Eq. (3.2). They are deep because the imaginary part of M is small. For large values of $-t$ the real integral in Eq. (3.2) dominates. This integral vanishes at $t=-1.07\ \text{GeV}^2$ and $t=-0.95\ \text{GeV}^2$ at $s=550\ \text{GeV}^2$ and $s=2790\ \text{GeV}^2$, respectively, leading to the second minimum visible in Fig. 2. It can be seen that while the Pomeranchukon has a strong effect at large $-t$, a forward peak associated with the Regge pole persists at these energies. Of course, the results of Sec. II show that the real integral in Eq. (3.2) exactly cancels the Regge pole at infinitely high energy.

It should be noted that the secondary peaks are much more prominent in Figs. 2 and 3 than they are in the elastic angular distributions presented in Ref. 9. This is due largely to the absorptive Pomeranchukon we have used, which maximizes the corrections to the Regge pole. The proper way to obtain the corrections is to use a Pomeranchukon that fits the elastic angular distributions. Since the forward peak associated with Regge-pole exchange is evident in the extreme case studied here, we expect it to be a feature of the data.

IV. DISCUSSION

We have seen that a black-disk Pomeranchukon will completely absorb an ordinary Regge pole. A pole with a linear trajectory will be forced off the physical sheet in the momentum transfer range $|t|<(r_0/2\alpha')^2$. The reason for this dramatic effect is easy to see in impact-parameter space. The Regge pole falls off significantly for impact parameters greater than $2\alpha' y^{1/2}$, whereas we have assumed that the Pomeranchukon is perfectly absorbing out to a radius of $r_0 y$. Although we have used the absorption model in our detailed calculations, we expect this basic effect to be quite general. It appears to depend only on the fact that the Pomeranchukon is black out to a radius which grows faster than $y^{1/2}$.

If there are no particles on the Regge trajectory with masses less than $r_0/2\alpha'$, then for negative t the rightmost singularities in the l plane are the branch points at $\alpha_c^\pm=\alpha-r_0^2/4\alpha'\pm i r_0\sqrt{-t}$, and they control the large s behavior of the charge-exchange amplitude $M(s,t)$. On the other hand, if the first N particles on the trajectory have masses less than $r_0/2\alpha'$, the asymptotic behavior of $M(s,t)$ is determined by the branch points at $\alpha_N^\pm=r_0(\pm i\sqrt{-t}$

$-m_N)+2N$. In this case the particles that have been knocked off the original Regge trajectory form the singular tips of branch cuts for noninteger values of l; however, the physical partial-wave amplitudes $f(t, 2n)$ still have the particle poles in t. If one wishes to require that all particles in nature lie on isolated Regge trajectories, then one has an upper bound on the radius of the black disk: $r_0 < 2\alpha'_\pi m_\pi \simeq .06$ fermi for $\alpha'_\pi \simeq 1$ GeV^{-2}. This bound is indeed satisfied by the fit to the ISR data in Sec. III.

The calculations that we have presented here can be extended to baryon trajectories. We write the trajectories of the positive- and negative-parity MacDowell partners in the form

$$\alpha^\pm(\sqrt{u}) = \alpha \pm \epsilon\sqrt{u} + \alpha' u\,, \qquad (4.1)$$

and for definiteness take $\epsilon > 0$. We then find branch points similar to $\alpha_c^\pm$ at $l = \alpha - (r_0 \pm \epsilon)^2/4\alpha' \pm r_0\sqrt{u}$. The negative-parity pole is on the physical sheet of the l plane for $\sqrt{u} > (r_0+\epsilon)/2\alpha'$, but not for $\sqrt{u} < (r_0+\epsilon)/2\alpha'$ or $u<0$. For $r_0 > \epsilon$ the positive-parity pole is present only for $\sqrt{u} > (r_0 - \epsilon)/2\alpha'$, while for $r_0 < \epsilon$ it is always present. As in the case of meson exchange, the l-plane structure of the amplitude is quite complicated. It seems that one is more likely to gain physical insight by studying the amplitude as a function of energy and impact parameter. For example, the new features of baryon exchange can all be traced to the fact that the $\epsilon\sqrt{u}$ term in the trajectory function makes the Regge-pole amplitude appreciable out to impact parameters of order ϵy.

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