trajectory of each signature exists.
[9]Stanley Mandelstam, Phys. Rev. D 1, 1745 (1970).
[10]J. L. Rosner, Phys. Rev. D 7, 172 (1973).
[11]The harmonic-oscillator quark-model spectrum is given by G. Karl and E. Obryk [Nucl. Phys. B8, 609 (1968)]; a simpler description of the leading trajectories is given by P. G. O. Freund and Ronald Waltz [Phys. Rev. 188, 2270 (1969)].
[12]The experimental data used here are taken from the compilation of the Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).
[13]Convenient tables of SU(6) Clebsch-Gordan coefficients are given by C. L. Cook and G. Murtaza [Nuovo Cimento 39, 531 (1965)].
[14]The signs in Eq. (A5) are opposite for quark-spin up and down, because the π is a member of a W-spin triplet.

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Charged- and Neutral-Particle Correlations at the Critical Point*

Gerald H. Thomas
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 26 January 1973)

Motivated by the gas-liquid analog, a critical-point theory for charged multiplicities is generalized to describe the production of more than one type of final-state particle. The distribution in the total number is independent of the number of types created. As a specific application, a theory for charged and neutral particles results. It is found that the theory accounts for the observed linear rise of the mean number of neutrals $E\{n_0|n_{ch}\}$ versus the number of charged particles n_{ch}. An extension to neutral-K production is given and it is found that $E\{n_K^0|n_{ch}\}/E\{n_0|n_{ch}\}$ is independent of n_{ch}.

I. INTRODUCTION

Recently a phenomenological theory was proposed for the number distribution of charged particles in very-high-energy collisions.[1,2] Contained in this theory is the new idea that charged particles behave in a manner analogous to a 1-dimensional fluid at the critical point. For such a fluid the relative fluctuation of the number density

$$\frac{D}{\langle n\rangle} \equiv \frac{\langle (n-\langle n\rangle)^2\rangle^{1/2}}{\langle n\rangle}$$

decreases more slowly than the usual $\langle n\rangle^{-1/2}$, in agreement with recent experimental evidence.[3] Encouraged by the qualitative success of this idea applied to charged-particle distributions, we suggest in this paper that the idea can be applied more generally to all of the produced particles. At the critical point not only will the number distribution for each particle type be characterized by large number density fluctuations, but also there will be strong correlations between different types of particles. As an illustration of this, we work out a no-parameter theory for the correlations between the average number of neutral particles produced as a function of the number of produced charged particles. This theory accounts in a novel way for the observed qualitative linear correlation.[4]

In Sec. II a specific theory is proposed for the asymptotic joint number distribution of k types of produced particles, $k=1,2,\ldots$. For $k=1$, the theory of Ref. 1 is obtained; for $k=2$, a theory for charged- and neutral-pion production is obtained; with the proper choice of k and appropriate subsidiary assumptions, a theory for π, K, η, etc. production can also be obtained. In Sec. III the theory of charged- and neutral-pion production is detailed and compared with the trend of recent data. Additional experimental checks are suggested. In Sec. IV other possible applications are discussed.

II. CRITICAL-POINT THEORY FOR MIXTURES

The theory for producing n particles in the final state in which there are n_i particles of type i is to be constructed as in Ref. 1, by writing a simple ansatz (motivated by the gas-liquid analog) for the partial cross sections which allows for a critical point. The parameters of the theory are subsequently fixed according to certain reasonable physical assumptions.

The ansatz which we start with is, with $n=n_1+\cdots+n_k$,

$$\sigma_{n_1 n_2 \cdots n_k} \equiv \sigma_0(Y) Q_{n_1\cdots n_k}(Y) \tag{2.1}$$

$$= \sigma_0(Y)\frac{g_1^{n_1}}{n_1!}\cdots\frac{g_k^{n_k}}{n_k!}(Y-nb)^n e^{an^2/Y}, \quad n<Y/b$$
$$=0, \quad n>Y/b \tag{2.2}$$

where, as in Ref. 1, the volume of the 1-dimensional box is approximately

$$Y = \ln s - 1.5\,, \qquad (2.3)$$

and

$$\sigma_0(Y) \cong A s^{2a-2} \text{ (modulo logarithms)}\,. \qquad (2.4)$$

The following meanings are assigned to the various factors in (2.2): The bare cross section σ_0 describes the inelastic process with two particles in the final state; the production mechanism is assumed to be dominated by the Regge exchange α. If this Regge-pole exchange dominates the $n_1 \cdots n_k$ particle amplitude as in the simplest multi-Regge models, then σ_0 is a common factor to each $\sigma_{n_1 \cdots n_k}$. The factors

$$\frac{g_1^{n_k}}{n_1!}, \ldots, \frac{g_k^{n_k}}{n_k!}$$

are also expected in such models; indeed for multiperipheral models with no correlations we obtain (2.2) with $a = b = 0$. The parameters a and b represent the Van der Waals parametrization of two physical effects necessary to give a critical point. First, the parameter b represents a pairwise exclusive repulsion which we assume, in first approximation, to be independent of particle type. Second, the parameter a represents a pairwise long-range attraction which is also assumed to be independent of particle type. The significance of these parameters is discussed below in more detail.

In order to make the formal connection between (2.2) and gas-liquid theory, we interpret $Q_{n_1 \cdots n_k}(Y)$ as the canonical partition function,

$$\Omega(z_1, \ldots, z_k, Y) = \sum_{n_1 \cdots n_k} Q_{n_1 \cdots n_k}(Y) z_1^{n_1} \cdots z_k^{n_k}$$

as the grand partition function as a function of the fugacities $z_1, \ldots, z_k$,

$$p(z_1, \ldots, z_k) = \lim_{Y \to \infty} \frac{\Omega(z_1, \ldots, z_k, Y)}{Y}$$

as the pressure, and

$$\rho_j(z_1, \ldots, z_k) = z_j \frac{\partial}{\partial z_j} p(z_1, \ldots, z_k) = \lim_{Y \to \infty} \frac{\langle n_j \rangle}{Y}$$

as the density for the particle of type j, for $j = 1, \ldots, k$. The equation of state for the analog fluid is that which relates $p(z_1, \ldots, z_k)$ to the $v_i(z_1, \ldots, z_k) \equiv 1/\rho_i(z_1, \ldots, z_k)$, eliminating all explicit dependence on $z_1, \ldots, z_k$.

The equation of state can be obtained either by explicit calculation or immediately by noting that the grand partition function

$$\Omega(z_1, \ldots, z_k, Y) = \sum_{n=0}^{\infty} \frac{(g_1 z_1 + \cdots + g_k z_k)^n}{n!} \times (Y - nb)^n e^{an^2/Y}$$

considered as a function of $z_1 g_1 + \cdots + z_k g_k$ is identical to the distribution in Ref. 1 considered as a function of zg. Defining $\rho = \sum_{i=1}^{k} \rho_i$, $v = 1/\rho$, the resultant equation of state is the Van der Waals (VDW) equation

$$p = \frac{1}{v - b} - \frac{a}{v^2}\,. \qquad (2.5)$$

The connection between $z_1 g_1 + \cdots + z_k g_k$ and ρ is

$$z_1 g_1 + \cdots + z_k g_k = \frac{\rho}{1 - \rho b} \exp\left(\frac{\rho b}{1 - \rho b} - 2a\rho\right). \qquad (2.6)$$

It follows that

$$p(z_1, \ldots, z_k) = p_{\text{VDW}}(z_1 g_1 + \cdots + z_k g_k)$$

and hence

$$\rho_j = \frac{z_j g_j}{z_1 g_1 + \cdots + z_k g_k} \rho \qquad (2.7)$$

or equivalently

$$z_j g_j = \frac{\rho_j}{1 - \rho b} \exp\left(\frac{\rho b}{1 - \rho b} - 2a\rho\right), \qquad (2.8)$$

for $j = 1, \ldots, k$.

Before determining the parameters of the mixture, it is worthwhile to repeat a few general remarks made in Ref. 1. The value of the Van der Waals equation has been that it is a useful way of describing real gases even though such gases are not described by (2.5) in detail. It is the simplest theory of a fluid which allows for exactly one critical point. In (2.5) the parameter b characterizes the "hard core," and the term with a results from a mean-field approximation applied to a long-range attractive potential.[5] Independent of the analogy, both the short- and long-range forces might reasonably be expected in particle-physics theories. The short-range repulsive core can be derived in multiperipheral models.[6] The parameter a can be thought of as representing the effect of positive cuts acting between all pairs of particles (which would appear as long-range forces in rapidity space). Thus, independent of the critical-point hypothesis, the Van der Waals equation may be a useful way to explore theories in the fluid analog generally, without relying on the details of any specific theory. Assuming the critical-point hypothesis, an example of a new class of theories for particle physics is obtained which can be tested experimentally. In this article, we argue that there is evidence that hadrons behave like a critical-point fluid. In the future, it may happen nevertheless that data decide against the critical-point hypothesis, and so the possibility of a noncritical hadron fluid approximated by (2.5) should be kept in mind.

Making the assumptions of Ref. 1 [namely, that (1) the basic exchange in (2.4) is $\alpha=\frac{1}{2}$, (2) the fluid is at the critical point ($\partial p/\partial v=0$, $\partial^2 p/\partial v^2=0$) when $z_1=\cdots=z_k=1$, and (3) the total inelastic cross section is constant up to logarithms], the Van der Waals parameters a and b are determined, as well as the coupling constant $g=g_1+\cdots+g_k$ and the density ρ:

$$\begin{aligned} &\rho_c=\tfrac{8}{3}\,,\\ &a=\tfrac{27}{64}\,,\\ &b=\tfrac{1}{8}\,,\\ &g=4\exp(-\tfrac{7}{4})\cong 0.695\,. \end{aligned} \tag{2.9}$$

From (2.7) or (2.8), it follows that there are $k-1$ free parameters

$$\frac{g_j}{g_1}=\frac{\rho_j(z_1=1,\ldots,z_k=1)}{\rho_1(z_1=1,\ldots,z_k=1)}\,. \tag{2.10}$$

In specific examples these parameters may be fixed by additional theoretical assumptions or by experimental input.

It is apparent that the innovative content of (2.2) in gas-liquid language is that a fluid described by a Van der Waals equation may be a mixture of several different types of particles. For example, it seems a reasonable improvement over Ref. 1 to assume that in addition to charged particles (n_1), neutral particles (n_2) are also components of the analog fluid ($n=n_1+n_2$; $k=2$). One consequence is that whereas the distribution in the total number of particles obeys a Van der Waals equation, the distribution in n_1 (summed over n_2) does not. To sum inclusively over n_2 is equivalent to fixing $z_2=1$ in Eq. (2.8), i.e.,

$$g_2=\frac{\rho_2}{1-(\rho_1+\rho_2)b}\exp\left[\frac{(\rho_1+\rho_2)b}{1-(\rho_1+\rho_2)b}-2a(\rho_1+\rho_2)\right]. \tag{2.11}$$

This implies (correctly) that the pressure depends only upon ρ_1, since (2.11) determines ρ_2 as a function of ρ_1. There is thus a quantitative difference between the resultant equation of state and a Van der Waals equation. A further study shows that if there is a critical point in (2.5) as a function of ρ, there will also be one as a function of ρ_1. The position of the critical point occurs for $\rho_1=\rho_1(z_1=1, z_2=1)$ if the total fluid has a critical point at $z_1=z_2=1$ (true also for $k>2$). Qualitatively, the change in σ_{n_1} is that the mean is at $[g_1/(g_1+g_2)]\rho_c Y$ asymptotically, and that the cutoff at large n_1 is sharper than the simple Van der Waals $(Y-n_1 b)^{n_1}$.

In Sec. III a numerical evaluation of the two theories shows that between them there is no qualitative difference in comparison with present data; quantitatively, the mixture theory gives a better description.

III. CHARGED- AND NEUTRAL-PARTICLE CORRELATIONS

It is important to be able to account for neutral-particle production as well as charged-particle production, not only for completeness but also because in many experiments neutral particles such as π^0's give rise to Dalitz pairs which can contaminate the charged-particle sample. Moreover, it has been recently emphasized that the study of the average number of neutrals $E\{n_0|n_{ch}\}$, subject to there being n_{ch} charged particles, can give nontrivial information about underlying dynamics.[4] Specifically, it has been known for some time that uncorrelated multiperipheral models for charged and neutral pions which have the approximate form

$$\sigma(n_+,n_-,n_0)=\frac{g_+^{\,n_+}}{n_+!}\frac{g_-^{\,n_-}}{n_-!}\frac{g_0^{\,n_0}}{n_0!}Y^n \tag{3.1}$$

lead to $E\{n_0|n_{ch}\}$ being $\langle n_0\rangle$, a constant as a function of n_{ch}.[7] This result is only slightly changed by imposing on (3.1) charge-conservation constraints (e.g., $n_+=n_-$) or, more generally, isospin constraints (e.g., using Cerulus coefficients[8]). On the other hand, recent data, though not yet very accurate, suggest a trend[9]:

$$E\{n_0|n_{ch}\}_{exp}\approx\tfrac{1}{2}n_{ch}. \tag{3.2}$$

In the multiperipheral framework, the common explanation of this has been to require the basic produced cluster in the chain to be ρ- or ω-like: A nonconstant $E\{n_0|n_{ch}\}$ results since $\pi^\pm\pi^0$ or $\pi^+\pi^-\pi^0$ clusters necessarily correlate charged- and neutral-particle production. Limiting-fragmentation models which have been studied all give the behavior (3.2).[4]

We now propose a new explanation for $E\{n_0|n_{ch}\}$ as well as for $\sigma_{n_{ch}n_0}$ which, in spirit, is of the multiperipheral type (3.1). Our theory is Eq. (2.2) for $k=2$, $n_{ch}=n_1$, and $n_0=n_2$. The one undetermined parameter is the relative coupling between neutral and charged particles, which is fixed to be

$$g_0/g_{ch}=\tfrac{1}{2} \tag{3.3}$$

assuming (1) that inclusive single-particle production in the central region is described by a discontinuity of a three-to-three amplitude given by double-Pomeron exchange,[10] and (2) that pion production is dominant. At asymptotic energies, the equalities $\langle n_{\pi^+}\rangle=\langle n_{\pi^-}\rangle=\langle n_{\pi^0}\rangle$ then follow, and hence also $\langle n_{ch}\rangle = 2\langle n_0\rangle$. Since $\langle n_{ch}\rangle/\langle n_0\rangle$ approaches ρ_{ch}/ρ_0 asymptotically, the result (3.3) follows from (2.10).

In Ref. 1 the function

$$L_{exp}(n_{ch}, Y) \equiv \left[\ln\left(\frac{n_{ch}!\,\sigma_{n_{ch}}}{\sigma_{inelastic}}\right) - n_{ch}\ln Y\right]\Big/ Y \quad (3.4)$$

was convenient for comparing the qualitative features of the data with the single-fluid critical-point theory. It was shown that the Van der Waals distribution

$$L_{VDW}(n_{ch}, Y) = -1 + \frac{n_{ch}}{Y}\ln g + a\left(\frac{n_{ch}}{Y}\right)^2 + \frac{n_{ch}}{Y}\ln\left(1 - \frac{n_{ch}}{Y}b\right) \quad (3.5)$$

was in substantial agreement with the data. It should be remarked that general arguments lead to having L depend only on n/Y for large n and Y and fixed n/Y.[11] The motivation for introducing L is that the cross section written as

$$\sigma_n = A\frac{Y^n}{n!}e^{YL} = Ae^{-Y}Q_n$$

[see Eqs. (2.1), (2.4)] displays the simple factors we expect to be present, and lets L represent the complications. In a gas, $\ln Q_n$ grows like Y times a function of n/Y and the result for L follows. Note that the use of the canonical pressure $(\partial/\partial Y)\ln Q_n$ is an alternative and formally equivalent way to calculate the equation of state. The equation

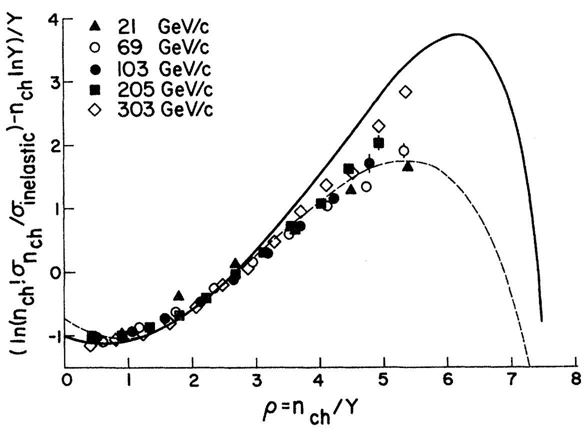

FIG. 1. The quantity $L_{exp}(n_{ch}/Y)$, Eq. (3.4), vs. $\rho_{ch} = n_{ch}/Y$, and the single-fluid prediction (3.5), are taken from Fig. 1 of Ref. 1. The dashed curve is the two-fluid theory of Sec. III, Eq. (3.8). The error bars on the data for $\rho_{ch} > 4$ correspond to the minimum and maximum values of $\sigma_{n_{ch}}$ for a variation of one standard deviation. If a standard deviation of 2 is allowed, many of the data points in this region would have $\sigma_{n_{ch}}$ consistent with zero, i.e., $L_{exp} = -\infty$.

$$p = \frac{\partial}{\partial Y}\ln Q_n = L(\rho) - \rho L'(\rho) + \rho + 1$$

shows how $L(\rho)$ can be used to compute the pressure, and allows a physical interpretation of L.

In comparing the general form of (3.5) with data (Fig. 1), note that certain simple choices of a, b, and g are ruled out. There is positive curvature in the data, and hence empirical evidence for a nonzero value for positive a. By contrast a Poisson distribution $\sigma_n = Ae^{-Y}(g^nY^n/n!)$, having $L(\rho) = \rho\ln g - 1$, is clearly ruled out. [It is further ruled out if σ_{inel} is required to be constant, since then $g=1$ and $L(\rho) = -1$.] A Poisson-like distribution with only a repulsive core does no better. We have attempted a fit of (3.5) to experiment, varying a, b, g, and $L(0)$; values near the critical-point values of Ref. 1 are found and do somewhat better than the critical ones.

We now investigate whether the mixture theory at the critical point

$$\sigma_{n_{ch}n_0} = Ae^{-Y}\frac{g_0^{n_0}}{n_0!}\frac{g_{ch}^{n_{ch}}}{n_{ch}!}(Y - nb)^n e^{an^2/Y}, \quad n < Y/b$$
$$= 0, \quad n > Y/b$$
$$g_0 = \tfrac{1}{3}g, \quad g_{ch} = \tfrac{2}{3}g \quad (3.6)$$

can provide a better description of data than the simple Van der Waals equation at the critical point. We compute first the L function

$$L(n_{ch}, z_0, Y) = \left[\ln\left(\frac{n_{ch}!\sum_{n_0}\sigma_{n_{ch}n_0}z_0^{n_0}}{\sum_{n_0 n_{ch}}\sigma_{n_{ch}n_0}}\right) - n_{ch}\ln Y\right]\Big/ Y, \quad (3.7)$$

which reduces to (3.4) when $z_0 = 1$; derivatives of (3.7) with respect to z_0 at $z_0 = 1$ give moments (scaled by Y) of the neutral-particle distribution for a fixed number of charged particles. At finite volume Y, the function $L(n_{ch}, z_0, Y)$ can be quite complicated analytically. As $Y\to\infty$, this function simplifies. The asymptotic form can be computed by first replacing the sum $\sum_{n_0}$ in (3.7) by an integral, and second using the method of steepest descents. The result is, writing $\rho_{ch} = n_{ch}/Y$ and $\rho = \rho_0 + \rho_{ch}$,

$$L(\rho_{ch}, z_0) = -1 + \frac{\rho_0}{1-\rho b} - a\rho_0^2 + \rho_{ch}\ln g_{ch} + a\rho_{ch}^2 + \rho_{ch}\ln(1-\rho b), \quad (3.8)$$

where for fixed z_0 and ρ_{ch}, ρ_0 is determined by (2.11):

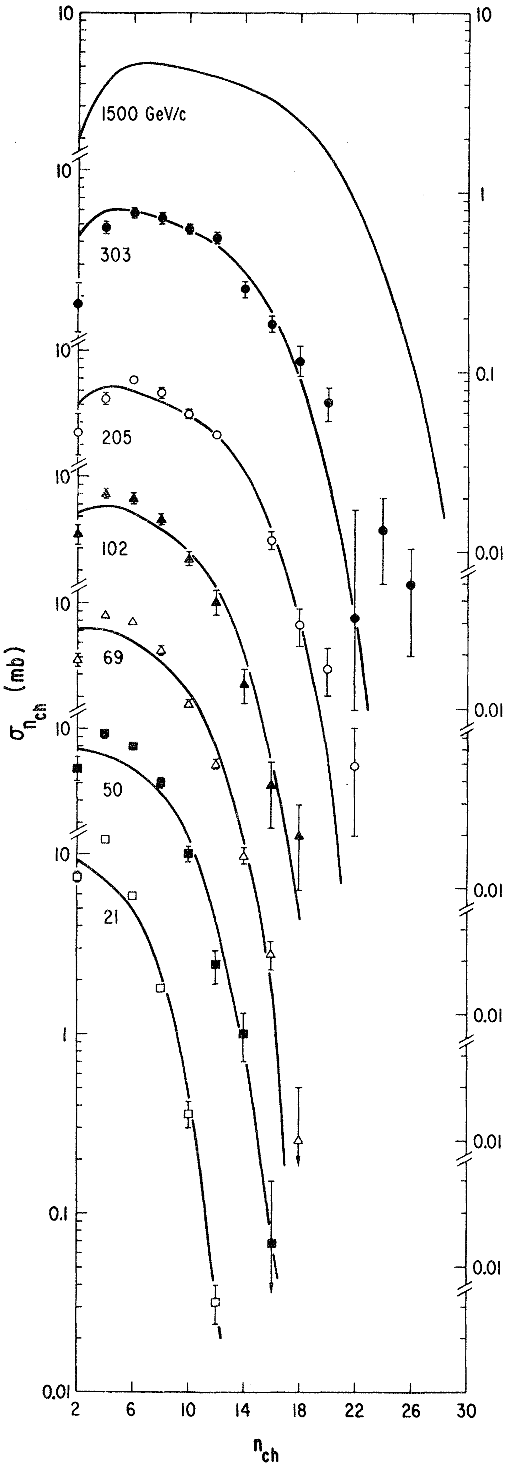

FIG. 2. Using (3.6) and $A = 31$ mb as the over-all normalization constant at all energies, $\sigma_{n_{ch}}$ vs. n_{ch} is given at 21, 50, 69, 102, 205, 303, and 1500 GeV/c and compared with data (Ref. 12) (at 1500 GeV/c reliable data do not yet appear to exist).

$$\frac{\rho_0}{1-\rho b} = z_0 g_0 \exp\left(\frac{-\rho b}{1-\rho b} + 2a\rho\right). \tag{3.9}$$

The parameters a, b, g_0, and g_{ch} are given by (2.9) and (3.3). It is obvious that (3.8) and (3.5) give *quantitatively* different functions; Fig. 1 shows, however, that both reproduce about the same *qualitative* features of the data. To test the mixture model more extensively, we give a detailed comparison of (3.6) with data[12] in Fig. 2. The good agreement justifies our taking the two-fluid theory seriously and extending the investigation to charged- and neutral-particle correlations. Note that a constant $A = 31$ mb reproduces the normalization in the data from 21 to 303 GeV/c. This should perhaps not be taken seriously since logarithms have been left out of σ_0. Since $\sigma_{\text{inel}} \sim AY^{1/4}$,[1] the maximum growth of A allowed in the critical-point theory is $Y^{7/4}$ in order not to violate the Froissart bound.

The first derivative

$$Y\frac{\partial}{\partial z_0} L(\rho_{ch}, z_0)\bigg|_{z_0=1}$$

gives the average number of neutral particles $E\{n_0|n_{ch}\}$ as a function of n_{ch}. Asymptotically the derivative can be computed using (3.8) and (3.9); the result is, for fixed $\rho_0 = n_0/Y$ and $\rho_{ch} = n_{ch}/Y$, and for $n_0, n_{ch}, Y \to \infty$,

$$E\{\rho_0|\rho_{ch}\} \equiv \lim \frac{1}{Y} E\{n_0|n_{ch}\} \to \rho_0, \tag{3.10}$$

which is implicitly a function of ρ_{ch} and is plotted in Fig. 3 along with the observed trend of recent

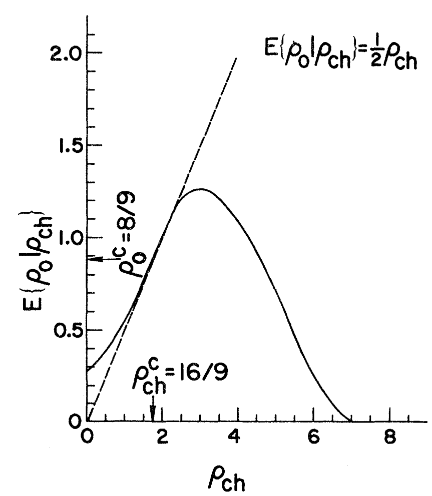

FIG. 3. The solid curve is the asymptotic average density of neutrals $E\{\rho_0|\rho_{ch}\}$ vs. density of charged particles (3.10). For comparison, the trend of recent data is included (dashed line). We assert that the data, though still very crude, behave like $\frac{1}{2}\rho_{ch}$ for $\rho_{ch} \lesssim 3$ (Ref. 9).

data (3.2). (We note that noncritical values of a, b, g_0, and g_{ch} may also reproduce the trend of present data. We obtain a no-parameter prediction of the correlation only by using the critical-point hypothesis.) For $\rho \lesssim 3$ the trend of high-energy data and theory are in reasonable agreement; for $\rho > 3$ no statement is possible since very few data exist in this region, although simple phase-space considerations require $E\{n_0|n_{ch}\}$ to decrease to zero for sufficiently large n_{ch}. Since a wide class of gas models, including the present theory, require a sharper cutoff than that required by the relation $n \lesssim \sqrt{s}/m$, measurements in the high-density region provide a test of the idea that there are strong short-range repulsive forces (as parametrized here by b). We realize that experiments in this region may be difficult. Nevertheless, we feel such experiments are important and should be considered.

It is worth emphasizing that (3.10) is an asymptotic prediction, which has the theoretical advantage that it is not sensitive to many types of possible finite volume corrections to $\sigma_{n_{ch} n_0}$; the particular form (3.10) or one not too different could remain in a large class of theories which give a critical point. For phenomenology, however, the appropriate way to calculate $E\{n_0|n_{ch}\}$ is to start from a theory which includes finite volume corrections. Although many choices are possible, it may be a good first approximation to use the explicit form (3.6) in order to calculate $E\{n_0|n_{ch}\}$. This will become particularly relevant when this correlation is measured accurately; at even fairly low lab momenta (10–20 GeV/c), qualitative features of the model might be checked. We claim that not only is evidence of a linear rise significant, but so is any evidence for the way in which $E\{n_0|n_{ch}\}$ tends to zero for large n_{ch}. We give a set of illustrative curves at various energies in Fig. 4 using (3.6).

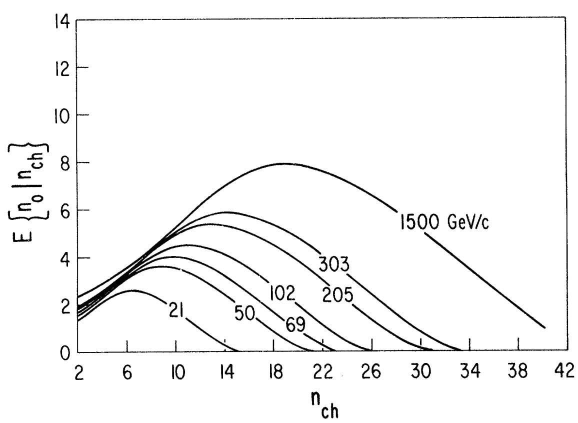

FIG. 4. The family of curves represent $E\{n_0|n_{ch}\}$ vs. n_{ch} at 21, 50, 69, 102, 205, 303, and 1500 GeV/c computed from Eq. (3.6).

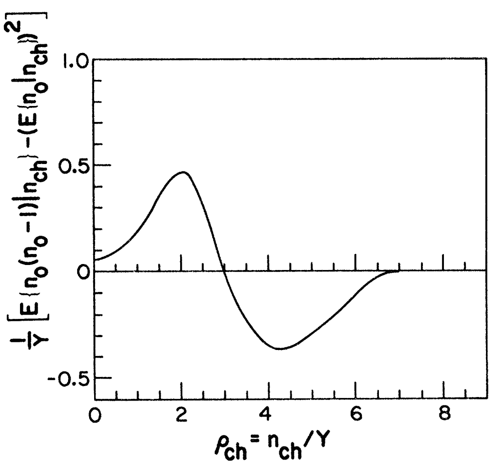

FIG. 5. The second correlation moment (corresponding to f_2) for the neutral-particle density distribution as a function of charged particles [Eqs. (3.11)–(3.12)].

An additional test is to measure the second correlation moment[13]

$$E\{n_0(n_0-1)|n_{ch}\} - (E\{n_0|n_{ch}\})^2 = Y \frac{\partial^2}{\partial z_0^2} L(n_{ch}, z_0)\bigg|_{z_0=1}, \tag{3.11}$$

which can be compared with the theory as computed from the asymptotic form (3.8) or from the "finite volume" form (3.6). The asymptotic form yields

$$\frac{\partial^2}{\partial z_0^2} L(n_{ch}, z_0)\bigg|_{z_0=1} = \frac{\rho_0}{[(1-\rho_{ch}b)^2 + \rho_0\rho_{ch}b^2]/(1-\rho b)^2 - 2a\rho_0} - \rho_0, \tag{3.12}$$

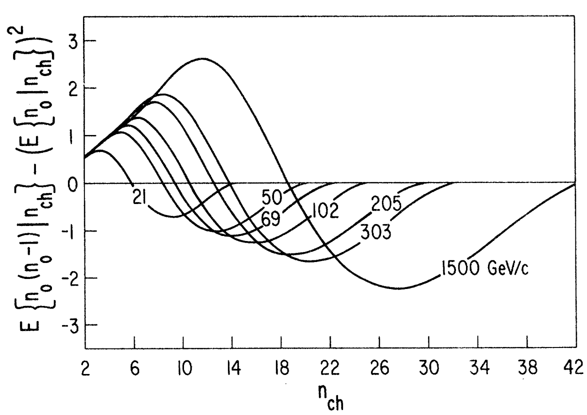

FIG. 6. The second correlation moment $E\{n_0(n_0-1)|n_{ch}\} - (E\{n_0|n_{ch}\})^2$ vs. n_{ch} is computed from (3.6) at 21, 50, 69, 102, 205, 303, and 1500 GeV/c.

which gives, in conjunction with the behavior of ρ_0 (see Fig. 3) a specific prediction for (3.11). This result is plotted in Fig. 5. Results for the finite-volume form (3.6) are given in Fig. 6.

IV. DISCUSSION AND CONCLUSIONS

The theory detailed in Sec. II has many possible applications in addition to the charged-neutral correlations in Sec. III. For example, the average number of neutral K's, $E\{n_K^0|n_{ch}\}$, as a function of n_{ch}, can be obtained from the theory (2.2) if we set $k=4$:

$$n_1 = n_{\pi^+} + n_{\pi^-},$$
$$n_2 = n_{\pi^0},$$
$$n_3 = n_{K^+} + n_{K^-},$$
$$n_4 = n_{K^0} + n_{\bar{K}^0} \equiv n_K^0 .$$

Equality of $\langle n_1\rangle = 2\langle n_2\rangle$ and $\langle n_3\rangle = \langle n_4\rangle$ follows from a Mueller analysis asymptotically. The $K^\pm/\pi^\pm$ ratio is taken from experiment[14] to be about 1/7. From (2.10) it follows then that

$$g_2/g_1 = \tfrac{1}{2}, \quad g_3/g_1 = \tfrac{1}{7}, \quad g_4/g_1 = \tfrac{1}{7}. \tag{4.1}$$

Thus a one-parameter theory is obtained for charged- and neutral-K and -π production where g_3/g_1 is the one experimentally determined parameter. The resultant theory has many consequences, one of which is a prediction for $E\{n_K^0|n_{ch}\}$. It can be shown, as in Sec. III, that asymptotically

$$E\{n_K^0|n_{ch}\} \to YE\{\rho_K^0|\rho_{ch}\} = Y\rho_K^0, \tag{4.2}$$

which is implicitly a function of ρ_{ch}. Moreover, because of the relationships (2.8),

$$\rho_K^0 = \frac{\rho_K^0}{\rho_\pi^0}\rho_\pi^0 = \frac{g_4}{g_2}\rho_\pi^0 \cong \tfrac{2}{7}\rho_\pi^0, \tag{4.3}$$

where ρ_0 has the approximate behavior shown in Fig. 3 (there is now an additional small correction since $\langle n_{ch}\rangle/\langle n_0\rangle$ is no longer precisely 2).

In an analogous manner, the theory can be extended to include the production of other particles. The following theoretical difficulty should be noted. With K's (or η's), it may still be reasonable to assume that the theory has a multiperipheral-like limit with the basic exchange $\alpha_0 = \frac{1}{2}$. If baryon and antibaryon production are to be included, this may no longer be a reasonable approximation.

We conclude from this study that the mixture theory provides a fairly comprehensive phenomenological framework in which to discuss experimental number distributions even when several types of particles are produced. For example, a novel explanation for a linear correlation in $E\{n_0|n_{ch}\}$ has been given, as well as a similar prediction for neutral K production. The theory is much more detailed than this and in fact gives the joint distributions allowing for extensive experimental tests. Also, it provides an approximate model for calculating distributions for proposed experiments. Therefore, we hope that the model presented here will be useful to experimentalists as well as to theorists by making more precise the way in which the gas analog can be applied phenomenologically.

ACKNOWLEDGMENTS

It is with pleasure that I thank Dr. Richard Arnold for several conversations which led to the formulation of the fluid theory for mixtures.

---

*Work performed under the auspices of U. S. Atomic Energy Commission.

[1] R. C. Arnold and G. H. Thomas, Phys. Lett. (to be published).

[2] The analog picture has been discussed in the literature by a number of people. See, e.g., R. P. Feynman, in *Particle Physics*, proceedings of the Conference on Particle Physics, Irvine, California, 1971, edited by M. Bander, G. L. Shaw, and D. Y. Wong (A.I.P., New York, 1972), p. 183; A. H. Mueller, Phys. Rev. D 4, 150 (1971); K. Wilson, Cornell Report No. CLNS-131, 1970 (unpublished); J. D. Bjorken, in *Particles and Fields–1972,* (proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971), p. 110; M. Bander, Phys. Rev. D 6, 164 (1972); Phys. Rev. Lett. 30, 460 (1973); Phys. Rev. D 7, 2256 (1973); R. C. Arnold, *ibid.* 5, 1724 (1972); ANL Report No. ANL/HEP 7241 (unpublished); T. D. Lee, Phys. Rev. D 6, 3617 (1972).

[3] E.g., A. Wroblewski, Warsaw University Report No. IFD/72/2, 1972 (unpublished).

[4] For a collection of recent data and a summary of various theoretical interpretations, see E. L. Berger, D. Horn, and G. H. Thomas, Phys. Rev. D 7, 1412 (1973).

[5] M. Kac, G. E. Uhlenbeck, and P. C. Hemmer, J. Math. Phys. 4, 216 (1963).

[6] J. Dash, Argonne Report No. 7304 (unpublished); G. F. Chew and D. Snider, Phys. Lett. 31B, 75 (1970).

[7] L. Caneschi and A. Schwimmer, Phys. Lett. 33B, 577 (1970); more recently, B. R. Webber, Nucl. Phys. B43, 541 (1972).

[8] F. Cerulus, Nuovo Cimento 19, 528 (1961). The comparison of Cerulus coefficients with (3.1) using $n_+ = n_-$ is made in Ref. 3 for $E\{n_0|n_{ch}\}$.

[9] See Ref. 4. National Accelerator Laboratory (NAL) pp data are consistent with $E\{n_0|n_{ch}\} = \frac{1}{2}(n_{ch}-2)$ as well as a constant. CERN Intersecting Storage Rings (ISR) data

have a much more convincing rise, but the data are taken in only a small solid angle.

[10]A. Mueller, Phys. Rev. D 2, 2963 (1970).

[11]For a discussion of the canonical partition function, see any standard text such as K. Huang, *Statistical Mechanics* (Wiley, New York, 1963).

[12]21 GeV/c: D. Smith, thesis, LBL Report No. UCRL-20632, 1971 (unpublished). 50, 69 GeV/c: V. V. Ammosov *et al.*, Phys. Lett. 42B, 519 (1972). 102 GeV/c: J. W. Chapman *et al.*, Phys. Rev. Lett. 29, 1686 (1972). 205 GeV/c: G. Charlton *et al.*, Phys. Rev. Lett. 29, 515 (1972). 303 GeV/c: F. T. Dao *et al.*, Phys. Rev. Lett. 29, 1627 (1972).

[13]D. Horn and A. Schwimmer, Nucl. Phys. B52, 627 (1973).

[14]The value of $\frac{1}{7}$ is only nominal. At ISR reported ratios are $K^-/\pi^- = 8.2 \pm 1.0\%$ and $K^+/\pi^+ = 13 \pm 1\%$, for $0.1 < x < 0.4$ and $0.2 < p_T < 0.9$ GeV/c. See J. C. Sens, in *Proceedings of the Fourth International Conference on High Energy Collisions, Oxford, 1972,* edited by J. R. Smith (Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England, 1972), p. 177.

PHYSICAL REVIEW D VOLUME 8, NUMBER 9 1 NOVEMBER 1973

Absorptive Pomeranchukon and Quantum-Number Exchange*

J. B. Bronzan
Department of Physics, Rutgers University, New Brunswick, New Jersey 08903

R. L. Sugar
Department of Physics, University of California, Santa Barbara, California 93106
(Received 18 June 1973)

Quantum-number-exchange scattering is studied in the presence of a black-disk Pomeranchukon whose radius grows like the log of the energy. We find that there is a range of momentum transfers for which the Regge pole carrying the quantum numbers is completely absorbed and leaves the physical sheet of the angular momentum plane. The high-energy behavior of the scattering amplitude is controlled not by the Regge pole, but by a branch cut in the angular momentum plane.

I. INTRODUCTION

If total cross sections rise with energy, as suggested by CERN Intersecting Storage Rings (ISR) data,[1,2] the Regge description of processes involving quantum-number exchange may have some very peculiar features. These features arise if the Pomeranchukon is absorptive and saturates the Froissart bound at very high energies. For such a Pomeranchukon the s-channel partial-wave amplitude is

$$a_P(s,b) = \tfrac{1}{2}\theta(r_0 y - b)\,, \tag{1.1}$$

where the s channel is the high-energy channel, $y = \ln(s/s_0)$, and the partial-wave series has been replaced by the familiar integral over the impact parameter b. The corresponding invariant amplitude and t-channel partial-wave amplitude are

$$M(s,t) = 4s\int_0^\infty db\, bJ_0(b\sqrt{-t})\,a_P(s,b) = \frac{2ir_0 ys}{\sqrt{-t}}J_1(r_0 y\sqrt{-t})\,, \tag{1.2}$$

$$f(t,l) = \frac{2s_0 r_0^{\,2}}{[(l-1)^2 - r_0^{\,2}t]^{3/2}}\,. \tag{1.3}$$

This absorptive Pomeranchukon is consistent with the meagre evidence currently available. On the experimental side, the work of Yodh *et al.*[3] suggests that total cross sections rise like y^2 through cosmic-ray energies, and on the theoretical side, an absorptive Pomeranchukon is suggested by the Regge-eikonal model, and by work on the asymptotic behavior of cross sections in electrodynamics.[4]

An aspect of the Regge model for quantum-number exchange is that an appropriate Regge pole is exchanged, accompanied by Regge cuts involving the exchange of the pole and the Pomeranchuk singularity. The cuts are important because they lie to the right of the pole for $t < 0$; it is through the cuts that the character of the Pomeranchukon influences quantum-number exchange. The simplest recipe for calculating these cuts is the absorption model,[5] and we shall follow this recipe here. The absorption model has the virtues that it presents a physically attractive picture in impact-parameter space, and that the cuts it generates have the trajectories and threshold behavior stipulated by Reggeon unitarity.[6] It has also been criticized, and is, undoubtedly, an approximation.[7]