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Duality and the Baryon Spectrum*

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(Received 18 June 1973)

Various authors have found solutions to consistency equations based on duality for meson-meson and meson-baryon scattering amplitudes. It is pointed out that there are three simple solutions for the baryon spectrum and interactions that accommodate the observed lightest baryons. The differences in these solutions are discussed. Experimental data involving the third and fourth quark-model levels can be used to test which, if any, of these solutions is approximately correct.

I. INTRODUCTION

In the past several years two complimentary approaches have been used to predict the spectrum of of hadrons and hadron resonances, and their principal decay amplitudes. In the first, bootstrap conditions based on duality are applied to meson-meson and meson-baryon scattering amplitudes. In the second, a quark model is constructed in such a way as to satisfy certain consistency conditions.

Both approaches lead to the prediction of a quark-model spectrum, with hadron-hadron-hadron interactions corresponding approximately to the symmetry $SU(6)_W \otimes O(2)_{L_z}$.[1] (The symbol L denotes the total quark-model internal orbital angular momentum.) However, different solutions to the various consistency conditions differ in the baryon representations expected to exist, and in many of the baryon-baryon-meson interactions.

If the only baryons considered are those of the two lightest supermultiplets, the $(\underline{56},\underline{1})$ and $(\underline{70},\underline{3})$ of $SU(6)\otimes O(3)$, it is impossible to distinguish between some of the solutions, because the symmetry determines all the interaction ratios. Fortunately, data concerning heavier baryon resonances are beginning to accumulate now. The purpose of this paper is to show that such data can distinguish between solutions to the consistency conditions. We point out some measurements that may be crucial.

The consistency conditions and solutions are discussed in Sec. II. Although different authors have used slightly different sets of conditions, most sets have in common the same three simple solutions that fit the lightest hadrons. The experimental ways of distinguishing these three solutions are discussed in Sec. III.

II. THE DUALITY CONDITIONS AND SOLUTIONS

We consider a hadron-hadron scattering amplitude in the channel of the Mandelstam variable s, identifying the forward and backward directions as the regions of small t and u, respectively. The duality condition is that an appreciable energy region exists where both the Regge and resonance representations are valid for the imaginary part of the amplitude T. Thus,

$$\langle \mathrm{Im}\, T_t^{\mathrm{Regge}} \rangle = \langle \mathrm{Im}\, T_f^{\mathrm{res}} \rangle , \tag{1a}$$

$$\langle \mathrm{Im}\, T_u^{\mathrm{Regge}} \rangle = \langle \mathrm{Im}\, T_b^{\mathrm{res}} \rangle , \tag{1b}$$

where ⟨ ⟩ denotes some suitable average over the dynamical variables, res denotes resonances, Regge refers to all trajectories other than the Pomeranchukon, and f and b denote angles near or at the forward and backward directions. Many authors have used these conditions only when no resonances exist in the s channel.[2,3] We call these the "exotic duality conditions". We use the term "full conditions" to refer to application to amplitudes both with and without s-channel resonances. We do *not* assume that the s channel corresponds to any particular baryon number. Every two-hadron→two-hadron amplitude may be associated with the s channel, so no limitation is implied by the association of the resonances with the s channel in the above definitions.

One attempts to assume as little as possible about the quantum numbers of the resonances and trajectories, and to obtain as much as possible from the consistency conditions. We limit attention to solutions in which resonances correspond to a finite number of sets of internal quantum numbers, since this type of solution is suggested by present experimental evidence.[4]

In MB (meson-baryon) scattering, the trajectories in the channel of baryon-number zero are mesonic trajectories, coupled to mesons at one vertex and baryons at the other. Thus, the solutions depend on the MMM couplings. For this reason, we consider first the duality conditions for MM scattering. There is only one solution to the full duality conditions for MM scattering. The mesons of both parities correspond to the singlet and regular representations of an SU(n) group, with SU$(n)_W$ interaction symmetry.[5] The solution corresponds to the quark model, in agreement with experiment. We now proceed to MB scattering, taking the MMM vertices from this quark-model solution.

If only the exotic duality conditions are assumed, a symmetry of the SU(n) type for baryon interactions is not predicted, but must be assumed. Several authors have listed the solutions corresponding to SU(3) or SU(6).[3,6] We will consider first SU(3) symmetry and use the notation of Ref. 6. If the external baryons considered belong to the octet and decuplet, there are only three solutions involving as few as four baryon-trajectory multiplets.[6] We list these below; the appropriateness of the names will become clear later:

(a) The "opposite-symmetry solution". The baryons of one parity correspond to the representation $\underline{8}+\underline{10}$ and those of the other to $\underline{8}+\underline{1}$. One F/D ratio is arbitrary; if it is a fixed constant, the ratios of residues must be independent of momentum transfer.

There are two solutions in which the baryons of each parity correspond to the representation $\underline{8}+\underline{10}$. They are:

(b) The "same-symmetry solution". One F/D ratio is arbitrary; if it is a fixed constant, the ratios of residues must be independent of momentum transfer.

(c) The "harmonic-oscillator solution". The ratios of residues need not be independent of momentum transfer, but all F/D ratios are fixed (at the value $-\frac{1}{3}$).

These solutions may be generalized to SU(6) symmetry, in which case the multiplets corresponding to the $\underline{10}$, $\underline{8}$, and $\underline{1}$ of SU(3) are the $\underline{56}$, $\underline{70}$, and $\underline{20}$. The external baryons in these solutions may be placed on Regge trajectories, if the symmetry of the baryon states is generalized to SU$(n)_W \otimes$O$(2)_{L_z}$.

The full duality conditions are more restrictive, of course. When one applies them to a complete set of amplitudes, every $MB\to MB$ amplitude occurs in both the s and u channels. Thus, the results depend on the relation between the residues of the trajectories in the Regge and resonance regions. One may obtain simple results by assuming that the ratios of residues of degenerate trajectories are independent of momentum transfer, and that the ratio of the contributions of two such trajectories in the u channel is the same as the ratio of their contributions to $\langle \mathrm{Im}T_b^{\mathrm{res}}\rangle$ of the crossed amplitude.

When this proportionality assumption is made, the full duality conditions require that the baryons correspond to representations of the same SU(n) group that applies to the mesons.[7] It is shown in Ref. 7 that only two types of solutions are possible. In the first, the baryons of opposite parities correspond to the same representations, and the vertices of mesons with baryons of opposite parities are proportional to the corresponding vertices with baryons of the same parities. The baryon representation must be the direct product of a single "active" quark and some passive representation; the interactions depend only on the active quark. If the passive representation is the symmetric two-quark representation, the solution is a special case of the "same-symmetry solution" mentioned above. All interaction ratios and F/D values are fixed.[8]

In the other solution, the baryons interact as composites of two active quarks and some passive representation. The baryons of opposite parities correspond to states of opposite symmetry in the two quarks. If the passive representation is a single quark, this solution corresponds to the "opposite-symmetry solution", with all interaction ratios and F/D values fixed.

Experimentally, the lightest baryons of even and odd parities correspond to the SU(6) representations $\underline{56}$ and $\underline{70}$, respectively. Hence neither the same-symmetry nor opposite-symmetry solution can be exactly right, since each predicts two representations for the states of each parity. On the other hand, Mandelstam[9] and Rosner[10] have shown that if the residue ratios are not independent of momentum transfer, a special solution to the exotic duality conditions may be obtained involving the multiplets $\underline{56}^+$, $\underline{70}^+$, $\underline{56}^-$, and $\underline{70}^-$, where the superscript is the parity. The solution is constructed so that the residue of the $\underline{70}^+$ trajectory vanishes when the total quark orbital angular momentum L is zero, and that of the $\underline{56}^-$ vanishes when $L=1$. This solution is a special case of the "harmonic oscillator solution" mentioned above, and is so called because the predicted spectrum is similar to that of the quark model with harmonic-oscillator forces.[11]

Since this solution fits (by construction) the observed $L=0$ and 1 states for baryons, many people assume it is the correct solution to the duality conditions. We believe that this conclusion is premature. One defect of the harmonic-oscillator solution is the lack of proportionality of the residues, for this proportionality is suggested by the full duality conditions, provided that the leading trajectories dominate the $\langle \mathrm{Im} T^{\mathrm{Regge}} \rangle$ in Eqs. (1a) and (1b). If we take these trajectories to be degenerate and associate the t channel with baryon-number 0, then the leading u-channel Regge terms for any amplitude are of the form

$$T_u^{\mathrm{Regge}} = \beta(u) f(s,u) \{ 1 + \eta \exp[-i\pi\alpha(u) - \tfrac{1}{2}] \} \times \left(\frac{s}{s_0}\right)^{[\alpha(u) - n/2]}, \qquad (2)$$

where s_0 is a constant, n is an odd integer, $\alpha(u)$ is the trajectory, $\eta = \pm 1$ is the signature, $f(s,u)$ is a simple kinematic factor that is the same for all amplitudes of the same spin and helicity structure, and $\beta(u)$ is the residue. We consider the ratio of the imaginary parts of two amplitudes of even parity and the same spin structure corresponding to the SU(6) representations $\underline{70}$ and $\underline{56}$. In the harmonic-oscillator solution, such a ratio vanishes at the energy of the $L=0$ levels, but does not vanish at the energy of the $L=2$ levels. This violates Eq. (1b), if the s dependence of the Regge terms is that of Eq. (2). A similar argument applies to the t-channel trajectories. Clearly, the full duality conditions are not satisfied exactly, no matter what solution is most accurate.

It is pointed out in Ref. 7 that in the opposite-symmetry solution, the $\underline{70}^+$ and $\underline{20}^-$ multiplets are coupled relatively weakly by the mesonic interactions to the $\underline{56}^+$ and $\underline{70}^-$ multiplets. Therefore, a small perturbation on the solution might cause the lowest quark-model levels of the $\underline{70}^+$ and $\underline{20}^-$ trajectories to be pushed to higher mass or to vanish. This solution cannot be discarded on the basis of present evidence. On the other hand, we neglect the same-symmetry solution, since the prediction that baryons of opposite parities have the same symmetry properties is in striking disagreement with experiment.

III. EXPERIMENTAL COMPARISON OF HARMONIC-OSCILLATOR AND OPPOSITE-SYMMETRY SOLUTIONS

We will consider only the leading SU(6) states, defined as the states of a particular quark-model level in which the total quark angular momentum L is a maximum. These states are expected to dominate the duality conditions. Particularly important are the leading SU(3) multiplets, defined as the SU(3) multiplets of maximum total angular momentum for the quark-model level. These SU(3) multiplets correspond to quark-spin $\frac{3}{2}$. Within the SU(6) multiplets $\underline{56}$, $\underline{70}$, and $\underline{20}$, the leading SU(3) multiplets are the $\underline{10}$, $\underline{8}$, and $\underline{1}$, respectively. With any SU(6) solution of the duality conditions, the leading SU(3) multiplets correspond to the analogous SU(3) solution.

The full duality conditions have not been applied to external baryons of high spins, so we will make little use of the full conditions. In the harmonic-oscillator (HO) solution, we take the parameter left arbitrary by the exotic conditions to correspond to the quark-model prescription of Rosner, discussed in Sec. II.[10] In the opposite-symmetry (OS) solution, we take the one arbitrary F/D ratio from the full-duality solution of Ref. 7.

The leading, $L=2$ and $L=3$, SU(6) states predicted by the two solutions are [7,11]:

for the OS case:

$$\begin{array}{ll} L=2 \text{ (all even } L), & \underline{56} \text{ and } \underline{70} \\ L=3 \text{ (all odd } L), & \underline{70} \text{ and } \underline{20} \end{array} \qquad (3)$$

for the HO case:

$$\begin{array}{ll} L=2, & \underline{56} \text{ and } \underline{70} \\ L=3, & \underline{70},\ \underline{56}, \text{ and } \underline{20}\,. \end{array} \qquad (4)$$

Experimental evidence exists now for the $L=2$, $\underline{70}$ state. The strongest evidence is the identification of two particular j^P (spin-parity) $\frac{7}{2}^+$ states, the $N(1990)$ and the $\Lambda(2020)$.[12] These states cannot belong to an $L=2$, $\underline{56}$ multiplet. The $L=2$, $\underline{70}$ is predicted by both the OS and HO solutions, and so its discovery is evidence for the validity of duality.

The obvious way to distinguish between the two solutions is to look for the $L=3$, $\underline{56}$. However, since data on the $L=3$ level are scarce, we examine the differences in predictions concerning the decays of $L=2$ states, both into $L=0$ and $L=1$ baryons. We compare only decays involving the same MB orbital angular momentum l, although the $SU(6)_W \otimes O(2)_{L_z}$ symmetry of the solutions predicts certain relations between partial waves.[1] For each l, a partial width Γ_i of the $B \to MB$ decay labeled by i is related to the interaction constant g_i by the formula

$$\Gamma_i = \frac{g_i p_i^{2l+1}}{M_i^2}, \tag{5}$$

where p and M are the decay momentum and the mass of the decaying baryon resonance. We do not give any calculations in this section, but only results. Those results that concern the leading SU(3) multiplets in the HO solution may be obtained from Ref. 10. The other results given here may be obtained by using the method outlined in the Appendix.

We consider first the decays $B(L=2) \to MB$, where the final baryon is a member of the nucleon octet. The only ratio of such decays that does not follow from the $SU(6)_W \otimes O(2)$ symmetry is that relating $\underline{56}$ and $\underline{70}$, $L=2$ states. In order to avoid the uncertainties of configuration mixing of quark-model states, we consider decays of $j^P = \frac{7}{2}^+$ baryons, which must correspond to quark-spin $\frac{3}{2}$. A convenient comparison ratio is $R = g^2[N(1990) \to \pi N]/g^2[\Delta(1950) \to \pi N]$, since the $N(1990)$ and $\Delta(1950)$ are members of the $\underline{70}$ and $\underline{56}$, $L=2$ multiplets, respectively. The $\underline{\text{HO}}$ and $\underline{\text{OS}}$ solutions predict values of R of $\frac{1}{8}$ and $\frac{1}{16}$, respectively. If we take the πN partial width of the $\Delta(1950)$ to be 100 MeV, and use the phase-space factor of Eq. (5) for these f-wave decays, the predicted πN partial widths of the $N(1990)$ are 15 and 7.5 MeV in the two solutions. Experimentally, the total width of the $N(1990)$ is about 225 MeV, and two preliminary values of the πN branching fraction are 0.09 and 0.15.[12] This data supports the harmonic-oscillator solution better, but this ratio is not a good way of distinguishing the two solutions, since they both predict small $N(1990) \to \pi N$ partial widths.

In both solutions the F/D ratio of the decays of the $\frac{7}{2}^+$ octet into the nucleon octet is given by the $SU(6)_W$ symmetry to be $-\frac{1}{3}$.[13] This leads to predicted values of zero for the $\Lambda(2020) \to \bar{K}N$ partial width, and 3 for the ratio $g^2[\Lambda(2020) \to \pi\Sigma]/g^2[N(1990) \to \pi N]$. These partial widths of the $\Lambda(2020)$ are not yet measured.

Finally, we consider decays into $L=1$ baryon states. Decays of the type $\underline{70}(L=2) \to \underline{70}(L=1) + M$ are not predicted by the $SU(6)_W$ symmetry, since there are two $\underline{70}$-$\underline{70}$-$\underline{35}$ interactions allowed. The predicted linear combination of these interactions is quite different in the HO and OS solutions. The resulting SU(3), F/D ratios corresponding to initial- and final-octet baryons are given in Table I. The symbols $\frac{3}{2}^{\pm}$ and $\frac{1}{2}^{\pm}$ refer to states of quark-spins $\frac{3}{2}$ and $\frac{1}{2}$ and parities ±. The F/D ratio is defined in the usual way, such that the $NN\pi$ interaction is proportional to $F+D$.

TABLE I. F/D ratios for $\underline{70}$ $(L=2) \to \underline{70}$ $(L=1) + M$ transitions involving octets.

Quark spins and parities	HO solution	OS solution
$\frac{3}{2}^+ \to \frac{3}{2}^-$	$-\frac{1}{3}$	$\frac{1}{3}$
$\frac{3}{2}^+ \to \frac{1}{2}^-$	$-\frac{1}{3}$	∞
$\frac{1}{2}^+ \to \frac{3}{2}^-$	$-\frac{1}{3}$	∞
$\frac{1}{2}^+ \to \frac{1}{2}^-$	$\frac{2}{3}$	$\frac{1}{3}$

The leading SU(3) multiplets (those of $j^P = \frac{7}{2}^+$ for $L=2$ and $\frac{5}{2}^-$ for $L=1$) correspond to quark-spin $\frac{3}{2}$, while configuration mixing may be present for lower values of j. However, it is seen from Table I that in the harmonic-oscillator solution, the F/D ratio is $-\frac{1}{3}$ if at least one of the baryons is a quark-spin $\frac{3}{2}$ state. This solution predicts zero rates for the decays $\Lambda(2020) \to \bar{K}N$ corresponding to all N states in the $L=0$ and 1 levels.

The decay ratios of the OS solution shown in Table I are quite different. The $\Lambda(2020)$-decay branching ratio $\pi\Sigma(L=1)/\bar{K}N(L=1)$, uncorrected for phase space, is predicted to be zero for a quark-spin $\frac{3}{2} \to \frac{1}{2}$ transition and $\frac{3}{2}$ for a quark-spin $\frac{3}{2} \to \frac{3}{2}$ transition.

The relative rates of decay of the $\underline{56}$ and $\underline{70}$, $L=2$ states into leading $L=1$ states are also quite different in the two solutions. We illustrate this by considering decays into the state $\pi N(1670)$, since this N is of $j^P = \frac{5}{2}^-$. The predicted relative coupling rates (g^2 ratio) of the $\frac{7}{2}^+$ particles $N(1990)$ and $\Delta(1950)$ into this state are $\frac{1}{8}$ and 1 in the HO and OS solutions.

Thus, the decays of $L=2$ baryons into $L=1$ baryons are quite different in the harmonic-oscillator and opposite-symmetry solutions. Even after the $L=3$ spectrum is measured and compared with Eqs. (3) and (4), these decays should be measured in order to test the duality predictions.

ACKNOWLEDGMENT

The author would like to thank Dr. David Faiman for pointing out an error in the original manuscript.

APPENDIX: CALCULATION OF SOME INTERACTION RATIOS

When $SU(6)_W \otimes O(2)_{L_z}$ symmetry is applied, the component of quark orbital angular momentum along the z axis (direction of the interaction) is conserved.[1] Thus, we need consider only the quark spin and internal quantum numbers of the various baryon states.[1] Each baryon state may be considered to be a composite of three quarks, labeled A, B, C. The subscripts 1, 3, and 5 denote the quark states of spin up corresponding to the proton, neutron, and Λ quarks, while 2, 4, and 6 denote the corresponding spin-down states. Thus, A_2 is the quark A in the spin-down state of the proton quark.

In the opposite-symmetry solution, the baryon states of even and odd parity are symmetric and antisymmetric, respectively, with respect to the active quarks A and B.[7] It is straightforward to use raising and lowering operators to construct the various states. To illustrate the results, we list below the spin-up proton states of quark-spin $\frac{1}{2}$ in the various representations:

$$\psi(p)_{56^+} = (\tfrac{1}{18})^{1/2}[2A_1B_1C_4 + (1+\mathcal{P}_{AB})(2A_1B_4C_1 - A_1B_2C_3 - A_2B_3C_1 - A_3B_1C_2)], \quad (A1)$$

$$\psi(p)_{70^+} = (\tfrac{1}{18})^{1/2}[2A_1B_1C_4 + (1+\mathcal{P}_{AB})(-A_1B_4C_1 - A_1B_2C_3 + 2A_2B_3C_1 - A_3B_1C_2)], \quad (A2)$$

$$\psi(p)_{70^-} = (\tfrac{1}{6})^{1/2}(1-\mathcal{P}_{AB})(A_1B_4C_1 - A_1B_2C_3 + A_3B_1C_2), \quad (A3)$$

$$\psi(p)_{20^-} = (\tfrac{1}{6})^{1/2}(1-\mathcal{P}_{AB})(A_1B_2C_3 + A_2B_3C_1 + A_3B_1C_2). \quad (A4)$$

Here $\psi(p)_{i^\pm}$ is the wave function corresponding to the SU(6) representation i and parity $\pm$, and $\mathcal{P}_{AB}$ is the operator that transposes A and B.

In the opposite-symmetry solution, the interaction $\psi_i \to \pi^0\psi_j$ is proportional to the matrix element $\langle\psi_j|O(\pi^0)|\psi_i\rangle$, where[14]

$$O(\pi^0) = a_1^\dagger a_1 - a_2^\dagger a_2 - a_3^\dagger a_3 + a_4^\dagger a_4. \quad (A5)$$

The operators $a_i^\dagger$ and a_i are creation and annihilation operators for the quark A in the state i. Both A and B are active quarks, but the symmetry allows the interaction to be written in terms of A alone.[7]

The F/D ratios may be computed from the fact that the interaction ratio $p \to p+\pi^0/\Xi^- \to \Xi^- + \pi^0$ (where the p and Ξ^- are in spin-up states) is equal to $(D+F)/(D-F)$.

The F/D ratios corresponding to the harmonic-oscillator solution in Table I are the same as the corresponding F/D ratios of the same-symmetry solution. These may be obtained from the symmetric wave functions, such as those of Eqs. (A1) and (A2), by regarding C as the active quark. The operator $O(\pi^0)$ is given by Eq. (A5), if the annihilation and creation operators are replaced by those corresponding to the quark C.

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*Work supported in part by the U. S. Atomic Energy Commission.

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Charged- and Neutral-Particle Correlations at the Critical Point*

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Motivated by the gas-liquid analog, a critical-point theory for charged multiplicities is generalized to describe the production of more than one type of final-state particle. The distribution in the total number is independent of the number of types created. As a specific application, a theory for charged and neutral particles results. It is found that the theory accounts for the observed linear rise of the mean number of neutrals $E\{n_0|n_{ch}\}$ versus the number of charged particles n_{ch}. An extension to neutral-K production is given and it is found that $E\{n_K^0|n_{ch}\}/E\{n_0|n_{ch}\}$ is independent of n_{ch}.

I. INTRODUCTION

Recently a phenomenological theory was proposed for the number distribution of charged particles in very-high-energy collisions.[1,2] Contained in this theory is the new idea that charged particles behave in a manner analogous to a 1-dimensional fluid at the critical point. For such a fluid the relative fluctuation of the number density

$$\frac{D}{\langle n\rangle} \equiv \frac{\langle (n-\langle n\rangle)^2\rangle^{1/2}}{\langle n\rangle}$$

decreases more slowly than the usual $\langle n\rangle^{-1/2}$, in agreement with recent experimental evidence.[3] Encouraged by the qualitative success of this idea applied to charged-particle distributions, we suggest in this paper that the idea can be applied more generally to all of the produced particles. At the critical point not only will the number distribution for each particle type be characterized by large number density fluctuations, but also there will be strong correlations between different types of particles. As an illustration of this, we work out a no-parameter theory for the correlations between the average number of neutral particles produced as a function of the number of produced charged particles. This theory accounts in a novel way for the observed qualitative linear correlation.[4]

In Sec. II a specific theory is proposed for the asymptotic joint number distribution of k types of produced particles, $k=1,2,\ldots$. For $k=1$, the theory of Ref. 1 is obtained; for $k=2$, a theory for charged- and neutral-pion production is obtained; with the proper choice of k and appropriate subsidiary assumptions, a theory for π, K, η, etc. production can also be obtained. In Sec. III the theory of charged- and neutral-pion production is detailed and compared with the trend of recent data. Additional experimental checks are suggested. In Sec. IV other possible applications are discussed.

II. CRITICAL-POINT THEORY FOR MIXTURES

The theory for producing n particles in the final state in which there are n_i particles of type i is to be constructed as in Ref. 1, by writing a simple ansatz (motivated by the gas-liquid analog) for the partial cross sections which allows for a critical point. The parameters of the theory are subsequently fixed according to certain reasonable physical assumptions.

The ansatz which we start with is, with $n=n_1+\cdots+n_k$,

$$\sigma_{n_1 n_2\cdots n_k} \equiv \sigma_0(Y)Q_{n_1\cdots n_k}(Y) \tag{2.1}$$

$$\begin{aligned} &= \sigma_0(Y)\frac{g_1^{n_1}}{n_1!}\cdots\frac{g_k^{n_k}}{n_k!}(Y-nb)^n e^{an^2/Y}, && n<Y/b \\ &=0, && n>Y/b \end{aligned} \tag{2.2}$$