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Proton Electromagnetic Form Factor—Data Analysis and Asymptotic Behavior

B. B. Deo and M. K. Parida
Physics Department, Utkal University, Bhubaneswar-4, Orissa, India
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Existing data on the proton electromagnetic form factor are analyzed with a view to extrapolating to the vector-meson resonance region and also to suggesting a method of verifying bounds predicted by composite models. An N/D method is suggested. The D and the N functions are assumed to represent the elastic and inelastic cut contributions, respectively. We find that the existing data are consistent with the asymptotic behavior $[\ln Q^2]^c/(Q^2)^{(p+1)/2}$, with either $c=4$, $p=3$ or $c=4$, $p=5$. An unbiased extrapolation to the timelike region shows a resonance at $m_V=708$ MeV having the width $\Gamma_V=25$ MeV for the case $c=4$, $p=3$ and the resonance shifts to $m_V=680$ MeV with $\Gamma_V=120$ MeV for the case $c=4$, $p=5$. The latter fit extrapolates to the value $|G_M^p|=0.25$ for $t=(2.1\text{ GeV})^2$ compared with the Frascati datum point $|G_M^p|=0.27\pm 0.04$.

I. INTRODUCTION

A growing amount of experimental information on electromagnetic form factors at very high momentum transfer ($t=q^2=-Q^2$) has spurred an increasing interest in the problem of finding an appropriate description of form factors in the asymptotic limit $Q^2\to\infty$. There has been an extensive theoretical investigation of the asymptotic behavior of hadron form factors from the considerations of analyticity and consequent dispersion relations. A successful model has also been developed to calculate the hadron form factors by treating them as bound states in the ladder-approximation to the two-body Bethe-Salpeter equation. Both these types of investigations sometimes lead to different results. Experimental data on proton form factors based on some models seem to suggest that the nucleon is a composite particle, a bound state of a bare nucleon and a spinless scalar gluon. However, till now no definite analysis of data has been made to extract the likely asymptotic behavior which can throw light on this important subject.

Almost thirteen years back Frazer and Fulco[1] conjectured that the observed large value of the electromagnetic radius of the proton may be due to the exchange of a pion-pion p-wave resonance. They predicted that the pion-pion resonance should occur at the square of momentum transfer values from 0.20 GeV^2 to 0.32 GeV^2. These ideas are the consequences of the vector-dominance hypothesis. Thus a good theory should give a good description of the experimental data and when extrapolated to the timelike region should predict the resonance structures whose number in general can be more than one. According to the idea of Frazer and Fulco[1] the two-pion cut contribution could be sufficient to describe the nucleon structure. In the present paper we find that by incorporating the two-pion and the three-pion cuts in a suitable way one can construct an analytic expression for the proton form factor which saturates various predicted asymptotic bounds. The parametrization gives a very good fit to the experimental data available in the spacelike momentum-transfer region. When extrapolated to the timelike side we get a resonance whose peak positions in two favorable cases are $t=0.46$ GeV^2 and $t=0.52$ GeV^2 with the widths 120 MeV and 25 MeV, respectively. Extrapolating to Frascati energy region we find that the $|G_M^p|$ values are much larger than given by the simple dipole formula $G_M^D=\mu_p/(1-t/0.71)^2$, with t in units of GeV^2.

We plan this paper as follows: In Secs. II and III we review some of the previous bounds on proton form factors and discuss some recent investigations on extrapolation problems. In Sec. IV we show how from consideration of the analyticity properties of the proton form factor the recent results on bounds can be combined with the pion-pion p-wave structure of the proton. In Sec. V we discuss our results.

II. ASYMPTOTIC BOUNDS

An investigation to obtain a lower bound on the proton form factor was first carried out by Martin[2] who, under the assumption of analyticity and certain bounds on the growth of the form factor on the cut, arrived at a result previously established by Wu and Yang.[3] Later on this result was rederived by Jaffe[4] from the basic principles of local quantum field theory and by Harte[5] as a consequence of a nonlinear bound-state model for the wave function in the crossing-symmetric bootstrap model. This result is

$$\lim_{|t|\to\infty} |F(t)| > e^{-b|t|^{1/2}}, \quad \text{for } t<0. \tag{1}$$

Improvements on the exponential asymptotic lower bound have been suggested[6] from various considerations.

Available experimental data on the proton form factor seem to suggest a falloff like t^{-2} of the dipole type. Theoretical results on the dipolelike behavior are extensive[7] and are rather model-dependent. The most attractive of the recent interesting results are those due to Broadhurst[8] and Drell and Lee.[9] From a consideration of inelastic electron-nucleon scattering and the sidewise dispersion relations for the nucleon electromagnetic form factors, Broadhurst has shown that the Drell-Yan-West[10] relation is the extremum of an inequality imposed by unitarity and analyticity and specifically that the proton's Dirac form factor is bounded by

$$\lim_{Q^2\to\infty} |F_1(Q^2)| \leq (\ln Q^2)^c/(Q^2)^{(p+1)/2} \tag{2}$$

provided that the inelastic structure function W_2 obeys in the Bjorken scaling limit, a threshold relation of the form

$$\lim_{Q^2\to\infty} \nu W_2 = F_2(\omega) \simeq (\omega-1)^p \tag{3}$$

near $\omega=1$. In Eq. (2), c is an arbitrary positive constant. Electroproduction experiments support the value $p=3$. Thus the analysis of Broadhurst is compatible with the dipolelike asymptotic behavior. By treating the physical nucleon specifically as a bound state of a bare nucleon and a bare meson (or a few bare mesons) Drell and Lee,[9] in addition to providing a fully relativistic generalization to the parton model, have correlated the scaling property in the inelastic processes with the rapid decrease of form factors in the elastic scattering. Their model predicts

$$F_1(Q^2) \sim (\ln Q^2)^2/Q^4$$

and

$$\lim_{Q^2\to\infty} \nu W_2 = F_2(\omega) \to (\omega-1)^3,$$

which is consistent with Eqs. (2) and (3) with $c=2$ and $p=3$. Bounds of the Broadhurst type are more general and contain the theoretical predictions from many other composite models[8] also. We will incorporate the asymptotic behavior (2) into our scheme of parametrization for analyzing the proton form factor data.

III. EXTRAPOLATION PROBLEM

There have been some investigations of the extrapolation of the proton form factor into the timelike region using dispersion relations. Since the pioneering work of Frazer and Fulco some authors have tried to shift the peak position towards the ρ-mass region. Bowcock, Cottingham, and Lurié[11] have obtained a ρ peak near $20\, m_\pi^2$. Ball and Wong[12] have shown that the large width of the vector meson tends to shift the π-π resonance to the lower mass region (600–650 MeV). A method based on the inversion of the dispersion integral has been suggested by Pfister.[13,14] Although an imaginary part of the form factor with a zero in the mass region between 840 and 970 MeV is obtained no peak is observed in the region of the ρ meson. It is also observed that the scaling law $G_E^p = G_M^p/\mu_p$ is violated in the timelike region. Recently a sophisticated model-independent analysis of the proton form factor, employing powerful data-analysis methods initiated by Cutkosky and Deo[15] and Ciulli[16] and using a technique of convergence test function suggested by Cutkosky and Deo,[17] Chao,[18] and Cutkosky,[19] has been carried out by Cheung.[20] In this analysis a dipolelike asymptotic behavior, a p-wave structure at the two-pion threshold and two ρ poles in the second sheet of the cut t plane have been imposed on the theory of the proton form factor. Her result confirms a ρ peak in the region $t = 0.38$ to $0.56\ \text{GeV}^2$ with negative bumps and kinks in the spectral function which become negligible when a hypergeometric kernel is used in an appropriately defined Hilbert space. Although considerable improvement of the χ^2 fit is obtained when the form-factor data from various experimental groups are renormalized, a large amount of uncertainty is introduced into the ρ parameters. Further it is found that the formula incorporating the p-wave structure, the two-sheet conformal mapping and the two ρ poles gives the best fit although there is no considerable effect of p-wave threshold structure on the χ^2.

IV. FORM-FACTOR PARAMETRIZATION

As has been discussed by Cheung, owing to the difficulty in determining the proton electric form factor G_E^p for $-t \gg 4m^2$, where m is the proton mass, we treat the magnetic form factor G_M^p only.[21] As is well known, $G_M^p(t)$ is analytic in the cut t plane with a right-hand cut starting at $t_c = 4m_\pi^2$. The next branch point is at $(3m_\pi)^2$ corresponding to the isoscalar contributions. We propose to parametrize G_M^p as

$$G_M^p(t) = N(t)/D(t), \tag{4}$$

where the denominator $D(t)$ contains the right-hand elastic cut and $N(t)$ the cut in the region $(3m_\pi)^2 \leq t < \infty$. In Eq. (4) the most convenient way of parametrization is to include poles of $G_M^p(t)$ in the function $D(t)$ and zeros in $N(t)$. We do not adopt the rigorous way of solving the coupled integral equation as is adopted in the usual N/D method, but rather make some simple approximations. As suggested by Frazer and Fulco[1] and Gounaris and Sakurai[22] we explicitly introduce the p-wave structure into the D function by parametrizing it in the form of an effective-range formula of Chew-Mandelstam[23] type where

$$D(t) = L(t) + h(t) + m_\pi^2/\pi, \tag{5}$$

where

$$h(t) = \frac{2}{\pi}\frac{q^3}{\sqrt{t}} \ln[(t/4m_\pi^2)^{1/2} + (t/4m_\pi^2 - 1)^{1/2}] - \frac{iq^3}{\sqrt{t}}, \tag{6}$$

with

$$q = (\tfrac{1}{4}t - m_\pi^2)^{1/2}$$

and $L(t)$ is a polynomial in t which should be rapidly convergent in the whole t plane without any cuts in the first sheet. The value of $h(t)$ at $t=0$ is $-m_\pi^2/\pi$, m_π being the pion mass. The fact that the D function includes the threshold structure of the p-wave $\pi\pi$ contribution can be verified by observing that $h(t)$ satisfies the twice-subtracted dispersion relation

$$h(t) = \frac{1}{\pi}\left(-m_\pi^2 + \tfrac{1}{3}t - t^2 \int_{4m_\pi^2}^{\infty} \frac{(\frac{1}{4}t' - m_\pi^2)^{3/2}}{t'^{5/2}(t'-t)}\, dt'\right). \tag{7}$$

Further, the extrapolation to a possible pole in the second sheet will be very convenient. To construct an N function which takes into account the inelastic cut, we adopt the ideas of the analytic approximation theory of data analysis[15,16] and use a conformally mapped variable z suggested by us[24] to describe high-energy phenomena. We map the right-hand inelastic cut in the t plane into the branches of a parabola with the origin as the focus in the z plane, the physical region for electron-proton scattering $-\infty \leq t \leq 0$, being mapped into the right half of the real axis $0 \leq \mathrm{Re}\, z < \infty$. The whole plane of analyticity is thus mapped into the interior of the parabola. The explicit form of z is

$$z(t) = \{\ln[(-t/9m_\pi^2)^{1/2} + (-t/9m_\pi^2 + 1)^{1/2}]\}^2. \tag{8}$$

We observe that

$$|z(t)| \underset{t\to\infty}{\sim} (\ln t)^2.$$

Now we approximate $N(t)$ by a power series expansion in z, and using the expression (4) and (5) represent the proton form factor as

$$G_E^p(t)=G_M^p(t)/\mu_p = \frac{\sum_n g_n z^n(t)}{\sum_n a_n t^n + h(t) + m_\pi^2/\pi}\,. \tag{9}$$

The form factor is normalized to unity at $t=0$. Thus $g_0=a_0$. Formula (9) is now in a form which can saturate asymptotic bounds of the Broadhurst type except for the case when the polynomial in the denominator is linear in t, for in this case the denominator behaves as $t\ln t$ and the $\ln t$ behavior comes from the dominant kinematical factor due to the p wave. It has been suggested by Roos and Pišút[25] for $\pi\pi$ scattering that the precise form of parametrization is unimportant so long as the dominant p-wave kinematical factor is present. Thus when $L(t)$ is linear in t we replace $h(t)$ by $H(t)$, where

TABLE I. Total χ^2 values for different c and p values where $G_M^p(t) \underset{t\to\infty}{\sim} (\ln t)^c/t^{(p+1)/2}$ with the formula (9) and for 89 data points.

p \ c	0	2	4
1	63 276.5	4845.3	198.5
3	151.7	90.7	86.3
5	94.7	91.02	86.248

$$H(t)=h(t)-\frac{2}{\pi}\frac{(\frac{1}{4}t-m^2)^{3/2}}{\sqrt{t}}\ln[(t/4m^2)^{1/2}+(t/4m^2-1)^{1/2}]+\frac{i(\frac{1}{4}t-m^2)^{3/2}}{\sqrt{t}}-\frac{m^2}{\pi}$$
$$=\frac{1}{\pi}\left(-m_\pi^2+t^2\int_{4m^2}^{\infty}\frac{(\frac{1}{4}t'-m^2)^{3/2}}{t'^{5/2}(t'-t)}dt'-t^2\int_{4m_\pi^2}^{\infty}\frac{(\frac{1}{4}t'-m_\pi^2)^{3/2}}{t'^{5/2}(t'-t)}dt'\right)\,, \tag{10}$$

where m is the proton mass. The function $H(t)$ in addition to removing the dominant asymptotic behavior takes into account the two-nucleon cut in a nonrigorous way. With this modification we observe that the dominant large-t behavior comes from $L(t)$ in the denominator and formula (9) gives all the Broadhurst types of bounds corresponding to the even integral values of c and odd integral values of p. For example, when terms in the numerator with power of z higher than unity and those in the denominator involving powers of t higher than two are zero, we have $g_2=g_3=\cdots=0$ and $a_3=a_4=\cdots=0$, and we get the Drell-Lee asymptotic behavior which corresponds to Broadhurst-type bounds with $c=2$ and $p=3$. We will use Eq. (9), with the modification given by Eq. (10) for the specific case $p=1$, to analyze the experimental data on $G_M^p(t)$.

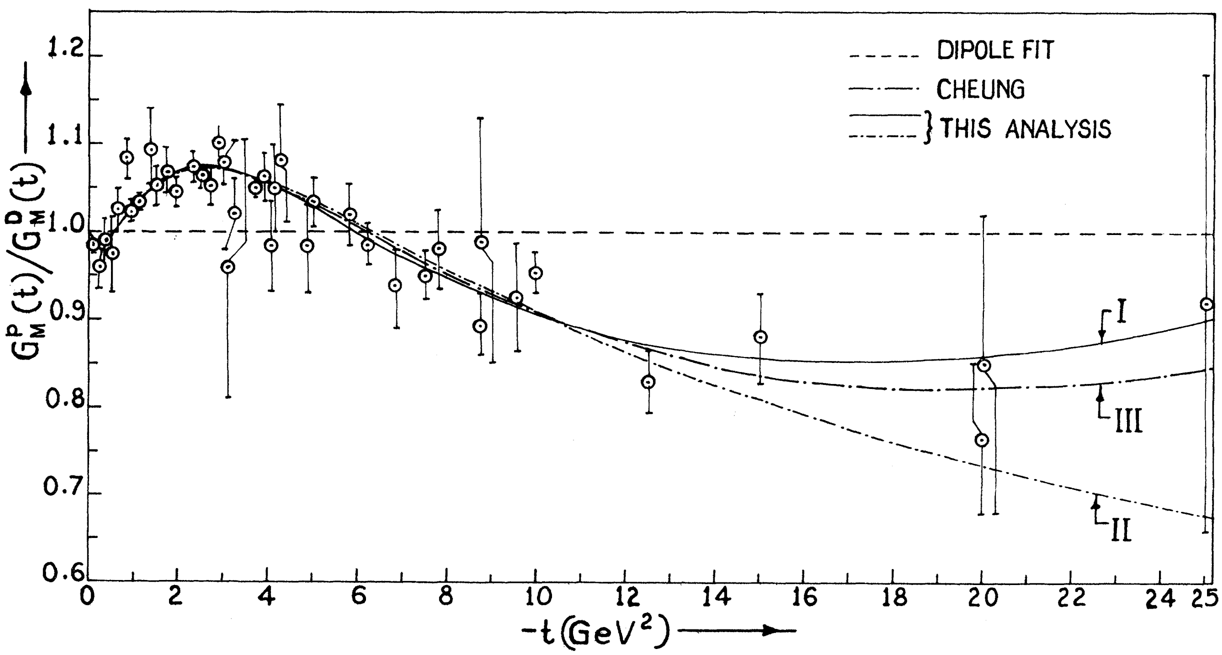

FIG. 1. Fit to the proton form-factor data. The ordinate represents $G_M^p(t)/G_M^D(t)$, where $G_M^D(t)=\mu_p/(1-t/0.71)^2$. The solid line (curve I) shows the fit for the case $c=4$, $p=3$. Curve II is the fit corresponding to the case $c=4$, $p=5$. Curve III is the fit due to Cheung (Ref. 20) with the prime data. The dashed line is the dipole fit $G_M^D(t)/\mu_p$. The data points are from the experimental groups given in Ref. 20.

TABLE II. Values of parameters for the two different χ^2 as reported in Table I.

Parameters	$c=4$ $p=3$	$c=4$ $p=5$
a_0	2.720	0.537
a_1 (GeV^{-2})	−5.968	−1.605
a_2 (GeV^{-4})	0.909	0.675
c_3 (GeV^{-6})	0.0	−0.0635
$g_0=a_0$	2.720	0.537
g_1	−0.465	−0.035
g_2	0.024	0.005

V. RESULTS AND DISCUSSION

We now report the analysis of the proton form-factor data by the formula (9) and (10). The experimental data have been taken from experimental groups described in Ref. 20. Unlike Cheung we have not renormalized the data from various sources, but used the prime data. As has been pointed out earlier the renormalization of data might have led to the large uncertainties in the resonance parameters. To study the asymptotic region we chose $c=0, 2, 4$. Table I shows nine different sets of χ^2 results with $c=0, 2, 4$ and $p=1, 3, 5$ with 89 data points. Figure 1 shows the fits to the experimental data for the two values of χ^2 (86.3 and 86.248) representing good fits. The solid line (curve I) corresponds to $c=4$ and $p=3$ which gives the asymptotic behavior $(\ln t)^4/t^2$. Curve II corresponds to $c=4$ and $p=5$ and gives the asymptotic behavior $(\ln t)^4/t^3$. Curve III corresponds to the best fit of Cheung with unrenormalized data points (see Fig. 9 of this reference). We find that our fits at lower values of $|t|$ are analogous to that of Lohrman[30] as reported at the Lund Conference, 1969. The values of parameters for the two good fits are given in Table II.

Figures 2 and 3 give the real and imaginary parts corresponding to the fits I and II of Fig. 1 and extrapolated to the timelike region. It is seen that there is a negative bump in the imaginary part near $t=0.46\ \mathrm{GeV}^2$. Figure 4 is a plot of the magnitude of $G_M^p(t)/\mu_p$ vs t as extrapolated into the timelike region. It is found that the theory extrapolates smoothly into the timelike region. The solid line corresponds to the solid-line fit of Fig. 1 and gives the resonance parameters as 708 MeV and 25 MeV for the ρ mass and width, respectively. The dot-dashed line corresponds to curve II of

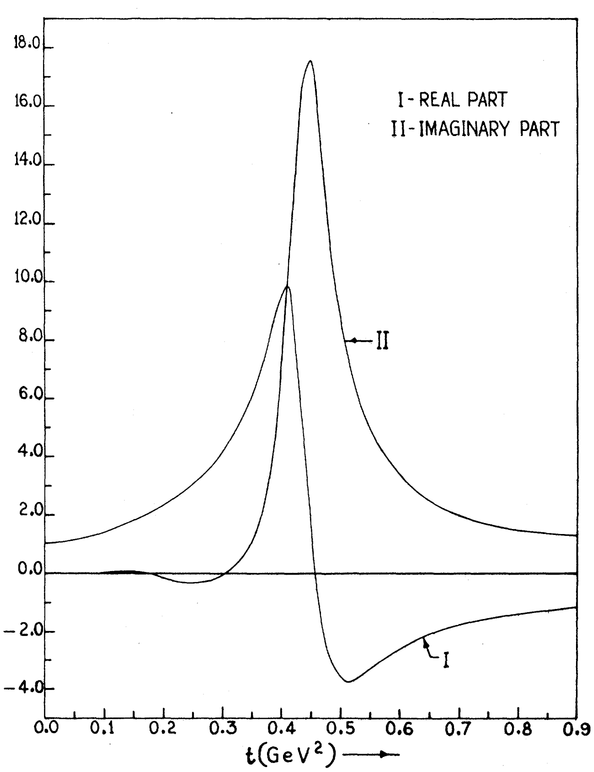

FIG. 2. Real part and imaginary part of the proton form factor $G_M^p(t)/\mu_p$ when extrapolated into the timelike region for the case $c=4$, $p=3$ (curve I, Fig. 1).

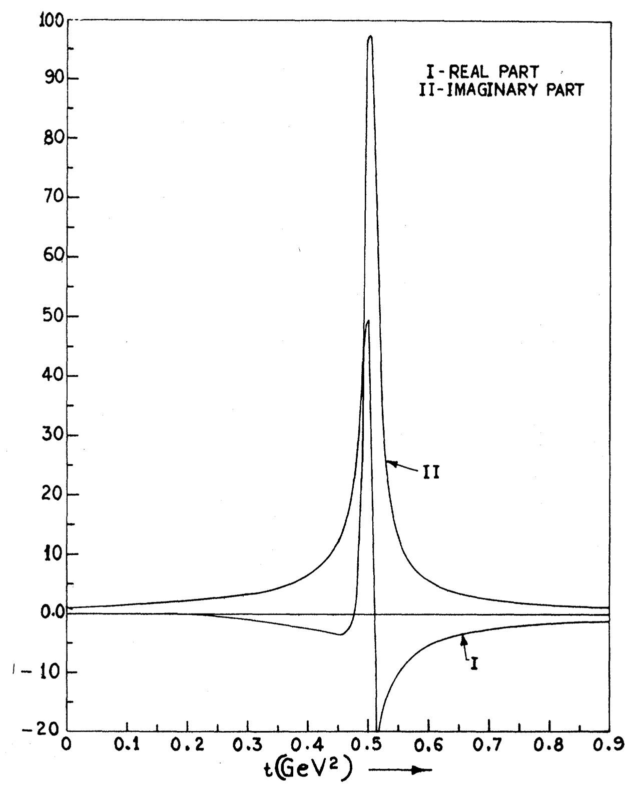

FIG. 3. Real and imaginary parts of the proton form factor when extrapolated into the timelike region for the case $c=4$, $p=5$ (curve II, Fig. 1).

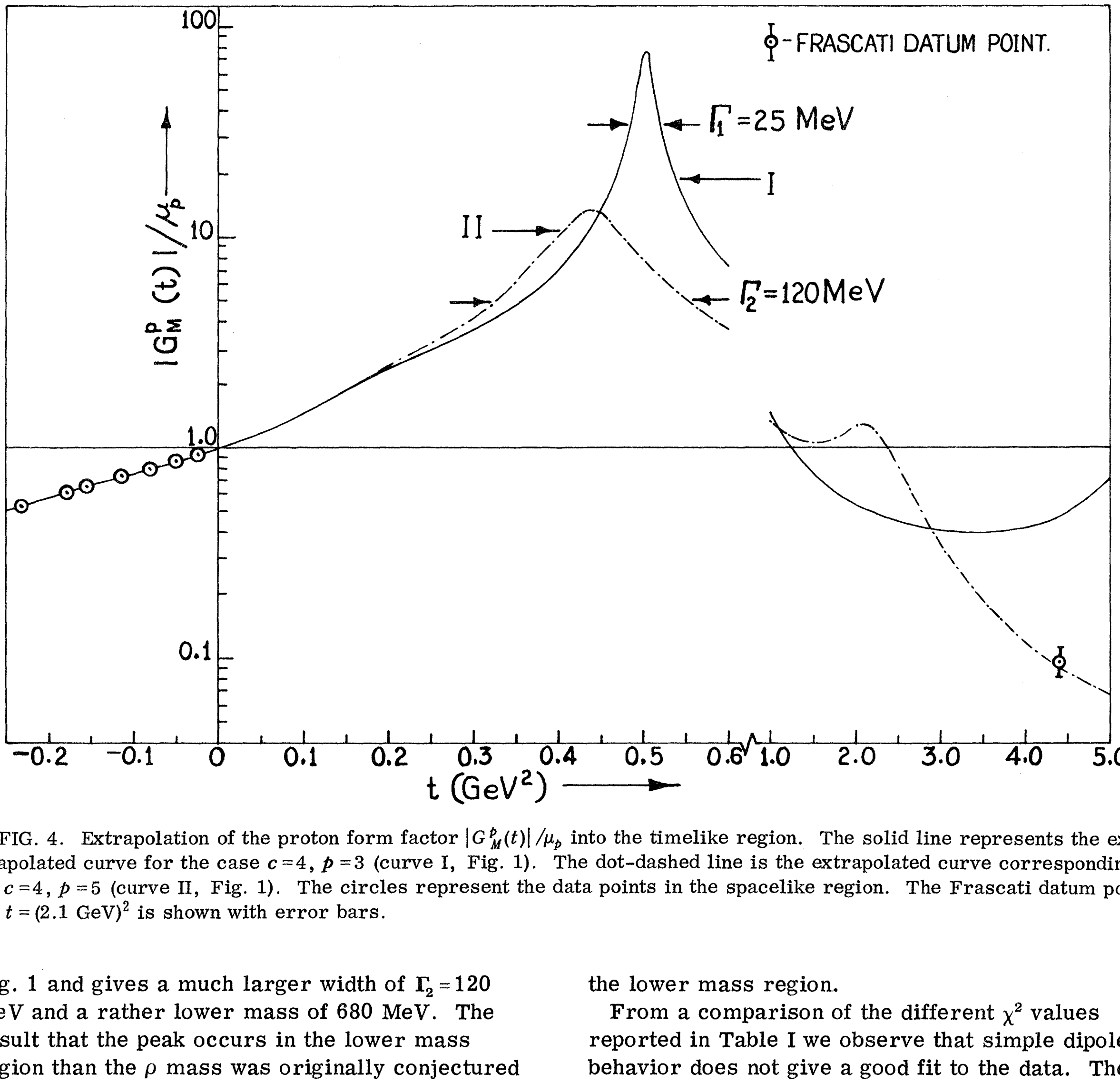

FIG. 4. Extrapolation of the proton form factor $|G_M^p(t)|/\mu_p$ into the timelike region. The solid line represents the extrapolated curve for the case $c=4$, $p=3$ (curve I, Fig. 1). The dot-dashed line is the extrapolated curve corresponding to $c=4$, $p=5$ (curve II, Fig. 1). The circles represent the data points in the spacelike region. The Frascati datum point at $t=(2.1\text{ GeV})^2$ is shown with error bars.

Fig. 1 and gives a much larger width of $\Gamma_2=120$ MeV and a rather lower mass of 680 MeV. The result that the peak occurs in the lower mass region than the ρ mass was originally conjectured by Frazer and Fulco and agrees with the analysis of Ball and Wong. But our analysis has shifted the peak towards the ρ mass. The sophisticated analysis of Cheung yields the peak at 748 MeV. That the peak position occurs at a position lower than the ρ mass can be explained in the light of the result of an important analysis by Nielsen, Petersen, Pietarinen, and Hamilton[26] and it has been pointed out by Pfister. If we believe that $G_M^p(t)$ is dominated by the 2π intermediate state for t as high as m_ρ^2, then

$$|G_M^p(t)|\propto|F_\pi(t)|\,|T(t)|\,,$$

where $F_\pi(t)$ is the pion form factor and $T(t)$ the p-wave scattering amplitude for $p\bar{p}\to 2\pi$. Recent analysis[26] shows that $\mathrm{Re}\,T(t)$ and $\mathrm{Im}\,T(t)$ have a common zero a little above the ρ mass. This zero of $T(t)$ may be responsible for shifting our peak to the lower mass region.

From a comparison of the different χ^2 values reported in Table I we observe that simple dipole behavior does not give a good fit to the data. The presence of the logarithmic forms in the N function or equivalently a cubic term in t in the D function tremendously improves the χ^2 value. But the present experimental data can also be adequately described by the asymptotic behavior of the type $(\ln t)^4/t^2$ even without going to terms containing higher powers of t in the denominator.

However, fit II gives a much better ρ signal when extrapolated into the timelike region. The falloff as $t\to-\infty$ is faster than a dipole in accord with recent observations of Chanowitz and Drell.[27] There exists a Frascati datum point[28] $|G_M^p|=0.27\pm0.04$ for $t=(2.1\text{ GeV})^2$. The dipole formula gives the value as only 0.1. So the experimental result is surprisingly large as this assumes $|G_E^p|=|G_M^p|$ contrary to the scaling relation for $t<0$, but consistent with the conspiracy relation $G_E^p=G_M^p$ at $t=4m^2$. Extrapolated values obtained from our fits are

fit I: $|G_M^p| = 1.5$,

fit II: $|G_M^p| = 0.25$.

The fit II value is in excellent agreement with experiment. So fit II is to be preferred over fit I. For this fit a small but distinct peak is also seen at $t \simeq (1.45\text{ GeV})^2$ in the ρ'-mass region. (One has to be cautious in drawing very definite conclusions when extrapolating to such higher positive t values.)

This scheme of parametrization has also been carried out by us[29] for the pion form factor by incorporating the elastic cut contribution into the D function and the inelastic cut contribution into the N function through a parabolic conformal mapping. The formula also contains similar asymptotic behavior. However, we have shown that the asymptotic behavior of the type $(\ln t)^2/t^2$ is sufficient to explain the existing data for the pion.

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[21]Our definition for spacelike and timelike momentum transfer is as follows: The square of the four-momentum transfer which we denote as t $(=q^2=-Q^2)$ is timelike if $t>0$. It is spacelike if $t<0$. The accuracy of G_M^p data is restricted by the assumption on $G_E^p = G_M^p/\mu_p$ used in separation of the form factor from cross-section data at high Q^2. Also see Ref. 20.

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