studied this system in detail; however, we expect that it is of more than academic importance. There happens to be a moderately broad $\frac{3}{2}^+$ state at 1860 MeV, which is the same distance above the threshold for these nucleon resonance + pion combinations as the $A_1$ is above the $\rho\pi$ threshold. Detailed analysis of the $\pi\pi N$ region near this effect could well run into ambiguities similar to those discussed here.

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# Symmetry Relations Among Pionization Cross Sections

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From Mueller's Regge analysis of the *pionization region*, symmetry relations to order $s^{-1/4}$ among invariant cross sections at $x=0$, in which a final pion is observed, are derived. SU(3) and quark universality are assumed to relate the Regge residues at the projectile-Reggeon vertex. There is good agreement with experiment to 10–15% indicating that the approximate symmetry pattern at nonasymptotic energies is that of total cross sections. The rising of the invariant cross sections to their asymptotic forms as $c-bs^{-1/4}$ can be understood in terms of *known* $j$-plane singularities if the $f$-Pomeron central vertex $g_{fP}^{\pi^+}(q_\perp^2)<0$. Symmetry relations valid for final kaons and nucleons and for both the pionization and the target-fragmentation regions are also given.

## I. INTRODUCTION

Relations among total cross sections derived from symmetries and quark models,[1-3] such as the Johnson-Treiman relation,[4] are known to be in good agreement with experiment in the range 25–55 GeV/$c$.[5] Previous applications of internal symmetries to inclusive reactions using Mueller's analysis[6] have concentrated on the fragmentation region.[7] Recently we have applied the additive quark model to the pionization region to obtain relations[8] among invariant cross sections at $x=0$, in which a final pion is observed, for different projectiles at nonasymptotic energies (see Appendix A).

In this paper we show that these symmetry relations among pionization cross sections also follow to order $s^{-1/4}$ from the conventional Regge analysis of the pionization region. The derivation exploits $G$ parity to restrict the relevant Regge exchanges to only $P$, $f$, and $\rho$. The symmetry input is through the well-known quark model relations[1-3,9] among the Regge residues $\beta_i^b$ at the projectile-Reggeon vertex. We obtain good agreement with existing experiments indicating that the approximate symmetry pattern in the pionization region is the same as that of two-body total cross sections—i.e., SU(3) and quark universality.

## II. SYMMETRY RELATIONS AMONG PIONIZATION CROSS SECTIONS

Mueller's analysis[6] of inclusive reactions leads to an analog of the optical theorem that relates the inclusive reaction, $a+b\to c+$ anything, to the discontinuity in $(p_a+p_b-q)^2$ of the forward amplitude, $a+b+\bar{c}\to a+b+\bar{c}$. Following Mueller, in the pionization region it is convenient to work in the rest frame of particle $c$ where

$$
\begin{aligned}
p_a &= m_a(\cosh\xi_a,\ \sinh\xi_a\cos\varphi,\ \sinh\xi_a\sin\varphi,\ 0),\\
p_b &= m_b(\cosh\xi_b,\ -\sinh\xi_b,\ 0,\ 0),\\
q &= m_c(1,0,0,0).
\end{aligned}
\tag{2.1}
$$

For the pionization region, we consider $s=(p_a+p_b)^2$ increasing to infinity with $q_\perp$ and $q_\parallel^*=(m_c^2+q_\perp^2)^{1/2}\times\sinh y^*$ fixed and small in the center-of-mass system, so $x=2q_\parallel^*/\sqrt{s}\to 0$. In this limit the invariants

$$q\cdot p_{a,b}=m_c m_{a,b}\cosh\xi_{a,b}\to s^{1/2}(m_c^2+q_\perp^2)^{1/2}\tfrac{1}{2}e^{\mp y^*}$$

become large and

$$\frac{(q\cdot p_a)(q\cdot p_b)}{p_a\cdot p_b}\to\frac{m_c^2}{1+\cos\varphi}\to\tfrac{1}{2}(m_c^2+q_\perp^2)$$

remains finite. If the singularities in the $a\bar{a}$ and $b\bar{b}$ channels are Regge poles, the invariant cross section is (see Fig. 1)

$$E\frac{d^3\sigma}{d^3q}\equiv F(ab\to c;q_\perp^2)\,,\tag{2.2}$$

where

$$F(ab\to c;q_\perp{}^2)|_{x=0}=\beta_P^a\beta_P^b g_{PP}^c(q_\perp{}^2)+\sum_{i\neq P}\beta_i^a\beta_P^b\tau_i g_{iP}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_i(0)]/2}+\sum_{j\neq P}\beta_P^a\beta_j^b\tau_j g_{Pj}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_j(0)]/2}$$
$$+\sum_{\substack{i\neq P\\ j\neq P}}\beta_i^a\beta_j^b\tau_i\tau_j g_{ij}^c(q_\perp{}^2)(s/s_0)^{-\{1-[\alpha_i(0)+\alpha_j(0)]/2\}} \qquad (2.3)$$

The sums are over the non-Pomeron $j$-plane singularities in the $a\bar{a}(b\bar{b})$ channels with intercepts $\alpha_i(0)\,[\alpha_j(0)]$. The signature of the $i$ Reggeon is $\tau_i$. In writing (2.3) we have set $x=0$ and have assumed factorization for all the Reggeons. The Pomeron is assumed to have $\alpha_P(0)=1$.

For very large $s$, since the leading singularities are Pomerons, we have scaling and $F(ab\to c)/\sigma_T(ab)\to\tilde{g}^c(q_\perp{}^2)$, where $\tilde{g}^c(q_\perp{}^2)$ is independent of both particles $a$ and $b$. However, assuming each non-Pomeron has $\alpha_i(0)=\frac{1}{2}$, the next term in this expansion is expected to be $s^{-1/4}$, so that the limit will be approached slowly and hence ideas based on the asymptotic form, such as the independence of $\tilde{g}^c(q_\perp{}^2)$ from the quantum numbers of $a$ and $b$, are not expected to be valid for all reactions in existing experiments (< 30 GeV/$c$). Ferbel[10] has shown that existing experimental data are consistent with $\int_0^\infty F(ab\to c;q_\perp{}^2)dq_\perp{}^2$ rising as $c-bs^{-1/4}$ to its asymptotic form.

Therefore, we consider only the terms in the Regge expansion to order $s^{-1/4}$. First it is important to note from Fig. 1, that for $c$ a final-pion $G$ parity restricts the possible non-Pomeron exchanges to this order to $f$ and $\rho$. Second, since the additive quark model gives good results at nonasymptotic energies for a variety of two-body reactions, it seems natural to apply it here to relate the Regge residues. Of course, depending on one's point of view, these residues can be related by different symmetry arguments, such as exchange degeneracy and $\rho$-$\omega$ universality, or simply determined from fits to two-body data.[11] For definiteness, we take the quark model where[1-3,9]

$$\beta_P^{\pi^+}=\beta_P^{K^+}=\tfrac{2}{3}\beta_P^P,$$

$$\beta_f^{\pi^+}=2\beta_f^{K^+}=\tfrac{2}{3}\beta_f^P, \qquad (2.4)$$
$$\beta_\rho^{\pi^+}=2\beta_\rho^{K^+}=2\beta_\rho^P.$$

The first equality amounts to SU(3) and the second is quark universality, i.e., that quarks in mesons and baryons are equivalent.

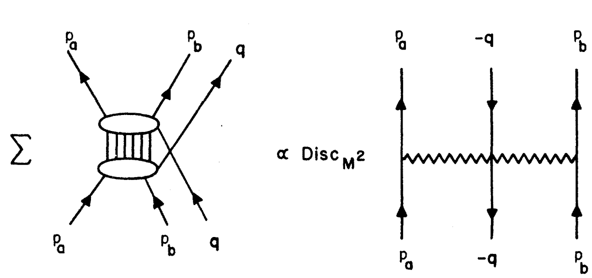


FIG. 1. Generalized unitarity relation for the pionization region.

We fix $c$ to be either a $\pi^\pm$ or $\pi^0$. Factorization at the $b\bar{b}$ vertex gives

$$F(ab\to c;q_\perp{}^2)|_{x=0}=\beta_P^b R_P^{ac}+\beta_f^b R_f^{ac}-\beta_\rho^b R_\rho^{ac}+O(s^{-1/2}), \qquad (2.5)$$

with the unknowns

$$R_P^{ac}=\beta_P^a g_{PP}^c(q_\perp{}^2)+\beta_f^a g_{Pf}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_f(0)]/2}-\beta_\rho^a g_{P\rho}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_\rho(0)]/2},$$
$$R_f^{ac}=\beta_P^a g_{fP}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_f(0)]/2}, \qquad (2.6)$$
$$R_\rho^{ac}=\beta_P^a g_{\rho P}^c(q_\perp{}^2)(s/s_0)^{-[1-\alpha_\rho(0)]/2}.$$

Ignoring the $O(s^{-1/2})$ terms, we obtain using Eq. (2.4) the following independent linear relations between invariant cross sections ($c=\pi^\pm$ or $\pi^0$), where $a$ is any target:

$$2F(pa\to c)=2F(\pi^+a\to c)+F(\pi^-a\to c), \qquad (2.7a)$$
$$2F(\bar{p}a\to c)=2F(\pi^-a\to c)+F(\pi^+a\to c), \qquad (2.7b)$$
$$2F(K^-a\to c)+F(\pi^+a\to c)=2F(K^+a\to c)+F(\pi^-a\to c). \qquad (2.7c)$$

The cross sections in these sum rules are to be evaluated at the same energy.

Some of these sum rules can be tested with present data in the pionization region. By charge conjugation $F(p\bar{p}\to\pi^+)=F(p\bar{p}\to\pi^-)$, so we obtain for proton targets

$$2F(pp\to\pi^\pm)=2F(\pi^+p\to\pi^\pm)+F(\pi^-p\to\pi^\pm), \qquad (2.8a)$$
$$2F(\pi^-p\to\pi^+)+F(\pi^+p\to\pi^+)=2F(\pi^-p\to\pi^-)+F(\pi^+p\to\pi^-), \qquad (2.8b)$$
$$2F(K^-p\to\pi^\pm)+F(\pi^+p\to\pi^\pm)=2F(K^+p\to\pi^\pm)+F(\pi^-p\to\pi^\pm), \qquad (2.8c)$$
$$2F(\bar{p}p\to\pi^\pm)=2F(\pi^-p\to\pi^\pm)+F(\pi^+p\to\pi^\pm), \qquad (2.8d)$$

where only one of the sum rules (2.8d) is independent as the other follows from (2.8b). These sum rules[8] were first obtained by an alternative argument based on the additive quark model (see Appendix A).

The model can be extended[12] to photon-induced inclusive processes by vector-meson dominance. For instance,

$$F(\gamma p\to\pi^{\pm})=\tfrac{1}{4}\alpha\sum_{\rho,\omega,\varphi}\frac{4\pi}{\gamma_V^{\,2}}F(Vp\to\pi^{\pm}), \tag{2.9}$$

with

$$\begin{aligned}F(\rho^0p\to\pi^{\pm})&=F(\omega p\to\pi^{\pm})\\&=\tfrac{1}{2}[F(\pi^+p\to\pi^{\pm})+F(\pi^-p\to\pi^{\pm})],\\F(\varphi p\to\pi^{\pm})&=F(K^+p\to\pi^{\pm})+F(K^-p\to\pi^{\pm})\\&\quad-\tfrac{1}{2}[F(\pi^+p\to\pi^{\pm})+F(\pi^-p\to\pi^{\pm})].\end{aligned} \tag{2.10}$$

To probe the contributions of particular pairs of Reggeons, it is useful to consider the sum and difference relations. The $\rho$-$P$ pair with a $C$-odd meson exchange is isolated by considering[13]

$$\begin{aligned}\Delta(ab)&\equiv F(ab\to\pi^+)-F(ab\to\pi^-)\\&=(\beta_\rho^a\beta_P^b+\beta_P^a\beta_\rho^b)g_{\rho P}^{\pi^+}(q_\perp^{\,2})(s/s_0)^{-[1-\alpha_\rho(0)]/2}\\&\quad+O(s^{-1/2}),\end{aligned} \tag{2.11}$$

and for the quark-model residues the predictions are

$$\begin{aligned}\tfrac{4}{3}\Delta(pp)&=\Delta(\pi^+p)\\&=-2\Delta(\pi^-p)\\&=\tfrac{8}{5}\Delta(K^+p)\\&=-8\Delta(K^-p).\end{aligned} \tag{2.12}$$

Similarly the $C$-even exchanges can be isolated by defining

$$\Sigma(ab)=F(ab\to\pi^+)+F(ab\to\pi^-), \tag{2.13}$$

which contains contributions from both the $P$-$P$ and $f$-$P$ pairs. Charge conjugation gives for proton targets $\Sigma(p\bar{b})=\Sigma(pb)$, where $b$ is any projectile, and with the quark-model residues

$$\begin{aligned}\tfrac{2}{3}\Sigma(pp)&=\Sigma(\pi^+p)\\&=\Sigma(\pi^-p).\end{aligned} \tag{2.14}$$

To isolate the $f$-$P$ pair, we consider

$$\begin{aligned}\Sigma(ab)&=\Sigma(ab)|_{s\to\infty}+\overline{\Sigma}(ab)(s/s_0)^{-[1-\alpha_f(0)]/2}\\&\quad+O(s^{-1/2}),\\\overline{\Sigma}(ab)&=(\beta_f^a\beta_P^b+\beta_P^a\beta_f^b)g_{fP}^{\pi^+}(q_\perp^{\,2}),\end{aligned} \tag{2.15}$$

so $\overline{\Sigma}(ab)$ is the coefficient of the $s^{-1/4}$ contribution. For the quark-model residues the predictions are

$$\begin{aligned}\tfrac{2}{3}\overline{\Sigma}(pp)&=\overline{\Sigma}(\pi^+p)\\&=\tfrac{4}{3}\overline{\Sigma}(K^+p)\end{aligned} \tag{2.16}$$

and, of course, $\overline{\Sigma}(p\bar{b})=\overline{\Sigma}(pb)$.

It should be noted that we are assuming that the usual Regge-pole singularities in the $j$-plane are sufficient for describing the behavior of one-particle invariant inclusive cross sections in the pionization region. We are ignoring the contributions of cuts and other possible singularities, such as a $Q$ trajectory.[13] If such singularities should play an important role in the pionization region, then agreement of these sum rules with experiment would indicate that their couplings to hadrons, i.e., their residues, also manifest the SU(3) and quark-universality symmetries of the quark-model. In this circumstance, the conclusions, see Sec. III, from present data that $g_{PP}^{\pi^+}(q_\perp^{\,2}), g_{\rho P}^{\pi^+}(q_\perp^{\,2})>0$ and $g_{fP}^{\pi^+}(q_\perp^{\,2})<0$ would need reinterpretation in terms of these additional singularities. It should be stressed that these conclusions are also relative to the convention that $\beta_P^{\pi^+}$, $\beta_f^{\pi^+}$, and $\beta_\rho^{\pi^+}$ are positive, e.g., $g_{PP}^{\pi^+}(q_\perp^{\,2})$, $g_{\rho P}^{\pi^+}(q_\perp^{\,2})$, and $g_{fP}^{\pi^+}(q_\perp^{\,2})$ could all be positive if $\beta_P^{\pi^+}\gtrless 0$, $\beta_\rho^{\pi^+}\gtrless 0$, and $\beta_f^{\pi^+}\lessgtr 0$. In Regge phenomenology of quasi-two-body reactions, amplitudes are proportional to $\beta_j^a\beta_j^b$; hence, only the relative signs of the $\beta_j^b$'s are determined there, not their absolute signs.

Finally, if other trajectories are included, the derivation can be extended to final kaons and nucleons. Simultaneously, this *also* extends the derivation to include terms of order $s^{-1/2}$ in the pionization region and to terms of order $s^{-1}$ in the target-fragmentation region. For example, for the $\omega$ and $A_2$ trajectories the quark-model residues are[1-3,9]

$$\begin{aligned}\beta_\omega^{K^+}&=\tfrac{1}{3}\beta_\omega^p,\\\beta_A^{K^+}&=\beta_A^p\end{aligned} \tag{2.17}$$

and we obtain for any $a$ and $c$ the sum rules

$$\begin{aligned}&F(\bar{p}a\to c)+F(na\to c)=2F(\pi^-a\to c)+F(\pi^+a\to c),\\&F(pa\to c)+F(\bar{n}a\to c)=2F(\pi^+a\to c)+F(\pi^-a\to c),\\&3F(K^+a\to c)+F(\bar{p}a\to c)+F(\pi^-a\to c)\\&\qquad=3F(K^-a\to c)+F(pa\to c)+F(\pi^+a\to c),\end{aligned}$$

where the second follows from the first by charge conjugation. (The third requires $\beta_\omega^{K^+}=\frac{1}{3}\beta_\omega^p$; none requires $\beta_A^{K^+}=\beta_A^p$.) Further knowledge or assumptions about the relations of different $g_{ij}^c(q_\perp^{\,2})$'s would yield additional sum rules [cf. Eq. (A3)].

## III. COMPARISON WITH EXPERIMENT

We must compare our sum rules with experiment[11] to test whether the approximate symmetry

TABLE I. Tests of relations among invariant pionization cross sections.[16]

| | Sum rule | $F(ab \to c) = \left.\frac{E d^2\sigma}{\pi dq_{\parallel} dq_{\perp}^2}\right\rvert_{x=0}$ [mb/(GeV/$c$)$^2$] | $P_{\text{lab}}$ (GeV/$c$) | References |
|---|---|---|---|---|
| plus mode of (2.8a) | $2F(\pi^+p \to \pi^+) + F(\pi^-p \to \pi^+)$ | 81 | 16 | 14 [b] |
| | $2F(pp \to \pi^+)$ | 88, 90 | 19.2, 12 | 27, [c] 28 [d] |
| negative mode of (2.8a) | $2F(\pi^+p \to \pi^-) + F(\pi^-p \to \pi^-)$ | 58 | 16 | 14 [b] |
| | $2F(pp \to \pi^-)$ | 50, 60 | 19.2, 12 | 27, [c] 28 [d] |
| (2.8b) | $2F(\pi^-p \to \pi^+) + F(\pi^+p \to \pi^+)$ | 71 | 16 | 14 [b] |
| | $2F(\pi^-p \to \pi^-) + F(\pi^+p \to \pi^-)$ | 63 | 16 | 14 [b] |
| negative mode of (2.8c) | $2F(K^-p \to \pi^-) + F(\pi^+p \to \pi^-)$ | 7.4 mb [a] | 10($Kp$), 16($\pi p$) | 20, 14 |
| | $2F(K^+p \to \pi^-) + F(\pi^-p \to \pi^-)$ | 7.3 [a] | 12.7($Kp$), 16($\pi p$) | 21, 14 |

[a] For the negative mode of (2.8c) the value listed is for $\int_0^\infty F(ab \to c;\ q_\perp^2) dq_\perp^2$.
[b] Bin size $0 < q_\perp^2 < 0.06$ (GeV/$c$)$^2$.
[c] Value for $q_\perp^2 = 0$ from fit to data of Ref. 17.
[d] Bin size $0 < q_\perp^2 < 0.04$ (GeV)$^2$.

pattern at nonasymptotic energies in the pionization region is that of the quark model, i.e., SU(3) and quark universality.

Ideally the relations, Eq. (2.8), should be tested by comparing the invariant cross sections $F(q_\perp^2)$ at $q_\perp^2 = 0$, or by comparing $\int_0^\infty F(q_\perp^2) dq_\perp^2$, using experiments done at the same energy by the same experimental group to minimize normalization errors. Unfortunately, only (2.8b) can be tested (Table I) under these conditions by using the 16-GeV/$c$ bubble chamber data from the collaboration[14] of ABBCCHLVW. Relations (2.8a) and (2.8c) can be tested by comparing the available results of different groups at slightly different energies. Notice that on a scale $s^{-1/4}$ this difference in energy is only about 10% which is less than the scatter and error bars of the data. For the negative mode of (2.8c) it is necessary to compare[15] $\int_0^\infty F(q_\perp^2) dq_\perp^2$. We were unable to locate the necessary data for testing relations (2.8c) (plus mode) and (2.8d) in which final $\pi^\pm$ are observed. Experimental error bars are about 15% and unknown normalization errors may be serious in some cases. The results are given in Table I (see Ref. 16). There is good agreement to 10–15%. The agreement is unchanged if comparison is made for $\int_0^\infty F(q_\perp^2) dq_\perp^2$ instead of $F(q_\perp^2)$ at $q_\perp^2 = 0$. This is the accuracy one would expect, we think, by comparison with applications of symmetries to two-body total cross sections.

From photon-induced inclusive processes, using relations in Eq. (2.10), a value for $\gamma_\rho^2/4\pi$ can be obtained: For the ratios of the $\gamma$-$V$ coupling constants, SU(6) predicts[18] $\gamma_\rho^{-2}:\ \gamma_\omega^{-2}:\ \gamma_\varphi^{-2} = 9:1:2$ and symmetry breaking changes these ratios to 9:1.2:1.[19] From the data [14,20,21] by relations (2.10) we find $F(\varphi p \to \pi^-):\ F(\rho^0 p \to \pi^-) = 0.21$. Since applications[22] of vector-meson dominance indicate that the $\varphi$ contribution is suppressed, we neglect it and accept the SU(6) value for the ratio $\gamma_\rho^{-2}:\ \gamma_\omega^{-2}$. So, from $F(\gamma p \to \pi^-) = \int_0^\infty F(q_\perp^2) dq_\perp^2 = 17.1 \pm 0.7\ \mu$b of Moffeit *et al.*[23] and Ref. 14, the $\gamma$-$\rho^0$ coupling constant is $\gamma_\rho^2/4\pi = 0.32$. This agrees with the values obtained[21] from two-body photo-production data which are smaller by a factor of two from those from $e^+e^-$ annihilation.

We also note that it is amusing that the negative mode of (2.8a) provides a possible explanation of the analysis of Chen *et al.*[24] to verify factorization experimentally by comparing $(1/\sigma_T) d\sigma/dq_\parallel^{\text{lab}}$ in the laboratory frame: They find the average values for this quantity for the region $q_\parallel^{\text{lab}} < 0.5$ GeV/$c$ for $\pi^+p \to \pi^-$, $K^+p \to \pi^-$, $pp \to \pi^-$, and $\pi^-p \to \pi^-$ are, respectively, $0.23 \pm 0.02$, $0.20 \pm 0.02$, $0.23 \pm 0.02$, and $0.32 \pm 0.02$, thus making $\pi^-p \to \pi^-$ *anomalous*. (The distribution for $\pi^-p \to \pi^-$ is an exception as a function of $q_{\text{lab}}$ even if the average is not taken.) However, the negative mode of sum rule (2.8a) gives, when the $\sigma_{\text{tot}}$ dependence[24] of these numbers is removed by multiplication ($\hat{F}$ = average $d\sigma/dq_\parallel^{\text{lab}}$ for $q_\parallel^{\text{lab}} < 0.5$ GeV/$c$),

$$\begin{aligned} &2\hat{F}(pp \to \pi^-) = 18 \text{ mb}, \\ &2\hat{F}(\pi^+p \to \pi^-) + \hat{F}(\pi^-p \to \pi^-) = 19 \text{ mb}, \end{aligned} \tag{3.1}$$

well within experimental errors. It must be stressed that this agreement is only suggestive and *should not* be taken too seriously since $q_\parallel^{\text{lab}} < 0.5$ GeV/$c$ includes *both* the pionization and target-fragmentation regions. In the target-fragmentation region the negative mode of (2.8a) should only hold for the contribution from the $G$-parity-even exchanges. Of course, by exploiting factorization in the $\bar{b}b$, channel sum rules, e.g., Eq. (2.18), can be derived which should hold in both the pionization and target-fragmentation regions.

Finally, we consider the sum and difference relations to study the above agreement as a function of energy and as a function of the $C$ quantum number of the exchanges. The difference relations, Eqs. (2.12) which isolate the $\rho$-$P$ pair, can be tested (Fig. 2) (see Ref. 25) by comparing

$$\Delta \int_0^\infty F(q_\perp^2)dq_\perp^2 = \int_0^\infty [F(ab\to\pi^+) - F(ab\to\pi^-)]\,dq^2 .$$

All available experimental data[25] is plotted as a function of $(p_{\rm lab})^{-1/4}$ except for the CERN ISR data[26] for $\frac{4}{3}\Delta(pp)$, which is 0.0 mb at $(p_{\rm lab})^{-1/4}=0.162$ GeV$^{-1/4}$, 0.5 at 0.176, 0.1 at 0.212, and 0.3 at 0.244, but with very large error bars of $\pm 2.5$ mb so we have excluded these data. The straight line through the $\Delta(\pi^+p)$ points is an eye estimate. While better data is badly needed, the difference relations are not inconsistent. Clearly $g_{\rho P}^{\pi^+}(q_\perp^2)>0$ below ISR energies.

The sum relations, Eqs. (2.14) which contain both the $P$-$P$ and $f$-$P$ pairs, can be tested at about 16 GeV by comparing $\Sigma(ab)=F(ab\to\pi^+)+F(ab\to\pi^-)$ at $q_\perp^2=0$[14,27,28]:

$$\tfrac{2}{3}\Sigma(pp)=48\pm4,\quad 50\pm4\ \text{mb}/(\text{GeV}/c)^2$$

$$\Sigma(\pi^+p)=46\pm4\,,$$

$$\Sigma(\pi^-p)=43\pm3.$$

However, a plot of

$$\Sigma \int_0^\infty F(q_\perp^2)dq_\perp^2 = \int_0^\infty [F(ab\to\pi^+) + F(ab\to\pi^-)]\,dq_\perp^2$$

as a function of $(p_{\rm lab})^{-1/4}$ shows that while $\frac{2}{3}\Sigma(pp)$ agrees with $\Sigma(\pi^+p)$ and $\Sigma(\pi^-p)$ at about 16 GeV, or $(p_{\rm lab})^{-1/4}=0.5$ GeV$^{-1/4}$, it is much smaller than $\Sigma(\pi^+p)$ at lower energies. Thus, as with sum rules for two-body total cross-sections, the sum rules for $C$-odd exchanges agree much better at low energies than do those for $C$ even. Here this disagreement may also be due to effects from the neglected $M$-$M$ terms of order $s^{-1/2}$. For $p_{\rm lab}>16$ GeV, comparing[29] the ISR data[26] for $\frac{2}{3}\Sigma(pp)$ with the lower-energy data[25] shows that $g_{PP}^{\pi^+}(q_\perp^2)>0$, but that $g_{fP}^{\pi^+}(q_\perp^2)<0$ (see Sec. III). The latter is the apparent reason that $\int_0^\infty F(ab\to c;\, q_\perp^2)dq_\perp^2$ rises[10] as $c-bs^{-1/4}$ to its asymptotic form.

## IV. CONCLUSIONS

The various sum rules that relate invariant cross sections at $x=0$ for different projectiles at nonasymptotic energies agree with experiment to 10–15%. This agreement indicates that the approximate symmetry pattern of $F(ab\to c)$ is the same as that of two-body total cross sections—i.e., SU(3) and quark universality. While the sum rules were derived for $F(ab\to\pi)$ in which a single final pion is observed, they clearly also follow for the corresponding invariant cross sections[30] for $n$ final pions in which all are in the pionization region. The rising of $\int_0^\infty F(ab\to c;\, q_\perp^2)\,dq_\perp^2$ to its asymptotic form as $c-bs^{-1/4}$ can be understood in terms of known $j$-plane singularities if $g_{fP}^{\pi^+}(q_\perp^2)<0$. Hence, there is no need for other possible singularities, such as a $Q$ trajectory. Hopefully, understanding the approximate symmetry pattern will be a useful step towards discovering the dynamics of production processes in the pionization region.

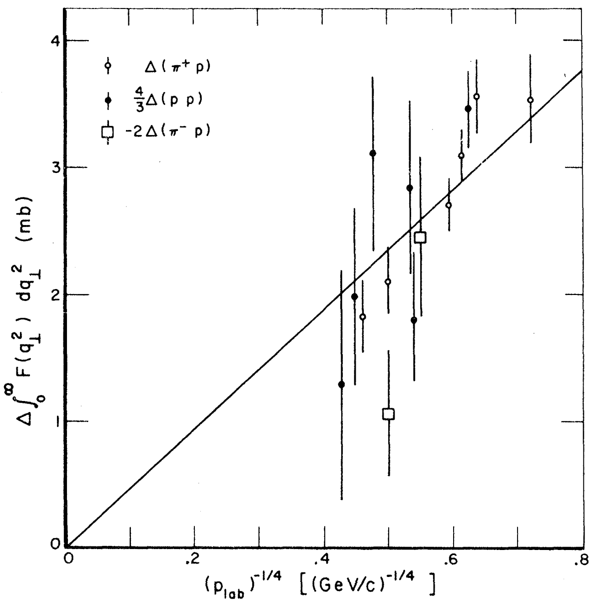


FIG. 2. Data for test of symmetry relations $\Delta(\pi^+p) = \frac{4}{3}\Delta(pp) = -2\Delta(\pi^-p)$. (See Ref. 25.) For ISR data see text.


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## APPENDIX A: ADDITIVE QUARK MODEL

In this appendix we present the *alternative derivation*[8] of the sum rules based on the additive quark model.

For the two-body elastic scattering amplitude $T_{ab}$, the additivity assumption can be expressed by writing it as the sum of all possible elastic amplitudes $t_{ij}$ for the scattering of a quark or antiquark

$i$ in $a$ with a quark or antiquark $j$ in $b$:

$$T_{ab}(s,t)=\sum_{i,j}F_i^a(t)F_j^b(t)t_{ij}(s,t),$$

where $F_i^a(0)=F_j^b(0)=1$. The quark-quark amplitudes are asymptotic below 30 GeV/$c$, but the quark-antiquark amplitudes are not.[31] Therefore, we follow the approach of Levinson, Wall, and Lipkin[32] and assume that these amplitudes are given by

$$\begin{aligned}&(\mathcal{P}\mathcal{P})=(\mathfrak{N}\mathfrak{N})=P, && (\bar{\mathcal{P}}\mathfrak{N})=(\bar{\mathfrak{N}}\mathcal{P})=P+A',\\ &(\mathcal{P}\mathfrak{N})=P', && (\mathcal{P}\lambda)=(\mathfrak{N}\lambda)=P-S,\\ &(\bar{\mathcal{P}}\mathcal{P})=(\bar{\mathfrak{N}}\mathfrak{N})=P+A, && (\bar{\lambda}\mathcal{P})=(\bar{\lambda}\mathfrak{N})=P-S',\end{aligned} \tag{A1}$$

This parametrization,[1] it should be stressed, *only depends* on charge conjugation and isospin, and it holds for the *full* $qq$ and $q\bar{q}$ scattering amplitudes, not just for the imaginary parts. The power of the Mueller analysis is that it relates a sum over many complicated processes to a much simpler one, forward 3-to-3 scattering. Since $t$ and $u$ both go as $-\mu\sqrt{s}$ in the pionization region and since the additive quark model gives good results at nonasymptotic energies for a variety of two-body reactions, it seems natural to apply it here. We view Fig. 1 as a two-step process splitting the Mueller amplitude into two successive two-body scatterings. If, as in Sec. II, we restrict ourselves to the asymptotic ($P^2$, $P'^2$, etc.) and $s^{-1/4}$ terms ($PA$, $PA'$, etc.), then the intermediate particle must have the same quark content as $\bar{c}$. Therefore we take

$$\mathfrak{M}=\gamma_{a\bar{c}}(u)\gamma_{b\bar{c}}(t)g^c(q_\perp^2), \tag{A2}$$

where the $\gamma$'s are calculated by analogy with the two-body case, giving $\mathfrak{M}$ as a product of quark-quark scattering amplitudes. Thus, the reaction $pp\to\pi^+ +$anything, is given by

$$\mathfrak{M}(pp\to\pi^+)=(2P'+4P+2A+A')^2g^{\pi^+}(q_\perp^2),$$

where $P'$, $P$, $A$, and $A'$ are defined as in Eq. (A1). By charge conjugation $\mathfrak{M}(p\bar{p}\to\pi^+)=\mathfrak{M}(\bar{p}p\to\pi^-)$, so $g^{\pi^+}(q_\perp^2)=g^{\pi^-}(q_\perp^2)$. By making similar expansions for other reactions involved and combining terms, we obtain the sum rules given in Eq. (2.8).

This model, Eq. (A2), only uses the symmetry of the quark model to classify and count the $\gamma$'s and not for their $s,t,u$ physics. Additivity is assumed for the $\gamma$'s, *but not* with respect to the intermediate particle as a whole. Since $t$ and $u$ both go as $-\mu\sqrt{s}$, it seems unlikely that additivity corrections to $a$ and $b$, e.g., rescattering of $a\bar{c}$ again before or after $b\bar{c}$ scatters, will be important. Double scattering effects are small in two-body scattering.

At $x=0$ for final kaons, $c=K^\pm$, we obtain the first and second sum rules listed in Eq. (2.18) and also the third if $S=S'$ and $(\lambda\bar{\lambda})=(\bar{\lambda}\bar{\lambda})$. Formally these also follow for final nucleons [and Eqs. (A3) and following, with $K^+\to p$, $K^-\to\bar{p}$]; however, the $\gamma$'s then involve baryon-antibaryon annihilation so they should not hold in the quark model. Since the invariant cross sections involving neutrons will be difficult to measure, we set $P=P'$ and obtain

$$F(\bar{p}a\to K^+)+F(pa\to K^+)=2F(\pi^-a\to K^+)+F(\pi^+a\to K^+), \tag{A3}$$

$$F(pa\to K^-)+F(\bar{p}a\to K^-)=2F(\pi^+a\to K^-)+F(\pi^-a\to K^-),$$

since $F(\bar{p}a\to K^+)=F(\bar{n}a\to K^+)$ and $F(pa\to K^-)=F(na\to K^-)$ [in the Regge approach $F(\bar{p}a\to c)+F(pa\to c)=2F(\pi^-a\to c)+F(\pi^+a\to c)$ since $F(pa\to c)=F(na\to c)$ if the $\rho-A_2$ exchange degeneracy relation for the residues, $\beta_\rho^b=\beta_A^b$, can be extended to $g_{\rho j}^c(q_\perp^2)=g_{Aj}^c(q_\perp^2)$]. Other interesting equalities are ($d=$deuteron)

$$\begin{aligned}&F(\bar{p}a\to\pi^\pm)=F(na\to\pi^\pm),\\ &F(\pi^\mp d\to\pi^+)=F(\pi^\pm d\to\pi^-),\\ &F(np\to\pi^+)=F(np\to\pi^-),\\ &F(K^-d\to\pi^+)=F(K^+d\to\pi^-),\\ &F(\bar{p}p\to K^+)=F(\bar{p}p\to K^-),\end{aligned} \tag{A4}$$

if $P=P'$. We do not list the other $P=P'$ relations, e.g., for $c=K^0$, $\bar{K}^0$, $\pi^0$, $\bar{p}$, and $\bar{\Lambda}$.

## APPENDIX B: PHENOMENOLOGICAL SUM RULES

In this appendix the analogous sum rules based on phenomenological two-body residues are given and compared with experiment. We take[30]

$$\begin{aligned}&\beta_P^{\pi^+}=3.6,\quad \beta_P^p=6.1,\quad \beta_P^{K^+}=2.9,\\ &\beta_F^{\pi^+}=\beta_\rho^{\pi^+}=2.9,\\ &\beta_F^p=\beta_\rho^{\pi^+}=6.3,\quad \beta_\rho^p=\beta_A^p=1.4,\\ &\beta_F^{K^+}=\beta_\rho^{K^+}=\beta_\omega^{K^+}=\beta_A^{K^+}=1.5,\end{aligned} \tag{B1}$$

and to order $s^{-1/4}$ obtain

$$2F(pp\to\pi^\pm)+3.3F(K^-p\to\pi^\pm)=2.7F(\pi^+\to\pi^\pm)+3.4F(\pi^-p\to\pi^\pm), \tag{B2a}$$

$$1.8F(\pi^-p\to\pi^+)+F(\pi^+p\to\pi^+)=1.8F(\pi^-p\to\pi^-)+F(\pi^+p\to\pi^-), \tag{B2b}$$

$$1.9F(K^-p\to\pi^\pm)+F(\pi^+p\to\pi^\pm)=1.9F(K^+p\to\pi^\pm)+F(\pi^-p\to\pi^\pm), \tag{B2c}$$

TABLE II. Tests of sum rules based on phenomenological residues.[33]

| | Sum rule | $\int_0^\infty F(ab\to c;\, q_\perp^2)dq_\perp^2$ (mb) | $P_{\rm lab}$ (GeV/$c$) | References |
|---|---|---|---|---|
| plus mode of (B2a) | $2.7F(\pi^+p\to\pi^+)+3.4F(\pi^-p\to\pi^+)$ | 25 | 16 | 14 |
| | $2F(pp\to\pi^+)+3.3F(K^-p\to\pi^+)$ | 20, 17 | 24, 12($pp$); 10($Kp$) | 28; 20 |
| negative mode of (B2a) | $2.7F(\pi^+p\to\pi^-)+3.4F(\pi^-p\to\pi^-)$ | 20 | 16 | 14 |
| | $2F(pp\to\pi^-)+3.3F(K^-p\to\pi^-)$ | 16, 14 | 24, 12($pp$); 10($Kp$) | 28; 20 |
| (B2b) | $1.8F(\pi^-p\to\pi^+)+F(\pi^+p\to\pi^+)$ | 11 | 16 | 14 |
| | $1.8F(\pi^-p\to\pi^-)+F(\pi^+p\to\pi^-)$ | 10 | 16 | 14 |
| negative mode of (B2c) | $1.9F(K^-p\to\pi^-)+F(\pi^+p\to\pi^-)$ | 7.3 | 10($Kp$); 16($\pi p$) | 20; 14 |
| | $1.9F(K^+p\to\pi^-)+F(\pi^-p\to\pi^-)$ | 7.2 | 12.7($Kp$); 16($\pi p$) | 21; 14 |

$$2F(\bar{p}p\to\pi^\pm)+3.3F(K^+p\to\pi^\pm)=2.7F(\pi^-a\to\pi^\pm)+3.4F(\pi^+p\to\pi^\pm). \tag{B2d}$$

In Table II (see Ref. 33) comparison with experiment is made for $\int_0^\infty F(q_\perp^2)dq_\perp^2$. There is good agreement except for (B2a) where the agreement is worse than for the analogous symmetry relation (2.8a) of Table I. Because of experimental error bars, possible normalization errors, and the neglect of $O(s^{-1/2})$ terms, we do not think this discrepancy is serious.

From Eq (2.11) the phenomenological difference relations for the $\rho$-$P$ contribution are

$$\begin{aligned}\tfrac{4}{3}\Delta(pp)&=\Delta(\pi^+p)\\&=-1.8\Delta(\pi^-p)\\&=1.7\Delta(K^+p)\\&=-4.5\Delta(K^-p),\end{aligned} \tag{B3}$$

where the coefficient of $\Delta(K^-p)$ is about half that of Eq. (2.12). Figure 2 is unchanged except that now the $\pi^-p$ points are lowered by 10%. We have from Eq. (2.15) sizable changes for the $f$-$P$ contribution

$$\begin{aligned}0.46\overline{\Sigma}(pp)&=\overline{\Sigma}(\pi^+p)\\&=1.9\overline{\Sigma}(K^+p),\end{aligned} \tag{B4}$$

and since $\Sigma(pp)$ and $\Sigma(\pi^+p)$ are now not proportional,

$$\begin{aligned}&0.60\Sigma(pp)+\Sigma(K^-p)=1.8\Sigma(\pi^+p),\\&\Sigma(\pi^+p)=\Sigma(\pi^-p).\end{aligned} \tag{B5}$$

Experimental data at about 16 GeV give[14,20,28]

$$0.60\Sigma(pp)+\Sigma(K^-p)=10\text{ mb},$$

$$1.8\Sigma(\pi^+p)=14\text{ mb}$$

and at $q_\perp^2=0$

$$\Sigma(\pi^+p)=46\text{ mb}/(\text{GeV}/c)^2,$$

$$\Sigma(\pi^-p)=43\text{ mb}/(\text{GeV}/c)^2.$$

For final kaons and nucleons, and to terms of order $s^{-1/2}$ in the pionization region and of order $s^{-1}$ in the target-fragmentation region, we also have the prediction

$$4.2F(K^+a\to c)+1.7F(\pi^-a\to c)+F(\bar{p}a\to c)=4.2F(K^-a\to c)+F(pa\to c)+1.7F(\pi^+a\to c). \tag{B6}$$

The analog of the first sum rule of (2.18) involves $b=K^0$ or $\bar{K}^0$ unless further assumptions are made.

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[1] E. M. Levin and L. L. Frankfurt, Zh. Eksp. Teor. Fiz. Pis'ma Red. 2, 105 (1965) [JETP Lett. 2, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Lett. 16, 71 (1966).

[3] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966).

[4] K. Johnson and S. B. Treiman, Phys. Rev. Lett. 14, 189 (1965); V. Barger and M. H. Rubin, Phys. Rev. 140, B1365 (1965).

[5] For a recent test with the Serpukhov data see J. Coyne and S. Meshkov, Phys. Rev. D 6, 399 (1972).

[6] A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[7] A. Białas and O. Czyżewski, Phys. Lett. 35B, 576 (1971); R. C. Hwa, Phys. Lett. 37B, 405 (1971);

D. Horn, Nucl. Phys. B47, 559 (1972). For work on internal symmetries and inclusive reactions see the reference cited in H. J. Lipkin, in *Particle Physics*, proceedings of the Conference on Particle Physics, Irvine, California, 1971, edited by M. Bander, G. L. Shaw, and D. Y. Wong (A.I.P., New York, 1972); Report No. THY-44 (unpublished).

[8]C. A. Nelson and David Steele, NBS Report No. NBS-PUB-10/4/72 (unpublished).

[9]See, e.g., V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966); N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Lett. 22, 336 (1966), and later work by these authors.

[10]T. Ferbel, Phys. Rev. Lett. 29, 448 (1972).

[11]See the Appendix where similar sum rules based on phenomenological residues are compared with experiment.

[12]H. Joos, Phys. Lett. 24B, 103 (1967); K. Kajantie and J. S. Trefil, *ibid.* 24B, 106 (1967); F. Buccella and M. Colocci, *ibid.* 24B, 61 (1967).

[13]Chan Hong-Mo, H. I. Miettinen, D. P. Roy, and P. Hoyer, Phys. Lett. 40B, 406 (1972); Chan Hong-Mo, Rutherford High Energy Laboratory Report No. RPP/T/17, 1972 (unpublished).

[14]Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-London-Vienna-Warsaw Collaboration (ABBCCHLVW), report presented at the Fourth International Conference on High Energy Collisions, Oxford, England, 1972.

[15]We used $\sigma_T(K^-p) = 20.1$ mb. Using the data of Ref. 14 and checking such a comparison by using (2.8b) gives 12 mb and 11 mb to be compared with the entires for (2.8b) in Table I.

[16]Respectively, $F(\pi^+p \to \pi^+)$, $F(\pi^-p \to \pi^+)$, $F(\pi^+p \to \pi^-)$, and $F(\pi^-p \to \pi^-)$ are 30, 21, 18, 22 mb/(GeV/$c$)$^2$, and $\int dq_\perp^2$ for $F(K^-p \to \pi^-)$, $F(K^+p \to \pi^-)$, $F(\pi^+p \to \pi^-)$, and $F(\pi^-p \to \pi^-)$ are 2.4, 1.6, 2.6, 4.1 mb.

[17]J. V. Allaby *et al.*, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968,* edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[18]C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Lett. 7, 81 (1963); $\cos\lambda = (\frac{2}{3})^{1/2}$ by SU(6).

[19]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 19, 470 (1967).

[20]Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw Collaboration and Aachen-Berlin-CERN-London-Vienna Collaboration, as cited in footnote 53 of M. Deutschmann, in *Proceedings of the Amsterdam International Conference on Elementary Particles, 1971,* edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1972).

[21]S. Stone, T. Ferbel, P. Slattery, and B. Werner, Phys. Rev. D 5, 1621 (1972).

[22]G. Wolf, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972).

[23]K. C. Moffeit *et al.*, Phys. Rev. D 5, 1603 (1972).

[24]M. S. Chen *et al.*, Phys. Rev. Lett. 26, 1585 (1971). The values for $\sigma_{\rm tot}$ used by these authors for $\pi^+p$, $K^+p$, $pp$, and $\pi^-p$, respectively are 23.4, 17.4, 39.8, and 24.9 mb.

[25]The data used in Fig. 2 are from the following sources: $\pi^+p$ data from Ref. 14 and M. Alston-Garnjost *et al.*, Phys. Lett. 39B, 402 (1972); J. V. Beaupre *et al.*, *ibid.* 37B, 432 (1971); D. J. Crennell *et al.*, Phys. Rev. Lett. 28, 643 (1972); $pp$ data from Refs. 27 and 28 and E. R. Gellert, LBL Report No. LBL-749, 1972 (unpublished); $\pi^-p$ data from Ref. 14 and T. Ferbel, U. of Rochester Report No. COO-3065-25, 1972 (unpublished).

[26]M. Banner *et al.* (Saclay, Strasbourg), *in Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 344.

[27]N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Lett. 25, 557 (1970).

[28]H. J. Mück *etal.*, Phys. Lett. 39B, 303 (1972).

[29]The CERN ISR data, Ref. 26, for $\frac{2}{3}\Sigma(pp)$ are 11.6 mb at $(p_{\rm lab})^{1/4} = 0.162$ GeV$^{-1/4}$, 11.6 at 0.176, 11.7 at 0.212, and 10.2 at 0.244 with error bars of $\pm 0.8$ mb. At lower energies, $\frac{2}{3}\Sigma(pp)$ is $5.4 \pm 0.4$ at 0.427 (Ref. 27), 6.4 $\pm 0.4$ at 0.453 (Ref. 28), and $5.7 \pm 0.4$ at 0.477 (Ref. 27).

[30]For recent work on two-particle inclusive processes in the *fragmentation regions*, see R. C. Brower, R. N. Cahn, and J. Ellis, Phys. Rev. D 7, 2080 (1973).

[31]H. J. Lipkin, Phys. Rev. Lett. 16, 1015 (1966).

[32]C. A. Levinson, N. S. Wall, and H. J. Lipkin, Phys. Rev. Lett. 17, 1122 (1966); also P. G. O. Freund, *ibid.* 15, 929 (1965); J. J. J. Kokkedee, Phys. Lett. 22, 88 (1966).

[33]Respectively, $\int dq_\perp^2$ for $F(\pi^+p \to \pi^+)$, $F(\pi^-p \to \pi^+)$, $F(\pi^+p \to \pi^-)$, $F(\pi^-p \to \pi^-)$, $F(K^-p \to \pi^+)$, $F(K^-p \to \pi^-)$, $F(K^+p \to \pi^-)$; $F(pp \to \pi^+)$; $F(pp \to \pi^-)$, are 4.9, 3.6, 2.6, 4.0, 2.5, 2.4, 1.6; 5.5, 4.4; 4.0, 3.0 mb.