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Eikonal Models for Diffraction Dissociation on Nuclei*

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An upper bound to the cross section for diffraction dissociation on a nucleus is derived by combining the fundamental viewpoint of Good and Walker with the eikonal model. The bound is consistent with measurements of $\pi A \to (3\pi)A$ in the coherent region. A number of the observed features of diffractive production are shown to be understandable by assuming the existence of a complete set of states which are diagonal in nuclear matter, and which are absorbed approximately equally, as would be expected according to a naive quark model. A possible explanation for the apparent smallness of the absorption of 3π and 5π sytems in nuclei is given. Finally, speculations are made on how these results might apply to diffractive production on proton targets.

I. INTRODUCTION

According to the eikonal model, the elastic scattering amplitude for a hadron on a nucleus at very high energy is given by

$$M(t) = -4\pi i \int_0^\infty b\,db\, J_0(b\sqrt{-t})[e^{i\chi(b)} - 1], \tag{1}$$

$$\chi(b) = \tfrac{1}{2}F(0)D(b).$$

$F(0)$ is the forward elastic amplitude on individual nucleons, and $D(b)$ is the nuclear density at impact parameter b:

$$D(b) = \int_{-\infty}^{\infty} \rho(b,z)\,dz, \qquad A = 2\pi\int_0^\infty b\,db\,D(b). \tag{2}$$

(A is the number of nucleons and ρ is the nuclear density. The normalization is given by $\sigma_{\text{total}} = \text{Im}\,M(0)$ and $d\sigma/dt = |M(t)|^2/16\pi$. The possibility of energy dependence is suppressed.) The eikonal model can be derived from Glauber's multiple scattering theory, in the limit of many nucleons and negligible correlations.[1] It has been used with good success to predict total cross sections of neutrons[2] and K_L mesons[3] on nuclei.

The success of the eikonal model appears to be misleading, when one allows for the structure of the incident hadron. For, as has been emphasized by Goldhaber,[4] the system which rescatters inside the nucleus is not an asymptotic state, but one which in its rest frame is essentially newborn. Its forward scattering amplitude might therefore not be equal to $F(0)$. The additive-quark model,[5] which accounts correctly for the systematics of total cross sections on nucleons, predicts that forward scattering depends only on the internal quantum numbers of the state involved (quark content), and not on its detailed structure (quark wave function). The quark model could therefore be invoked to explain the success of the eikonal model for total cross sections on nuclei.

Another aspect of the structure of the incident hadron is the existence of additional states with the same quantum numbers. Equation (1) can be generalized to cover this possibility, by interpreting $F(0)$ as a matrix of forward scattering amplitudes among the various channels.[6] The off-diagonal matrix elements of M correspond to diffraction dissociation.[7] The diagonal elements contain contributions from coherent inelastic channels.[8] Historically, the two-channel analysis was applied to determine the ρ-nucleon total cross section from ρ^0 photoproduction.[9] The result, $\sigma_{\rho N} \simeq \sigma_{\pi N}$, cannot be interpreted as a true measurement of $\sigma_{\rho N}$,[10] but can nevertheless be understood on the basis of the quark model, as discussed above.

The final states produced by diffraction dissociation of pions ($3\pi, 5\pi$) (see Refs. 11 and 12) or nucleons ($N\pi, N\pi\pi$) (see Ref. 13) consist mostly or entirely of a continuum, rather than a small number of discrete states. Attempts have therefore been made recently to interpret $\chi(b)$ as a continuous "matrix",[14] but they suffer from uncertainty in the behavior to be expected of χ. From the experimental side, if one naively analyzes the low-mass 3π continuum in $\pi A \to 3\pi A$ as if it were a single second channel, the measured A dependence yields $\sigma_{(3\pi)N} \simeq \sigma_{\pi N}$.[11] This result is consonant with the quark model, since the 3π system has the same quantum numbers, except for spin and parity, as the π. The actual breakup into three distinct pions (six quarks) does not occur inside the nucleus, because of time dilation. Only after a time $\propto p_{\text{lab}}$ should one expect $\sigma_{(3\pi)N} \simeq 3\sigma_{\pi N}$.

In Sec. II, we examine general properties of eikonal models which are consistent with the fundamental viewpoint of Good and Walker.[7] In particular, we derive a bound on the magnitude of diffraction dissociation in such models. The models are free from the objections discussed above, because they require no specific assumptions about the states which propagate inside the nucleus. They rely strongly on unitarity, as models for elastic scattering and diffraction dissociation ought to. In Sec. III, we apply the intuition from the quark model to obtain more specific results.

II. BOUNDS ON DIFFRACTION DISSOCIATION

Let us assume there are an unspecified, but finite, number of states with the same internal quantum numbers as the incident particle. New physics would not be expected to creep into the continuum limit. Since M is linearly related to the unitary S matrix, it can be diagonalized by a unitary transformation. By analogy with (1), we can therefore write

$$\begin{aligned} M_{fj}(t) &= -4\pi i \int_0^\infty b\,db\, J_0(b\sqrt{-t})G_{fj}(b)\,, \\ G_{fj}(b) &= \sum_n U^*_{nf}\, U_{nj}(e^{i\chi_n}-1), \\ \delta_{fj} &= \sum_n U^*_{nf}\, U_{nj}\,. \end{aligned} \tag{3}$$

Both U and χ may vary with the impact parameter b. This formula embodies the viewpoint of Good and Walker.[7]

For a fixed initial state j, let us define $P_n = |U_{nj}|^2$ to be the probability associated with the nth diagonal channel. ($\sum_n P_n = 1$ and $0 \leq P_n \leq 1$.) The elastic cross section, integrated over t, is

$$\begin{aligned} \sigma_{\rm el} &= 2\pi \int_0^\infty b\,db\, \sigma_{\rm el}(b)\,, \\ \sigma_{\rm el}(b) &= |G_{jj}(b)|^2 \\ &= \Big|\sum_n P_n(e^{i\chi_n}-1)\Big|^2\,. \end{aligned} \tag{4}$$

The cross section for diffraction dissociation, integrated over t and summed over final states, is

$$\begin{aligned} \sigma_{\rm dd} &= 2\pi \int_0^\infty b\,db\, \sigma_{\rm dd}(b)\,, \\ \sigma_{\rm dd}(b) &= \sum_{f\neq j} |G_{fj}(b)|^2 \\ &= \sum_n P_n |e^{i\chi_n}-1|^2 - \sigma_{\rm el}(b)\,. \end{aligned} \tag{5}$$

The total cross section is

$$\begin{aligned} \sigma_{\rm total} &= 2\pi \int_0^\infty b\,db\, \sigma_{\rm total}(b)\,, \\ \sigma_{\rm total}(b) &= -2{\rm Re}\sum_n P_n(e^{i\chi_n}-1)\,. \end{aligned} \tag{6}$$

$T_n = (e^{i\chi_n}-1)/2i$ lies inside the standard unitary circle

$$\text{Im}T_n - |T_n|^2 = I_n \geq 0\,, \tag{7}$$

where I_n represents the contribution of nondiffractive inelastic states to the unitarity sum. Thus, $0 \leq \text{Im}T_n \leq 1$ by unitarity alone. On physical grounds, we expect absorptive effects to dominate, so that $0 \leq \text{Im}T_n \leq \frac{1}{2}$ and $\text{Re}T_n \simeq 0$. This can be seen by writing (7) as

$$\text{Im}T_n = \tfrac{1}{2} \pm [\tfrac{1}{4} - (\text{Re}T_n)^2 - I_n]^{1/2}\,, \tag{8}$$

assuming T_n is driven by I_n, and appealing to continuity in b or in the strength of the interactions to choose the lower sign. We therefore assume

$$A_n = 1 - e^{i\chi_n}\,, \quad \text{Im}A_n = 0\,, \quad 0 \leq A_n \leq 1\,. \tag{9}$$

A_n represents the fraction of the nth eigenchannel which is absorbed. This is easily shown (see the Appendix) to imply

$$\begin{aligned} &0 \leq \sigma_{\rm el}(b) \leq 1\,, \\ &0 \leq \sigma_{\rm dd}(b) \leq [\sigma_{\rm el}(b)]^{1/2} - \sigma_{\rm el}(b)\,. \end{aligned} \tag{10}$$

The upper bound on $\sigma_{\rm dd}$ can be written as $\sigma_{\rm el}(b) + \sigma_{\rm dd}(b) \leq \frac{1}{2}\sigma_{\rm total}(b)$, and is thus a generalization of the familiar total absorption limit. $\sigma_{\rm dd}$ can easily be made zero by making all of the χ_n's equal, or by making all but one of the P_n's equal to zero.

To examine the upper bound given by (10), let us assume that elastic scattering can be approximated by an eikonal form

$$\sigma_{\rm el}(b) = (e^{-\lambda D(b)} - 1)^2\,, \tag{11}$$

where $\lambda = \frac{1}{2}\sigma_0$, and σ_0 is the cross section on a single nucleon. Using $\sigma_0 = 25$ mb, and a nuclear density

$$\rho(\vec{r}) = \rho_0/[1 + \exp((|\vec{r}| - r_0A^{1/3})/d)]\,, \tag{12}$$

with $r_0 = 1.12$ F, $d = 0.545$ F as in Ref. 11, we obtain the upper bound in Fig. 1. Also shown are the 15-GeV measurements of coherent 3π production, in the region $m_{3\pi} \leq 1.9$ GeV.[11,12] The curve shown has been corrected for the suppression due to minimum momentum transfer, using the two-channel eikonal model with $\sigma_\pi = \sigma_{3\pi} = 25$ mb, as an approximation. It is thus an estimate of the cross section at infinite energy. It has also been corrected approximately for the undetected $\pi^-\pi^0\pi^0$ channel, by a factor of two. (This factor should be 2 for pure $\rho\pi$ production and 1.5 for pure $f\pi$ or $\epsilon\pi$, provided that Bose statistics for the nonreso-

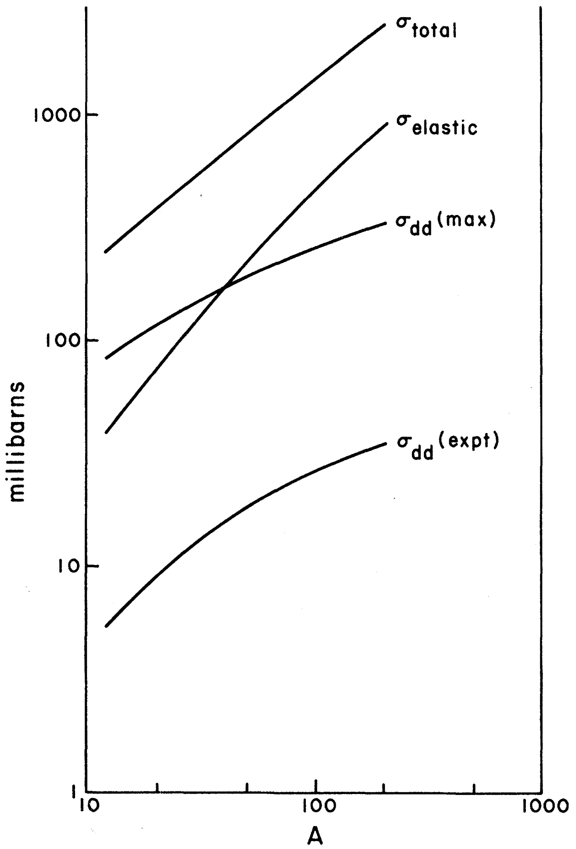

FIG. 1. Cross sections as a function of nucleon number A. σ_{total} and σ_{el} are obtained from the one-channel eikonal model ($\sigma_{nucleon}=25$ mb). σ_{dd}(max) is the upper bound (10). σ_{dd}(expt) is an estimate of the diffraction-dissociation cross section based on Ref. 12 (see text).

nant pions can be ignored.) The 3π mass distribution (not shown) falls by a factor of 5 between 1 and 2 GeV, even after the t_{min} correction is applied.[12] The cross section for $m_{3\pi}\geq 1.9$ GeV is, therefore, probably not large even at very high energy. The cross section for 5π, and presumably 7π, etc., production is also not large. It therefore appears that the total diffraction-dissociation cross section is considerably smaller than the calculated upper bound, and will remain so at very high energy.

The upper bound (10) can be attained only if some eigenchannel is totally absorbed ($A_n=1$) and another is totally transmitted ($A_{n'}=0$). If we assume a stronger condition

$$A_L \leq A_n \leq A_H \,, \tag{13}$$

then, as shown in the Appendix,

$$\begin{aligned} &A_L \leq [\sigma_{el}(b)]^{1/2} \leq A_H \\ &\sigma_{dd}(b) \leq \{A_H - [\sigma_{el}(b)]^{1/2}\}\{[\sigma_{el}(b)]^{1/2} - A_L\}. \end{aligned} \tag{14}$$

To employ this bound, let us assume an eikonal form for the absorption of each eigenchannel:

$$e^{i\chi_n(b)} - 1 = e^{-\lambda_n D(b)} - 1\,, \quad \lambda_L \leq \lambda_n \leq \lambda_H\,. \tag{15}$$

If we retain the approximation (10) for elastic scattering,

$$\sigma_{dd}(b) \leq [e^{-\lambda D(b)} - e^{-\lambda_H D(b)}][e^{-\lambda_L D(b)} - e^{-\lambda D(b)}]\,, \quad \lambda_L \leq \lambda \leq \lambda_H\,. \tag{16}$$

According to the quark-model arguments of Sec. I, we expect all of the absorption coefficients to be about equal. However, we find that the bound (16), calculated with $2\lambda_L=15$ mb, $2\lambda=25$ mb, $2\lambda_H=40$ mb, is approximately equal to the experimental estimate. Thus, a relatively large range of λ_n is required.

III. A "QUARK" MODEL

Now let us assume the existence of a set of basis states which propagate in diagonal fashion through nuclear matter, and which are absorbed according to the eikonal model:

$$\begin{aligned} M_{fj}(t) &= -4\pi i\int_0^\infty b\,db\,J_0(b\sqrt{-t}) \times \sum_n U^*_{nf}U_{nj}[e^{-\lambda_n D(b)}-1]\,, \\ \delta_{fj} &= \sum_n U^*_{nf}U_{nj}\,, \end{aligned} \tag{17}$$

with $U_{nn'}$ and λ_n independent of b. We require $\mathrm{Re}\lambda_n \geq 0$, and assume $\mathrm{Im}\lambda_n=0$ (pure absorption) for simplicity.

According to the quark picture discussed in Sec. I, we expect λ_n to be nearly independent of n. This independence is restricted by the bound (16). However, $D(b)$ is less than 0.2 mb^{-1} even for Pb at $b=0$, so it is a fair approximation to set $\lambda_n=\frac{1}{2}\sigma_0+\alpha_n$ and work to lowest order in α_n:

$$M_{fj}(t) = 4\pi i\int_0^\infty b\,db\,J_0(b\sqrt{-t}) \times \begin{cases} 1-e^{-\sigma_0 D(b)/2}, & \text{if } f=j \\ D(b)e^{-\sigma_0 D(b)/2}\sum_n U^*_{nf}U_{nj}\alpha_n, & \text{if } f\neq j. \end{cases} \tag{18}$$

Elastic scattering has the original eikonal form (1), which agrees with measurements of total cross sections on nuclei.[2,3] The shape of the mass spectrum in diffraction dissociation is buried in the unknown factor $\sum_n U^*_{nf}U_{nj}\alpha_n$. The total magnitude will be small compared to the bound given by (10), in agreement with data. The variation of the cross section with target nucleus is completely determined, and is equivalent to the two-channel eikonal model in the limit where the two channels are absorbed equally.[6,11] The dependence on A therefore agrees with the data for $\pi A \to (3\pi)A$.[11] The prediction that the mass distribution is independent of A at high energy agrees with the same data.

On a large nucleus, (18) predicts that elastic scattering is *central*, i.e., largest at $b=0$; while diffraction dissociation is *peripheral*, i.e., largest at $b \simeq$ nuclear radius. We therefore expect the momentum-transfer distribution to be somewhat steeper for diffraction dissociation than for elastic scattering. To evaluate this effect, we use $\sigma_0 = 25$ mb and the nuclear density (12). Defining $d\sigma/dt \propto e^{Bt}$ for $t \to 0$, with B in GeV^{-2}, we find for $A=12$, 64, 208 that $B=75$, 162, 317 for elastic scattering, and 83, 197, 422 for diffraction dissociation. The effect is thus quite large on heavy nuclei. The actual slopes should be slightly larger because of finite hadron size.

IV. CONCLUSION

We have derived an upper bound to the coherent production of inelastic states (diffraction dissociation) on nuclei, by combining the viewpoint of Good and Walker[7] with the eikonal model.[1] The bound is considerably larger than the data for $\pi A \to (3\pi, 5\pi)A$ in the mass region which is accessible to experiment at 15 GeV,[11] and is therefore consistent with it.

The smallness of diffraction dissociation, as compared to the upper bound, can be understood by assuming the existence of a complete set of states which propagate in a diagonal fashion through nuclear matter, and which are absorbed *approximately* equally, as would be expected on a naive quark model. The absorption coefficients must, however, vary by at least a factor of 2–3 according to the bound (16). This picture also explains the observed A dependence of the cross section, and the A independence of the shape of the mass distribution. Data for other coherent production reactions on nuclei[12] are less precise, but also consistent with this picture.

The smallness of diffraction dissociation could instead be accounted for by assuming that the transformation between the physical states and the diagonal ones [$U_{nn'}$ in (17)] is approximately the identity, rather than by assuming that the absorption strengths (λ_n) are approximately equal.[15] This assumption would be motivated by the Deck model.[16] It would not lead to the correct A dependence of $\pi A \to (3\pi)A$, or to the observed A independence of the mass distribution. These criticisms of the Deck model can thus be added to those of Ref. 16.

We have neglected the longitudinal momentum transfer which is required kinematically, at finite beam energy. Roughly speaking, its effect must be to reduce the coherent cross section, because the four-momentum transfer in the forward direction is $|t| = |t|_{\min} \simeq [(m_f^2 - m_i^2)/2E_{\mathrm{lab}}]^2$, while the cross section $d\sigma/dt$ falls off on the scale of (nuclear radius)$^{-2}$. The exact behavior is unclear, however, since it depends on the masses of all of the intermediate states. This was, of course, ignored in the two-channel model which was used in Ref. 11 to fit $\pi A \to (3\pi, 5\pi)A$, and *perhaps accounts for the anomalously small effective cross sections* ($\simeq$15 mb) obtained for 5π and higher-mass 3π systems, especially at 9 GeV.[12]

The point of view we have taken does not predict the shape of the mass distribution. Experimentally, diffraction dissociation seems to be confined rather strongly to the production of low-mass states, even at the energies of the CERN Intersecting Storage Rings, where the minimum momentum transfer is completely negligible.[17] Qualitatively, this might be explained by saying that the quark wave function for the incident particle gets changed only slightly by differential absorption, and as a result, corresponds mainly to states which are similar to it. We also do not predict the details of the energy dependence, although we expect the cross section to be approximately constant.

It is tempting to apply our results to diffractive production on proton targets. This cannot be done rigorously, because the various states may have different distributions in impact parameter, and therefore the effective $D(b)$ may vary with n. For production on a large nucleus, we were able to ignore this effect. For production on protons, it is probably significant, as it offers a natural explanation for the observed variation of diffractive slope with final-state mass.[18] Also, helicity flip may be important for proton targets.[19] Proceeding in spite of these objections, let us parametrize elastic scattering as pure imaginary, with $d\sigma/dt \propto e^{Bt}$. The upper bound (10) states that $\sigma_{\mathrm{dd}} < (2/x - 1)\,\sigma_{\mathrm{el}}$, where $\sigma_{\mathrm{el}} = \frac{1}{4} x \sigma_{\mathrm{total}}$, and $x = \sigma_{\mathrm{total}}/4\pi B$ is the fraction absorbed at zero impact parameter. For πp scattering, we might take $\sigma_{\mathrm{total}} = 25$ mb and $B = 9\ \mathrm{GeV}^{-2}$. Then $x = 0.57$, $\sigma_{\mathrm{el}} = 3.6$ mb, and $\sigma_{\mathrm{dd}} \leq 8.9$ mb. This bound should presumably be applied separately to the diffractive breakup of the pion and of the proton, and therefore elastic scattering plus single dissociation should account for at most 85% of the total cross section. In the quark-type model of Sec. III, diffraction dissociation would be considerably smaller. As a function of impact parameter, it would have the form $\chi(b) e^{i\chi(b)}$, where $1 - e^{i\chi(b)} = x e^{-b^2/2B}$. This form is slightly more peripheral than elastic scattering. Using the above numbers, it predicts a slightly steeper slope, $d\sigma/dt \propto e^{10.0t}$ at $t=0$, and a first diffraction zero at $t=0.7$, in units where GeV = 1.

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APPENDIX

We wish to maximize $\sigma_{dd}(b)$, where

$$\begin{aligned}
&\sigma_{el}(b) = \left(\sum P_n A_n\right)^2 , \\
&\sigma_{dd}(b) = \sum P_n A_n{}^2 - \sigma_{el}(b) , \\
&1 = \sum P_n , \\
&A_L \le A_n \le A_H .
\end{aligned} \tag{A1}$$

Using Lagrange multipliers to enforce the constraints, we write

$$L = \sum P_n A_n{}^2 + \alpha\left\{\sum P_n A_n - [\sigma_{el}(b)]^{1/2}\right\} + \beta\left(\sum P_n - 1\right) . \tag{A2}$$

Varying L with respect to P_n, we obtain

$$0 = A_n{}^2 + \alpha A_n + \beta . \tag{A3}$$

If A_j lies in the interior region $A_L < A_j < A_H$, then varying L with respect to A_j gives

$$0 = P_j(2A_j + \alpha) . \tag{A4}$$

Assuming $P_j \ne 0$ would enable us to evaluate α and β, and find that $A_n = A_j$ for all n, so $\sigma_{dd}(b) = 0$. We conclude that to maximize $\sigma_{dd}(b)$, A_j can lie in the interior only if $P_j = 0$. Defining P to be the sum of P_n's over the set of n for which $A_n = A_L$, we obtain

$$\begin{aligned}
&\sigma_{el}(b) = [PA_L + (1-P)A_H]^2 , \\
&\sigma_{dd}(b) = P(1-P)(A_H - A_L)^2 , \\
&0 \le P \le 1 .
\end{aligned} \tag{A5}$$

Eliminating P yields

$$\begin{aligned}
&A_L \le [\sigma_{el}(b)]^{1/2} \le A_H , \\
&0 \le \sigma_{dd}(b) \le \{A_H - [\sigma_{el}(b)]^{1/2}\}\{[\sigma_{el}(b)]^{1/2} - A_L\} .
\end{aligned} \tag{A6}$$

Substituting the extreme values $A_L = 0$ and $A_H = 1$ yields (10).

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[10] Of course, $\sigma_{\rho\eta}$ cannot be precisely defined because the ρ decays strongly. But one would prefer to let it age for a time $\sim 1/\Gamma$ after production, in order for it to become approximately "asymptotic".

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