

factory than the larger one, for which  $\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0) \approx 630$  eV.

It is important to note that if we were to set the  $(3, 3^*)$  mass term to zero (i.e.,  $A_3 = B_3 = 0$ ), then the parameters  $A_8$  and  $B_8$  would be completely determined from the mass relations of Eq. (19) [ $B_8 = 0$  for the  $(8, 8)$ ]. The values of  $h'$ ,  $a_0$ ,  $a_2$  would then be fixed, and moreover they would all be highly unsatisfactory ( $h' \approx -4$ ,  $a_0 \approx -1.5 m_\pi^{-1}$ , and

$a_2 \approx -0.7 m_\pi^{-1}$ ). We may therefore summarize what we have learned from the  $\pi^+ \pi^- \pi^0$  mode of  $\eta$ -decay by saying that the kinetic Lagrangian must break  $SU(3) \times SU(3)$  [but not  $SU(3)$ ], and that the mass Lagrangian must contain the  $(3, 3^*) \oplus (3^*, 3)$  representation and one other representation such as the  $(8, 8)$ . To find out more about this other representation we shall need accurate data on  $\pi$ - $\pi$ ,  $K$ - $\pi$ , and  $K$ - $K$  scattering.

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## Pion-Nucleon Charge-Exchange Scattering in a Regge-Regge-Cut Model

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We have obtained a fit to most of the available data on pion-nucleon charge-exchange scattering using a model with Regge-Regge cuts. This shows quantitatively the importance of the double-Regge cuts, as opposed to the Regge-Pomeron cuts. We make comments on a recent work by Worden claiming that, in certain idealized limits, the Regge-Regge cuts in this process cancel. It is suggested that because of the strong assumptions in that work, many of which are probably not valid, the conclusions are not necessary. Comparisons are also made with several other works on this process.

### I. INTRODUCTION

For a long time pion-nucleon charge-exchange scattering has been a favorite reaction for testing models of Regge poles and cuts. Recent polarization data<sup>1</sup> have, however, shown strong disagreement with the original predictions of two of the

currently popular Regge-cut models, namely, the weak-cut<sup>2</sup> and the strong-cut models.<sup>3</sup> Both of these models in their unmodified form basically include the contributions of the Regge cut due to the simultaneous exchange of the  $\rho$  and the Pomeron trajectories and predict large negative polarization in the vicinity of  $|t| \approx 0.4$  (GeV/c)<sup>2</sup>.

There is a disagreement between the Argonne and the CERN experimental data mentioned in Ref. 1. But both of these clearly rule out large negative values of polarization in this region. Some time ago, one of us (K.V.) suggested that the non-Pomeron cuts could also play an important role in high-energy scattering.<sup>4,5</sup> The importance of double-particle (Regge) exchange was also suggested by a number of authors with somewhat different points of view.<sup>6</sup> In the present work, we have successfully fitted most of the available data in this reaction for the pion lab energy region from 4.8 GeV to 60 GeV and up to a very large momentum transfer [ $\sim 4$  (GeV/c)<sup>2</sup>] using that model. The fits establish quantitatively the importance of the non-Pomeron cut contributions.

In a recent work Worden<sup>7</sup> has claimed that, in a model with SU(3) symmetry, strong exchange degeneracy, duality, etc., the Regge cuts due to  $\rho$ - $P'$  and  $\omega$ - $A_2$  exchange cancel, and hence the pion-nucleon charge-exchange process cannot be explained in terms of these cuts. We will point out in the following that in the real world with broken symmetries there will be nonvanishing effects of double-Regge cuts due to the exchange of a whole set of vector-tensor meson trajectories.

To calculate Regge-cut contributions from first principles has been found to be extremely involved and intractable from a practical point of view. Several absorption models based on the eikonal-convolution method,<sup>3</sup> box diagrams, etc., have been used from time to time. Many of these do not have complete theoretical justification. In particular, doubts have been expressed about the validity of the absorption model.<sup>8</sup> It may be that the absorption model is not valid to calculate both real and imaginary parts. In order to avoid these ambiguities, we have simply replaced the cuts by effective poles at the branch points, having the same signature and the nonsense-choosing mechanisms at the negative integers. The residues are to be regarded as some effective average quantities. It is true that this procedure introduces more parameters than the various phenomenological prescriptions to calculate the cuts from the pole parameters. On the other hand, we will be able to take the residue structure as extremely simple (just one exponentially decreasing function for each relevant contribution). This is to be contrasted with a number of previous works on the subject where quite complicated residue functions have to be chosen in order to fit the data. *Thus the total number of parameters used is comparable to the other existing models which attempt to fit the data quantitatively.* In addition, we fit most of the available world data for a wide range of energy and momentum transfer. Many of the existing

works just consider a limited momentum transfer region. Thus our fits will be relatively free from ambiguities of various models and prescriptions to calculate the cut contributions and cover an extensive range of data. In Sec. II we discuss the model and present the results. In Sec. III comparison is made with other works on the subject and comments are made on Worden's work.

## II. DETAILS OF THE MODEL AND RESULTS

In this section we will consider explicitly only the  $\rho$ ,  $P'$ , and  $P$  trajectories and the cuts generated by them.  $\omega A_2$  and  $K^* \bar{K}^{**}$  cuts will be considered later on. We assume weak exchange degeneracy ( $\alpha_\rho = \alpha_{P'}$ ) for the sake of simplicity and take the  $\rho$ ,  $P'$  trajectories as

$$\alpha_1(t) = \alpha_0 + \alpha' t, \quad \alpha' = (1 - \alpha_0)/m_\rho^2. \quad (1)$$

The Pomeron trajectory is taken to be

$$\alpha_p(t) = 1 + \alpha'_p t. \quad (2)$$

The non-Pomeron branch points [ $\rho P'$ ,  $\rho \rho P'$  (or  $\rho P' P'$ ), etc.] are given by

$$\alpha_2(t) = 2\alpha_0 - 1 + \alpha' t/2, \quad (3)$$

$$\alpha_3(t) = 3\alpha_0 - 2 + \alpha' t/3, \quad (4)$$

etc.

We simulate possible absorptive or diffractive corrections by considering the  $\rho P$  branch point

$$\bar{\alpha}(t) = \alpha_0 + \frac{\alpha' \alpha'_p}{\alpha' + \alpha'_p} t. \quad (5)$$

The last one is just one of the several possibilities that can be considered and our model is not particularly dependent on this way of representing absorption effects. The Pomeron trajectory also could be given a  $\sqrt{-t}$  form, so that multi-Pomerons give rise to the same trajectory. The main idea in the present model is that the  $\rho P'$  cut makes significant contribution for low values of  $t$  and the various non-Pomeron cuts dominate different  $t$  regions. In addition the Pomeron-absorptive correction could be expected to fall rapidly as a function of  $t$ .

We use the standard  $A'$  and  $B$  amplitudes defined by Singh.<sup>9</sup> The differential cross section for the charge-exchange reaction, the polarization of the recoil nucleon, and the difference of the  $\pi^- p$  and  $\pi^+ p$  total cross sections are given by

$$\frac{d\sigma}{dt} = \frac{m^2}{16\pi s q^2} \left[ \left(1 - \frac{t}{4m^2}\right) |A'|^2 + \frac{t}{4m^2} \left( s - \frac{(m+\omega)^2}{(1-t/4m^2)} \right) |B|^2 \right], \quad (6)$$

$$P = -\frac{\sin\theta}{16\pi\sqrt{s}} \frac{\text{Im}[A'B^*]}{d\sigma/dt}, \quad (7)$$

and

$$\Delta\sigma = -\frac{\sqrt{2} \text{Im}A'(s, t=0)}{p}, \quad (8)$$

where  $\omega$  and  $p$  are the pion lab energy and momentum,  $q$  is the pion c.m. momentum, and  $s$  and  $t$  are the usual kinematic variables.  $\theta$  is the c.m. scattering angle, and  $m$  is the proton mass.

Various contributions to the amplitudes are given by

$$A'_i(s, t) = \gamma_i e^{\beta_i t} (1 - e^{-i\pi\alpha_i(t)}) \Gamma(1 - \alpha_i(t)) \left(\frac{\nu}{\nu_i}\right)^{\alpha_i(t)}, \quad (9)$$

$$B_i(s, t) = \beta_i e^{\alpha_i t} (1 - e^{-i\pi\alpha_i(t)}) \times \Gamma(1 - \alpha_i(t)) \left(\frac{\nu}{\nu_i}\right)^{\alpha_i(t)-1}, \quad (10)$$

where  $\nu = (s - u)/4m$ . Here the index  $i$  runs over various terms under consideration and  $A' = -\sqrt{2} \sum_i A'_i$ .  $\gamma_i, \beta_i$  are constants. We take the nonsense-choosing mechanisms for both  $A'$  and  $B$ . The usual logarithmic terms associated with the cuts are omitted for the following reason. Various theoretical models give different constants in the denominator along with the  $\ln(\nu/\nu_0)$  term and are such that the logarithmic dependence may become noticeable only at asymptotic energies.

The scale factors  $\nu_i$  can be all taken to be the same or preferably related to  $\nu_1$  (scale factor for the  $\rho$  pole) by some theoretical relations. Then the dependence on  $t$  can be absorbed in the exponentials. A Veneziano type of ansatz<sup>10</sup> for the effective cut contributions, for example, gives  $\nu_1 = 1/2m\alpha'$ ,  $\nu_2 = 2\nu_1$ ,  $\nu_3 = 3\nu_1$ ,  $\bar{\nu} = [(\alpha' + \alpha'_b)/\alpha'_b]\nu_1$ , etc. During the course of fitting we varied  $\alpha_0$ , but found that in all cases it settled to a value close to 0.48. Similarly,  $\nu_1$  was kept as a variable parameter initially but, amazingly enough, the fitted value came extremely close to the value given above.

The experimental data are taken from the sources mentioned in Refs. 1 and 11. Some of the fits are shown in Figs. 1, 2, 3(a), 3(b), and 3(c). Altogether, we have used 86 data points for differential cross sections (including 12 points from the Case-Western-Reserve data at large  $|t|$ ), 15 points for polarization (Argonne data), 16 points for polarization (CERN data), and 17 points for the difference of  $\pi^-p$  and  $\pi^+p$  total cross sections ( $\Delta\sigma$ ). There is an obvious disagreement between the Argonne and the CERN polarization data at 5 GeV. The Argonne data have been claimed to have higher precision. However, we have included both sets

in our fits.

First of all we excluded the Case-Western-Reserve (CWR) data at large  $|t|$ . Then with the terms  $\rho(\gamma_1, \beta_1, p_1, q_1)$ ,  $\rho P'(\gamma_2, \beta_2, p_2, q_2)$ , and  $\rho P(\bar{\gamma}, \bar{\beta}, \bar{p}, \bar{q})$ , we get a good fit with  $\chi^2(d\sigma/dt) = 156$ ,  $\chi^2(\text{pol}) = 14.6$ , and  $\chi^2(\Delta\sigma) = 13.4$ . For polarization we consider the 8-GeV CERN data with either 5-GeV Argonne data or 5-GeV CERN data. It turns out that polarization is extremely sensitive to the values of the parameters and either set can be fitted. However, the fit to the CERN data gives slightly lower value of  $\chi^2(d\sigma/dt) = 144$ . In general, both the  $\rho P'$  cut and the  $\rho P$  cut terms are necessary for a good fit. However, the  $\rho P'$  term [ $\alpha_2(0) \approx 0$ ] is seen to be very important for charge-exchange scattering. For example, it was possible to get a fit with  $\chi^2(d\sigma/dt) = 337$  without the  $\rho P$  cut. If we kept only the  $\rho P$  cut term instead of the  $\rho P'$  term,  $\chi^2(d\sigma/dt)$  exceeded 800. These results establish the importance of the double-Regge term and verify within the context of our model the fact that both the weak- and the strong-cut models fare poorly in the explanation of the complete set of data. Our results are in agreement with the qualitative discussion given in Ref. 4. Since the cut parameters are left free here, it turns out that efforts to prevent a negative spike in polarization resulting from the usual  $\rho$ -Pomeron cut term result in an enormously large value of  $\chi^2$  for the differential cross sections. Alternately, if we demand good fit to the differential cross sections, a negative spike in the polarization becomes inevitable with only the  $\rho$ -Pomeron term.

With the above terms only, the CWR data cannot be fitted well. There are some normalization problems since these data do not go smoothly into the low-energy data. In addition, from the discussion in Ref. 4 we realize that we may be approaching the  $t$  region, where the triple-Regge cut ( $\rho P'P'$  or  $\rho\rho P'$ ) can be expected to make significant contribution. We add such a term with two additional parameters ( $\gamma_3, p_3$ ) and find a fit with additional value of  $\chi^2(d\sigma/dt)$  of about 20. This case is shown in Figs. 1-3.<sup>12</sup> For very large value of  $|t|$ , it may be necessary to include  $u$ -channel exchanges also.

We do not plot our amplitudes here, but just mention that  $\text{Im}A'(s, t)$  passes through zero around  $|t| = 0.2$  (GeV/c)<sup>2</sup>. As is well known, this behavior produces the well-established crossover effect when the pion-nucleon elastic scattering is considered. This zero comes out in an interesting way in our model. Both the coefficient of the  $\rho$ -pole term ( $\gamma_1$ ) and the  $\rho P'$  cut term ( $\gamma_2$ ) are positive. But since  $\alpha_2(t)$  is negative, the imaginary part of the cut term becomes negative and cancels the pole term in the crossover region. This fact

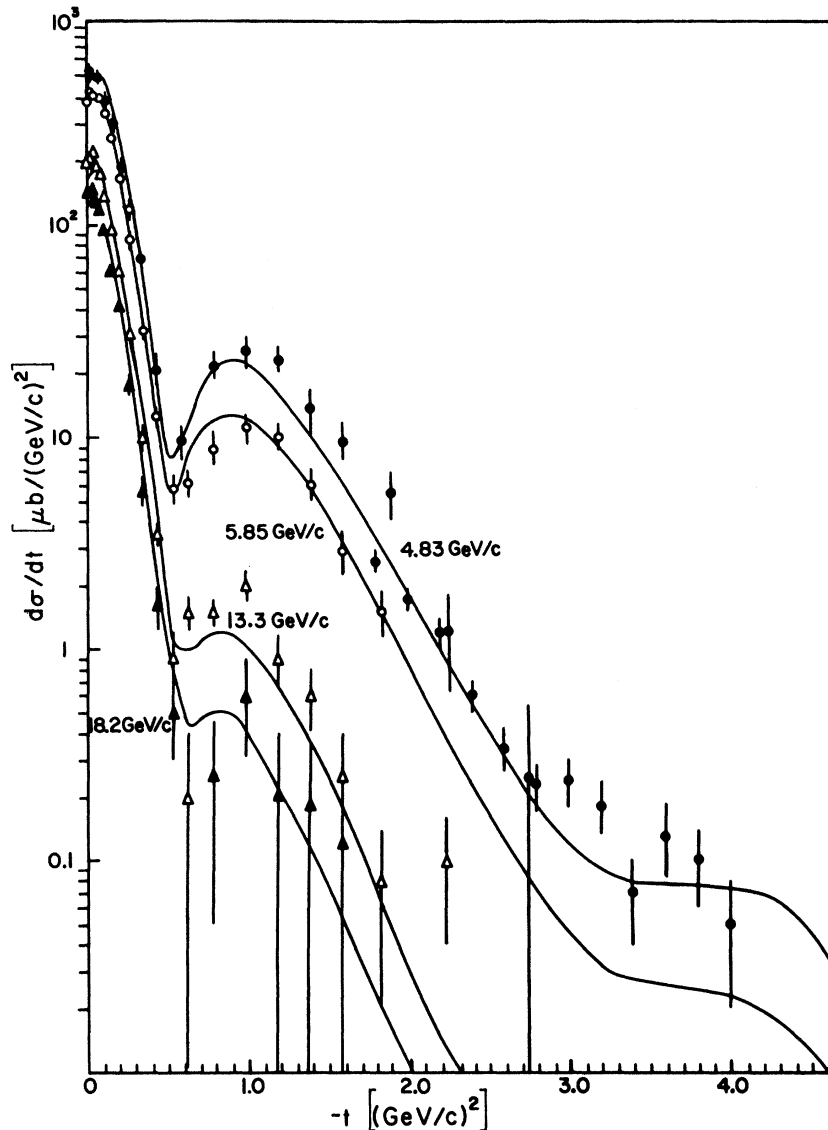


FIG. 1. Fit to the charge-exchange differential cross sections.

could have deeper significance for the question of relative sign of the pole and the cut term. It should also be mentioned that, in the present model, the crossover point moves very slowly to slightly larger value of  $|t|$  as the energy increases.

It is clear that some improvements in our  $\chi^2$  values are possible if we introduce more complicated residue functions, e.g., those with linear and higher terms in  $t$ . A large part of  $\chi^2(d\sigma/dt)$  does come from the low- $t$  region. Another possibility is to introduce an extra phase difference between the pole and the cut term. Eikonal models, for example, give this phase difference to be proportional to  $\pi/[2 \ln(\nu/\nu_0)]$ . In Sec. III we make

brief comparisons with other models and discuss Worden's work.

### III. COMPARISON WITH OTHER MODELS

Now we make brief comparisons with some of the recent works on this subject. Barger and Phillips<sup>13</sup> introduce a zero in the residue function for the  $\rho$  to produce the crossover effect and also assume existence of a  $\rho'$  trajectory with zero intercept and slope equal to that of the  $\rho$ . Unless both of these are considered as effective poles representing the combined effects of poles and cuts, the former fact leads to the factorization difficulty,

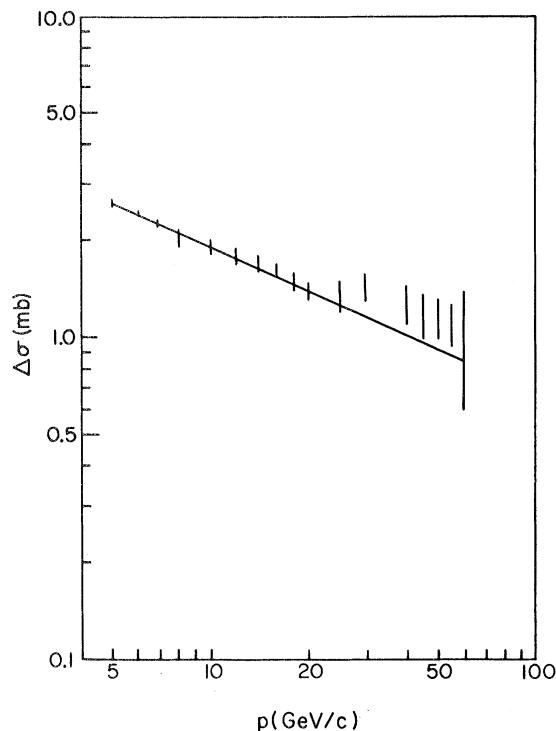


FIG. 2. Fit to the cross section difference ( $\Delta\sigma = \sigma_{\pi^-\rho} - \sigma_{\pi^+\rho}$ ).

and the latter to the problem of identification of a particle lying on the  $\rho'$  trajectory. We prefer to parameterize the pole and cut terms separately so that at least in some approximation, we can make contact with theory. Furthermore, the Barger and Phillips model requires  $\rho$  as sense-choosing, whereas we have the nonsense-choosing mechanism. Finally their model would predict a rapid and  $t$ -independent decrease of polarization with energy, whereas the present model would predict a slow decrease with energy which is dependent on the momentum transfer. Future experiments could test this.

Recently Leader and Nicolescu<sup>14</sup> have proposed a model with the  $\rho'$  having an intercept close to zero and slope less than half of that of  $\rho$ . They identify a recently discovered resonance of mass 1968 MeV as a particle lying on this trajectory. Since the intercept and the slope of this  $\rho'$  are quite close to the corresponding parameters of our  $\rho$ - $P'$  cut, evidently the two models give very similar results. The points of view are different however. Note that, if these authors take the  $\rho'$  slope as similar to that of the  $\rho$ , the  $t=0$  intercept will lie too low to fit the data. Regardless of the fact whether such a  $\rho'$  trajectory with correct quantum numbers is established in the future or not, the

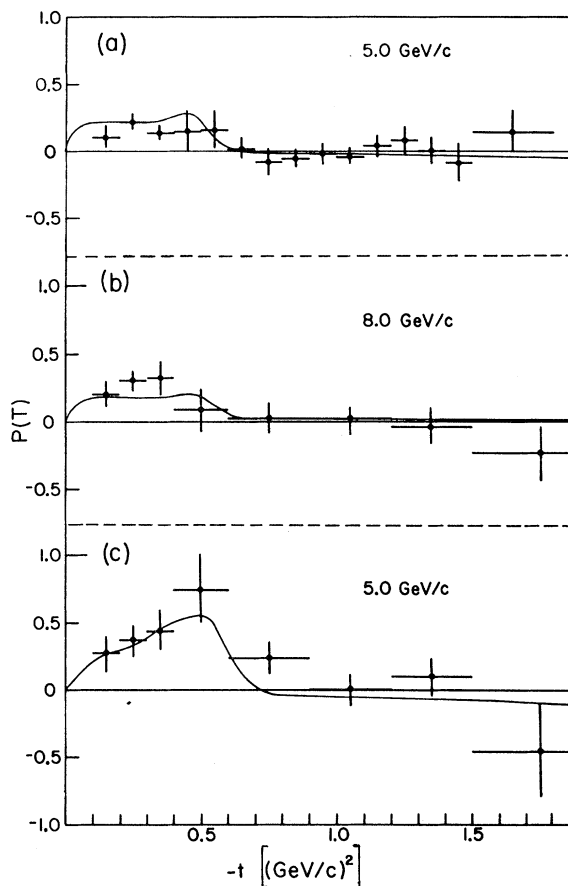


FIG. 3. Fit to the charge-exchange polarization. (a) 5-GeV/c Argonne data, (b) 8-GeV/c CERN data, (c) 5-GeV/c CERN data.

question of a  $\rho P'$ -cut contribution will still remain. In Regge theory, once the existence of the pole trajectory is granted, one has to accept the existence of the cuts. In addition we find the notion of trajectories having universal slope (except for the Pomeron which is special anyway) as too attractive to give up, unless one has to. Finally, in the Leader-Nicolescu model the  $I=1$  amplitude does not vanish for very low values of  $t$ . Thus unless the  $I=0$  amplitude has a subtle structure, their model would have difficulty in explaining the elastic-scattering crossover phenomenon. In our case, the crossover zero was not imposed but was the natural result of fitting other data.

Another recent model including a background term representing a fixed pole has been proposed by Kogitz and Logan.<sup>15</sup> They use a complicated residue function chosen precisely to peak at the correct position to produce dip-bump structure. Their work does show that the background term is indeed necessary to produce such a structure.

But it seems to be preferable to parametrize the background directly in terms of effective-cut contributions, in order to make greater connection with theory.

In a recent paper, Tuan *et al.*<sup>16</sup> consider various convolution models for double-Regge cuts and find that the  $\rho P'$  cuts are important. Most of their fits were made before the polarization data of Ref. 1 became available and show a first zero of polarization near  $|t| \approx 0.35 \text{ GeV}^2$  in disagreement with experiments. By changing the cut parameters they can move the zero to about  $0.55 \text{ GeV}^2$ , but the polarization values thereafter seem to be in disagreement with the experiments. This may indicate a failure of convolution models for larger values of  $t$ . An important question is the presence of cuts in the helicity-flip amplitude, which is neglected by Tuan *et al.* The polarization data of Ref. 1 show a zero at somewhat larger value of  $|t|$  and also possibly moving with energy. Our model has Regge-Regge cuts in the helicity-flip amplitude and is in agreement with these data.

Somewhat similar results are found in a model by Hartley and Kane,<sup>17</sup> namely, the large positive value of polarization after the zeros around  $|t| = 0.5 \text{ GeV}^2$ . These authors also use the absorption model.

In a recent paper, Girardi *et al.*<sup>18</sup> introduce a diagram with Regge-Pomeron-Regge cut. This may well exist, but from the phenomenological point of view, there is not much difference between it and the Regge-Regge cut. These authors use the absorption model which, as mentioned above, may have troubles.

Now we consider Worden's recent work<sup>7</sup> mentioned earlier. Assuming resonance saturation, Regge-resonance duality, strong exchange degeneracy, SU(3) symmetry, etc., he claims to show that for small values of  $t$ , the contributions of the  $\rho-P'$  and the  $A_2-\omega$  cuts cancel each other exactly in the helicity-nonflip amplitude of the pion-nucleon charge-exchange scattering. For the helicity-flip part the cancellation is believed to be only approximate. We believe that the assumptions are quite strong and hence in the real world with broken symmetries, there will be some nonvanishing effects from the Regge-Regge cuts due to the simultaneous exchange of vector-tensor meson trajectories, especially away from the forward direction. Note that in our case the Regge-Regge cut terms become vanishingly small at  $t=0$  anyway because of the factor  $(1 - e^{-t\pi\alpha_2(t)})$  and  $\alpha_2(0) \approx 0$ .

More specifically, our arguments are as follows. It is well known that in  $\pi N$  scattering, the helicity-nonflip amplitude made out of resonances just does not have a zero at the place where the  $\rho$ -trajectory

contribution becomes zero. Thus strong-cut corrections are required, which break the strict Regge-resonance duality. Since Regge cuts are corrections to pole contributions, it may be dangerous to throw away contributions which might seem to be canceled in some idealized limits. An important question is the breaking of strong exchange degeneracy which seems to be the case. Furthermore Regge-cuts naturally arise in a nonlinear relation (the unitarity condition), which gives corrections to the strict Regge-resonance duality results. Strict duality may be only a linear constraint.

In addition, if one wants to talk about SU(3) limits, one should take into account the cut due to  $K^*\bar{K}^{**}$  exchange also. This does not cancel even in the idealized limit. The only reason for not taking this cut into account may be that simple quark-duality diagrams cannot be drawn. Again the pitfalls of the latter approach in the strict form are well known. We suggest that applying these ideas to Regge-cut corrections might result in throwing away important contributions. Since even in the idealized limits, the cancellations are not exact for the helicity-flip amplitudes, one has to be cautious in the present case where the helicity-flip amplitude plays an important role in polarization phenomena producing nonzero polarization. As we have already mentioned above, the movement of the zero of polarization away from  $|t| \approx 0.6$  suggests an important role for the double-Regge cuts in the flip amplitudes.

Thus we conclude that there will be nonvanishing effects from double-Regge cuts especially away from the forward direction. We have parameterized the contributions of the  $\rho-P$  cut in our fits. We could include  $\omega-A_2$  and  $K^*\bar{K}^{**}$  cuts in the same way. However, in the case when the strong exchange degeneracy and the SU(3) symmetry are broken, this will introduce many more parameters. We already got good results with the present number of parameters. Hence inclusion of such additional terms may not be meaningful. Thus we find it appropriate to regard the parameterized  $\rho-P'$  contribution as an effective contribution from the cuts due to exchanges of various vector-tensor meson trajectories. Only when more stringent data like  $A$  and  $R$  parameters for a range of energy and momentum transfer are available, such a separation of  $\rho-P'$ ,  $\omega-A_2$ , and  $K^*\bar{K}^{**}$  terms may be meaningful.

In summary, we have described the world data on  $\pi-N$  charge-exchange scattering in terms of Regge poles and effective poles representing various cuts. We have retained the power behavior, signature factors, etc., given by the usually accepted theoretical considerations, but have not

chosen any particular model to represent Regge-cut effects. The fits have strong implications for the relative contributions of various terms. It will be interesting to extend such an analysis to  $\pi N$

elastic,  $KN$  elastic, and charge-exchange scattering. In these processes data extending to large momentum transfers are available and interesting structures have been already seen.<sup>4</sup>

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<sup>12</sup>The values of the parameters in the appropriate units with  $\hbar = c = \text{GeV} = 1$  are  $\gamma_1 = 2.79$ ,  $\gamma_2 = 39.07$ ,  $\gamma_3 = 0.11$ ,  $\bar{\gamma} = 3.14$ ,  $\beta_1 = 61.42$ ,  $\beta_2 = 35.97$ ,  $\bar{\beta} = 6.66$ ,  $p_1 = 7.30$ ,  $p_2 = 7.38$ ,  $p_3 = -0.27$ ,  $\bar{p} = 7.8$ ,  $q_1 = 1.43$ ,  $q_2 = 1.19$ ,  $\bar{q} = 1.61$ ,  $\alpha_0 = 0.48$ ,  $\alpha'_p = 0.49$ ,  $\nu_1 = 0.64$ . These parameters give the fits shown in Figs. 1, 2, 3(a) and 3(b). Figure 3(c) requires somewhat different parameters.  $\gamma_3$  and  $p_3$  are required only to fit the CWR data. Also note that values of  $p_1$ ,  $p_2$ , and  $\bar{p}$ , and  $q_1$ ,  $q_2$ , and  $\bar{q}$  are quite close. They could have been taken to be the same without changing the quality of the fits appreciably. Thus most of the data can be fitted with nine parameters.

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