Statistical Approach to Differential Cross Sections near 90°*

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Differential cross sections near $\theta_{\text{c.m.}} = 90°$ are calculated using an incoherent sum of direct-channel resonances with exponentially increasing density. Good agreement with the data is obtained for the $\pi^{\pm}p$, K^-p, and $\bar{p}p$ elastic cross sections. Two-body inelastic processes and the exotic pp and K^+p elastic cross sections near 90° are discussed.

INTRODUCTION

Many two-body reactions show very similar energy dependence at or near 90° in the c.m. coordinate system for $E_{\text{c.m.}} > 2$ GeV. More remarkably, for a large number of such hadron-hadron reactions the cross sections are even similar in absolute value. This includes the s-channel nonexotic reactions π^+p, π^-p, K^-p, and $\bar{p}p$ elastic scattering (see Refs. 1, 2, 3, and 4, respectively), as well as π^-p charge exchange,[5,6] $\bar{K}^0p \to \pi^+\Lambda^0$, and $\bar{K}^0p \to \pi^+\Sigma^0$.[6] The variation in absolute value of the cross sections is, within the experimental accuracy, not more than a factor of 2 or 3. Photoproduction of pions also shows similar energy dependence.[7] Since the similarity persists down into the discrete resonance region we examine the possibility that a superposition of resonances, dominantly incoherent in the central region near 90° in the c.m. system, can explain this behavior. Since there are presumably more and more resonances as we go to higher energies in such a picture, we shall invoke statistical considerations to describe the density of hadronic states at high excitation as well as the widths of these states.

Appropriate formulas for incoherent resonance superposition cross sections have been discussed recently.[8,9] These cross sections are characteristically essentially symmetric with respect to 90° and decrease exponentially with center-of-mass total energy. This is in contrast to the forward- and backward-peaked coherent two-body differential cross sections which fall as a power of the energy in the forward and backward directions (except for forward diffractive processes which are constant to within logarithms of the energy) and which then behave in a manner characteristic of the reaction as the angle increases, eventually falling into the central region under discussion here.

There has been much discussion using t-channel pictures[10] of how an exponential in momentum transfer squared, which crudely represents the small-angle behavior of many reactions, converts into an exponential behavior in momentum transfer as one goes from small to large momentum transfer. This corresponds to a transition from exponential falloff in c.m. energy squared to exponential falloff in c.m. energy at fixed angle as one approaches the central region. The explanation of this transition in t-channel language usually involves a small-angle assumption. We are interested here in large angles. At any rate we do not examine any possible connection between this t-channel description and the statistical s-channel considerations described in this paper.

Returning to statistical considerations, Fast *et al.*[11] some time ago considered the possibility that p-p scattering near 90° is statistical in nature. Although we are mainly concerned in this paper with nonexotic s-channel situations, we will discuss the exotic case to some extent below.

Parton models have been studied extensively for predictions of large-angle scattering and characteristically yield behavior at fixed angle falling as a high inverse power of c.m. energy.[12] This behavior results from a power-law behavior in t for the electromagnetic form factors. In both our treatment and in parton models Regge-type contributions are negligible owing to the small distances involved. The specific method of treating the remaining contributions is different here from that used in parton models; partons are assumed to be exchanged in models of the latter type. Ultimately experiments at high enough energy will answer the question of whether inverse-power or exponential decrease prevails. It is of course possible that there may be a region of energies where exponential behavior prevails and another region where inverse-power behavior takes over.

STATISTICAL CROSS SECTIONS FROM RESONANCE SUPERPOSITION

We propose here to describe differential cross sections near 90° in terms of an incoherent superposition of a large number of resonant amplitudes. The interference terms are assumed to vanish near 90° because of the large number of resonances of many angular momenta. Related studies have been made recently and appear in the literature.[8,9]

Consider the two-body process

$$a+b\to c+d\ . \tag{1}$$

We can write the differential cross sections for this reaction as

$$\frac{d\sigma}{d\Omega}=\frac{1}{64\pi^2 s}\,\frac{\lambda(s,m_c^2,m_d^2)}{\lambda(s,m_a^2,m_b^2)}\,|T_{ab;cd}(s,\Omega)|^2\,, \tag{2}$$

where the masses of the particles a, b, c, and d are m_a, m_b, m_c, and m_d, respectively, s is the total center-of-mass energy squared, Ω represents center-of-mass scattering angles for particle c, $T_{ab;cd}$ is the invariant amplitude, and

$$\lambda(s,m_a^2,m_b^2)=(s^2+m_a^4+m_b^4-2m_a^2 s-2m_b^2 s-2m_a^2 m_b^2)^{1/2}\,.$$

We ignore spins here, and if for the moment we also ignore orbital angular momentum then according to the incoherence assumption we write

$$|T_{ab;cd}|^2=\sum_i \frac{(\gamma_{ab}^i)^2(\gamma_{cd}^i)^2}{(\sqrt{s}-m_i)^2+\frac{1}{4}\Gamma_i^2}\ . \tag{3}$$

Here m_i is the mass of the ith resonance, whose total width is Γ_i, and the couplings to the incoming and outgoing channels are γ_{ab}^i and γ_{cd}^i, respectively. We now assume a high density of resonances so that we can approximate the sum in (3) by an integral. Assuming the integral to be peaked at the maximum in the Breit-Wigner component we find

$$|T_{ab;cd}|^2\simeq\frac{2\pi\gamma_{ab}^2(\sqrt{s}\,)\gamma_{cd}^2(\sqrt{s}\,)\rho(\sqrt{s}\,)}{\Gamma(\sqrt{s}\,)}\ . \tag{4}$$

Here the widths and couplings are mean values taken at the peak in the Breit-Wigner component of the integral.

We then have for the incoherent contribution to the differential cross section

$$\frac{d\sigma}{d\Omega}\simeq\frac{1}{32\pi s}\,\frac{\gamma_{ab}^2(\sqrt{s}\,)\gamma_{cd}^2(\sqrt{s}\,)\rho(\sqrt{s}\,)}{\Gamma(\sqrt{s}\,)}\,\frac{\lambda(s,m_c^2,m_d^2)}{\lambda(s,m_a^2,m_b^2)}\ . \tag{5}$$

This cross section is spherically symmetric obviously since we have ignored angular momentum. We can modify this formula approximately to include angular factors by multiplying by[8]

$$\xi(s,\theta)=\frac{\sum_0^{kR}(2l+1)^2P_l^2(\cos\theta)}{\sum_0^{kR}(2l+1)}\ , \tag{6}$$

where R is the hadronic radius and θ is the c.m. scattering angle. A more detailed treatment of angular momentum requires information on the J dependence of the density of states and of the couplings. However, (6) is only weakly dependent on angle and is of order unity near 90°.

The formulas (5) and (6) will yield a detailed prediction of statistical cross sections given the total width Γ and the couplings γ_{ab} and γ_{cd}. For elastic scattering $\gamma_{ab}=\gamma_{cd}$ of course. We can get expressions for the couplings γ_{ab} and γ_{cd} at high enough energies using the statistical bootstrap model.[13,14] This model yields the following equation for the total density of hadronic states[14]:

$$\rho(m)=\sum_{k=2}^{\infty}\left[\frac{V}{(2\pi)^3}\right]^{k-1}\frac{1}{k!}\prod_{l=1}^{k}\int d^3p_l\,dm_l\,\rho(m_l)\times\delta(m-\textstyle\sum E_l)\delta^3(\sum\vec{p}_l)\,, \tag{7}$$

where V is the hadronic volume taken to be a sphere with radius 1 to 1.4 F. The integration is over all particle and resonance masses that conserve energy. E_l and p_l are the energy and momentum of the particle of mass m_l with $E_l=(m_l^2+p_l^2)^{1/2}$.

The solution of this equation for m large enough is[14,15]

$$\rho(m)=(a/m^3)e^{m/T}\,,$$

where a and T depend on the value of V. One finds values for T in the range 140 to 170 MeV.[15] Since the above solution holds for $m\to\infty$, one corrects it for small m to read[16]

$$\rho(m)=\frac{am^{1/2}}{(m+m_0)^{7/2}}e^{m/T}\,, \tag{8}$$

where $m_0\simeq 500$ MeV.[16] We will come back to the question of values for these constants below.

The coupling γ_{ab} is clearly related to the partial width in channel ab. The total width is given in terms of partial widths at high enough energies through the equation

$$\Gamma(m)=\sum_{k=2}^{\infty}\frac{1}{k!}\prod_{l=1}^{k}\int\frac{d^3p_l}{2E_l}\,dm_l\rho(m_l)\delta(m-\textstyle\sum E_l)\times\delta^3(\sum\vec{p}_l)\Gamma(p_1,m_1,\ldots,p_k,m_k)\ . \tag{9}$$

The sum is over k-body states, where $k\geq 2$. The choice of form for $\Gamma(p_1,m_1,\ldots,p_k,m_k)$ is motivated by the bootstrap equation (7). Clearly (9) and (7) are satisfied if

$$\Gamma(p_1,m_1,\ldots,p_k,m_k)=\left[\frac{V}{(2\pi)^3}\right]^{k-1}\frac{\prod_{j=1}^{k}(2E_j)_{\text{c.m.}}\Gamma(m)}{\rho(m)}\ . \tag{10}$$

We then have at high enough energy

$$\Gamma_{ab}(m, m_a, m_b) = \frac{V}{(2\pi)^3} \frac{\Gamma(m)}{\rho(m)} \left[\frac{m^4 - (m_b^2 - m_a^2)^2}{m^2} \right]. \tag{11}$$

It is readily verified that in order for (3) to correspond to Breit-Wigner amplitudes with the correct strength at high energy, we must have

$$\gamma_{ab}^2 = 8\pi\Gamma_{ab} \ . \tag{12}$$

The last factor on the right side of (11) is the ratio of invariant to noninvariant phase space and results from the fact that the statistical bootstrap equation (8) involves noninvariant phase space. We assume now that the expression for γ_{ab} that results from (11) and (12) can be used in the energy region of interest to obtain the two-body differential cross section near 90° for reaction (1).

Using (5), (6), (11), and (12) we find then

$$\frac{d\sigma}{d\Omega} \simeq \left[\frac{V}{(2\pi)^3}\right]^2 \frac{\lambda(s, m_c^2, m_d^2)}{\lambda(s, m_a^2, m_b^2)} \frac{\Gamma(\sqrt{s}\,)}{\rho(\sqrt{s})} \frac{2\pi}{s} \times \left[\frac{s^2 - (m_b^2 - m_a^2)}{s}\right]\left[\frac{s^2 - (m_d^2 - m_c^2)}{s}\right] \xi(s, \theta), \tag{13}$$

where $\xi(s, \theta)$ is given by Eq. (6). For energies sufficiently above threshold this cross section then falls with energy according to

$$\frac{d\sigma}{d\Omega} \simeq \frac{s\Gamma(\sqrt{s}\,)}{\rho(\sqrt{s}\,)} \ . \tag{14}$$

DATA ANALYSIS

We will apply formula (13) to fit $\pi^{\pm}p$, $K^{-}p$, and $\bar{p}p$ elastic scattering data at 90° for $s \geq 4$ GeV^2 assuming that the total width Γ is constant, $V = \frac{4}{3}\pi R^3$, where R is the hadronic matter radius, and $\rho(\sqrt{s}\,)$ is given by Eq. (8). The resulting fits are given for temperatures $T = 140$ and 160 MeV in Figs. 1–4. The corresponding values for $\Gamma V^2/a$ are given in Table I, and those for a, assuming $\Gamma = 200$ MeV, in Table II. The radii $R = 1.1$ and 1.3 F correspond to those for which the statistical bootstrap yields temperatures $T = 160$ and 140 MeV, respectively.[15] The most extensive data are for $\pi^{-}p$ scattering, which seems to prefer a temperature near 160 MeV, corresponding to a hadronic interaction radius of 1.1 F. The density of *all* hadronic states as determined by Hagedorn and Ranft[16] (see Table II) yields a value of $a \sim 1.5 \times 10^6$ MeV^2 (for $T = 147.5$ MeV). The values in Table II are about three orders of magnitude less. This is quite reasonable since one is only able to excite states of a particular charge and with one or two values of the z component of angular momentum. In Hagedorn's fits the isotopic and angular momentum multiplicity due to magnetic degeneracy is included. Also other quantum numbers must be conserved. It is to be noted, although this is rather unimportant, that since the z component of angular momentum is severely limited it would be more correct to have added another half power to the $1/m^3$ term in the expression (8) for $\rho(m)$, i.e., the density would perhaps better be written $\rho(m) \sim a m^{-7/2} e^{m/T}$. Finally, we find that baryon resonances are two or three times more plentiful than boson resonances. We do not know whether this apparent difference is significant. Further, strange baryon resonances are determined here to be as plentiful as nonstrange baryon resonances.

Although we do not analyze inelastic reactions here, we point out again that in the few-GeV region near 90° they behave in a manner very similar to elastic scattering,[6,7] both in energy dependence and absolute value. This can be understood very well in the picture described above. For example the reaction $\pi^{-}p \to \pi^0 n$ will have a differential cross section near 90° given by

$$\frac{d\sigma_{\mathrm{CE}}}{d\Omega} \simeq \frac{1}{2}\left(\frac{d\sigma^{\mathrm{el}}_{\pi^- p}}{d\Omega} + \frac{d\sigma^{\mathrm{el}}_{\pi^+ p}}{d\Omega}\right) , \tag{15}$$

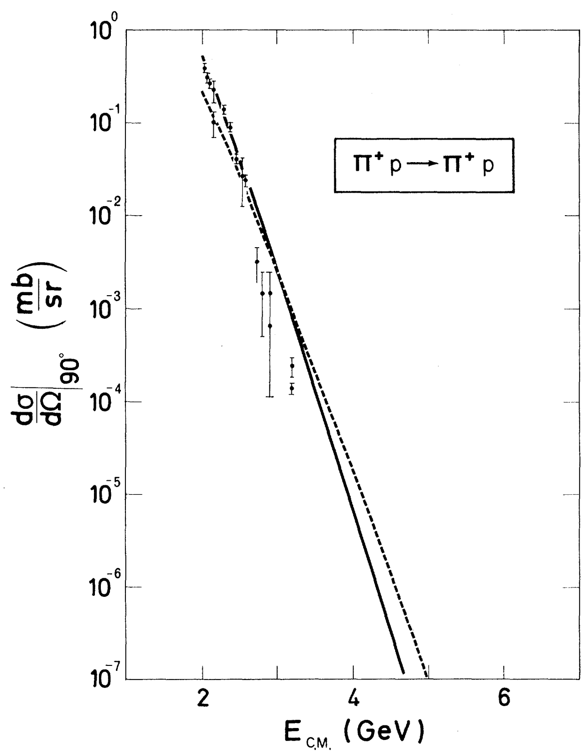

FIG. 1. π^+p elastic scattering data from Ref. 1 and theoretical predictions with temperatures $T = 140$ MeV (solid line) and 160 MeV (dashed line), using formulas (8) and (13).

TABLE I. The factor $V^2\Gamma/a$ determined by fitting elastic scattering data taking the hadronic temperature T =140 and 160 MeV corresponding to hadronic radii R =1.3 and 1.1 F, respectively.

		$V^2\Gamma/a$ (mb/MeV^5)			
T (MeV)	R (F)	π^-p	π^+p	$\bar{p}p$	K^-p
160	1.1	1.3×10^{-8}	1.3×10^{-8}	3.8×10^{-8}	1.3×10^{-8}
140	1.3	2.0×10^{-7}	2.0×10^{-7}	3.8×10^{-7}	2.0×10^{-7}

where we have invoked the incoherence assumption to eliminate the interference term between π^-p and π^+p elastic scattering. Other reactions similarly related to elastic scattering by symmetry considerations can be expected to behave like the corresponding elastic amplitudes near 90°.

REMARKS ON EXOTIC CASES

Proton-proton elastic scattering involves an exotic s-channel situation. Provided exotic resonances exist, the considerations discussed above would be valid. It is interesting to note that the measured 90° p-p cross section is many times that for the nonexotic cases described above.

TABLE II. The level-density parameter a for elastic scattering processes determined from the data. Γ=200 MeV.

		a (MeV^2)			
T (MeV)	R (F)	π^-p	π^+p	$\bar{p}p$	K^-p
160	1.1	2.9×10^3	2.9×10^3	1.0×10^3	2.9×10^3
140	1.3	5.4×10^2	5.4×10^2	2.8×10^2	5.4×10^2

K^+p scattering is also somewhat larger than K^-p scattering near 90°.[17] This could indicate a lower density of exotic resonances according to formula (14). However, there is no clear evidence for exotic resonances. Further, there is no evidence for statistical fluctuation effects in p-p scattering.

Alternatively we may look at Eq. (14) again with a view to interpreting its significance and possibly generalizing our considerations to exotic cases, assuming there are no exotic resonances. The dominant feature of (14) is its inverse dependence on the density of resonances (or, to use the language of statistical theories, fireballs). Indeed, the widths $\Gamma(\sqrt{s})$ have a behavior with energy which is not really known, but the s dependence is presumably weak, given the agreement with experiment found here under this assumption.

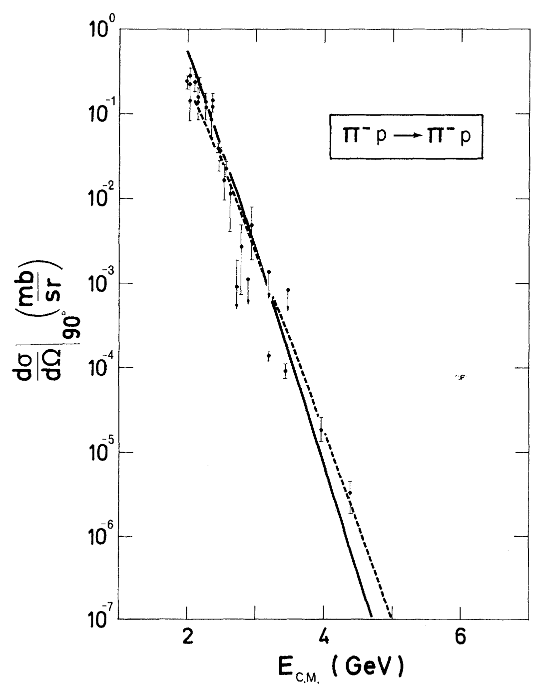

FIG. 2. π^-p elastic scattering data from Ref. 2 and theoretical predictions with temperatures T =140 MeV (solid line) and 160 MeV (dashed line) using formulas (8) and (13).

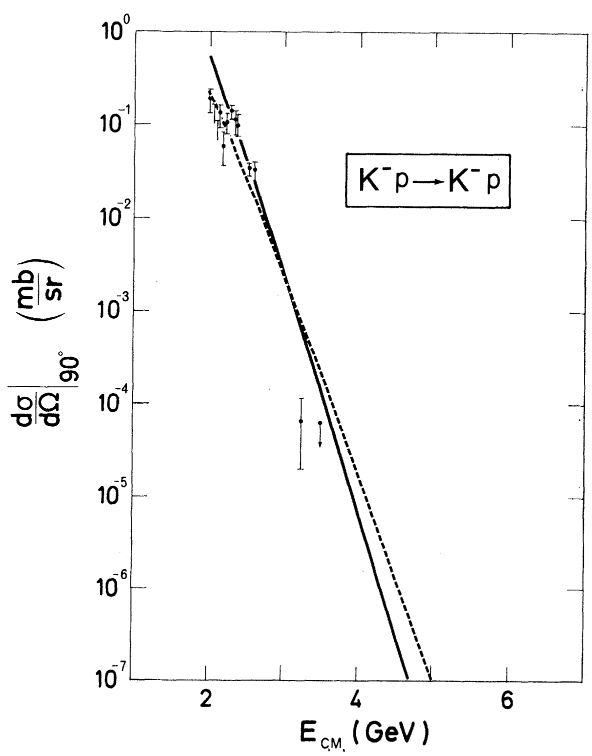

FIG. 3. K^-p elastic scattering data from Ref. 3 and theoretical predictions with temperatures T =140 MeV (solid line) and 160 MeV (dashed line) using formulas (8) and (13).

Provided there are no exotic resonances, proton-proton scattering could still be statistical in nature near 90° if it involved the formation of two or more heavy fireballs at rest or nearly so, followed by their decay. If in fact the density of two-fireball states governs the 90° cross section we will then get a larger cross section for exotic situations if the density of two-fireball states at a given mass is less than for one-fireball states. This in fact seems to be the case, as can be established using formulas (7) and (8). From these formulas it follows that the ratio of two-fireball to one-fireball densities at rest is

$$\frac{\rho^{(2)}(M)}{\rho^{(1)}(M)}=\frac{1}{2}\,\frac{\int_{M_0}^{M-M_0}\rho(M')\rho(M-M')dM'}{\rho(M)}\,, \qquad (16)$$

where M_0 is the lowest possible mass of the fireballs. Using the form (8) for the density $\rho(M)$ we find

$$\frac{\rho^{(2)}(M)}{\rho^{(1)}(M)}=\frac{2^5 a}{M^2}\int_0^{1-2M_0/M}\frac{dv}{(1-v^2)^3}$$

$$=\frac{4a}{M^2}\left\{\frac{2(1-2M_0/M)}{[(4M_0/M)(1-M_0/M)]^2}+\frac{3(1-2M_0/M)}{(4M_0/M)(1-M_0/M)}+\frac{3}{2}\ln\left(\frac{M-M_0}{M_0}\right)\right\}. \qquad (17)$$

This expression is slowly varying and has the value $a/2M_0^2$ as M approaches infinity. Using the results of Table II we find that the ratio (17) is roughly 10^{-3} at $M=4$ GeV. On the other hand, using the total hadronic density of states ($a\sim 1.5\times10^6$ MeV^2) yields a value for (17) of the order of unity. The value of the normalization constant a to be used in the present discussion depends on dynamics to some extent. At any rate, it should not be nearly so large as that for the total density of hadronic states. It is therefore reasonable from these considerations that p-p scattering should be larger than $\bar{p}$-p at 90°. Similarly we expect K^+p scattering to be larger than K^-p scattering near 90°. The data indicate that this is so.[17]

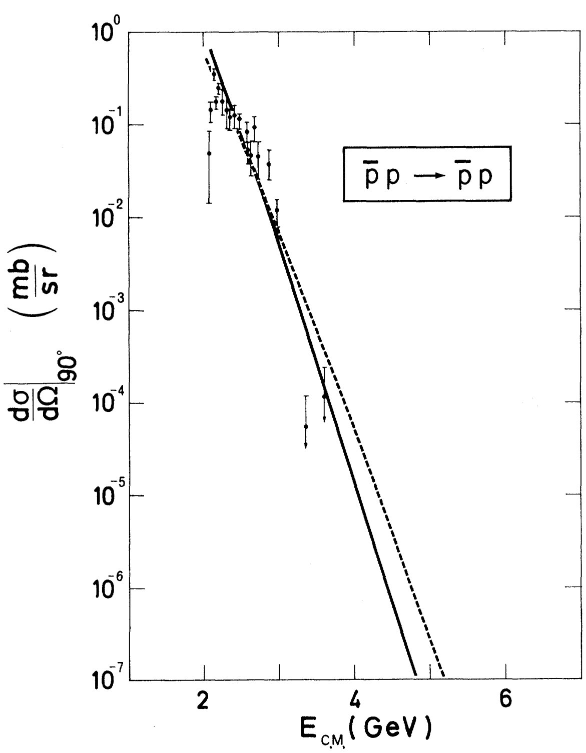

FIG. 4. $\bar{p}p$ elastic scattering data from Ref. 4 and theoretical predictions with temperatures $T=140$ MeV (solid line) and 160 MeV (dashed line) using formulas (8) and (13).

Since the density of hadronic states or fireballs is intimately connected with the number of channels available,[8,14] we can perhaps look at the exotic-vs-nonexotic situation as being determined by the number of these channels. Competition from a larger number of channels tends to make the 90° cross sections smaller in nonexotic s-channel reactions.

It is to be noted, however, that p-p 90° elastic scattering data[18] show a break in slope near $s\simeq 20$ GeV^2, followed by a somewhat less steep decrease with increasing energy. We present no explanation of this phenomenon.

CONCLUSIONS

We have presented a statistical model of differential two-body cross sections which allows us to explain in detail the energy dependence near 90° for nonexotic cases. The angular distributions should be relatively flat in this region and roughly symmetric with respect to 90°. One can expect fluctuations to some extent with respect to the statistical cross-section formulas presented here. Indeed at low enough energies where there are not too many resonances, coherent effects may be dominant at 90°. The main characteristic of the statistical model is an exponential decrease with c.m. energy of the differential cross section. This in essence is the result of competition from other channels. The large number of results explained by the statistical model is particularly encouraging. It can be expected from the above study that a statistical picture is extremely relevant for central collisions. It will be interesting to see whether this is so at higher energies or whether some other mechanism becomes dominant. It is worth noting that parton models[12] usually predict

$$\left.\frac{d\sigma}{d\Omega}\right|_{90^\circ} \sim s^{-n} ,$$

with n of the order of 10, while we have here

$$\left.\frac{d\sigma}{d\Omega}\right|_{90^\circ} \sim s \exp(-\sqrt{s}/T).$$

It is difficult with the present data to distinguish between these two predicted behaviors. Furthermore, a different n for $\bar{p}p \to \bar{p}p$ and $\pi^-p \to \pi^-p$ is predicted in parton models,[12] while a similar behavior for both processes is predicted here.

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